Sum Signed Graphs, Parity Signed Graphs and Cordial Graphs

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***Abstract*—Signed graphs are graphs with every edge is signed either positive or negative. Given an n vertex graph, the vertices are bijectively labelled from 1 to n. A signed graph is a sum signed graph if and only if every edge is signed negative whenever the sum of the vertex labels exceeds n and every edge is signed positive whenever the sum of the vertex labels does not exceed n. For a parity signed graph, an edge receives a negative sign, if the end vertices are of opposite parity and a positive sign otherwise. Cordial signed graphs are the ones with the difference between the total number of negative edges and the positive ones is at most 1. We discuss the connection between sum signed labeling with parity signed labeling and cordial labeling. The absolute cordial condition for graphs satisfying sum signed labeling will be analyzed.**

***Index Terms*—signed graph, sum signed graph, parity signed graph, cordial labeling, *rna* number.**

I. Introduction

GRAPHS are models of structures. When it comes to discrete structures, they are also known as networks. Graph models are essential in the development of algorithms, computation of complexity of algorithms, laying of networks, developing techniques of optimization, efficient coding[7], understanding patterns[6], estimating physio-chemical properties [3], [9], [8] etc. In recent decades, the concept of signed graphs has gained a tremendous amount of attention among researchers. The notion of signed graphs originated primarily to depict psycho-social scenarios. Here, people are represented as vertices, and the relationship between every two of them is portrayed using positive or negative edges[5]. Many variations of signed graphs, including placing conditions on the relation between the edges and labeling of vertices, have come into existence. The concept of sum signed graphs deals with assigning signs of edges based on the sum of the labels of the vertices, as the name indicates.

A graph, $S = (G,\sigma)$ is a signed graph where $G = (V, E)$ is the underlying unsigned graph and σ: $E(G) \to \{+,-\}$. An edge that receives a $+(-)$ sign is called a positive (negative) edge of S. The set $E^+(S)(E^-(S))$ is the set of positive(negative) edges of S and $E(S) = E^+(S) \bigcup E^-(S)$. If the signed graph consists of only positive (negative) edges then the graph is an all-positive (negative) signed graph. Such graphs are also known as homogeneous signed graphs. Otherwise, the signed graph is a heterogeneous signed graph. All the graphs considered in this paper are simple graphs with no loops. For terminologies of graphs, we refer to [13], and for signed graphs, we refer to [15].

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II. Sum Signed Graphs

Definition 2.1: A sum signed graph is a trio, $S = (G, f, \sigma)$ where G is a graph, $f : V(G) \longrightarrow \{1, 2, \dots, |V(G)|\}$ is a bijective function and $\sigma : E(G) \longrightarrow \{+,-\}$ is a mapping such that $\sigma(uv) = +$, whenever $f(u)+f(v) \leq n$ and $\sigma(uv) = -$, whenever $f(u)+f(v) > n$.

In a company, the CEO decides to conduct a discussion with the top seven employees of his company. He asks his assistant to draw up a seating chart. The assistant draws a graph depicting the seating arrangement based on the employees' preferences, with the employees and the CEO as the vertices. The assistant decides that a positive edge between the vertices indicates a good employee relationship. The scenario in which there is a lack of edge between the employees indicates that the employees do not have a good relationship with each other. Also, it was decided to mark the CEO as a vertex having only negative edges as he does not have any preference. See Fig. 1 for a graph representing the seating arrangement of eight persons. The thick line represents the positive edge, and the dashed line represents the negative edge.

