Improving Models Accuracy Using Kalman Filter and Holt-Winters Approaches Based on ARFIMA Models

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Abstract—**The analysis, modeling, and forecast of oil prices are among the most important studies related to global and local economic trends. Such studies are necessary to increase investments and reduce risks because oil prices exert a significant impact on supply and demand in global markets. In the current work, four models are proposed, namely, autoregressive fractionally integrated moving average (ARFIMA), ARFIMA with additive Holt–Winters (ARFIMA-AHW), ARFIMA with multiplicative Holt–Winters (ARFIMA-MHW), and ARFIMA with Kalman filter (ARFIMA-KF), for modeling monthly Brent crude oil prices. Accordingly, this study aims to extend the researchers' previous work by comparing the performance of the proposed statistical methods to provide an accurate individual or hybrid model for the efficient and reliable modeling of these prices. In addition, the characteristics of the optimal and most accurate method are identified to refinement the prediction outcomes of the ARFIMA model by using the Kalman filter and Holt–Winters methods in hybridization. The capabilities of these proposed models are evaluated in view of the root-mean-square error and by conducting the autoregressive conditional heteroscedasticity with Lagrange multiplier and Ljung–Box tests. This study shows that the ARFIMA (2,0.3589648,2)-KF model outperforms the other proposed models on the basis of the test results.**

Index Terms—**ARFIMA, KF, HW, Hybrid Approach, Modeling.**

I. Introduction

At present, the analysis of time series data set continues to be an important topic in many fields, the main ones are for example, economics, business, and the stock market. Moreover, modeling, simulation and forecasting methods are being increasingly used by researchers. Popular examples of these methods include autoregressive fractionally integrated moving average (ARFIMA), Kalman Filter (KF), Holt–Winters (HW), and hybrid methods. The ARFIMA models have been implemented in diverse fields, such as in prices. The HW methods have been applied to know the production and prices also. The KF methods have been used in crude oil prices. That is, all these methods contribute to the modeling and forecasting field. So, they have their own strengths and weaknesses, as will be discussed in later sections.

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Different methods and approaches have been used to improve modeling and forecasting. The ARFIMA model is utilized to fit time series data set, make sense the behavior of it, or forecast the future. The importance of forecasts in the financial economics field is extremely significant at the national, and international levels because forecasts can help investors to increase profits and decrease financial risks despite the fluctuations of the international economy. Another method, KF, can be used to obtain optimal and high accuracy forecasts [1]. The KF approach was initially verified to be most useful in engineering and space technology, and then it started being used in the field of statistics [2], [3]. In the same studies, the researchers mentioned that the primary feature of this method is its capability to update system knowledge after receiving a new observation, minimizing error terms and time (t) by filtering noisy terms. Consequently, KF is widely used in stationary and nonstationary data analysis due to its appropriate performance; it is also computationally efficient, requiring only an extremely small storage capacity [1].

With numerous competing models for obtaining the best forecast, selecting a suitable one has become a problem [2]. Researchers may find choosing the appropriate model for their study difficult because many methods are available. Another challenge is the difficulty in applying methods. The HW method is a simple, fast, and inexpensive procedure that is widely used in forecasting; it can cope with trends and seasonal variations [4], [5]. This technique differs from other forecasting methods because it does not rely on fitting for any statistical modeling procedure; it also employs repeated steps to obtain the future values of the forecast [6]. The HW method has two versions, depending on whether the seasonal pattern in the series is modeled in an additive or multiplicative process. Moreover, the HW method is not a special case of the Box–Jenkins procedure for all practical purposes [4].

Another strategy for obtaining accurate forecasts is using a hybrid method to obtain the future values of the forecast and overcome the disadvantages and inefficiencies of individual models, such as the presence of non-normal residuals. Hybrid procedures can solve many problems of both linear and nonlinear time series structures. These procedures are known as hybrid models. In the current paper, the KF and HW methods are hybridized with the ARFIMA model to demonstrate the aforementioned phenomenon. This study aims to determine if the ARFIMA model can be improved by integrating either the KF or HW method or both. Accordingly, a general problem is highlighted through empirical comparisons of the datasets used in this study, as discussed in later sections.

Crude oil is a highly essential commodity for all countries globally; it impacts everyone's daily life in different aspects [3]. Given the increasing request for crude oil in all aspects of life [2], its pricing poses a challenge to the world's economy and to domestic and international investments. Therefore, developing the best strategic plans for understanding the price changes of crude oil, particularly Brent crude oil, is imperative.

However, despite the numerous studies on Brent crude oil prices, the following question remains unanswered: which hybridization approach is the most appropriate for modeling the problem of non-normal residual distribution in the ARFIMA model?

