The Dynamic Behaviors of Nonselective Harvesting Lotka-Volterra Predator-Prey System With Partial Closure for Populations and the Fear Effect of the Prey Species

Xiaoran Li, Qin Yue, and Fengde Chen

Abstract—**This paper proposes and investigates a nonselective harvesting Lotka-Volterra predator-prey system that incorporates population closure and the fear effect of the prey. The boundary equilibrium and positive equilibrium are studied in terms of their local and global stability characteristics. Our research indicates that the proportion of commodities designated for harvesting has a significant impact on the dynamic behavior of the system. Meanwhile, dynamic behavior of the system is not affected by the fear effect of the prey species. To demonstrate the viability of the key findings, numerical simulations are performed.**

Index Terms—**predator, prey, harvesting, stability, fear effect**

I. Introduction

THIS paper tries to figure out how the Lotka-Volterra predator-prey system with non-selective harvesting, partial population closure, and the fear effect of the prey changes over time. The model is as follows:

$$\begin{aligned}\frac{du}{dt} &= r_0 u f(k,v) - du - au^2 \\ &\quad -puv - q_1 E m_1 u, \\ \frac{dv}{dt} &= cpuv - mv - q_2 E m_1 v,\end{aligned} \tag{1}$$

where u and v represent the density of prey species and predator species, respectively, at time t. p represents the intensity of capture; For the biological meaning of r_0, d, a, m and c, one could refer to system Wang, Zanette, and Zou[1]. E is the combined fishing effort used to harvest; $m_1 (0 < m_1 < 1)$ is the fraction of the stock available for harvesting; k is the level of fear caused by the anti-predator behaviors of the prey; $f(k,v)$ is consistent with the following hypotheses:

$$\begin{aligned}& f(0,v) = 1, \quad f(k,0) = 1, \\ & \lim_{k\to+\infty} f(k,v) = 0, \lim_{v\to+\infty} f(k,v) = 0, \\ & \frac{\partial f(k,v)}{\partial k} < 0, \quad \frac{\partial f(k,v)}{\partial v} < 0.\end{aligned} \tag{2}$$

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Scholars hold the predator-prey relationship in high regard due to its prevalence([1]-[30]). In recent years, the fear effect of prey species has become one of the most significant aspects of predator-prey system research[1]-[13]. According to their research, not only can predators kill their prey directly, but they can also influence prey behavior, which is more lethal. Wang, Zanette, and Zou[1] proposed for the first time the following predator-prey system with fear effect:

$$\begin{aligned}\frac{du}{dt} &= r_0 u f(k,v) - du - au^2 - puv, \\ \frac{dv}{dt} &= cpuv - mv.\end{aligned} \tag{3}$$

The authors explore the stability of each equilibrium point of the system.

Meanwhile, the study of resource management, which includes wildlife management, forestry and fisheries, is extremely important. See references [31]-[47] for research on the influence of harvesting on ecological modeling. As Chakraborty, Das, and Kar[46] pointed out, in order to achieve long-term ecological sustainability and conservation of the species, harvesting must be regulated so that the species can be harvested in a sustainable manner. Since the works of Chakraborty, Das, and Kar[46], many scholars ([42]-[47]) have conducted research in this area. For instance, an investigation has been conducted by Lin[45] on the dynamics of the following two species commensal symbiosis model:

$$\begin{aligned}\frac{dx}{dt} &= x\Big(a_1 - b_1 x + \frac{c_1 y}{d_1 + y^2}\Big) - q_1 E m x, \\ \frac{dy}{dt} &= y(a_2 - b_2 y) - q_2 E m y.\end{aligned} \tag{4}$$

According to his findings, depending on the value of m, the system can collapse, survive partially, or coexist in a stable state.

Xiao and Lei[43] studied the following single species stage structure system with nonselective harvesting and partial population closure.

$$\begin{aligned}\frac{dx_1}{dt} &= \alpha x_2 - \beta x_1 - \delta_1 x_1 - q_1 E m x_1, \\ \frac{dx_2}{dt} &= \beta x_1 - \delta_2 x_2 - \gamma x_2^2 - q_2 E m x_2,\end{aligned} \tag{5}$$

where $x_1(t)$ and $x_2(t)$ are the densities of the embryonic and mature species at time t, respectively. Their research demonstrated that the proportion of harvestable populations play crucial roles in the dynamic behaviors of the system.

