New Information Model MGRM (1, n) Based on Vector Continued Fractions and Reciprocal Accumulation Generation Operation

Xiaogao Yang, Deqiong Ding*, Xiaozhong Zhang

***Abstract*—In engineering applications and scientific research, the parameters of multivariable research systems affect and restrict each other, resulting in inaccurate descriptions of the established analysis models and low accuracy of analysis data. Based on vector continued fraction theory and reciprocal accumulation generation, a multivariable grey MGRM (1, n) model is established in this paper. The model uses extrapolation and the trapezoidal formula of rational interpolation and numerical integration to construct the background value of the model. A component of the variable data generated by reciprocal accumulation is taken as the initial value of the solution of the grey differential equation. Through experimental analysis and comparative research, the results show that the non-equidistant multivariable innovation model MGRM (1, n) based on vector continued fraction theory and reciprocal accumulation has the characteristics of high accuracy and ease of use. The model does not need to consider the monotonicity of the data sequence, which has significant application prospects and is of theoretical value to the scientific and engineering fields.**

***Index Terms*—Innovation model, Multivariable grey model, Reciprocal accumulation generation, Vector continued fractions**

I. INTRODUCTION

Scientific research and practical engineering problem analysis succeed or fail depending on whether the model can accurately describe the research object [1]-[3]. Therefore, many researchers have established analysis models of research objects, conducted in-depth studies, and achieved many scientific developments [4]-[8]. Grey system theory, founded by Professor Deng, studies the uncertainty of little data and poor information [9], which has been widely used in grey system analysis, modeling, prediction, decision-making, and control. As an essential part of grey system theory, the grey prediction model can find data rules during data processing. After decades of development, a new information priority principle [10]-[13] and continued fraction [14]-[18] have been proposed. These mitigate the shortcomings of existing data mining methods and provide a new approach for data prediction and analysis. MGM (1, n) (Multi variable Grey Model) introduces N-dimensional differential equations into grey theory to describe the relationship between variables' mutual constraints and influences [19]. However, the model is only applicable to equally spaced data variables. By means of a homogeneous exponential function to fit the background value, Wang established multivariable MGM (1, n) for a non-equidistant data. This model is suitable for a research object with a non-homogeneous exponential function but does not have universal applicability [20]. Xiong et al. established a multivariate non-equidistant MGM (1, n) [21] by using the method of producing background values from mean. Due to the low accuracy of data fitting, which introduced non-homogeneous exponential function to improve the model. A non-equidistant multivariable GM (1, n) is established by means of a non-homogeneous exponential function to fit the background value. The results of using the model for data analysis showed that the analysis accuracy had been significantly improved [22].

When using a grey prediction model to analyze nonnegative discrete data series, the monotonicity of the generated series must be consistent with the original data. The original grey theory model often adopts an increasing or decreasing algorithm, which makes the model show monotonic attributes. If the raw data increase monotonically, the binary analysis model algorithm adopts a monotonically decreasing algorithm, which produces a systematic error. Many researchers have attempted to reduce this systematic error. Based on reverse accumulation, Song and Deng proposed the grey model GOM (1, 1) with the method of reverse accumulation generation for data analysis [23]. Introducing reciprocal into grey theory, the established GRM (1,1) and CGRM (1,1) models further expand the application of grey theory [24-25]. The use of reciprocal product or reciprocal accumulation to generate background values significantly improves the accuracy of the fitted background values in monotonically decreasing sequences. The original data can be fitted with a monotonically decreasing analysis model to obtain the analysis data. When the model value of the analysis data is restored to the prediction value, there is no prediction error (such as traditional accumulation and subtraction) to improve the modeling accuracy.