To answer this question, the purpose of this work is as follows. Firstly, the primary target is to compare the performance of statistical methods, namely, ARFIMA, ARFIMA-KF, ARFIMA-AHW, and ARFIMA-MHW, to obtain an effective and accurate individual model or hybrid model for the reliable modeling of Brent crude prices. Secondly, the characteristics of the optimal and most accurate method to increase the effectiveness and quality of the modeling procedure for the ARFIMA model using the KF or HW hybridization method will be determined. Thirdly, this study also intends to check the following hypothesis: the model that has the best Akaike information criterion (AIC) is not necessarily the best forecast model for a dataset.

II. Literature Review

Several researchers have studied the ARFIMA model for modeling and forecasting crude oil prices. For example, [7]. While, [2] explained the KF procedure by comparing it with the ARIMA, generalized autoregressive conditional heteroskedastic, and ARIMA-KF models. They concluded that the performance of ARIMA-KF is considerably better than other models. Thereafter, [8] suggested using the KF-ARIMA (1,1,1) model to get accurate predictions relevant to the COVID-19 pandemic in Pakistan during the period from February 26, 2020 until to April 30, 2020. The experimental result showed that the proposed model achieved the lowest mean absolute percentage error (MAPE) compared with the SutteARIMA and HW models.

Various techniques have been utilized in modeling and forecasting in which the primary focus is on trend, seasonality, or a combination of both in certain time series dataset. One of these techniques is the HW method. The HW literature is extensive, and the application of this method has spread over many scientific areas. [9] used the ARIMA, ARFIMA, and HW smoothing procedures to evaluate and forecast the air quality in Chandigarh City from 2009 until to 2010. The ARFIMA (2,0.3051,2) model was suitable and better than the other models. [10] found that the MHW model outperformed the Seasonal ARIMA (0,1,1)(0,1,1)12 model when using monthly dataset on India's inbound tourism from January 2001 until to June 2018 based on the mean absolute error (MAE), MAPE, and mean-square error (MSE). [11] studied the Box–Jenkins (ARIMA), MHW, and AHW methods for Potato prices covering the period from January 2005 until to July 2019. The results indicated that the ARIMA (1,1,2) method achieved better forecasting accuracy based on root-mean-square error (RMSE), MAPE, and mean absolute deviation (MAD) compared with other methods.

The hybrid method is frequently used in practical prediction applications for several fields. [12] examined and compared numerous individual and hybrid models, namely, ARIMA, SARIMA, ARFIMA, HW, singular spectrum analysis, ARIMA-wavelet, ARFIMA-wavelet, SARIMA-wavelet, ARIMA-KF, ARFIMA-KF, and SARIMA-KF, for predicting the future workload of CPU, RAM, and network. The SARIMA-KF hybrid model outperformed the other models and achieved extremely high forecasting accuracy based on MAPE.

Many researchers have attempted to model and forecast Brent crude oil prices using time series, individual, and hybrid statistical models [2], [13]–[22]. Forecasting oil prices exerts a considerable impact on supply and demand in global markets. Moreover, oil price expectations remain highly important for investors and researchers. They pose a challenging problem to them by reason of the special characteristics of oil prices and their remarkable effect on several economic and financial sectors in the world, particularly in the current situation due to the COVID-19 pandemic. We choose this type of data for our study because of the aforementioned reason.

III. Methodology of Research

This section presents all methods and the dataset used in this study by employing the real-time series dataset. The dataset, methods, tests, criteria, and accuracy measures are introduced as follows.

A. Dataset

The monthly Brent crude oil prices datasets have been used in this study which are obtained from the website: www.indexmundi.com/commodities/?commodity=crude-oil-brent. The monthly datasets were selected from January 1979 until to July 2019, totaling 487 observations. The long period covered allows considering historical observations, i.e., after the 1973 oil crisis, as described by [23]. The preceding statement justifies our selected period for our dataset. Thus, the dataset covering the period from January 1979 until to July 2018, totaling 475 observations, were used as a training dataset. While the remaining observations were used as the testing dataset. R software (version 3.5.3) has been used to implement all statistical analyses in this work.

B. Long Memory

In this section, we use the same methodology that used by the authors in [19] to propose and find a new hybridization method based on ARFIMA models, thus improving the performance of the forecasting model.

The existence of the long memory behavior is detected if the autocorrelation function (ACF) decreases more slowly than the exponential decrease as explained by [7]. In addition, the presence of this memory can be noted through a nonstationary structural break [24]. Therefore, testing the structural breaks of any dataset is necessary because it specifies that the long memory is present or imaginary [24]–[26]. In 1960, [27] introduced a test for determining the presence of structural breaks. Then in 2015, [28] modified it into a Quandt likelihood ratio (QLR) test (also called the supremum F-statistic) for structural breaks between t_0 and t_1, which was given by

$$Sup\ F = \max\{F(t_0), F(t_0+1), \ldots, F(t_1)\}, \tag{1}$$

where the supremum F-statistic is the largest.

Numerous statistical methods are used to verify the existence of the long memory feature as mentioned in [29]. These methods include the R/S Hurst, aggregated variance, and Higuchi methods. Moreover, numerous approaches are available for testing and estimating long memory parameters. These approaches are presented in detail in the following subsections.