To the best of our knowledge, no academics have studied the dynamic behaviors of a predator-prey system that incorporates both the fear effect of prey species and nonselective harvesting. This prompted us to propose the system (1).

This paper aims to provide a comprehensive analysis of the dynamic behaviours of the system (1). Additionally, we will consider the impact of partial closure and harvesting.

The rest of the paper is organized as follows. In the section that follows, we will investigate the existence and local stability of the equilibrium of the system (1). In Section 3, we will discuss the global stability of the boundary equilibria and the positive equilibrium. In Section 4, the effects of harvesting and partial closure will be discussed. In Section 5, numerical simulations demonstrating the viability of the principal results are presented. This paper concludes with a brief discussion.

II. Existence and local stability of the system's equilibria

Concerned with the existence of the system (1)'s equilibria, we obtain the following result.

Theorem 2.1. *System (1) always have the boundary equilibrium $E_0(0,0)$, if $r_0 > d+q_1Em_1$ holds, then the nonnegative boundary equilibrium $E_1\Big(\frac{r_0-d-q_1Em_1}{a},0\Big)$ exists. Also, there exists a unique positive equilibrium $E_2(u^*,v^*)$, if*

$$r_0 > d+q_1Em_1+\frac{a(m+q_2Em_1)}{cp} \tag{6}$$

holds, where $u^*=\frac{m+q_2Em_1}{cp}$ and v^* satisfies

$$r_0f(k,v^*)-d-au^*-pv^*-q_1Em_1=0.$$

Proof. System (1)'s equilibria satisfy the equation

$$\begin{aligned} r_0uf(k,v)-du-au^2-puv-q_1Em_1u &= 0,\\ cpuv-mv-q_2Em_1v &= 0. \end{aligned} \tag{7}$$

From the second equation of (7), $v=0$ or $u=\frac{m+q_2Em_1}{cp}$ is derived. Substituting $v=0$ into the first equation of (7) yields

$$r_0uf(k,0)-du-au^2-q_1Em_1u=0. \tag{8}$$

Equation (8) has solutions $u_1=0$ and $u_2=\frac{r_0-d-q_1Em_1}{a}$. System (1) therefore has the boundary equilibrium $E_0(0,0)$, and if $r_0 > d+q_1Em_1$ holds, then the nonnegative boundary equilibrium $E_1\Big(\frac{r_0-d-q_1Em_1}{a},0\Big)$ exists.

Substituting $u=\frac{m+q_2Em_1}{cp}$ to (7) yields

$$r_0f(k,v)-d-a\frac{m+q_2Em_1}{cp}-pv-q_1Em_1=0. \tag{9}$$

Under the assumption of (6), it is straightforward to observe that (9) admits a unique positive solution v^*, consequently, system (1) admits a unique positive equilibrium $E_2(u^*,v^*)$.

The proof of Theorem 2.1 is finished.

Theorem 2.2. *$E_0(0,0)$ is locally asymptotically stable if*

$$r_0 < d+q_1Em_1 \tag{10}$$

holds; $E_1\Big(\frac{r_0-d-q_1Em_1}{a},0\Big)$ is locally asymptotically stable if

$$d+q_1Em_1 < r_0 < d+q_1Em_1+\frac{a(m+q_2Em_1)}{cp} \tag{11}$$

holds; $E_2(u^*,v^*)$ is locally asymptotically stable if

$$r_0 > d+q_1Em_1+\frac{a(m+q_2Em_1)}{cp} \tag{12}$$

holds.

Proof. The system's Jacobian matrix is calculated as

$$J=\begin{pmatrix} J_{11} & J_{12}\\ J_{21} & J_{22}\end{pmatrix}, \tag{13}$$

where

$$\begin{aligned} J_{11} &= r_0f(k,v)-d-2au-pv-q_1Em_1,\\ J_{12} &= r_0u\frac{\partial f(k,v)}{\partial v}-pu,\\ J_{21} &= cpv,\\ J_{22} &= cpu-m-q_2Em_1. \end{aligned} \tag{14}$$

Then, the system's Jacobian matrix about $E_0(0,0)$ is

$$J\big(E_0(0,0)\big)=\begin{pmatrix} r_0-d-q_1Em_1 & 0\\ 0 & -m-q_2Em_1\end{pmatrix}. \tag{15}$$

The eigenvalues of $J(E_0)$ are $\lambda_1=r_0-d-q_1Em_1$, $\lambda_2=-m-q_2Em_1<0$. Thus, if $r_0<d+q_1Em_1$ holds, $\lambda_1<0$ and consequently, $E_0(0,0)$ is locally asymptotically stable.