For multivariable research objects, it is very difficult to

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establish an analytical model due to the mutual restriction between variables. Many researchers have attempted to solve the problem of multivariable research system modeling. Zhai and Sheng deduced a grey theory prediction model suitable for an equidistant multivariable system by establishing the N-element first-order differential equation [26]. Zhong and Yan applied the multivariable grey theory model MGR (1, 1) to predict the daily power generation of the photovoltaic power station at the Wuhan International Exhibition Center. The analysis results showed that the model obtained accurate predictions [27]. To analyze the influence of environment, temperature, and other factors on the fluctuation of photovoltaic power generation, Zhong and Yang proposed a multivariable grey theory model based on the particle swarm optimization algorithm, which reduced the average relative error of the prediction model from 7.14% to 3.53% [28]. Zeng and Duan established a multivariable grey theory prediction model by including dependent variable lag, linear correction, and random disturbance terms to the GM (1, n) model. The model was obtained by adjusting the model parameters and was compatible with the univariate GM (1, n) model [29]. It is clear that the fitting accuracy of previous multivariable grey prediction theory models is low and the monotonicity of the analysis object of the model has a great impact on the final results.

Based on vector continued fraction theory, this study constructs a model background value using the trapezoidal formula of rational interpolation and numerical integration and extrapolation methods. Taking the *m*-th component of the MGRM (1, n) model data as the initial value of the solution to the grey differential equation, a non-equidistant multivariable innovation model MGRM (1, n) is established based on reciprocal accumulation generation. The innovation model is suitable for equidistant and non-equidistant data sequences; however, it is unable to consider the monotonicity of data. The multivariable innovation model MGRM (1, n) established in this study is verified by examples. The results show that the data obtained by the model have high fitting accuracy, and the model is easy to use and has good theoretical value and application prospects.

II. MODEL DEVELOPMENT

To derive the non-equidistant multivariable grey innovation model MGRM (1, *n*) generated based on reciprocal accumulation, five definitions are required as follows.

Definition 1: Reciprocal sequence

If $\Delta t_j = t_j - t_{j-1} \neq constant$, a sequence can be expressed as $X_i^{(00)} = [x_i^{(00)}(t_1), \cdots x_i^{(00)}(t_j), \cdots x_i^{(00)}(t_m)]$, where $X_i^{(00)}$ is a non-equidistant sequence. In this equation, $i = 1, 2, \cdots n$; $j = 2, 3, \cdots m$; n is the number of variables; and m is the number of sequences for each variable.

Consider a variable $x_i^{(0)}(t_j)$ that can be expressed as $x_i^{(0)}(t_j) = \dfrac{1}{x_i^{(00)}(t_j)} (j = 1, 2, \cdots, m)$. Then, the reciprocal sequence of $x_i^{(00)}$ is $x_i^{(0)} = (x_i^{(0)}(t_1), \cdots, x_i^{(0)}(t_m))$.

Definition 2: If equations $x_i^{(1)}(t_j) = x_i^{(1)}(t_{j-1}) + x_i^{(0)}(t_j) \cdot \Delta t_j$ and $x_i^{(1)}(t_1) = x_i^{(0)}(t_1)$ (for $j= 2, \dots, m$, $i= 1, 2, \dots, n$, and $\Delta t_j = t_j - t_{j-1}$) hold, then $X_i^{(1)} = \{x_i^{(1)}(t_1), \cdots x_i^{(1)}(t_j), \cdots x_i^{(1)}(t_m)\}$ is the first-order accumulation generation of a non-equidistant sequence $X_i^{(0)}$.

From the sequence in Definition 2, a multivariable data matrix is derived:

$$X^{(0)} = \{X_1^{(0)}, \cdots X_n^{(0)}\}^T = \begin{bmatrix} x_1^{(0)}(t_1) & \cdots & x_1^{(0)}(t_m) \\ \vdots & \cdots & \vdots \\ x_n^{(0)}(t_1) & \cdots & x_n^{(0)}(t_m) \end{bmatrix}. \quad (1)$$

In (1), when $t_j - t_{j-1} \neq constant$, $X^{(0)}(t_j) = [x_1^{(0)}(t_j), \cdots x_n^{(0)}(t_j)]$ is the observed value of each variable of $X^{(0)}(t_j)$ at time t_j.