1) Hurst Exponent: This method depends on the range ($R_{(n)}$) of subtotals with values that deviate from their mean in the time series divided by the standard deviation ($S_{(n)}$), as explained in [29], [30]. It is symbolized as ($Q_{(n)}$) and written as:

$$\begin{aligned} Q_{(n)} &= \frac{R_{(n)}}{S_{(n)}} \\ &= \frac{\max\limits_{1\le k\le n} \sum\limits_{i=1}^{k} (X_i - \overline{X_n}) - \min\limits_{1\le k\le n} \sum\limits_{i=1}^{k} (X_i - \overline{X_n})}{\left(\sum\limits_{i=1}^{n} (X_i - \overline{X_n})^2\right)^{\frac{1}{2}}}, \end{aligned} \tag{2}$$

where

$$\overline{X_n} = \frac{1}{n}\sum_{i=1}^{n} X_i, \tag{3}$$

and (n) is the sample size.

2) Geweke and Porter–Hudak (GPH) : In 1983, [31] proposed the estimation for the parameter ($\hat{d}_n$) based on the regression equation (Y_i) in accordance with the following equation:

$$\hat{d}_n = -\left(\sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2\right)^{-1} \left(\sum_{i=1}^{n} \left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)\right), \tag{4}$$

where

$$Y_i = \alpha + \beta X_i + \varepsilon_i, \tag{5}$$

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i. \tag{6}$$

3) Smoothed Periodogram (Sperio) and Fractionally Differenced (fracdiff): By contrast, the Sperio and fracdiff are functions in R software that are employed to estimate the (d) value in accordance with the following paragraphs:

In 1994, [32] explained the first function, which estimates the (d) value in the ARFIMA(p, d, q) model. The Sperio function is denoted by $f_a(w)$ through the Parzen lag window, as it follows:

$$f_a\left(w_j\right) = \frac{1}{2\pi}\sum_{-m}^{m} k\left(\frac{a}{m}\right) R\left(a\right)\cos\left(a w_j\right), \tag{7}$$

where

$$k(v) = \left\{ \begin{array}{lll} 1 - 6v^2 + 6|v|^3 & , & |v| \le \frac{1}{2} \\ 2(1-|v|)^3 & , & -\frac{1}{2} < v \le 1 \\ 0 & , & v > 1 \end{array} \right\}. \tag{8}$$

$k(v)$ is called the Parzen lag window generator. (m) is the truncation point, and

$$\begin{aligned} R(a) &= \tfrac{1}{n}\left(\sum_{i=1}^{n-a} (X_i - \bar{X})(X_{i+a} - \bar{X})\right), \\ & \qquad a = 0, \pm 1, \ldots, \pm(n-1), \end{aligned} \tag{9}$$

which indicates the autocovariance function.

While in 1981, [33] explained the fracdiff operator, which employs the regression estimation method to estimate the (d) value for the ARFIMA model [34]. This factor (i.e., d value) is determined by a Binomial series, as:

$$\begin{aligned} \nabla^d &= (1-B)^d = \sum_{l=0}^{\infty} \binom{d}{l} \ (-B)^l \\ &= 1 - dB - \frac{1}{2}d\left(1-d\right)B^2 - \frac{1}{6}d\left(1-d\right)\left(2-d\right)B^3 - \ldots . \end{aligned} \tag{10}$$

C. ARFIMA Model

The general formula for the ARFIMA(p, d, q) model can be given as

$$\phi_p(B)(1-B)^d x_t = \theta_q\left(B\right)\epsilon_t \quad \text{for } 0 < d < 0.5, \tag{11}$$

the parameter (d) is a non-integer value and a nonseasonal difference order which is defined using Equation (10); $\{x_t\}$ is a time series variable at time (t); (ϵ_t) is a white noise; and $\phi_p\left(B\right)$ and $\theta_q\left(B\right)$ represent AR(p) autoregression for order (p) and $MA(q)$ moving average for order (q) components with backward shift operators (B), respectively (more details, see [30] and [35]).

D. Kalman Filter (KF)

KF is an optimum linear estimator, and it deduces model parameters from observations that are indirect, unconfirmed, and inaccurate as explained by [2], [3]. This procedure exhibits an important advantage that distinguishes it from other technologies, i.e., the system updates after receiving each new observation. Thus, the error is also reduced. Before learning about the KF technique, the presentation of state space (SS) modeling is essential in order to KF use SS model terminologies as demonstrated by [2], [3]. The procedure of the SS model, also called a dynamic linear model [3], consists of two phases. First, a state vector is formed to capture the significant components of the time series and then added until the end. The smallest vector (i.e., the state vector) summarizes the past behavior of the overall system by playing the key role in SS modeling, which determines the state vector [36]. The SS modeling approach explains the smoothing of the series in the format of two linear equations. Equation (12)), called the observation equation, shows the relationships between the present observation and the unobserved cases [2]. While equation (13), called the state equation, shows the progress in the states over time and updates the state vector continuously [2]. The state vector is updated continuously through the state equation (the state equation is a vector that contains the undetected components of a series, such as the trend, seasonality, and level of the

