A Novel TOPSIS Method for Multiple Attribute Decision Making Based on Single-Valued Neutrosophic Sets

Debin Liu* and Dongsheng Xu

***Abstract*—A single-valued neutrosophic set is practical to catch up incomplete information by employing three degrees namely membership degree, indeterminacy degree and non-membership degree, and thus has been widely used in recent years. Moreover, technique for order preference by similarity to an ideal solution (TOPSIS) is one of the effective methods for multiple attribute decision making problems (MADM). However, in some special cases, TOPSIS algorithms, which consider only the relative closeness coefficients (RCCs) of the alternatives with respect to the positive ideal solution (PIS), are not reliable. Therefore, in this paper, a novel TOPSIS method, which takes into consideration both the PIS closeness and the RCC, is proposed based on a new concept of closeness coefficient. In addition, two illustrative examples are presented to demonstrate the applicability and effectiveness of the proposed approach. Sensitivity analysis verifies the robustness of the proposed method.**

***Index Terms*—blending closeness coefficient, decision making, single-valued neutrosophic set, TOPSIS**

I. Introduction

MULTIPLE attribute decision making problems (MADM) play an important role in modern science of decision making. They are widely used in fields such as medical diagnosis, supplier selection, business, and so on. To process fuzzy information, Zadeh proposed the concept of fuzzy set (FS), in which the real number between 0 and 1 is used to represent the membership degree [1]. However, it is difficult to express the fuzzy information by the affiliation function only, so to express the non-membership degree, Atanassov proposed intuitionistic fuzzy set (IFS) on the basis of FS [2]. Further, Torra and Narukawa put forward hesitant fuzzy set (HFS) to express the degree of hesitation [3]. In the family of FS, the sum of several affiliation functions is 1, but this is not necessarily true in real life. Therefore, in order to express fuzzy information more flexibly, Smarandache added an independent uncertainty to IFS and proposed the neutrosophic set (NS) [4]. Subsequently, Wang *et al.* defined single-valued neutrosophic set (SVNS) based on NS, and proposed a MADM method [5]. In some cases, the membership degree, indeterminacy degree and non-membership degree of a problem can not be accurately described by single numerical values, so Wang *et al.* defined interval neutrosophic set (INS) [6]. Furthermore, Wang and Li proposed the concept of multi-valued neutrosophic set (MVNS), whose membership degree, indeterminacy degree and non-membership degree are all finite sets of discrete values [7].

Technique for order preference by similarity to an ideal solution (TOPSIS), which was proposed by Hwang and Yoon, is one of the popular decision making methods [8]. Abo-Sinna used TOPSIS method to solve multi-objective dynamic programming problem [9]. Deng *et al.* used TOPSIS method to solve the problem of comparison between companies [10]. Chen extended TOPSIS method to FS to solve MADM problems [11]. Boran *et al.* extended TOPSIS method to the problem of multiple attribute intuitionistic decision making [12]. Pramanik and Mukhopadhyaya extended TOPSIS method to solve the problem of teacher selection in intuitionistic fuzzy environment [13]. Chi and Liu discussed an extended TOPSIS method for INS MADM problems [14]. Biswas and Pramanik extended TOPSIS method to MADM problems based on SVNS [15]. Selvachandran *et al.* presented a modified TOPSIS method with maximizing deviation method based on SVNS [16]. Nancy and Garg developed a novel TOPSIS method for solving single-valued neutrosophic MADM with incomplete weight information [17]. Biswas *et al.* developed a nonlinear programming approach based on TOPSIS method to determine relative closeness intervals of alternatives [18]. Nancy and Garg extended TOPSIS method to solve the group decision making problems [19]. Karaaslan and Hunu developed a multiple attribute group decision making method based on TOPSIS approach under the type-2 single-valued neutrosophic environment [20]. Ashraf and Butt developed a single-valued neutrosophic N-soft TOPSIS method based on single-valued neutrosophic N-soft aggregate operators to cumulate the decisions of all experts according to their opinions and parameters related to each alternative [21]. Sun and Cai developed a flexible decision-making method for green supplier selection integrating TOPSIS and grey relational analysis [22].

The traditional, and existing expanded TOPSIS methods rank alternatives according to the relative closeness coefficients (RCCs) of the alternatives with respect to the positive ideal solution (PIS). They are effective in most cases. But, if the RCC of the optimal alternative is close to the RCC

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of the suboptimal alternative, the rankings of TOPSIS methods are not robust thus making it difficult to produce reliable results.

Based on the above gap, this paper will propose a novel technique, which considers both the PIS closeness and the RCC of the alternatives based on a new concept of closeness coefficient, to improve existing methods.

II. PRELIMINARIES

A. Basic Definition

In this section, some important definitions related to NS are provided to facilitate a better understanding of the paper.

Definition 1 [4]. Let X be a non-empty set. A neutrosophic set N over the universe of discourse X is defined as:

$$N = \left\{(x, T_N(x), I_N(x), F_N(x)) \middle| x \in X\right\}, \tag{1}$$

where $T_N(x)$, $I_N(x)$, $F_N(x)$: $X \to]^-0,1^+[$ are the degrees of "membership", "indeterminacy" and "non-membership" such that $^-0 \le \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \le 3^+$.