A new matrix is obtained by accumulating the matrix obtained by (1):

$$X^{(1)} = \{X_1^{(1)}, \cdots X_n^{(1)}\}^T = \begin{bmatrix} x_1^{(1)}(t_1) & \cdots & x_1^{(1)}(t_m) \\ \vdots & \cdots & \vdots \\ x_n^{(1)}(t_1) & \cdots & x_n^{(1)}(t_m) \end{bmatrix}, \quad (2)$$

where

$$x_i^{(1)}(t_j) = \begin{cases} \sum_{j=1}^{k} x_i^{(0)}(t_j)(t_j - t_{j-1}) & (k = 2, \cdots m) \\ x_i^{(0)}(t_1) & (k = 1) \end{cases}. \quad (3)$$

A. Expressing the Background Value

The multivariable non-equidistant MGRM (1, *n*) model generated based on reciprocal accumulation is a system of *N*-ary first-order differential equations,

$$\begin{cases} \dfrac{dx_1^{(1)}}{dt} = a_{11}x_1^{(1)} + a_{12}x_2^{(1)} + \cdots + a_{1n}x_n^{(1)} + b_1 \\ \vdots \qquad\qquad \vdots \\ \dfrac{dx_n^{(1)}}{dt} = a_{n1}x_1^{(1)} + a_{n2}x_2^{(1)} + \cdots + a_{nn}x_n^{(1)} + b_n \end{cases}. \quad (4)$$

Using A to represent the coefficient of component $x_i^{(1)}$ and B to represent the constant of differential equation b_i, we obtain

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}, \ B = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}.$$

Then, (4) can be expressed as

$$\frac{dX^{(1)}(t)}{dt} = AX^{(1)}(t) + B. \quad (5)$$

In the literature, the first component $x_i^{(1)}(t_1)$ of sequence $x_i^{(1)}(t_j)$ is often used as the initial condition of the grey differential equation. This processing method results in an insufficient utilization of new information. In this study, the *m*-th component $x_i^{(1)}(t_m)$ of the sequence is used as the initial condition of the grey differential equation to fully use the latest information.

The continuous-time response equation of (5) can be expressed as

$$X^{(1)}(t) = e^{At}X^{(1)}(t)(t_m) + A^{-1}(e^{At} - I)B, \quad (6)$$

where $e^{At} = I + \sum_{K=1}^{\infty} \frac{A^k}{K!} t^k$ and the letter I represents an identity matrix.

To distinguish between a and B, the following expression can be obtained by discretizing (4):

$$x_i^{(0)}(t_j) = \sum_{l=1}^{n} a_{il} \frac{\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt}{\Delta t_j} + b_i, \quad (7)$$

where i, l=1, 2, …n and j=2, 3, …m.

Suppose $z_i^{(1)}(t_j) = \frac{\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt}{\Delta t_j}$, then $z_i^{(1)}(t_j)$ can be expressed using the mean value to generate the background value as

$$z_i^{(1)}(t_j) = 0.5[x_i^{(1)}(t_{j-1}) + x_i^{(1)}(t_j)]. \quad (8)$$

L can be expressed is follows:

$$L = \begin{bmatrix} \frac{1}{2}(x_1^{(1)}(t_2) + x_1^{(1)}(t_1)) & \cdots & \frac{1}{2}(x_n^{(1)}(t_2) + x_n^{(1)}(t_1)) & 1 \\ \cdots & \cdots & \cdots & \cdots \\ \frac{1}{2}(x_1^{(1)}(t_m) + x_1^{(1)}(t_{m-1})) & \cdots & \frac{1}{2}(x_n^{(1)}(t_m) + x_n^{(1)}(t_{m-1})) & 1 \end{bmatrix}.$$

Let $a_i = (a_{i1}, a_{i2} \cdots a_{in}, b_i)^T$, then $\hat{a}_i$ can be expressed utilizing the least square method as

$$\hat{a}_i = (\hat{a}_{i1}, \hat{a}_{i2} \cdots \hat{a}_{in}, \hat{b}_i)^T = (L^T L)^{-1} L^T Y_i, \quad (9)$$

where $Y_i = [x_i^{(0)}(t_2), x_i^{(0)}(t_3), \cdots, x_i^{(0)}(t_m)]^T$.