AR and MA factors of a time series) as mentioned by [2]. Accordingly, the general formula for the SS model is presented as follows:

$$x_t = T' \ \gamma_t + \varepsilon_t, \tag{12}$$

$$\gamma_t = H' \ \gamma_{t-1} + F \ \omega_t, \tag{13}$$

where $\{x_t\}$ represents the observed vector of the variables, and $\{\gamma_t\}$ displays the vector of the unobserved variables. Moreover, $[T]$, $[H]$, and $[F]$ are parametric matrices; and(ε_t) and (ω_t) are the white noise terms with their covariance matrices $[Q]$ and $[R]$, respectively [37].

Thus, the filter is an algorithm used for solving and clarifying the linear SS models, while the equations for predicting and updating the system are called KF. The new part of the time series $\{x_t\}$ is called innovations and equivalent to the residuals of KF. KF forecasts the state estimates of the series $\{x_t\}$ recursively on the basis of past information with a variance of the prediction error. In addition, ε_t is the innovation at time (t), which is the new information in $\{x_t\}$ and not supposed to be forecasted from the previous information; it is indicated as a one-step-ahead forecasting error [38].

E. Holt-Winters (HW)

This method for exponential smoothing includes trend and seasonality, which is based on three smoothing equations: level, trend, and seasonality [5]. HW models have two types, depending on the seasonal pattern in a series of characteristics: an additive HW (AHW) model, which is employed when the seasonal component is constant, and a multiplicative HW (MHW) model, which is employed when the size of the seasonal component is proportional to the trend level [4]. A technical description of the two types is presented in the succeeding subsections, as explained by [5].

1) MHW: Seasonal MHW is inapplicable if the time series has zeros or negative values. Its equation is as follows:

$$f_{t+m} = (L_t + b_t m) S_{t-s+m}, \tag{14}$$

where L_t, b_t, and S_t are given by

$$L_t = \alpha \ \frac{y_t}{S_{t-s}} + (1-\alpha)(L_{t-1} + b_{t-1}), \tag{15}$$

$$b_t = \beta(L_t - L_{t-1}) + (1-\beta) b_{t-1}, \tag{16}$$

$$s_t = \gamma \ \frac{y_t}{L_t} + (1-\gamma)\, s_{t-s}, \tag{17}$$

such that (f_{t+m}) presents a forecast for m periods ahead of time (t), y_t is the observed series at time (t), (L_t) shows the level of the series at time (t), (b_t) shows the slope (trend) of the series at time (t), (s_t) shows the seasonal component of the series at time (t), and (s) shows the number of seasons in a year. The constants α, β and $\gamma \in [0, 1]$ represent the smoothing parameters.

2) AHW: Seasonal AHW differs from seasonal MHW in terms of the smoothing and forecast processes. Its equation is as follows:

$$f_{t+m} = L_t + b_t m + S_{t-s+m}, \tag{18}$$

where L_t, b_t and S_t are given by

$$L_t = \alpha \ (y_t - s_{t-s}) + (1-\alpha)(L_{t-1} + b_{t-1}), \tag{19}$$

$$b_t = \beta(L_t - L_{t-1}) + (1-\beta) b_{t-1}, \tag{20}$$

$$s_t = \gamma \ (y_t - L_t) + (1-\gamma)\, s_{t-s}, \tag{21}$$

where $(f_{t+m}), y_t, (L_t), (b_t), (s_t), (s), (\alpha), (\beta)$ and (γ) are defined in previous subsections.

F. Hybrid Methods

Various methods for time series modeling that use a combination of several models instead of only one model have been advanced over the previous decades to improve forecasting results. These composite models are called hybrid methods. The hybrid model shows the capability of being a reliable tool for increasing the accuracy of the final prediction and facilitating the capture of various kinds of behaviors in any time series dataset [17]. In this study, the first method uses KF and the second one uses HW. Both models are applied to the ARFIMA model, which functions as the basic model. Consequently, hybrid models are proposed for simultaneously modeling a time series dataset.

The hybrid ARFIMA-KF model uses the estimated values of the ARFIMA model as the starting values for KF repetition. This means, in the hybrid model, the ARFIMA model is first applied to obtain the parameter estimate of this model. Then, these estimated values are applied as the beginning values for the KF repetition. That is, when using KF based on Section (III-D), the matrices of $[F]$, $[H]$, $[Q]$, and $[R]$ must be estimated. Subsequently, the forecasts obtained using the two models are combined to determine the ultimate expected value of the suggested hybrid model. Thus, hybridizing KF with ARFIMA can provide better forecasts and improve the forecasting capability of the ARFIMA model.