The Jacobian matrix of the system (1) about $E_1\big(\overline{u},0\big)$ is

$$\begin{aligned} &J\Big(E_1\big(\overline{u},0\big)\Big)\\ &= \begin{pmatrix} -a\overline{u} & r_0\overline{u}\frac{\partial f(k,v)}{\partial v}|_{v=0}-p\overline{u}\\ 0 & cp\frac{r_0-d-q_1Em_1}{a}-m-q_2Em_1\end{pmatrix}. \end{aligned}$$

The eigenvalues of $J(E_1)$ are $\lambda_1=-a\overline{u}<0$, and $\lambda_2=cp\frac{r_0-d-q_1Em_1}{a}-m-q_2Em_1<0$ if the assumption (11) holds. Consequently, $E_1(\overline{u},0)$ is locally asymptotically stable.

The Jacobian matrix of the system (1) with respect to $E_2(u^*,v^*)$ is

$$\begin{aligned} &J\big(E_2(u^*,v^*)\big)\\ &= \begin{pmatrix} -au^* & r_0u^*\frac{\partial f(k,v)}{\partial v}|_{v=v^*}-pu^*\\ cpv^* & 0\end{pmatrix}. \end{aligned} \tag{16}$$

Then we have

$$\begin{aligned} &DetJ\big(E_2(u^*,v^*)\big)\\ &= -cpv^*\Big(r_0u^*\frac{\partial f(k,v)}{\partial v}|_{v=v^*}-pu^*\Big)\\ &> 0, \end{aligned}$$

and

$$TrJ\big(E_2(u^*,v^*)\big)=-au^*<0.$$

Therefore, both eigenvalues of $J\big(E_2(u^*,v^*)\big)$ have negative real parts, and $E_2(u^*,v^*)$ is locally asymptotically stable.

The proof of Theorem 2.2 is now over.

III. Global asymptotical stability

This section's objective is to investigate the global stability property of system equilibria. Indeed, the following is the outcome:

Theorem 3.1. *(i)* $E_0(0,0)$ *is globally asymptotically stable if*

$$r_0 < d + q_1 E m_1 \quad (17)$$

holds;

(ii) $E_1\Big(\dfrac{r_0 - d - q_1 E m_1}{a}, 0\Big)$ *is globally asymptotically stable if*

$$d + q_1 E m_1 < r_0 < d + q_1 E m_1 + \frac{a(m + q_2 E m_1)}{c(p + r_0 M)} \quad (18)$$

holds, where

$$M = \sup_{v\in[0,+\infty)} \Big|\frac{\partial f(k,v)}{\partial v}\Big|; \quad (19)$$

(iii) $E_2(u^*,v^*)$ *is globally asymptotically stable if*

$$r_0 > d + q_1 E m_1 + \frac{a(m + q_2 E m_1)}{cp} \quad (20)$$

holds.

Proof. (1) Consider the following Lyapunov function:

$$V_1(u,v) = u + \frac{1}{c}v. \quad (21)$$

Then the time derivative of V_1 along the trajectories of (1) is

$$\begin{aligned} & D^+V_1(t) \\ = \ & r_0 u f(k,v) - du - au^2 - puv - q_1 E m_1 u \\ & + \frac{1}{c}cpuv - \frac{m}{c}v - \frac{q_2 E m_1}{c}v \\ = \ & r_0 u f(k,v) - du - au^2 - q_1 E m_1 u \\ & - \frac{m}{c}v - \frac{q_2 E m_1}{c}v \\ < \ & (r_0 - d - q_1 E m_1)u - au^2 \\ & - \Big(\frac{m}{c} + \frac{q_2 E m_1}{c}\Big)v. \end{aligned}$$

Thus, $V_1(x,y)$ satisfies the Lyapunov asymptotic stability theorem, and $E_0(0,0)$ of the system (1) is globally asymptotically stable.