Definition 2 [5]. Let X be a non-empty set. A single-valued neutrosophic set N over the universe of discourse X is defined as:

$$N = \left\{(x, T_N(x), I_N(x), F_N(x)) \middle| x \in X\right\}, \tag{2}$$

where $T_N(x)$, $I_N(x)$, $F_N(x) \in [0,1]$ and for each $x \in X$.

The triplet $(T_N(x), I_N(x), F_N(x))$ is called single-valued neutrosophic number (SVNN).

Definition 3 [5]. For two SVNNs $N_1 = (T_1, I_1, F_1)$ and $N_2 = (T_2, I_2, F_2)$ and a positive real number λ, some operations are defined as:

(i) $N_1 \subseteq N_2$ if $T_1 \le T_2, I_1 \ge I_2, F_1 \ge F_2$.

(ii) $N_1 \cap N_2 = (\min(T_1, T_2), \max(I_1, I_2), \max(F_1, F_2))$.

(iii) $N_1 \cup N_2 = (\max(T_1, T_2), \min(I_1, I_2), \min(F_1, F_2))$.

(iv) $N_1 = N_2$ if and only if $N_1 \subseteq N_2$ and $N_2 \subseteq N_1$.

(v) $N_1{}^C = (F_1, 1 - I_1, T_1)$.

(vi) $N_1 \oplus N_2 = (T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2)$.

(vii) $N_1 \otimes N_2 = (T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2)$.

(viii) $\lambda N_1 = (1 - (1 - T_1)^\lambda, (I_1)^\lambda, (F_1)^\lambda)$.

(ix) $(N_1)^\lambda = ((T_1)^\lambda, 1 - (1 - I_1)^\lambda, 1 - (1 - F_1)^\lambda)$.

Definition 4 [23]. For two SVNSs $N_1 = \{(x, T_{N_1}(x), I_{N_1}(x), F_{N_1}(x)) | x \in X\}$ and $N_2 = \{(x, T_{N_2}(x), I_{N_2}(x), F_{N_2}(x)) | x \in X\}$ over a finite universe $X = \{x_1, x_2, \cdots; x_n\}$, two distance measures between N_1 and N_2 are defined as follows:

(i) The Hamming distance measure between N_1 and N_2 is defined as:

$$\begin{aligned} d_H(N_1, N_2) = \sum\nolimits_{i=1}^{n} \{ &| T_{N_1}(x_i) - T_{N_2}(x_i) | \\ &+ | I_{N_1}(x_i) - I_{N_2}(x_i) | + | F_{N_1}(x_i) - F_{N_2}(x_i) | \}. \end{aligned} \tag{3}$$

(ii) The normalized Hamming distance measure between N_1 and N_2 is defined as:

$$\begin{aligned} d_{HN}(N_1, N_2) = \frac{1}{3n} \sum\nolimits_{i=1}^{n} \{ &| T_{N_1}(x_i) - T_{N_2}(x_i) | \\ &+ | I_{N_1}(x_i) - I_{N_2}(x_i) | + | F_{N_1}(x_i) - F_{N_2}(x_i) | \}. \end{aligned} \tag{4}$$

(iii) The Euclidean distance measure between N_1 and N_2 is defined as:

$$d_E(N_1, N_2) = \sqrt{\begin{aligned} \sum\nolimits_{i=1}^{n} \{ &(T_{N_1}(x_i) - T_{N_2}(x_i))^2 \\ &+ (I_{N_1}(x_i) - I_{N_2}(x_i))^2 \\ &+ (F_{N_1}(x_i) - F_{N_2}(x_i))^2 \} \end{aligned}}\ . \tag{5}$$

(iv) The normalized Euclidean distance measure between N_1 and N_2 is defined as:

$$d_{EN}(N_1, N_2) = \sqrt{\begin{aligned} \frac{1}{3n} \sum\nolimits_{i=1}^{n} \{ &(T_{N_1}(x_i) - T_{N_2}(x_i))^2 \\ &+ (I_{N_1}(x_i) - I_{N_2}(x_i))^2 \\ &+ (F_{N_1}(x_i) - F_{N_2}(x_i))^2 \} \end{aligned}}\ . \tag{6}$$

B. TOPSIS Method

TOPSIS method, proposed by Hwang and Yoon in 1981, is commonly used to determine the optimal alternative. This method can make full use of the information of original data, and the results can accurately show the gaps between different alternatives. According to TOPSIS method, the best alternative should have the shortest distance to the PIS and the farthest distance to the negative ideal solution (NIS).

Let $A = \{A_1, A_2, \cdots; A_m\}$ be the set of alternatives, $C = \{C_1, C_2, \cdots; C_n\}$ be the set of attributes with the weight vector $W = \{w_1, w_2, \cdots; w_n\}$, and $D = (d_{ij})_{m \times n}$, $i = 1, 2, \cdots; m$, $j = 1, 2, \cdots; n$ be the decision matrix.