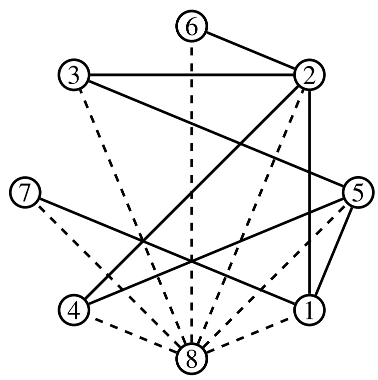

Fig. 1. An 8-person seating arrangement

The assistant found that each person is compatible with someone of the same gender. As a process of elimination and to increase the comfort of each person, he thought each person should sit with at least one person of the same gender. This idea was implemented by considering the edge between vertices with the same parity as persons having the same gender. Since the CEO will be in the discussion for a short period, the assistant neglected this condition. See Fig. 2 that represents the seating arrangement of eight persons in the discussion when the gender condition is also counted in. Here, when the adjacent vertices are of the same parity, the employees are considered to be of the same gender. If the adjacent vertices are of opposite parity, they are considered to be of a different gender.

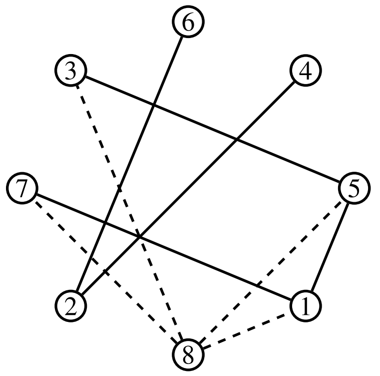

Fig. 2. An 8-person seating arrangement with gender condition

III. Parity Signed Graphs

The assignment of signs to edges based on parity of labels is known as parity signed labeling. It was introduced by Acharya and Kureethara [1] in 2020. The definition of parity signed labeling is as follows. See Fig. 2.

Definition 3.1: A signed graph $S = (G, \sigma)$ is a parity signed graph if there exists a bijection $f : V(G) \longrightarrow \{1, 2...n\}$ such that for an edge uv in G, $\sigma(uv) = +$ if $f(u)$ and $f(v)$ are of same parity and $\sigma(uv) = -$ if $f(u)$ and $f(v)$ are of opposite parity.

IV. Parity signed labeling and Sum signed labeling

The idea behind parity signed labeling was that if the adjacent vertices have labels of the same parity then the edge between them will be positive. If the adjacent vertices have labels of opposite parity then the edge between them will be negative. In other words, we can say that if the sum of labels of adjacent vertices is even (odd), then the edge between them will be positive (negative). Taking inspiration from the interpretation of parity signed labeling in terms of the sum of labels, the concept of sum signed labeling was introduced. There are some striking similarities between parity signed graphs and sum signed graphs. We list some of them now.

Theorem 4.1: A negative homogeneous sum signed graph of path P_n satisfies parity signed labeling if and only if n is even.

Proof: Consider a negative homogeneous sum signed path S of a path P_n having n number of vertices. If possible, assume that S also satisfies parity signed labeling. Then, the end vertices of every edge should receive labels of different parity. Since the sum signed path is negative homogeneous, the label 1 should be assigned to one of the end vertices, and the adjacent vertex should receive the maximum label n, which should be of opposite parity. This implies that n can only be even.

Assume that n is even. Then, there are $\frac{n}{2}$ odd and even integers. Let $v_1, v_2, ..., v_n$ be the n vertices of path such that v_1 and v_n are end vertices. Let the odd -indexed vertices $v_1, v_3, ..., v_{n-1}$ be assigned the labels $1, 3, ..., n-1$, respectively and the even -indexed vertices $v_2, v_4, ..., v_n$ be assigned the labels $n, n-2, ..., 2$, respectively. This implies that the sum of labels of any two adjacent vertices will be greater than n, making the graph satisfy the sum signed labeling. Since the adjacent vertices have labels of opposite parity, the graph also satisfies parity signed labeling. ■

Theorem 4.2: A sum signed graph of a cycle $C_n, n \geq 6$ satisfies parity signed labeling whenever the following conditions are sufficed.

- The label n should only be adjacent to labels of opposite parity
- The label *1* should be adjacent to at least one odd label.

Proof: Consider a cycle C_n with n number of vertices. Let S_1 be a sum signed graph of the cycle in which the vertex with label n is adjacent to vertices with labels of the same parity. By the parity signed labeling, the edge between these vertices should receive a positive sign as the labels are of the same parity. Since S_1 is a sum signed graph all the edges incident to the vertex having the label n will be negative in sign. This leads to the first condition.