Based on the above, the proposed hybrid algorithm (i.e., the ARFIMA with the KF) can select optimal parameters through a training and iteration process based on the period of the Brent crude oil dataset. Therefore, the results in this study will depend on the period of training and the frequency of observations, as discussed by [1] and [3]. Apart from the aforementioned hybrid KF procedure, this study also makes the following practical recommendations for a hybrid HW procedure, whether in additive or multiplicative form, with the ARFIMA model. The suggested hybrid model involves of two phases. The first one uses the ARFIMA model and its parameters, i.e., Equation (11). In the second phase, the HW method is used to analyze the residuals obtained in the first phase, i.e., Equation (14) or (18). Equation (22) shows the hybridized approach.

$$y_t = L_t + f_{t+m}, \tag{22}$$

where y_t refers to the real-time series dataset for the period (t), (L_t) is the linear portion estimated by the ARFIMA model, and (f_{t+m}) represents the AHW or MHW portion of the residual series. In this manner, the forecasting values of Equation (22) can be obtained as follows:

$$\hat{y}_t = \hat{L}_t + \hat{f}_{t+m}, \tag{23}$$

where $\hat{y}_t$ refers to the forecasting values of the real dataset for the time (t); $(\hat{L}_t)$ is the forecast values for the time (t) from the applied relationship in Equation (11); and $(\hat{f}_{t+m})$ is the forecast values for the time (t) from the applied relationship in Equation (14) or (18). Thereafter, the forecasts gotten from these models are collected to know the ultimate expected value of the suggested hybridization model. Being meticulous is necessary when selecting the correct hybrid HW model, either additive or multiplicative. Thus, determining whether the additive model is better than the multiplicative model, or vice versa, is beneficial for the hybridization process. Although hybrid HW modeling requires effort, such effort is considerably less than that required by the hybrid KF procedure. Moreover, practical considerations exclude the hybrid MHW procedure if negative observations or zeros are present.

G. Stationarity and Normality Tests

To check the stationarity of the dataset, augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are performed [39], [40]. Meanwhile, numerous methods are available for evaluating whether a dataset is normally distributed. These methods can be classified into two major categories: graphical [41], [42] and statistical [42]. The most common are the Jarque-Bera [43] and Shapiro-Wilk [44] tests, which are used in the current study.

H. Autoregressive Conditional Heteroscedasticity with Lagrange Multiplier (ARCH-LM) and Ljung–Box Tests

[45] recommended the Lagrange multiplier (LM) test to determine whether disturbances follow an autoregressive conditional heteroscedasticity (ARCH) process. This test is used to check if an error (ϵ_t) in the resulting model residuals is a heteroscedastic operation [28]. In addition, the Ljung-Box test is another necessary phase in checking the existence of the correlation among the residuals in the model [46].

I. Information Criterion and Accuracy Measure

The fit model selection depends on several criteria, such as AIC, as noted by [47] and [35]. This criterion is formulated in the following equation:

$$AIC = -2\ln(l) + 2k, \tag{24}$$

where (l) is a maximum likelihood for the model, and (k) is the total number of the parameters $(k = p + q)$ through Equation (11). Furthermore, RMSE is an accuracy measure used to evaluate a model's performance, as clarified by [46].

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(Y_t - \hat{Y}_t\right)^2} \tag{25}$$

where (Y_t) is the real value, and $(\hat{Y}_t)$ is the predicted value.

TABLE I
STRUCTURAL BREAK TEST

QLR	P-value
1190	$< 2.2 \times 10^{-16}$

Although there are many measures of forecasting accuracy, this study used only one, the RMSE, since this measure is considered more useful than other performance measures, according to the following explanations. The RMSE is better suited for understanding model performance than the MAE when the error distribution is expected to be Gaussian and enough samples are available (i.e., $n \geq 100$); thus, reconstructing the error distribution using the RMSE value is considered more reliable. Moreover, the RMSE satisfies the triangle inequality requirement for a distance measure as reported by [48]. They also stated that the RMSE penalizes variance because it gives errors with larger absolute values more weight than errors with smaller absolute values, whereas the MAE gives the same weight for all errors and is most commonly used for outliers due to its sensitivity. [48] also mentioned that the best statistics measures should provide a performance measure for the forecasting model with a representation of the error distribution simultaneously. Finally, they confirmed that when evaluating various models using one accuracy measure, differences in the error distributions become more significant. Therefore, we have chosen this type of accuracy measure for the previously mentioned reasons.

IV. DISCUSSION AND RESULTS

The monthly price ($/bbl) graph for the Brent crude oil series is shown in Fig.1. $\{x_t\}$ denotes the price, and (t) represents the time in all months of the period from January 1979 until to July 2019. This series indicates stable prices followed by a gradual fluctuation. That is, the series exhibits considerable fluctuations over time, particularly in 2008. The statistical measures for that series are as follows: the mean is 42.95, the median is 30.20, and the right skewness is 1.177466. Furthermore, all four structural breaks for the series are shown in Fig.1 in 1986, 1999, 2005, and 2013. That occurred in the 1980s, 1990s, 2000s, and 2010s refer to the oil crisis period since 1973. These break points correspond to that period, as described in Section III-A.