B. Vector Continued Fraction

The non-equidistant multivariable MGRM (1, n) model is generated based on reciprocal accumulation [25], and the mean is used to generate the background value. The analysis data obtained by this method is often inaccurate. To obtain a more accurate analysis, the present study develops a novel method to construct the background value. Based on vector continued fraction theory, the background value of the model can be constructed using extrapolation, by combining rational interpolation and the trapezoidal integral formula.

To facilitate constructing the background values, the following definitions and theorems are required.

Definition 3: A continued fraction and its n-order progressive fraction.

For real number columns $\{a_n\}$ and $\{b_n\}$, the n-th progressive fraction of the continued fraction can be expressed as

$$b_0 + \underset{n=1}{\overset{\infty}{K}}(a_n / b_n). \quad (10)$$

Definition 4: Assuming that the vector modulus of an n-dimensional vector $v = (v_1, v_2, \cdots v_n)$ represents $|v| = \sqrt{\sum_{j=1}^{n} v_j^2}$, the generalized inverse of the term is expressed as $v^{-1} = \frac{v}{|v|^2}$.

Definition 5: Vector deficit quotient of vector set V^m

If the following equations hold, then $\varphi[x_0, \cdots, x_j, x_k, x_l]$ is determined by the formula below as the deficit quotient of the l-order vector of V^m at $(x_0, x_1, \cdots, x_l)$.

$$\begin{cases} \varphi[x_i] = v_i, (i = 0, 1, \cdots l) \\ \varphi[x_p, x_q] = \dfrac{x_q - x_p}{\varphi[x_q] - \varphi[x_p]} \\ \varphi[x_0, \cdots, x_j, x_k, x_l] = \dfrac{x_l - x_k}{\varphi[x_0, \cdots, x_j, x_l] - \varphi[x_0, \cdots, x_j, x_k]} \end{cases}$$

Definition 6: Inverse quotient vector theorem

Suppose the following equation holds, then $R_n(x_i) = v_i = (x_1^{(1)}(i), x_2^{(1)}(i), \cdots, x_n^{(1)}(i))$ (for i = 0, 1, …, n), where $\varphi[x_0, x_1, x_2 \cdots x_n]$ is the deficit quotient of the order vector of vector set V at $(x_0, x_1, \cdots, x_l)$.

$$R_n(x) = \varphi[x_0] + \cfrac{x - x_0}{\varphi[x_0, x_1] + \cfrac{x - x_1}{\varphi[x_0, x_1, x_2]} + \ddots + \cfrac{x - x_{n-1}}{\varphi[x_0, x_1, x_2 \cdots x_n]}}$$

C. Constructing the New Information Model MGRM (1, n)

To facilitate constructing the background value of the grey derivative vector, the integral interval $[a, b]$ is divided into m_1 equal parts with step size $h = (b-a)/m_1$. Therefore, the generalized trapezoidal integral formula is

$$T_m = h\left[\frac{1}{2}f(a) + f(a+h) + \cdots + f(a + (m_1 - 1) + \frac{1}{2}f(b)\right]. \quad (11)$$

When m_1 = 4 and m_1 = 8, (11) can be expressed as

$$\begin{cases} zf^{(1)}(k+1) = \frac{1}{4}[\frac{1}{2}x^{(1)}(k) + \cdots + \frac{1}{2}x^{(1)}(k+1)] & m_1 = 4 \\ zf^{(1)}(k+1) = \frac{1}{8}[\frac{1}{2}x^{(1)}(k) + \cdots + \frac{1}{2}x^{(1)}(k+1)] & m_1 = 8 \end{cases}.$$