The procedures of TOPSIS method can be described as follows [15]:

Step 1. Normalization of the decision matrix

To eliminate the effects of different attributes, we need to normalize two types of attributes including benefit type and cost type.

Let d_{ij}^N be the normalized value, it can be calculated as follows:

$$d_{ij}^N = \begin{cases} \dfrac{d_{ij} - d_j^-}{d_j^+ - d_j^-}, & \textit{benefit type} \\ \dfrac{d_j^+ - d_{ij}}{d_j^+ - d_j^-}, & \textit{cost type} \end{cases}, \tag{7}$$

where $d_j^+ = \max_i(d_{ij})$ and $d_j^- = \min_i(d_{ij})$.

Step 2. Calculation of the weighted normalized decision matrix

The weighted normalized value v_{ij} is calculated as the following way:

$$v_{ij} = w_j \times d_{ij}^N, \tag{8}$$

where w_j is the weight of the j-th attribute such that $w_j \ge 0$ for $i = 1, 2, \cdots; m$, $j = 1, 2, \cdots; n$ and $\sum_{j=1}^{n} w_j = 1$.

Step 3. Determination of the PIS and the NIS

The PIS and the NIS are derived as follows:

$$\begin{aligned} \text{PIS} &= \{v_1^+, v_2^+, \cdots, v_n^+\} \\ &= \{(\max_i v_{ij} \mid j \in J_1), (\min_i v_{ij} \mid j \in J_2)\}, \end{aligned} \tag{9}$$

$$\begin{aligned} \text{NIS} &= \{v_1^-, v_2^-, \cdots, v_n^-\} \\ &= \{(\min_i v_{ij} \mid j \in J_1), (\max_i v_{ij} \mid j \in J_2)\}, \end{aligned} \tag{10}$$

where J_1 and J_2 are the sets of benefit type and cost type attributes, respectively.

Step 4. Calculation of the distance measures

The distance between the alternative A_i and the PIS can be measured by using the n-dimensional Euclidean distance, which is given as:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2}, i = 1, 2, \cdots, m. \quad (11)$$

Similarly, the distance measure between the alternative A_i and the NIS is calculated as:

$$D_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, i = 1, 2, \cdots, m. \quad (12)$$

Step 5. Calculation of the RCCs

The RCC of the alternative A_i with respect to the PIS is given as:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}, i = 1, 2, \cdots, m. \quad (13)$$

Step 6. Ranking the alternatives

Rank the alternatives according to the RCCs. The larger value of C_i indicates the better alternative A_i.

The above process is illustrated with the following example.

Example 1. An elementary school wants to order a batch of uniforms from one of five suppliers denoted by A_1, A_2, A_3, A_4, A_5. The selection is held on the basis of two different attributes, namely, quality (C_1) and price (C_2). C_1 is benefit type, and C_2 is cost type with the attribute weight vector $W = \{0.5, 0.5\}$. The decision matrix is given as:

$$D = \begin{bmatrix} 0.3 & 0.3 \\ 0.4 & 0.4 \\ 1 & 1 \\ 0 & 0 \\ 0.3 & 0.5 \end{bmatrix}.$$

Step 1. Normalized decision matrix is computed by using (7) as follows:

$$D^N = \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \\ 1 & 0 \\ 0 & 1 \\ 0.3 & 0.5 \end{bmatrix}.$$

Step 2. Weighted normalized decision matrix is calculated by using (8) as follows:

$$V = \begin{bmatrix} 0.15 & 0.35 \\ 0.2 & 0.3 \\ 0.5 & 0 \\ 0 & 0.5 \\ 0.15 & 0.25 \end{bmatrix}.$$

Step 3. Determination of the PIS and the NIS:

PIS = {0.5, 0.5}, NIS = {0, 0}.

Step 4. Calculation of the distance measures

Using (11) and (12), we compute the distance measures as follows: $D_1^+ = 0.3808$, $D_2^+ = 0.3606$, $D_3^+ = 0.5$, $D_4^+ = 0.5$, $D_5^+ = 0.4301$, $D_1^- = 0.3808$, $D_2^- = 0.3606$, $D_3^- = 0.5$, $D_4^- = 0.5$, $D_5^- = 0.2915$.

Step 5. Calculation of the RCCs

Using (13), we calculate the RCCs as follows:

$C_1 = C_2 = C_3 = C_4 = 0.5$, $C_5 = 0.404$.

Step 6. Ranking the alternatives

The RCCs of alternatives A_1, A_2, A_3, A_4 are the same, and larger than the value of A_5. So, it is difficult to determine the best alternative. As can be seen from this example, the above TOPSIS method fails to select the optimal alternative.

In what follows, we will propose a new approach to calculate the closeness coefficients of the alternatives.

III. Proposed TOPSIS method for SVNS

TOPSIS method ranks the alternatives according to the RCCs of the alternatives with respect to the PIS. If the RCCs of two alternatives are equal, the decision maker usually prefers the alternative which has the shortest distance to the PIS. By integrating the relative distance measure and the absolute distance measure, we introduce the blending closeness coefficient (BCC) to characterize the distance measure between the alternatives and the ideal solutions.