Table I presents the preliminary result of the QLR structural breaks test on all the dates. The null hypothesis for this test is not accepted because the supremum F-statistic is extremely large while the P-value is extremely small.

Moreover, the ACF (Fig.2) exhibits a slow decline, which is an ideal behavior of the long memory process. Thus, a preliminary conclusion is drawn that a long memory exist, and this conclusion is confirmed in Table II in accordance with several statistical methods. Table II presents the results of checking for an existing long memory. All the (H) values are higher than 0.5, providing a confirmed conclusion for the presence of a long memory feature in the series. Thus, based on a P-value ($< 2.2 \times 10^{-16}$) for the Jarque-Bera test and the skewness value above-mentioned, the null hypothesis for this test is rejected (i.e., this series is not normal).

Notably, the previous tables (i.e., Tables I and II) were taken from [19] for use in the hybrid proposed models in

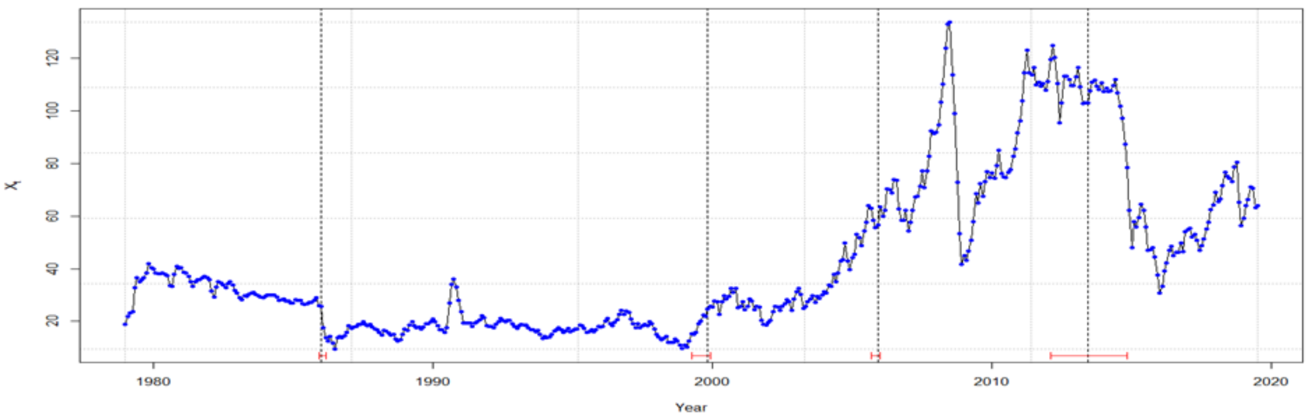

Fig. 1. Monthly Brent prices with all breaks and their confidence intervals

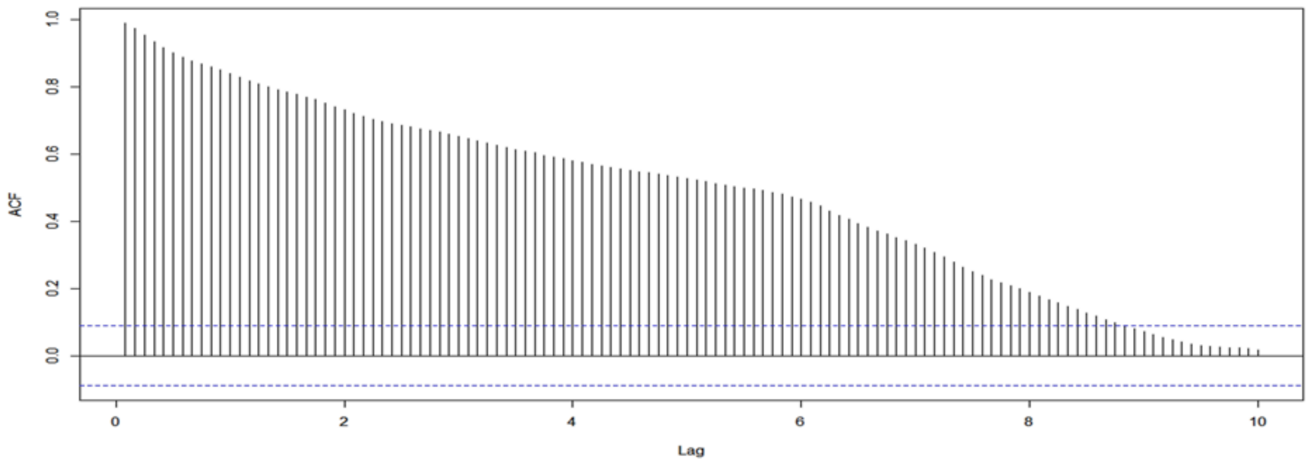

Fig. 2. ACF plot for the $\{x_t\}$ series

TABLE II
LONG MEMORY TESTS

R/S Analysis	Aggregated variance method	Higuchi method
H = 0.8531864	H = 0.7910981	H = 0.9578515

this study.