Combining the above formula creates a combination formula,

$$z^{(1)}(k+1) = \frac{4}{3}ze^{(1)}(k+1) - \frac{1}{3}zf^{(1)}(k+1), \quad (12)$$

where k=1, 2, …m-1. Modifying the variables in the above formula obtains the background value of the grey derivative vector, as shown in the following formula:

$$z^{(1)}(t_{j+1}) = \frac{4}{3}ze^{(1)}(t_{j+1}) - \frac{1}{3}zf^{(1)}(t_{j+1}), \quad (13)$$

where j=1, 2, … m-1. When the background value of the grey derivative vector is substituted into (9), L can be expressed as

$$\begin{bmatrix} \frac{4}{3}ze_1^{(1)}(t_2) - \frac{1}{3}zf_1^{(1)}(t_2) & \cdots & \frac{4}{3}ze_n^{(1)}(t_2) - \frac{1}{3}zf_n^{(1)}(t_2) & 1 \\ \cdots & \cdots & \cdots & \cdots \\ \frac{4}{3}ze_1^{(1)}(t_m) - \frac{1}{3}zf_1^{(1)}(t_m) & \cdots & \frac{4}{3}ze_n^{(1)}(t_m) - \frac{1}{3}zf_n^{(1)}(t_m) & 1 \end{bmatrix}.$$

After obtaining the estimated value $\hat{a}_i$ for a_i, the discrimination values of A and B are expressed as follows:

$$A=\begin{bmatrix}\hat{a}_{11} & \cdots & \hat{a}_{1n}\\ \vdots & \vdots & \vdots\\ \hat{a}_{n1} & \cdots & \hat{a}_{nn}\end{bmatrix},\ B=\begin{bmatrix}\hat{b}_1\\ \vdots\\ \hat{b}_n\end{bmatrix}. \quad (14)$$

The calculated value of the proposed MGRM (1, n) model generated based on reciprocal accumulation is expressed as

$$\hat{X}_i^{(1)}(t_j)=e^{\hat{A}(t_j-t_m)}X_i^{(1)}(t_m)+\hat{A}^{-1}(e^{\hat{A}(t_j-t_m)}-I)\hat{B}. \quad (15)$$

To optimize the use of the latest information, the m-th component of the above formula is taken as the initial condition of the grey differential equation. By restoring the original data, the fitting value of the data is obtained as

$$\hat{X}_i^{(0)}(t_j)=\begin{cases}\lim\limits_{\Delta t\to 0}\dfrac{X_i^{(1)}(t_1)-X_i^{(1)}(t_1-\Delta t)}{\Delta t} & j=1\\ (\hat{X}_i^{(1)}(t_j)-\hat{X}_i^{(1)}(t_{j-1}))/(t_j-t_{j-1}) & j=2,\cdots m\end{cases}. \quad (16)$$

Subsequently, Definition 1 can be used to obtain the model value of the original sequence $\hat{x}_i^{(00)}(t_j)$.

D. Calculation Error of Proposed MGRM (1, n)

Let $\hat{x}_i^{(00)}(t_j)-x_i^{(00)}(t_j)$ be the absolute error of the i-th variable, then the relative error is

$$e_i(t_j)=\frac{\hat{x}_i^{(00)}(t_j)-x_i^{(00)}(t_j)}{x_i^{(00)}(t_j)}\times 100\%. \quad (17)$$

The average value of the relative error of the i-th variable can be expressed as

$$\frac{1}{m}\sum_{j=1}^{m}\left|e_i(t_j)\right|, \quad (18)$$

and the average error of all the data is described as

$$f=\frac{1}{nm}\sum_{i=1}^{n}\left(\sum_{j=1}^{m}\left|e_i(t_j)\right|\right). \quad (19)$$

Therefore, when n = 1, the proposed innovation MGRM (1, n) model becomes an unequal innovation GRM (1, 1) model based on reciprocal accumulation, and when B = 0, the model is a combination of n GRM (1, n) models. The proposed MGRM (1, n) model can not only be used for modeling, but also for prediction or data fitting and processing. By setting n, the required unequal innovation MGRM (1, 2), MGRM (1, 3), MGRM (1, 4), and other models can be obtained.