Let S_2 be a sum signed graph of the cycle in which the vertex u with label 1 is adjacent to vertices with even label. If n is even and it is adjacent to u then, by sum signed labeling the edge will be negative. Also, since they are of opposite parity, the edge between them will be negative by parity signed labeling. If u is adjacent to another vertex with an even label then, the edge between them will be positive in S_1. By parity signed labeling, the edge will receive a negative sign as they are of opposite parity. This gives us the second condition.

In Fig. 3, we see three pairs of signed graphs of order 3, 4 and 5. In each of the pairs, the first signed graph satisfies sum signed labeling whereas the second signed graph with the same labelings satisfies parity signed labeling. Whenever $n < 6$, the sum signed graph of a cycle C_n does not satisfy the conditions for parity signed labeling. ■

Theorem 4.3: The sum signed graph of a complete graph never satisfies parity signed labeling.

Proof: Consider a complete graph K_n with n vertices. Let S be its unique sum signed graph and u be the vertex having the label 1. Then u is adjacent to all vertices having labels of the same and opposite parity with positive edge except the one vertex with label n. If n is odd then, the graph S will not satisfy parity signed labeling. If n is even, then there will exist another even labeled vertex adjacent to u with a positive edge. This implies that the sum signed graph of a complete graph will not satisfy parity signed labeling. ■

Theorem 4.4: For a sum signed graph to satisfy parity signed labeling the following conditions should hold.

1) The vertex with the minimum label can only be adjacent to vertices with labels of the same parity.
2) The vertex with the maximum label can be adjacent to the vertex with the minimum label whenever the former is even.
3) The vertex with the maximum label can only be adjacent to vertices with labels of opposite parity.

Proof: Let G be a graph with n number of vertices and S be one of its sum signed graphs. The vertices of the graph can be assigned labels from $\{1, 2, ..., n\}$. In sum signed labeling, any vertex adjacent to the vertex labeled 1 having a label other than n has a positive edge. In parity signed labeling an edge between two vertices receives a positive sign whenever they have labels of the same parity. Hence, the vertex with label 1 can only be adjacent to vertices with labels of the