Consequently, $\{x_t\}$ transformation must be implemented. Accordingly, the $\{Y_t\}$ series refers the growth rate for the $\{x_t\}$ series because the difference would be taken after that, as shown in the succeeding equation:

$$Y_t = \log(x_t). \tag{26}$$

The fractional difference (d) for the $\{Y_t\}$ series is appreciated through various methods and functions, as shown in Table III. The (d) value through the R/S Hurst analysis is 0.3589648, the Sperio estimate is 0.4984955, and the fractionally differenced estimate is 0.4994726. By contrast, we exclude GPH estimation because its value is greater than 0.5.

After calculating the fractional difference (d_i) applying Equation (26), the series is transformed as follows:

TABLE III
LONG MEMORY ESTIMATION FOR THE Y_t SERIES

Method / Function	d
R/S Hurst $(d = H - 0.5)$	$d_1 = 0.3589648$
Sperio (bandw.exp = 0.3, beta = 0.74)	$d_2 = 0.4984955$
Fractionally differenced (fracdiff)	$d_3 = 0.4994726$
GPH	$d_4 = 0.7676326$

$$Z_{t(di)} = diff\,(Y_t) = Y_t(1-B)^{di}, \tag{27}$$

where $d_i = d_1,\ d_2$, and d_3. The stationary results of the $Z_{t(di)}$ series are presented in Table IV. The P-values for the ADF and PP tests (Table IV) indicate that the series became stationary (i.e., the null hypothesis for these tests is rejected) after computing the fractional difference.

From Equation (24) and the results of the practical analysis of the aforementioned dataset, the best qualifying models are ARFIMA$(1, d_1, 0)$, ARFIMA$(2, d_1, 1)$, and ARFIMA$(2, d_1, 2)$ based on the lowest AIC criterion of $-962.91, -966.25$, and -966.07, respectively. It is noted that the three models belong to the lowest (d) value estimate.

Thus, the three models were selected and will be compared

TABLE IV
STATIONARY TEST FOR THE $Z_{t(di)}$ SERIES

Method / Function	Tests	Value
R/S Hurst	ADF test for the $Z_{t(d1)}$ series	-4.1727
	PP test for the $Z_{t(d1)}$ series	-82.923
Sperio	ADF test for the $Z_{t(d2)}$ series	-5.1927
	PP test for the $Z_{t(d2)}$ series	-151.34
Fracdiff	ADF test for the $Z_{t(d3)}$ series	-5.2001
	PP test for the $Z_{t(d3)}$ series	-151.89

TABLE V
NORMALITY TESTS OF THE RESIDUALS

Model	Ljung–Box test			Shapiro-Wilk normality test
	Lag (12)	Lag (24)	Lag (36)	
ARFIMA $(1, d_1, 0)$	0.02570	0.02195	0.04968	1.585×10^{-8}
ARFIMA $(2, d_1, 1)$	0.08763	0.06623	0.15880	1.919×10^{-9}
ARFIMA $(2, d_1, 2)$	0.12860	0.11040	0.22870	1.686×10^{-9}

to determine which is the best one. The succeeding step involves the testing of residuals [49]. Residual testing is a requisite step in this stage for examining any model by using different techniques, including ACF graphs and the Ljung–Box residual test. These techniques are also necessary when considering the correlations among residuals [46]. Table V indicates that these models do not exhibit the feature of the unit root for the residuals based on the P-values for the Ljung-Box test at different lags and on the P-values of the Shapiro-Wilk normality test. In addition, the P-value of the Jarque-Bera test for the residual's models is less than 2.2×10^{-16}. Thus, all previous results confirm that the residuals of these models are not normally distributed (i.e., rejecting the null hypothesis for these tests).

From the previous result, the procedure used in the next step should be eligible to address the problem of having non-normal structures in the selected time series type. This procedure is KF. Therefore, these three models can be hybridized with KF depending on the results of the first phase (ARFIMA modeling). Thus, the best training period models for our series are identified and their estimated coefficients are provided in Table VI. The ARFIMA-KF model uses the estimated values of the ARFIMA models given in Table VI as the initial values of KF recursion.

Table VI also shows that all the AR and MA coefficients of the three models are statistically significant because their P-values are 0.000, except for MA (2) of the last model, which is statistically insignificant because its P-value (0.1773) is greater than the typical significance level of 0.05. Thus, these coefficients will exert different effects on the accuracy of filtering, as shown in Table VII.