Definition 5. Let D_i^- be the distance measure between the i-th alternative and the PIS, D_i^+ be the distance measure between the i-th alternative and the NIS, $i = 1, 2, \cdots; m$, the PIS-closeness coefficient of the i-th alternative is defined as:

$$D_i^{PIS} = \frac{D_i^+}{\max_i D_i^+}, \quad (14)$$

the BCC of the i-th alternative is defined as:

$$BCC_i = \frac{D_i^-}{D_i^+ + D_i^-}\theta + (1 - D_i^{PIS})(1-\theta), \quad (15)$$

where $\theta \in [0,1]$ is called relative closeness level (RCL).

Remark 1. The BCC is reduced to the RCC with $\theta = 1$.

Remark 2. If the RCCs of alternatives are equal, the smaller absolute distance measure between the alternative and the PIS implies the larger BCC with fixed θ.

By definition 5, we can calculate the BCCs with $\theta = 0.8$ in Example 1 as follows: $BCC_1 = 0.4477$, $BCC_2 = 0.4558$, $BCC_3 = 0.4$, $BCC_4 = 0.4$, $BCC_5 = 0.3511$. BCC_2 is larger than the others, which indicates that the 2nd alternative A_2 is the optimal one.

Consider a MADM problem with m alternatives and n attributes. Let $A = \{A_1, A_2, \cdots; A_m\}$ be a discrete set of alternatives, and $C = \{C_1, C_2, \cdots; C_n\}$ be the set of attributes, with the weight parameters $W = \{w_1, w_2, \cdots; w_n\}$ fully or partially unknown satisfying $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Let $D = (d_{ij})_{m \times n}$ be the decision matrix, where $d_{ij} = (T_{ij}, I_{ij}, F_{ij})$ takes the form of SVNNs for alternative A_i with respect to attribute C_j. The values associated with the alternatives for MADM problems are presented in the following decision matrix:

$$D = (d_{ij})_{m \times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{bmatrix} \end{array}.$$

The procedures of TOPSIS method with SVNS based on the concept of BCC can be described as follows:

Step 1. Normalization of the decision matrix

Let $D^N = (r_{ij})_{m\times n}$ be the normalized decision matrix, the normalized value r_{ij} can be calculated as follows:

$$r_{ij} = \begin{cases} d_{ij}, & benefit\ type \\ d_{ij}^C, & cost\ type \end{cases}, \tag{16}$$

where d_{ij}^C is the complement of d_{ij}.

Step 2. Calculation of the attribute weights

Since the attribute weights are fully or partially unknown, we use the maximizing deviation method, which is proposed by Wang [24], to calculate the attribute weights. The basic idea is that an attribute will play a less important role with minor difference for all alternatives. According to the maximizing deviation method, the deviation values of alternative A_i to all the other alternatives under the attribute C_j are defined as:

$$D_{ij}(w_j) = \sum_{k=1}^{m} d(r_{ij}, r_{kj}) w_j, \tag{17}$$

the total deviation values of all alternatives to the other alternatives for the attribute C_j are defined as:

$$D_j(w_j) = \sum_{i=1}^{m} D_{ij}(w_j) = \sum_{i=1}^{m}\sum_{k=1}^{m} d(r_{ij}, r_{kj}) w_j. \tag{18}$$

An optimization model is constructed as follows:

$$\begin{cases} \max D(w_j) = \sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m} d(r_{ij}, r_{kj}) w_j \\ s.t. \quad \sum_{j=1}^{n} w_j^2 = 1 \end{cases}. \tag{19}$$

Solving the above model, we get

$$w_j = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} d(r_{ij}, r_{kj})}{\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} d^2(r_{ij}, r_{kj})}}, j = 1, 2, \cdots, n. \tag{20}$$

Normalizing the solutions, we get

$$w_j = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} d(r_{ij}, r_{kj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} d(r_{ij}, r_{kj})}, j = 1, 2, \cdots, n. \tag{21}$$

Step 3. Calculation of the weighted matrix

The weighted matrix can be calculated with the weight vector $W = \{w_1, w_2, \cdots; w_n\}$ as follows:

$$V = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \cdots & w_n r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_n r_{mn} \end{bmatrix}. \tag{22}$$

Step 4. Determination of the PIS and the NIS

$$\begin{aligned} \text{PIS} &= \{v_1^+, v_2^+, \cdots, v_n^+\} \\ &= \{(\max_i \overline{T}_{ij}, \min_i \overline{I}_{ij}, \min_i \overline{F}_{ij}) \mid j = 1, 2, \cdots, n\}, \end{aligned} \tag{23}$$

$$\begin{aligned} \text{NIS} &= \{v_1^-, v_2^-, \cdots, v_n^-\} \\ &= \{(\min_i \overline{T}_{ij}, \max_i \overline{I}_{ij}, \max_i \overline{F}_{ij}) \mid j = 1, 2, \cdots, n\}. \end{aligned} \tag{24}$$