(2) Consider the Lyapunov function as follows:

$$V_2(u,v) = u - \overline{u} - \overline{u}\ln\frac{u}{\overline{u}} + \frac{1}{c}v, \quad (22)$$

where $\overline{u} = \dfrac{r_0 - d - q_1 E m_1}{a}$.

It follows from (2) that

$$\frac{\partial f(k,\theta v)}{\partial v} < 0. \quad (23)$$

Also, from (18) one has

$$\begin{aligned} & \frac{m + q_2 E m_1}{c} - \Big(-r_0\frac{\partial f(k,\theta v)}{\partial v} + p\Big)\overline{u} \\ \geq \ & \frac{m + q_2 E m_1}{c} - \Big(r_0 M + p\Big)\frac{r_0 - d - q_1 E m_1}{a} \\ > \ & 0. \end{aligned} \quad (24)$$

Then the time derivative of V_2 along the trajectories of (1) is

$$\begin{aligned} & D^+V_2(t) \\ = \ & (u-\overline{u})\Big(r_0 f(k,v) - d - au - pv - q_1 E m_1\Big) \\ & + \frac{1}{c}cpuv - \frac{m}{c}v - \frac{q_2 E m_1}{c}v \\ = \ & (u-\overline{u})\Big(r_0 f(k,v) - r_0 + d + q_1 E m_1 \\ & + a\overline{u} - d - au - pv - q_1 E m_1\Big) \\ & + \frac{1}{c}cpuv - \frac{m}{c}v - \frac{q_2 E m_1}{c}v \\ = \ & (u-\overline{u})\Big(r_0\frac{\partial f(k,\theta v)}{\partial v}v - a(u-\overline{u}) - pv\Big) \\ & + \frac{1}{c}cpuv - \frac{m}{c}v - \frac{q_2 E m_1}{c}v \\ = \ & -a(u-\overline{u})^2 + r_0\frac{\partial f(k,\theta v)}{\partial v}uv \\ & - \Big[\frac{m + q_2 E m_1}{c} - \Big(-r_0\frac{\partial f(k,\theta v)}{\partial v} + p\Big)\overline{u}\Big]v \\ < \ & 0. \end{aligned} \quad (25)$$

Thus, $E_1\Big(\dfrac{r_0 - d - q_1 E m_1}{a}, 0\Big)$ of system (1) is globally asymptotically stable since $V_2(x,y)$ fulfills the Lyapunov asymptotic stability theorem.

(3) The system must not permit a limit cycle in the first quadrant in order to shown $E_2(u^*,v^*)$ to be globally asymptotically stable .

Consider first the Dulac function $B(u,v) = u^{-1}v^{-1}$, then

$$\frac{\partial(PB)}{\partial u} + \frac{\partial(QB)}{\partial v} = -\frac{a}{v} < 0, \quad (26)$$

where

$$\begin{aligned} P(u,v) &= r_0 u f(k,v) - du - au^2 \\ & \quad -puv - q_1 E m_1 u, \\ Q(u,v) &= cpuv - mv - q_2 E m_1 v. \end{aligned} \quad (27)$$

Therefore, $E_2(u^*,v^*)$ is globally asymptotically stable, since accoding to Dulac's theorem, system (1) has no closed orbit in the first quadrant.

This ends the proof of Theorem 3.1.

Remark 3.1 One might wonder why we prove (i) and (ii) by building the suitable Lyapunov function but use a different method to show that the positive equilibrium is global asymptotically stable. In fact, we have tried to prove (iii) by building the Lyapunov function:

$$V_3(u,v) = u - \overline{u} - \overline{u}\ln\frac{u}{\overline{u}} + k(u - \overline{v} - \overline{v}\ln\frac{v}{\overline{v}}), \quad (28)$$

In order to make the derivative of V definite, we have to give more complex conditions. At present, no one knows whether it is possible to prove the global asymptotic stability of the positive equilibrium by constructing the suitable Lyapunov function or not. We will conduct a more in-depth discussion in this direction.