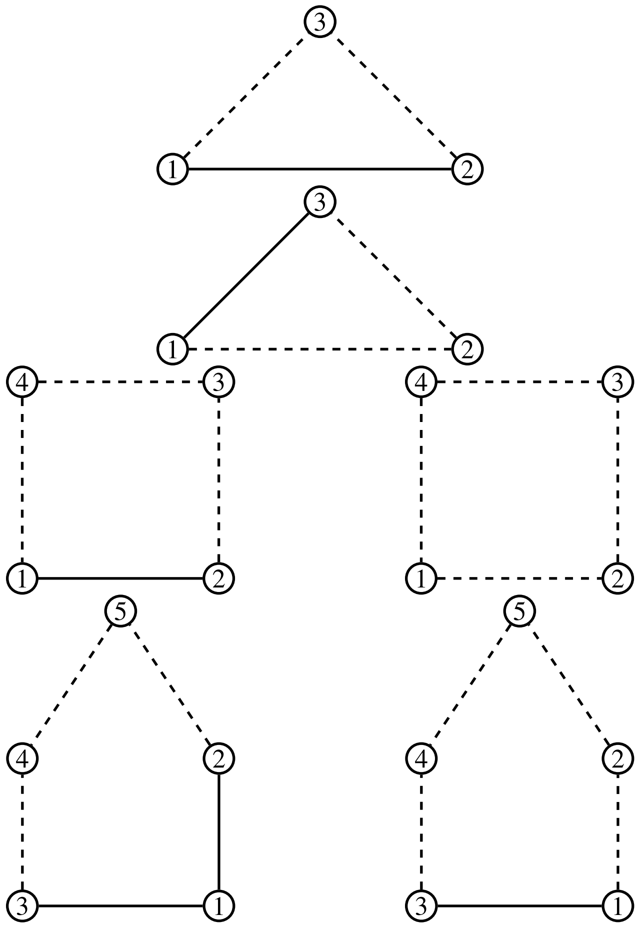

Fig. 3. Sum Signed Labeling and Parity Signed Labeling Comparison

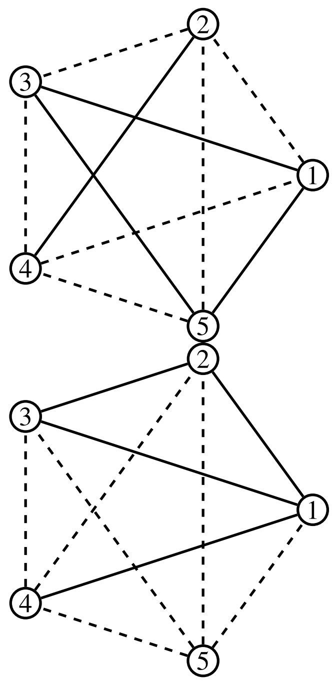

Fig. 4. Parity Signed Labeling and Sum Signed Labeling of K_5

same parity. In other words, it can only be adjacent to odd labels.

In sum signed labeling, the edge between the vertices with minimum label 1 and maximum label n will be a negative edge. But the two labels should be of opposite parity to satisfy parity signed labeling. Thus, the maximum label should be even since the minimum label is odd.

In sum signed labeling, any vertex adjacent to the vertex labeled n has a negative edge whereas in parity signed labeling an edge between two vertices receives a negative sign whenever they ate of opposite parity. Hence, the third condition. ■

An illustration of this result is seen in Fig. 4 in the case of K_5. The first graph is satisfying parity signed labeling whereas the second one is satisfying sum signed labeling. Though in both the graphs the labels are the same the signed graphs are different.

We have seen above the conditions for which some graphs satisfy both sum signed and parity signed labeling. Now, we will check which all labels can be adjacent in a graph such that they satisfy both labeling. It is vital to remember that we can restate the definition of parity signed labeling in terms of the sum of the labels also. If the sum of the labels is even, which is possible when the labels are of the same parity, the edge will receive a positive sign. Similarly, if the sum of labels is odd, which is possible when the labels are of opposite parity then, the edge will receive a negative.

Consider a graph G with n number of vertices. Then, the vertices $\{v_1, v_2, ..., v_n\}$ can be assigned labels from $\{1, 2, ..., n\}$. Let u be a vertex of G having the label $\frac{n}{2}$.

Case 1: *n is even*

Sub-case 1: $\frac{n}{2}$ is even

The label $\frac{n}{2}$ can be adjacent to either labels lower or higher than $\frac{n}{2}$. These labels can be either odd or even. The highest among the labels less than $\frac{n}{2}$ which is odd is $\frac{n}{2}-1$ and $\frac{n}{2}-2$ is even. The lowest among the labels greater than $\frac{n}{2}$ is $\frac{n}{2}+1$ and $\frac{n}{2}+2$ which is odd and even, respectively.

When u is adjacent to a vertex having an odd label lower than $\frac{n}{2}$,

$$\frac{n}{2} + \frac{n-2}{2} = n - 1.$$

The sum is odd and less than n. The edge between these vertices will receive a negative and a positive sign by parity and sum signed labeling, respectively. Hence, such a combination is not possible.

When u is adjacent to a vertex having an even label lower than $\frac{n}{2}$,

$$\frac{n}{2} + \frac{n-4}{2} = n - 2.$$

The sum is even and less than n. By parity signed and sum signed labeling the edge between the vertices will be positive. Hence, such a combination is possible.

When u is adjacent to a vertex having an odd label greater than $\frac{n}{2}$,

$$\frac{n}{2} + \frac{n+2}{2} = n + 1.$$

The sum is odd and greater than n. By parity signed and sum signed labeling the edge between these vertices will be negative. Hence, such a combination is possible.