III. Applications

To verify the effectiveness of the MGRM (1, n) model, two engineering examples were studied: metal cutting and nylon water-absorbent plastic. The feed rate and axial and radial forces of metal during cutting and the change in the material properties of nylon plastics due to water absorption are multivariable problems. The interaction and restriction between the variables of these research objects make describing the relationship between the variables complex, and it is difficult to establish prediction and analysis models.

A. Application I

When turning bar stock with a carbide cutter, there are axial and radial forces generated by the axial and radial feeds. These two forces and the cutting speed have a relationship with mutual influence and mutual restriction. Taking the general lathe model CA6140 and the cemented carbide YT14 turning tool (Shenyang No. 1 Lathe Works) for cylindrical machining, the proposed unequal innovation MGRM (1, n) was verified and analyzed.

For convenience, the geometric dimension and cutting speed of the tool remained unchanged during the cutting process. The corresponding relationship between the main cutting force (including the axial and radial cutting forces) and cutting depth was studied. The experimental data of the cutting depth and cutting force when the cutting speed was 0.02 mm/rev are shown in Table 1 [30].

TABLE I.
EXPERIMENTAL DATA OF CUTTING FORCE AND CUTTING DEPTH

Ordinal	Depth, a_p (mm)	Axial force, Fz (N)	Radial force, Fy (N)
1	1.00	838.98	255.10
2	1.25	1060.45	290.16
3	1.50	1261.79	355.22
4	1.75	1483.25	420.28
5	2.00	1704.72	469.08

To analyze the data, the non-equidistant multivariable grey innovation model MGRM (1, n) was used to establish a mathematical model. During modeling, a_p acted as t_k, F_z as x_1, and F_y as x_2. The data obtained using the proposed model are shown in Table II.

TABLE II.
FITTING DATA OF CUTTING FORCE

Depth, a_p (mm)	Axial force, Fz (N)	Radial force, Fy (N)	Predicted Fz, $\hat{F}_z$ (N)	Relative error of Fz, E_1 (%)	Predicted Fy, $\hat{F}_y$ (N)	Relative error of Fy, E_2 (%)
1.00	838.98	255.10	838.98	0	255.128	0.01
1.25	1060.45	290.16	1062.65	0.21	290.532	0.13
1.50	1261.79	355.22	1263.67	0.15	355.460	0.07
1.75	1483.25	420.28	1481.25	-0.13	420.034	-0.06
2.00	1704.72	469.08	1702.81	-0.11	470.019	0.20

According to the data in Table II, the average relative error of the predictions was 0.04%. The MGM model established in [30] and the GM model established in [31] were also used to predict the data in this study. The prediction results of the three models of the relationship between cutting depth and axial force in the process of a machine tool cutting a workpiece are shown in Fig. 1.

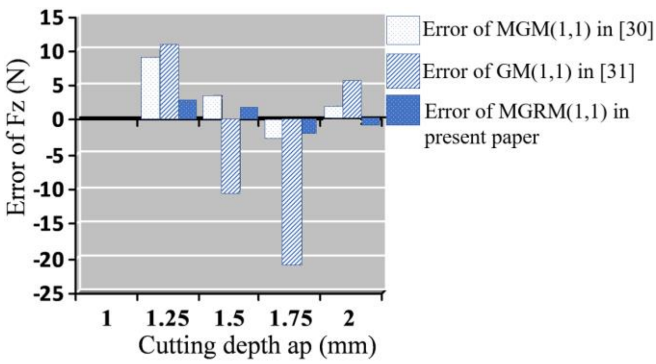

Fig. 1 Histogram of fitting error for three models of axial cutting force.

Fig. 1 shows the prediction errors in axial force by cutting depth for three models. The values predicted by GM (1, 1) and MGM (1, 1) at each cutting depth had a greater error than those obtained by MGRM (1, 1). The predicted values obtained by MGRM (1, 1) transitioned smoothly between cutting depth, which shows that the model has good prediction accuracy and good adaptability.