IV. The influence of partial closure and harvesting

We will discuss two aspects of this topic.

(1) The influence of partial closure and harvesting on the positive equilibrium.

Denote

$$\begin{aligned} F_1(u^*, v^*, E, m_1) &= r_0 f(k, v^*) - d - au^* \\ &\quad -pv^* - q_1 E m_1, \qquad (29) \\ F_2(u^*, v^*, E, m_1) &= cpu^* - m - q_2 E m_1. \end{aligned}$$

Then u^* and v^* satisfy the following equations:

$$\begin{cases} F_1(u^*, v^*, E, m_1) = 0, \\ F_2(u^*, v^*, E, m_1) = 0. \end{cases} \qquad (30)$$

And so,

$$\begin{aligned} J &= \frac{D(F_1, F_2)}{D(u^*, v^*)} \\ &= \begin{vmatrix} F_{1u^*} & F_{1v^*} \\ F_{2u^*} & F_{2v^*} \end{vmatrix} \\ &= \begin{vmatrix} -a & r_0 \frac{\partial f(k,v)}{\partial v}|_{v=v^*} - p \\ cp & 0 \end{vmatrix} \\ &= -cp\Big(r_0 \frac{\partial f(k,v)}{\partial v}|_{v=v^*} - p\Big) > 0 \end{aligned}$$

for all $u^* > 0, v^* > 0$, $m_1 \in (0,1)$ and $E \in [0, +\infty)$. Therefore, it follows from (30) that u^* and v^* could be the function of E and m_1,

$$u^* = u^*(E, m_1), \ \ v^* = v^*(E, m_1)$$

for all $m_1 \in (0,1)$ and $E \in [0, +\infty)$. Also,

$$\frac{\partial u^*}{\partial m_1} = -\frac{1}{J}\frac{D(F_1, F_2)}{D(m_1, v^*)}, \ \ \frac{\partial v^*}{\partial m_1} = -\frac{1}{J}\frac{D(F_1, F_2)}{D(u^*, m_1)},$$

$$\frac{\partial u^*}{\partial E} = -\frac{1}{J}\frac{D(F_1, F_2)}{D(E, v^*)}, \ \ \frac{\partial v^*}{\partial E} = -\frac{1}{J}\frac{D(F_1, F_2)}{D(u^*, E)}.$$

Mathematically, we have

(1) $\frac{\partial u^*}{\partial m_1} = \frac{q_2 E}{cp} > 0$. Thus, the prey density $u*$ increases as m_1 increases.

(2) $\frac{\partial v^*}{\partial m_1} = -\frac{1}{J}(aq_2 E + cpq_1 E) < 0$. Thus, the predator density $u*$ decreases as m_1 increases.

(3) $\frac{\partial u^*}{\partial E} = \frac{q_2 m_1}{cp} > 0$. Thus, the prey density $u*$ increases as E increases;

(4) $\frac{\partial v^*}{\partial E} = -\frac{1}{J}(aq_2 E + cpq_1 m_1) < 0$. Thus, the prey density $u*$ decreases as E increases;

(2) Three cases will be discussed now to illustrate the impact of partial closure:

Case 1. Assuming the inequality $r_0 < d$ is valid, it follows that for any $m_1 \in (0,1)$, the inequality (17) is satisfied. Specifically, in the event of extinction of the system without harvesting, the species in the system with harvesting, despite partial closure that prohibits harvesting, will inevitably face extinction. This outcome is attributed to a low birth rate of the prey species, which renders it vulnerable to extinction.

Case 2. Assuming that the inequality expressed in (31), namely

$$d < r_0 < d + \frac{am}{cp} \qquad (31)$$

is satisfied, Theorem A suggests that, absent harvesting, predator species will go extinct while prey species will persist. If the condition expressed in (32), namely

$$m_1 > \frac{r_0 - d}{q_1 E} \qquad (32)$$

is met, then Theorem 3.1 (i) implies that both predator and prey species will become extinct. In other words, if the proportion of the stock available for harvesting is excessively high, then both predator and prey species will face extinction.