When u is adjacent to a vertex having an even label greater than $\frac{n}{2}$,

$$\frac{n}{2} + \frac{n+4}{2} = n + 2.$$

The sum is even and greater than n. The edge between these vertices will receive a positive and a negative sign by parity and sum signed labeling respectively. Hence, such a combination is not possible.

Sub-case 2: $\frac{n}{2}$ is odd

Similarly, as above label $\frac{n-1}{2}$ can be adjacent to either labels lower or higher than $\frac{n-1}{2}$. These labels can be either odd or even. The highest among the labels lesser than $\frac{n}{2}$ which is odd is $\frac{n}{2}-2$ and $\frac{n}{2}-1$ is even. The lowest among the labels greater than $\frac{n-1}{2}$ is $\frac{n}{2}+2$ and $\frac{n}{2}+1$ which is odd and even, respectively.

When u is adjacent to a vertex having an odd label lower than $\frac{n-1}{2}$,

$$\frac{n-1}{2}+\frac{n-5}{2}=n-3.$$

The sum is odd and less than n. The edge between these vertices will receive a negative and a positive sign by parity and sum signed labeling respectively. Hence, such a combination is not possible.

When u is adjacent to a vertex having an even label lower than $\frac{n-1}{2}$,

$$\frac{n-1}{2}+\frac{n-3}{2}=n-2.$$

The sum is even and less than n. By parity signed and sum signed labeling the edge between these vertices will be positive. Hence, such a combination is possible.

When u is adjacent to a vertex having an odd label greater than $\frac{n-1}{2}$,

$$\frac{n-1}{2}+\frac{n+1}{2}=n.$$

The sum is even and equal to n. By parity signed and sum signed labeling the edge between these vertices will be positive. Hence, such a combination is possible.

When u is adjacent to a vertex having an even label greater than $\frac{n-1}{2}$,

$$\frac{n-1}{2}+\frac{n+3}{2}=n+1.$$

The sum is even and greater than n. The edge between these vertices will receive a positive and a negative sign by parity and sum signed labeling, respectively. Hence, such a combination is not possible.

V. Absolute Cordial Sum Signed Graph

The concept of cordial labeling was introduced by Cahit [4] in 1987. He introduced the labeling as a weaker version of graceful and harmonious labeling. Later many variations of the labeling came into existence. In 1997, Yilmaz and Cahit [14] initiated the study of edge cordial labeling as a weaker version of graceful labeling. Acharya and Kureethara [2] presented the idea of cordiality in parity signed graphs. They also defined the condition of absolute cordial in parity signed graphs. Here we will be redefining those concepts in the case of sum signed graphs.

There exist two important parameters that are useful in the study of absolute cordial sum signed graphs: the *rna* number and *adhika* number. These parameters have been discussed in [10], [11], [12] and the definition is as follows.

Definition 5.1: The smallest number of negative edges among all sum signed labeling of its underlying graph G is called the *rna* number. It is denoted by $\sigma^{-}(G)$.

Definition 5.2: The largest number of positive edges among all sum signed labeling of its underlying graph G is called the *adhika* number. It is denoted by $\sigma^{+}(G)$

Definition 5.3: A sum signed graph is cordial if $||E^{-}(S)|-|E^{+}(S)||\leq 1$. And, a sum signed graph is said to be absolutely cordial if $|\sigma^{-}(G)-\sigma^{+}(G)|\leq 1$.

Let G be a graph with n number of vertices and m number of edges. Let u and v represent the vertices having the minimum degree, $\delta(G)$ and maximum degree, $\Delta(G)$ of the graph respectively. For a graph to have the possibility of multiple sum signed labeling there should exist at least three vertices. We will be discussing a basic scheme to create sum signed graphs from an underlying graph having a different number of negative edges.