Step 5. Calculation of the distance measures

The distance measure between the alternative V_i and the PIS, denoted by D_i^+, and the distance measure between the alternative V_i and the NIS, denoted by D_i^-, can be calculated by using (6), (11), and (12):

$$\begin{cases} D_i^+ = \sum_{j=1}^{n} d_{EN}(v_{ij}, v_j^+) \\ D_i^- = \sum_{j=1}^{n} d_{EN}(v_{ij}, v_j^-) \end{cases}, i = 1, 2, \cdots, m. \tag{25}$$

Step 6. Calculation of the PIS-closeness coefficients

The PIS-closeness coefficient of the i-th alternative is given by definition 5 as follows:

$$D_i^{PIS} = \frac{D_i^+}{\max_i D_i^+}, i = 1, 2, \cdots, m.$$

Step 7. Calculation of the BCCs

Fix the real number $\theta \in [0,1]$, and the BCC of the alternative V_i is calculated by definition 5 as follows:

$$BCC_i = \frac{D_i^-}{D_i^+ + D_i^-}\theta + (1 - D_i^{PIS})(1-\theta), i = 1, 2, \cdots, m.$$

Step 8. Ranking the alternatives

The larger value of the BCC indicates the better alternative.

IV. Illustrative Examples

A. Selection of product suppliers (adapted from [25])

Example 2. A manufacturing company is going to select a product supplier from ten alternatives: A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}. Suppliers are evaluated based on six attributes: service quality, pricing and cost structure, financial stability, environmental regulation compliance, reliability, relevant experience, denoted by C_1, C_2, C_3, C_4, C_5, C_6.

Suppose the attribute weights are unknown. The final decision information can be presented by the SVNNs, and shown in Table I.

TABLE I (a)
Decision Matrix in Terms of SVNNs

A	C_1	C_2	C_3
A_1	(0.7,0.5,0.1)	(0.7,0.5,0.3)	(0.8,0.6,0.2)
A_2	(0.6,0.5,0.2)	(0.7,0.5,0.1)	(0.6,0.3,0.5)
A_3	(0.6,0.2,0.3)	(0.6,0.6,0.4)	(0.7,0.7,0.2)
A_4	(0.5,0.5,0.4)	(0.6,0.4,0.4)	(0.7,0.7,0.3)
A_5	(0.7,0.5,0.5)	(0.8,0.3,0.1)	(0.7,0.6,0.2)
A_6	(0.5,0.5,0.5)	(0.7,0.8,0.1)	(0.7,0.3,0.5)
A_7	(0.6,0.8,0.1)	(0.7,0.2,0.1)	(0.6,0.3,0.4)
A_8	(0.7,0.8,0.3)	(0.6,0.6,0.5)	(0.8,0,0.5)
A_9	(0.6,0.7,0.1)	(0.7,0,0.1)	(0.6,0.7,0)
A_{10}	(0.5,0.7,0.4)	(0.9,0,0.3)	(1,0,0)

TABLE I (b)
Decision Matrix in Terms of SVNNs

A	C_4	C_5	C_6
A_1	(0.9,0.4,0.2)	(0.6,0.4,0.7)	(0.6,0.5,0.4)
A_2	(0.6,0.4,0.3)	(0.7,0.5,0.4)	(0.7,0.8,0.9)
A_3	(0.5,0.5,0.3)	(0.6,0.8,0.6)	(0.7,0.2,0.5)
A_4	(0.9,0.4,0.2)	(0.7,0.3,0.5)	(0.6,0.4,0.4)
A_5	(0.7,0.5,0.2)	(0.7,0.5,0.6)	(0.6,0.7,0.8)
A_6	(0.4,0.8,0)	(0.7,0.4,0.2)	(0.6,0.6,0.3)
A_7	(0.3,0.5,0.1)	(0.6,0.3,0.6)	(0.5,0.2,0.6)
A_8	(0.7,0.3,0.6)	(0.6,0.8,0.5)	(0.6,0.2,0.4)
A_9	(0.7,0.4,0.3)	(0.6,0.6,0.7)	(0.7,0.3,0.2)
A_{10}	(0.5,0.6,0.7)	(0.5,0.2,0.7)	(0.8,0.4,0.1)

Step 1. Normalization of the decision matrix

As attributes are all benefit type, there is no need of normalization.

Step 2. Calculation of the attribute weights

The attribute weights calculated by (21) are as follows:

w_1 = 0.1379, w_2 = 0.1689, w_3 = 0.1935, w_4 = 0.1777, w_5 = 0.1392, w_6 = 0.1828.

Step 3. Calculation of the weighted matrix

Weighted matrix is calculated by using (22).

Step 4. Determination of the PIS and the NIS

PIS = {(0.1530, 0.8010, 0.7280), (0.3222, 0, 0.6778), (1, 0, 0), (0.3359, 0.8074, 0), (0.1543, 0.7993, 0.7993), (0.2549,0.7451, 0.6565)}, NIS = {(0.0912, 0.9697, 0.9088), (0.1434, 0.9630, 0.8895), (0.1625, 0.9333, 0.8745), (0.0614, 0.9611, 0.9386), (0.0919, 0.9694, 0.9516), (0.1190, 0.9600, 0.9809)}.