If the proportion of available stock for harvesting is moderate such that the following inequality holds.

$$\frac{r_0 - d - \dfrac{am}{c(p + r_0 M)}}{q_1 E + A} < m_1 < \min\Big\{\frac{r_0 - d}{q_1 E}, 1\Big\}, \qquad (33)$$

where A is defined by

$$A = \frac{aq_2 E}{c(p + r_0 M)} \qquad (34)$$

is met, according to Theorem 3.1 (ii), the predator species will become extinct, while the prey species will remain sustainable.

In this case, since

$$\overline{u} = \frac{r_0 - d - q_1 E m_1}{a}, \qquad (35)$$

one has

$$\frac{d\overline{u}}{dE} = \frac{-q_1 m_1}{a} < 0, \ \ \frac{d\overline{u}}{dm_1} = \frac{-q_1 E}{a} < 0. \qquad (36)$$

Thus, if there is a limited amount of stock available for harvesting, then prey species remain permanent; however, as harvesting and harvestable stock increase, the ultimate density of the prey species decreases.

Case 3. Now let's assume that

$$r_0 > d + \frac{am}{cp} \qquad (37)$$

holds. From Theorem A, we know that a system without harvesting has a unique positive equilibrium E_2 that is globally asymptotically stable.

Now, if

$$m_1 < \frac{r_0 - d - \dfrac{am}{cp}}{q_1 E + \dfrac{aq_2 E}{cp}} \qquad (38)$$

holds, consequently, Theorem 3.1 (iii) states that the system is stable, and the unique positive equilibrium is globally asymptotically stable. This allows predator and prey species

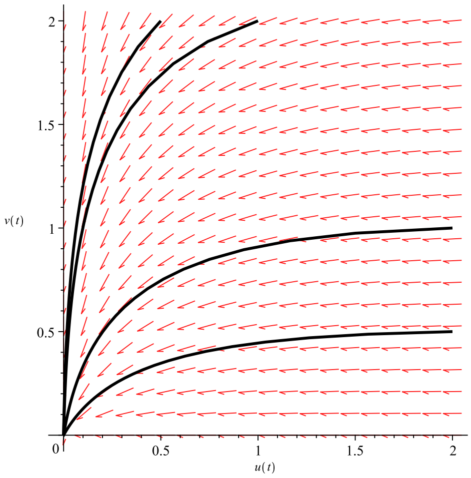

Fig. 1. The phase trajectory of the system (41), the initial condition $(u(0), v(0)) = (1, 2)$, $(2, 1)$, $(0.5, 2)$ and $(2, 0.5)$, respectively. $m_1 = 0.5$.

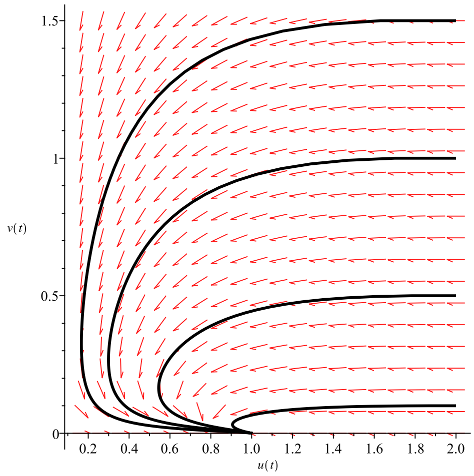

Fig. 2. The phase trajectory of the system (43), the initial condition $(u(0), v(0)) = (2, 0.5)$, $(2, 1)$, $(2, 1.5)$ and $(2, 0.1)$, respectively. $m_1 = 0$.

to persist in stable coexistence. In this case, from the above analysis, we have $\frac{\partial u^*}{\partial m_1} > 0$, $\frac{\partial v^*}{\partial m_1} < 0$, that is, with the increasing of the stock available for harvesting, the prey density u^* will increase, while the predator density v^* will be decreasing. The harvesting has a negative impact on predator species and a positive impact on prey species.

Similarly to the analysis of Case 2, if

$$\frac{r_0 - d - \frac{am}{c(p + r_0M)}}{q_1E + A} < m_1 < \min\left\{\frac{r_0 - d}{q_1E}, 1\right\}, \tag{39}$$

where

$$A = \frac{aq_2E}{c(p + r_0M)}, \tag{40}$$

then, predator species will become extinct while prey species will continue to exist. Thus, as the stock available for extraction increased, so did the likelihood that the predator would be driven to extinction, while prey species will still be permanent.