To get a sum signed graph with the minimum number of negative edges, we assign maximum label n to the vertex u and minimum label 1 to the vertex v. All the edges incident to u and v will be negative and positive respectively unless u and v are adjacent making the edge negative. If the vertex v is adjacent to x number of pendant vertices then, assign x number of maximum labels from the remaining set $\{2,3,...,n-1\}$. Thus, the edge between the pendant vertex and v will be positive. If there is no pendant vertex then assign the maximum labels from the set $\{2,3,...,n-1\}$ to the vertices adjacent to u depending on the degree that is the maximum label out of the set is assigned to the adjacent vertex of u with minimum degree. If there exists a pendant vertex to the vertices adjacent to v then, assign the maximum labels y from the set $\{2,3,...,n-1\}$ to those pendant vertices and assign the label $n-y$ to the common vertex of v and the pendant vertices. Taking into consideration all these things and many more we can maximize the number of positive edges in the graph giving us a sum signed graph with $\sigma^{-}(G)$ number of negative edges.

To get a sum signed graph with the maximum number of negative edges, we assign the maximum label n to the vertex v and minimum label 1 to the vertex u. If there exists two vertices in G with degree $\delta(G)$ of which one is adjacent to v then, assign the label 1 to that vertex. All the edges incident to u and v will be positive and negative respectively unless u and v are adjacent making the edge negative. Since the vertex v is adjacent to $\Delta(G)$ number of vertices then, assign $\Delta(G)$ number of minimum labels from the set $\{2,3,...,n-1\}$ to those vertices. Assign maximum labels from the remaining set of labels to the adjacent vertices of the $\Delta(G)$ vertices so that the edges between the vertices with minimum and maximum labels are negative. Taking into consideration all these things and many more we can minimize the number of positive edges in the graph giving us a sum signed graph with $\sigma_{+}(G)$ number of positive edges.

Since the number of negative and positive edges can be manipulated in the sum signed labeling of a graph, we can surely find at least one sum signed labeling of the graph which is cordial.

In Fig. 5, we can see that for a single graph, there exists different sum signed labeling whose negative edges vary in number. For the above graph G, six vertices and ten edges exist. In the third figure, we can see that the $||E^{-}(S_3)|-$

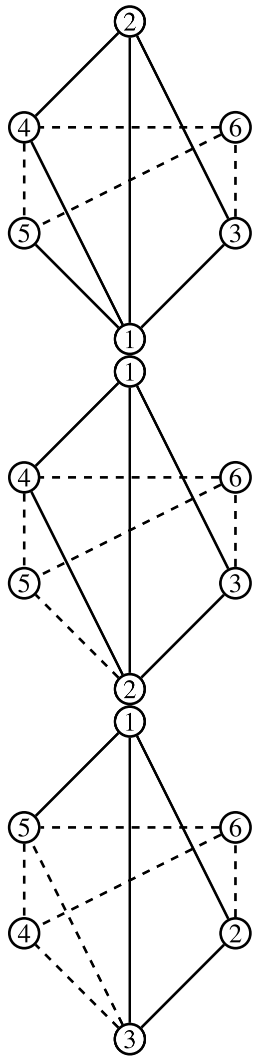

Fig. 5. Same graph but different labelings

$|E^+(S_4)|| \leq 1|$ which is the condition for cordial. Hence, we can conclude that every graph which has multiple sum signed labeling is cordial in nature.

The question which then arises in our mind is whether a graph having a single sum signed labeling will be cordial or not.

Theorem 5.4: The complete graph, K_n satisfying sum signed labeling is cordial whenever $n \leq 3$

Proof: Let K_n be a complete graph with n number of vertices and m number of edges. For, complete graph, K_n $m = \frac{n(n-1)}{2}$. Let S be its unique sum signed graph which is due to the fact that every vertex is adjacent to the other in the graph. Then, $|E^-(S)| = \sigma^-(K_n)$, hence, $\sigma^+(K_n) = m - \sigma^-(K_n)$. If possible assume that S satisfies the condition for cordial signed graph.