Step 5. Calculation of the distance measures

The distance measures calculated by (25) are as follows:

D_1^+ = 0.4343, D_2^+ = 0.4585, D_3^+ = 0.4572, D_4^+ = 0.4471, D_5^+ = 0.4385, D_6^+ = 0.4194, D_7^+ = 0.4273, D_8^+ = 0.4219, D_9^+ = 0.3610, D_{10}^+ = 0.2412, D_1^- = 0.1159, D_2^- = 0.0895, D_3^- = 0.0907, D_4^- = 0.1060, D_5^- = 0.0984, D_6^- = 0.2385, D_7^- = 0.1270, D_8^- = 0.2359, D_9^- = 0.3256, D_{10}^- = 0.4393.

Step 6. Calculation of the PIS-closeness coefficients

The PIS-closeness coefficients are given by (14) as follows:

D_1^{PIS} = 0.9474, D_2^{PIS} = 1, D_3^{PIS} = 0.9973, D_4^{PIS} = 0.9753, D_5^{PIS} = 0.9564, D_6^{PIS} = 0.9147, D_7^{PIS} = 0.9320, D_8^{PIS} = 0.9203, D_9^{PIS} = 0.7874, D_{10}^{PIS} = 0.5261.

Step 7. Calculation of the BCCs

The BCCs calculated by using (15) with $\theta = 0.8$ are as follows:

BCC_1 = 0.1791, BCC_2 = 0.1307, BCC_3 = 0.1330, BCC_4 = 0.1582, BCC_5 = 0.1554, BCC_6 = 0.3071, BCC_7 = 0.1969, BCC_8 = 0.3028, BCC_9 = 0.4219, BCC_{10} = 0.6113.

Step 8. Ranking the alternatives

The ranking of the alternatives is given as below:

$$A_{10} \succ A_9 \succ A_6 \succ A_8 \succ A_7 \succ A_1 \succ A_4 \succ A_5 \succ A_3 \succ A_2.$$

Therefore, the best alternative is supplier A_{10}.

TABLE II
COMPARISON FOR EXAMPLE 2

Method	Ranking
arithmetic operator [25]	$A_1 \succ A_4 \succ A_9 \succ A_5 \succ A_7 \succ A_2 \succ A_{10} \succ A_8 \succ A_3 \succ A_6$
geometric operator [25]	$A_{10} \succ A_9 \succ A_8 \succ A_1 \succ A_5 \succ A_7 \succ A_4 \succ A_2 \succ A_6 \succ A_3$
Biswas [26]	$A_{10} \succ A_9 \succ A_7 \succ A_1 \succ A_4 \succ A_6 \succ A_5 \succ A_8 \succ A_2 \succ A_3$
measure [16]	$A_1 \succ A_9 \succ A_4 \succ A_7 \succ A_5 \succ A_2 \succ A_{10} \succ A_6 \succ A_8 \succ A_3$
Ye [27]	$A_9 \succ A_7 \succ A_1 \succ A_4 \succ A_2 \succ A_{10} \succ A_5 \succ A_8 \succ A_3 \succ A_6$
our method	$A_{10} \succ A_9 \succ A_6 \succ A_8 \succ A_7 \succ A_1 \succ A_4 \succ A_5 \succ A_3 \succ A_2$

Some recent works in this area are tabulated in Table II. It can be seen that different methods result different rankings and optimal alternatives. The main reason for the difference is that the attribute weights are calculated differently. For example, the method proposed in [27] used the subjective weighting method to calculate the attribute weights, while the method in [25] used the weighted arithmetic average operator and geometric average operator, respectively. The approach proposed in this paper uses the BCC, which considers both the relative distance measure and the absolute distance measure, to rank the alternatives. Therefore, it is understandable that different ways of calculating weights lead to different results.

The value of θ determines the proportion of RCC in BCC. As θ decreases from 0.9 to 0.1, we find the best alternative remains the same, which implies the robustness of the proposed method.

For critical analysis, we apply the proposed approach to a problem about selection of the best seller.

B. *Selection of the best seller (adapted from [28])*

Example 3. A book publisher wants to publish the best seller. There are ten alternatives: *Java Language, C Language, C++ Language, Python Language, C# Language, PHP Language , Java-script Language, Visual Basic.NET Language, Perl Language, Assembly Language,* denoted by A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9 and A_{10}, respectively. Five attributes, popularity, innovation, price, interaction, readability, denoted by C_1, C_2, C_3, C_4, C_5, are evaluated. C_1, C_2, C_4, C_5 are benefit type, and C_3 is cost type. Suppose the attribute weights are completely unknown. The final decision information presented by the SVNNs is shown in Table III.