Finally, if $m_1 > \frac{r_0 - d}{q_1E}$, both predator and prey species will become extinct.

V. Numeric simulations

Example 5.1. Consider the following example:

$$\begin{aligned} \frac{du}{dt} &= \frac{u}{1+v} - 2u - u^2 - uv - m_1u, \\ \frac{dv}{dt} &= \tfrac{1}{2}uv - v - m_1v. \end{aligned} \tag{41}$$

Here, in accordance with system (1), we take $r_0 = 1, d = 2, k = a = p = q_1 = E = m = q_2 = 1$, $c = 0.5$. Then it became clear that

$$r_0 = 1 < 2 = d \tag{42}$$

holds true. Hence, it follows from Theorem 3.1 that for all $m_1 \in (0, 1)$, $E_0(0, 0)$ of system (41) is globally asymptotically stable. Fig.1 supports this assertion.

Example 5.2. Now contemplate the following illustration:

$$\begin{aligned} \frac{du}{dt} &= \frac{2u}{1+v} - u - u^2 - uv - 2m_1u, \\ \frac{dv}{dt} &= \tfrac{1}{2}uv - v - 2m_1v. \end{aligned} \tag{43}$$

According to system (1), here we choose $r_0 = 2, d = 1, k = a = p = q_1 = m = q_2 = 1$, $c = 0.5$, and $E = 2$. Then it could be seen that

$$d = 1 < r_0 = 2 < 3 = d + \frac{am}{cp} \tag{44}$$

is valid. Hence, it follows from Theorem 3.1 of Wang, Zanette, and Zou[1] that for the system without harvesting (i.e.,$m_1 = 0$), $E_1(1, 0)$ is globally asymptotically stable (see Fig.2). In addition, according to Theorem 3.1, (33) and (34), for

$$0.1 < m_1 < 0.5, \tag{45}$$

the boundary equilibrium $E_1(u(m_1), 0)$ of system (43) is globally asymptotically stable, and from (32), for

$$0.5 < m_1 < 1, \tag{46}$$

$E_0(0, 0)$ of the system (43) is globally asymptotically stable, as supported by numerical simulations (Fig. 3 and Fig. 4).

Example 5.3. Now contemplate the following example:

$$\begin{aligned} \frac{du}{dt} &= \frac{3u}{1+v} - u - u^2 - uv - m_1u, \\ \frac{dv}{dt} &= uv - v - m_1v. \end{aligned} \tag{47}$$

Here, in accordance with system (1), we set $r_0 = 3, d = 1, k = a = p = q_1 = m = q_2 = c$ and $E = 1$. Consequently, based on Theorem 3.1 and (38), for

$$m_1 < 0.5, \tag{48}$$

the positive equilibrium $E_2(u^*(m_1), v^*(m_1))$ of the system (47) is globally asymptotically stable, as supported by numerical simulation (Fig. 5).

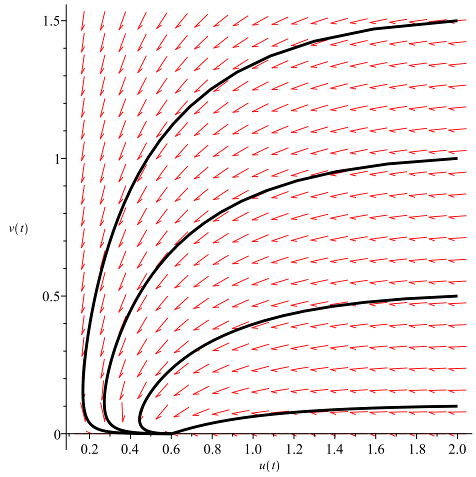

Fig. 3. The phase trajectory of the system (43), the initial condition $(u(0), v(0)) = (2, 0.5)$, $(2, 1)$, $(2, 1.5)$ and $(2, 0.1)$, respectively. $m_1 = 0.2$.