Case 1 *n is even*

When, K_n is having an even number of vertices, $|E^-(S)| = \frac{n^2}{4}$ [10]. Since, S is cordial in nature we have,

$$||E^-(S)| - |E^+(S)|| \leq 1$$

$$|\frac{n^2}{4} - (m - \frac{n^2}{4})| \leq 1$$

$$|\frac{n^2}{2} - \frac{n(n-1)}{2}| \leq 1 \Rightarrow n \leq 2$$

Case 2 *n is odd*

When, K_n is having an odd number of vertices, $|E^-(S)| = \frac{n^2-1}{4}$ [10]. Since, S is cordial sum signed in nature we have,

$$||E^-(S)| - |E^+(S)|| \leq 1$$

$$|\frac{n^2-1}{4} - (m - \frac{n^2-1}{4})| \leq 1$$

$$|\frac{n^2-1}{2} - \frac{n(n-1)}{2}| \leq 1 \Rightarrow n \leq 3$$

Thus, the condition for cordial signed graph is satisfied in complete sum signed graph only when $n \leq 3$. ■

Corollary 5.5: The complete graph, K_n satisfying sum signed labeling is absolute cordial whenever $n \leq 3$.

Theorem 5.6: A tree with n vertices satisfying sum signed labeling is absolutely cordial whenever $n \leq 4$.

Proof: Assume that every tree T with n number of vertices satisfying sum signed labeling is absolutely cordial. The total number of edges in a tree is $n-1$. From [10], it has been observed that the $\sigma^-(T) = 1$. Hence, $\sigma^+(T) = (n-1) - 1$. Since T is absolutely cordial the following equations can be considered.

$$|\sigma^-(G) - \sigma^+(G)| \leq 1$$

$$|1 - [(n-1) - 1]| \leq 1$$

$$|3 - n| \leq 1 \Rightarrow n \leq 4$$

Hence, every tree satisfying sum signed labeling with at least four vertices can be said to be absolutely cordial in nature. ■

Theorem 5.7: A cycle C_n satisfying sum signed labeling is absolutely cordial whenever $n \leq 5$

Proof: Assume that every cycle C_n with n number of vertices satisfying sum signed labeling is absolutely cordial. The total number of edges in a cycle is n. From [10], it has been observed that the $\sigma^-(C_n) = 2$. Hence, $\sigma^+(C_n) = n-1$. Since C_n is absolutely cordial the following equations can be considered.

$$|\sigma^-(G) - \sigma^+(G)| \leq 1$$

$$|2 - (n-2)| \leq 1$$

$$|4 - n| \leq 1 \Rightarrow n \leq 5$$

Hence, every cycle satisfying sum signed labeling with at least five vertices can be said to be absolutely cordial in nature. ■

Theorem 5.8: A graph G satisfying sum signed labeling is absolutely cordial whenever $\frac{m-1}{2} \leq \sigma^-(G) \leq \frac{m+1}{2}$ where n and m denote the number of vertices and edges respectively.

Proof: Let G be a graph with n number of vertices and m number of edges satisfying sum signed labeling. Let $\sigma^-(G)$ denote the minimum number of negative edges and $\sigma^+(G)$ denote the maximum number of positive edges out of all the sum signed labelings of G. Then, $\sigma^+(G) = m - \sigma^-(G)$. If G is absolutely cordial in nature then the following condition should hold.

$$|\sigma^-(G) - \sigma^+(G)| \leq 1$$

$$|\sigma^-(G) - (m - \sigma^-(G))| \leq 1$$

$$|2\sigma^-(G) - m| \leq 1$$
$$2\sigma^-(G) - m \leq 1 \;\&\; 2\sigma^-(G) - m \geq -1$$
$$\sigma^-(G) \leq \frac{m+1}{2} \;\&\; \sigma^-(G) \geq \frac{m-1}{2}$$
$$\Rightarrow \frac{m-1}{2} \leq \sigma^-(G) \leq \frac{m+1}{2}$$

Thus, a sum signed graph satisfying the above condition will satisfy the absolute cordial condition. ■

VI. CONCLUSION

In this paper, the sum signed graphs and the parity signed graphs are compared. The cordiality and the absolute cordiality of sum signed graphs and parity signed graphs are also briefly investigated. Both these signed graphs are relatively new. Applications of the sum signed graphs and the parity signed graphs in neuroscience, complex networks, data mining, clustering, social psychology, etc. are yet to be studied.

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