TABLE III (a)
DECISION MATRIX IN TERMS OF SVNNS

A	C_1	C_2	C_3
A_1	(0.6,0.7,0.8)	(0.4,0.5,0.8)	(0.4,0.7,0.2)
A_2	(0.5,0.6,0.8)	(0.5,0.6,0.7)	(0.4,0.8,0.1)
A_3	(0.6,0.7,0.9)	(0.5,0.7,0.8)	(0.3,0.6,0.1)
A_4	(0.6,0.8,0.7)	(0.5,0.5,0.7)	(0.4,0.7,0.3)
A_5	(0.5,0.8,0.9)	(0.6,0.5,0.7)	(0.5,0.7,0.2)
A_6	(0.6,0.7,0.8)	(0.4,0.5,0.8)	(0.4,0.6,0.2)
A_7	(0.4,0.6,0.7)	(0.5,0.6,0.7)	(0.4,0.8,0.1)
A_8	(0.6,0.6,0.9)	(0.4,0.7,0.8)	(0.3,0.5,0.1)
A_9	(0.7,0.8,0.7)	(0.5,0.5,0.6)	(0.3,0.7,0.2)
A_{10}	(0.6,0.8,0.9)	(0.6,0.6,0.7)	(0.5,0.7,0.2)

TABLE III (b)
DECISION MATRIX IN TERMS OF SVNNS

A	C_4	C_5
A_1	(0.5,0.7,0.8)	(0.6,0.2,0.3)
A_2	(0.4,0.6,0.9)	(0.5,0.1,0.3)
A_3	(0.4,0.6,0.9)	(0.4,0.2,0.2)
A_4	(0.6,0.8,0.7)	(0.7,0.1,0.4)
A_5	(0.5,0.7,0.7)	(0.6,0.2,0.3)
A_6	(0.5,0.7,0.8)	(0.6,0.2,0.3)
A_7	(0.5,0.6,0.9)	(0.6,0.1,0.3)
A_8	(0.4,0.7,0.9)	(0.6,0.2,0.2)
A_9	(0.6,0.8,0.8)	(0.7,0.2,0.4)
A_{10}	(0.5,0.6,0.7)	(0.6,0.2,0.3)

Step 1. Normalization of the decision matrix

Normalized decision matrix is listed in Table IV.

TABLE IV (a)
NORMALIZED DECISION MATRIX

A	C_1	C_2	C_3
A_1	(0.6,0.7,0.8)	(0.4,0.5,0.8)	(0.2,0.3,0.4)
A_2	(0.5,0.6,0.8)	(0.5,0.6,0.7)	(0.1,0.2,0.4)
A_3	(0.6,0.7,0.9)	(0.5,0.7,0.8)	(0.1,0.4,0.3)
A_4	(0.6,0.8,0.7)	(0.5,0.5,0.7)	(0.3,0.3,0.4)
A_5	(0.5,0.8,0.9)	(0.6,0.5,0.7)	(0.2,0.3,0.5)
A_6	(0.6,0.7,0.8)	(0.4,0.5,0.8)	(0.2,0.4,0.4)
A_7	(0.4,0.6,0.7)	(0.5,0.6,0.7)	(0.1,0.2,0.4)
A_8	(0.6,0.6,0.9)	(0.4,0.7,0.8)	(0.1,0.5,0.3)
A_9	(0.7,0.8,0.7)	(0.5,0.5,0.6)	(0.2,0.3,0.3)
A_{10}	(0.6,0.8,0.9)	(0.6,0.6,0.7)	(0.2,0.3,0.5)

TABLE IV (b)
NORMALIZED DECISION MATRIX

A	C_4	C_5
A_1	(0.5,0.7,0.8)	(0.6,0.2,0.3)
A_2	(0.4,0.6,0.9)	(0.5,0.1,0.3)
A_3	(0.4,0.6,0.9)	(0.4,0.2,0.2)
A_4	(0.6,0.8,0.7)	(0.7,0.1,0.4)
A_5	(0.5,0.7,0.7)	(0.6,0.2,0.3)
A_6	(0.5,0.7,0.8)	(0.6,0.2,0.3)
A_7	(0.5,0.6,0.9)	(0.6,0.1,0.3)
A_8	(0.4,0.7,0.9)	(0.6,0.2,0.2)
A_9	(0.6,0.8,0.8)	(0.7,0.2,0.4)
A_{10}	(0.5,0.6,0.7)	(0.6,0.2,0.3)

Step 2. Calculation of the attribute weights

The attribute weights calculated by using (21) are as follows:

$w_1 = 0.2255$, $w_2 = 0.1941$, $w_3 = 0.2028$, $w_4 = 0.2133$, $w_5 = 0.1643$.

Step 3. Calculation of the weighted matrix

Weighted matrix is calculated by using (22).

Step 4. Determination of the PIS and the NIS

The PIS and the NIS derived by (23) and (24) are as follows:

PIS = {(0.2378, 0.8912, 0.9227), (0.1629, 0.8741, 0.9056), (0.0698, 0.7215, 0.7834), (0.1775, 0.8968, 0.9267), (0.1795, 0.6850, 0.7676) }, NIS = {(0.1088, 0.9509, 0.9765), (0.0944, 0.9331, 0.9576), (0.0211, 0.8689, 0.8689), (0.1032, 0.9535, 0.9778), (0.0805, 0.7676, 0.8602) }.