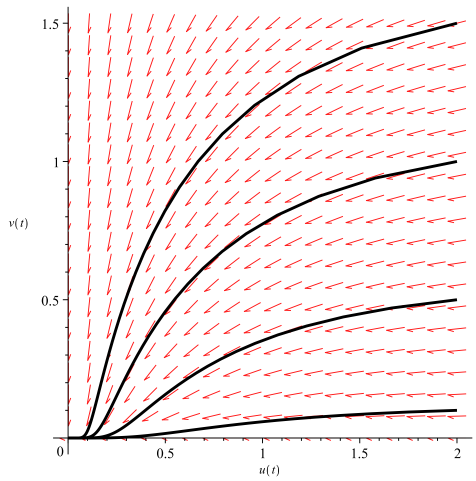

Fig. 4. The phase trajectory of the system (47), the initial condition $(u(0), v(0)) = (1, 1)$, $(0.5, 1)$ and $(1, 0.5)$, respectively. $m_1 = 0.6$.

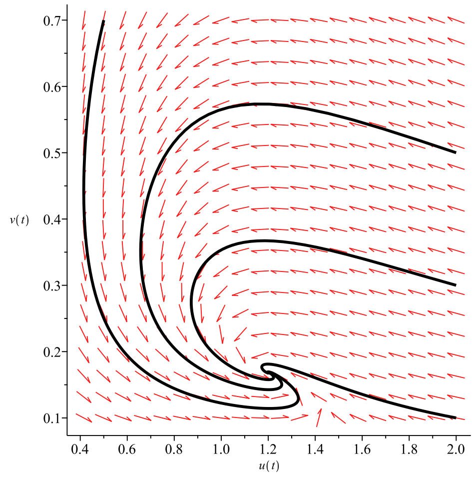

Fig. 5. The phase trajectory of the system (47), the initial condition $(u(0), v(0)) = (0.5, 0.7)$, $(2, 0.1)$, $(2, 0.5)$ and $(2, 0.3)$, respectively. $m_1 = 0.2$.

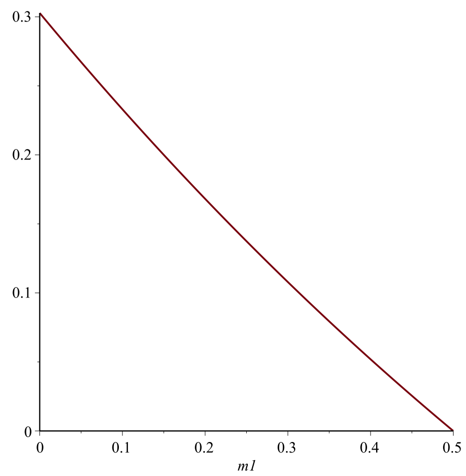

Fig. 6. Relationship of v^* and m_1.

The positive equilibrium of (47) satisfies the following equation:

$$\begin{aligned} &\frac{3u^*}{1+v^*} - u^* - (u^*)^2 - u^*v^* - m_1u^* = 0, \\ &u^*v^* - v^* - m_1v^* = 0. \end{aligned} \tag{49}$$

From (49), one could easily obtain that

$$\begin{aligned} u^* &= m_1 + 1; \\ v^* &= -m_1 - \tfrac{3}{2} + \tfrac{1}{2}\sqrt{4m_1^2 + 4m_1 + 3}. \end{aligned} \tag{50}$$

Clearly, u^* is the increasing function of m_1, while v^* is the decreasing function of m_1. as shown in Figure 6. This validates the prior analysis.

VI. Conclusion

Since the groundbreaking works of Chakraborty, Das and Kar[46], In ecological modeling, non-selective harvesting ecosystems with partial closure have been studied extensively for their dynamic behaviors([42]-[47]). On the other hand, recently, there has been considerable interest in the predator-prey system that incorporates the fear effect of prey species([1]-[13]). To the best of our knowledge, no academics have studied the influence of nonselective harvesting on the predator-prey system with fear effect to date; this prompted us to propose the system (1).

In this paper, we propose a nonselective harvesting Lotka-Volterra predator prey system that includes a partial population closure. Our research indicates that as the harvesting area expands, the ultimate density of predator species decreases, and may even result in the extinction of predator species or both. To guarantee the sustainable development of the ecosystem, it is necessary to restrict the harvesting area. Nonetheless, this study's main findings are independent of k, suggesting that the fear effect of the prey species is irrelevant to the dynamic behaviour of the system.

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