B. Application II

Nylon 66 plastic (PA66) is an engineering material widely used in current engineering fields. Due to the hydrophilic amide group in the molecular structure, PA66 has strong water absorption, which has a significant impact on the mechanical properties of the material. Different water absorption levels result in PA66 exhibiting different bending and tensile strengths. Therefore, water absorption significantly interferes in the accurate determination of various mechanical properties of PA66. The influence of water absorption on the mechanical properties of PA66 is a multivariable engineering problem. The mutual influence and restriction between parameters lead to an inaccurate description of the analysis model established in previous research [32], and the accuracy of the analysis data does not meet engineering and technical requirements.

To verify the proposed unequal innovation MGRM (1, *n*) model, the influence of water absorption on the mechanical properties of PA66 was modeled and analyzed. The experimental data of the change in flexural strength, flexural elastic modulus, and tensile strength with water absorption obtained from a mechanical property test of PA66 samples with different water absorption are shown in Table III [31]. The MGRM (1, n) established in this paper was used to analyze the data. The obtained mechanical properties of the nylon are shown in Table IV.

TABLE III.
EFFECT OF WATER ABSORPTION ON MECHANICAL PROPERTIES OF NYLON

Ordinal	Water absorption, t_j (%)	Bending strength, $X_1^{(0)}$ (MPa)	Flexural modulus of elasticity, $X_2^{(0)}$ (GPa)	Tensile strength, $X_3^{(0)}$ MPa)
1	0	83.4	2.63	84.2
2	0.06	84.9	2.64	84.4
3	0.11	84.5	2.61	86.3
4	0.17	84.2	2.65	84.3
5	0.21	84.4	2.66	81.3
6	0.43	78.4	2.52	74.9
7	0.52	75.4	2.32	75.7
8	0.85	59.5	1.90	73.2
9	0.98	54.1	1.72	66.9

By analyzing the nylon mechanical data obtained in Table IV, the average relative error of the three groups of data was 0.3%.

TABLE IV.
EFFECT OF WATER ABSORPTION ON MECHANICAL PROPERTIES OF NYLON

Ordinal	Water absorption, t_j (%)	Bending strength, $X_1^{(0)}$ (MPa)	Predicted $X_1^{(0)}$,	Flexural modulus of elasticity, $X_2^{(0)}$ (GPa)	Predicted $X_2^{(0)}$,	Tensile strength, $X_3^{(0)}$ (MPa)	Predicted $X_3^{(0)}$,
1	0	83.4	83.4	2.63	2.63	84.2	84.2
2	0.06	84.9	85.3	2.64	2.68	84.4	84.6
3	0.11	84.5	84.1	2.61	2.63	86.3	86.0
4	0.17	84.2	83.7	2.65	2.64	84.3	84.5
5	0.21	84.4	84.1	2.66	2.67	81.3	81.8
6	0.43	78.4	78.2	2.52	2.53	74.9	75.9
7	0.52	75.4	76.1	2.32	2.31	75.7	75.6
8	0.85	59.5	60.3	1.90	1.97	73.2	73.0
9	0.98	54.1	54.4	1.72	1.70	66.9	67.0

The data in Table IV were compared with data obtained from the MGM model established in [32], and a comparative analysis diagram was drawn. The change in flexural strength, flexural elastic modulus, and tensile strength with water absorption is shown in Fig. 2