Step 5. Calculation of the distance measures

$D_1^+ = 0.0468$, $D_2^+ = 0.0471$, $D_3^+ = 0.0595$, $D_4^+ = 0.0417$, $D_5^+ = 0.0525$, $D_6^+ = 0.0523$, $D_7^+ = 0.0468$, $D_8^+ = 0.0621$, $D_9^+ = 0.0438$, $D_{10}^+ = 0.0486$, $D_1^- = 0.0431$, $D_2^- = 0.0538$, $D_3^- = 0.0435$, $D_4^- = 0.0577$, $D_5^- = 0.0428$, $D_6^- = 0.0383$, $D_7^- = 0.0567$, $D_8^- = 0.0445$, $D_9^- = 0.0621$, $D_{10}^- = 0.0462$.

Step 6. Calculation of the PIS-closeness coefficients

The PIS-closeness coefficients are given by (14) as follows:

$D_1^{PIS} = 0.7541$, $D_2^{PIS} = 0.7578$, $D_3^{PIS} = 0.9571$, $D_4^{PIS} = 0.6720$, $D_5^{PIS} = 0.8455$, $D_6^{PIS} = 0.8411$, $D_7^{PIS} = 0.7541$, $D_8^{PIS} = 1$, $D_9^{PIS} = 0.7049$, $D_{10}^{PIS} = 0.7822$.

Step 7. Calculation of the BCCs

The BCCs calculated by using (15) with $\theta = 0.9$ are as follows:

$BCC_1 = 0.4559$, $BCC_2 = 0.5042$, $BCC_3 = 0.3847$, $BCC_4 = 0.5550$, $BCC_5 = 0.4196$, $BCC_6 = 0.3968$, $BCC_7 = 0.5176$, $BCC_8 = 0.3758$, $BCC_9 = 0.5572$, $BCC_{10} = 0.4605$.

Step 8. Ranking the alternatives

The ranking of the alternatives is given as below:

$A_9 \succ A_4 \succ A_7 \succ A_2 \succ A_{10} \succ A_1 \succ A_5 \succ A_6 \succ A_3 \succ A_8$.

Therefore, the optimal alternative is A_9: *Perl Language.*

TABLE V
COMPARISON FOR EXAMPLE 3

Method	Ranking
TOPSIS [28]	$A_9 \succ A_4 \succ A_5 \succ A_{10} \succ A_7 \succ A_1 \succ A_2 \succ A_6 \succ A_3 \succ A_8$
MABAC [28]	$A_9 \succ A_4 \succ A_7 \succ A_{10} \succ A_5 \succ A_2 \succ A_1 \succ A_8 \succ A_6 \succ A_3$
TOPSIS [15]	$A_9 \succ A_4 \succ A_5 \succ A_{10} \succ A_7 \succ A_1 \succ A_2 \succ A_6 \succ A_3 \succ A_8$
our method	$A_9 \succ A_4 \succ A_7 \succ A_2 \succ A_{10} \succ A_1 \succ A_5 \succ A_6 \succ A_3 \succ A_8$

We compare different methods and tabulate the results in Table V. As can be seen, our proposed method picks the same optimal alternative as the others although they have little differences in rankings. This shows that the proposed TOPSIS method is effective.

TABLE VI
SENSITIVITY ANALYSIS

θ	Ranking	The Optimal
0.9	$A_9 \succ A_4 \succ A_7 \succ A_2 \succ A_{10} \succ A_1 \succ A_5 \succ A_6 \succ A_3 \succ A_8$	A_9
0.8	$A_4 \succ A_9 \succ A_7 \succ A_2 \succ A_{10} \succ A_1 \succ A_5 \succ A_6 \succ A_3 \succ A_8$	A_4
0.7	$A_4 \succ A_9 \succ A_7 \succ A_2 \succ A_1 \succ A_{10} \succ A_5 \succ A_6 \succ A_3 \succ A_8$	A_4
0.2	$A_4 \succ A_9 \succ A_7 \succ A_2 \succ A_1 \succ A_{10} \succ A_5 \succ A_6 \succ A_3 \succ A_8$	A_4
0.1	$A_4 \succ A_9 \succ A_7 \succ A_2 \succ A_1 \succ A_{10} \succ A_6 \succ A_5 \succ A_3 \succ A_8$	A_4

Sensitivity analysis is tabulated in Table VI. As the value of θ decreases from 0.9 to 0.8, the optimal alternative changes from A_9 to A_4. This indicates that the critical value is between 0.8 and 0.9. As θ decreases from 0.8 to 0.1, the optimal alternative remains the same, which implies the robustness of the proposed method.

V. CONCLUSIONS

By integrating the relative distance measure and the absolute distance measure, we introduce the concept of BCC to measure the distance between each alternative and the PIS. An improved TOPSIS method is proposed on the basis of BCC. In some special cases, the decision maker will obtain different rankings as the value of RCL θ changes. By changing the value of θ, our method also enables the decision maker to test the robustness of the rankings.

Next, we shall extend the application of the proposed method to INS environment as well as multiple attribute group decision making problems.

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