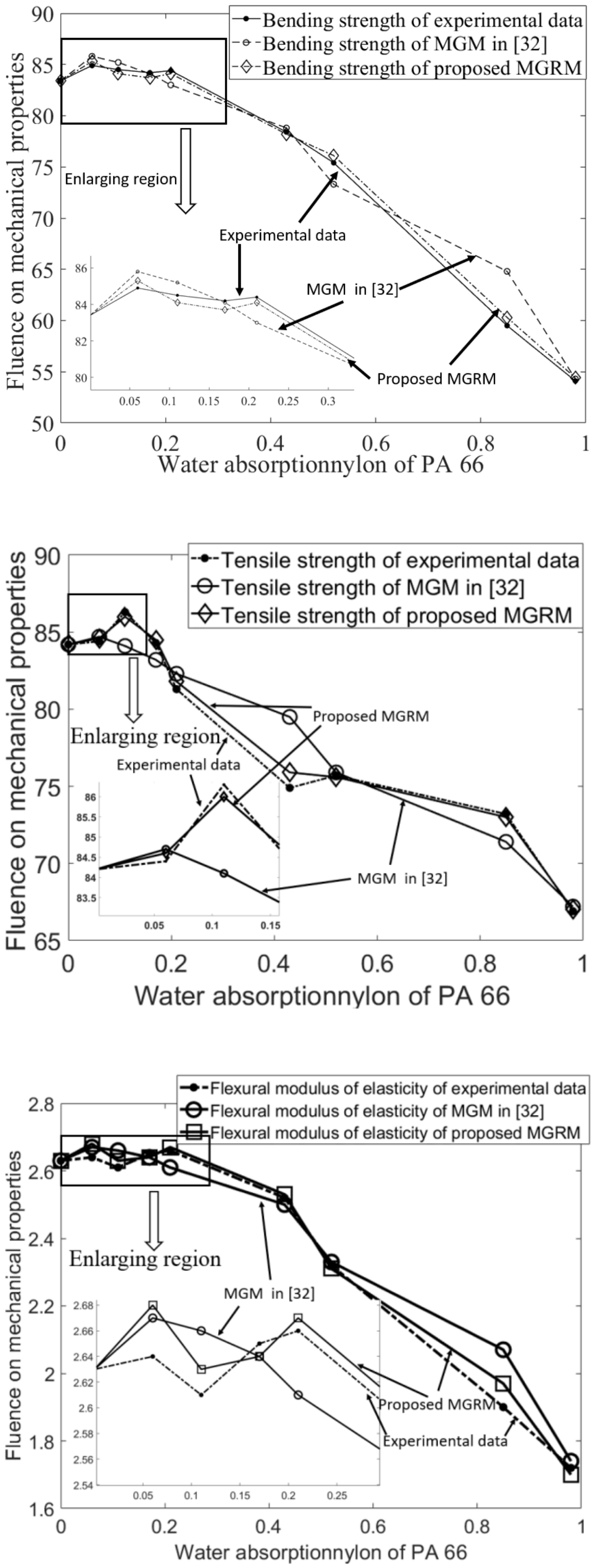

Fig. 2. Comparative analysis of mechanical property prediction of nylon between MGM and MGRM.

Fig. 2 shows a comparison of the experimental data, predicted data obtained using the MGM model established in [32], and the predicted data obtained using the proposed MGRM model. There is little difference between the flexural elastic modulus of PA66 obtained by the experiment, MGM model, and MGRM model. In the figure, under different water absorption rates, the bending elastic modulus of PA66 is parallel to the abscissa and the three lines almost coincide.

To observe the fit of the model to the data more clearly, the rectangular area in the figure was enlarged. From the inset figure, it is clear that the data obtained by the MGRM model are closer to the experimental data than those obtained by the MGM model, and the trend is consistent with the actual data.

IV. Conclusion

In engineering, there are many multivariable research objects that contain variables with mutual influence. Previous studies have not accurately described the relationship between parameters, which often leads to a large difference between the conclusion of the analysis and the actual situation. Based on the theory of vector continued fractions, the trapezoidal formula of rational interpolation, and numerical integration, this study used the extrapolation method to construct the background value of the model. Taking the m-th component of the MGRM (1, n) model data as the initial value of the solution of the grey differential equation, a non-equidistant multivariable innovation MGRM (1, n) model was established using reciprocal accumulation generation. The MGRM (1, n) model established in this study exhibited good practicability and popularization value.

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