Table VII summarizes the information related to the selected models in terms of AIC and RMSE. RMSE shows the performance of all models relative to the test set, and AIC presents the best ones. The experimental analysis emphasizes that the RMSE of these models is within 0.1 (i.e., close to the real values of the series). The table also exhibits the relationship among the ARFIMA models and the ARFIMA-KF hybrid models in term of the RMSE values for the test set, as the RMSE values in the hybrid models depends on the RMSE values of the ARFIMA models (i.e., the relationship between them is an incremental relationship for the same basic model). Thus, the best individual model is the ARFIMA $(2, d_1, 2)$ model and the best hybrid model is the ARFIMA $(2, d_1, 2)$-KF model, which have the smallest values for these measurements.

In Table VIII, the Ljung-Box test of the residuals for the first model shows that the residuals are not white noise and dependent [50], while these results differ from those of the hybrid model. Therefore, the ARFIMA $(2, d_1, 2)$-KF model is appropriate for studying the Brent series because its residuals are white noise. Furthermore, the model shows no evidence of the ARCH effect when the ARCH-LM test is used. Notably here, the ARFIMA $(2, d_1, 2)$-KF model has a good advantage, it takes 677 steps during processing to model. Conversely, this hybridization technique has an obvious disadvantage, it deals with linear estimators in the modeling.

The second major purpose of this paper is to determine if the ARFIMA results can be improved using the HW method through hybridization. From the previous results, the proposed models should be eligible to deal with the non-normality residuals problem for the real time series dataset. These proposed models are AHW and MHW. Therefore, the residuals obtained from the three models mentioned in the ARFIMA modeling phase are analyzed and hybridized with HW models (i.e., ARFIMA-AHW and ARFIMA-MHW). Table IX summarizes the RMSE results that related to the test dataset of all models. The experimental analysis proves that the performance of the RMSE for all models is within 0.1 (i.e., close to the real values of the time series). The ARFIMA $(2, d_1, 2)$ and ARFIMA $(2, d_1, 1)$-AHW models have the smallest values for this measurement. In agreement with the aforementioned results, the best hybrid model is the ARFIMA$(2, d_1, 1)$-AHW model, followed by the ARFIMA$(2, d_1, 2)$-AHW model. Moreover, the table shows a summary of the AIC values for the training dataset of these models. The model with the best AIC value does not produce the best forecast model for the dataset, which is not equivalent to the smallest RMSE value in the same model, as mentioned in [49]. In addition, Table IX shows that the MHW procedure is not applied to the residual series in our dataset because it is an inappropriate method for negative data.

The initial judgment on the AHW hybrid method is that it is accurate and efficient compared with other methods in accordance with RMSE (Table IX). However, this judgment is disproved when the result of the ARCH-LM test (Table X) is considered, although AHW hybrid forecasts are better.

The ARCH-LM and Ljung-Box tests of the residuals of different models are presented in Table X. Although the Ljung-Box test of the residuals for the ARFIMA$(2, d_1, 1)$-AHW and ARFIMA$(2, d_1, 2)$-AHW models confirms that the residuals are white noise and independent [50], the ARCH-LM test for these models is less than 0.05, indicating the occurrence of the heteroscedasticity effect. Meanwhile, the ARFIMA$(2, d_1, 2)$ model fails both tests. Thus, the ARFIMA-AHW hybrid model clearly does not meet all the model validation criteria. Accordingly, this hybrid model cannot be used to improve the ARFIMA model and to forecast the monthly Brent series.

The fitted ARFIMA and AHW models have common features based on the ARCH-LM test value for both model

TABLE VI
ESTIMATION RESULTS OF THE ARFIMA MODELS

Model	Coefficient	Estimate	Std. Error	z-value	P-value
ARFIMA $(1, d_1, 0)$	AR (1)	0.8893	0.0218	40.887	$< 2.2 \times 10^{-16}$
ARFIMA $(2, d_1, 1)$	AR (1)	1.8036	0.0413	43.668	$< 2.2 \times 10^{-16}$
	AR (2)	-0.8053	0.0403	-19.983	$< 2.2 \times 10^{-16}$
	MA (1)	-0.9646	0.0215	-44.858	$< 2.2 \times 10^{-16}$
ARFIMA $(2, d_1, 2)$	AR (1)	1.7593	0.0608	28.954	$< 2 \times 10^{-16}$
	AR (2)	-0.7617	0.0592	-12.8611	$< 2 \times 10^{-16}$
	MA (1)	-0.8682	0.0782	-11.0979	$< 2 \times 10^{-16}$
	MA (2)	-0.0844	0.0625	-1.3492	0.1773

TABLE VII
AIC AND RMSE VALUES

Model	Training set	Test set
	AIC	RMSE
ARFIMA $(1, d_1, 0)$	-962.91	0.08981462
ARFIMA $(2, d_1, 1)$	-966.25	0.08886256
ARFIMA $(2, d_1, 2)$	-966.07	0.08800826
ARFIMA $(1, d_1, 0)$-KF	-370.18	0.12225460
ARFIMA $(2, d_1, 1)$-KF	-280.01	0.11904290
ARFIMA $(2, d_1, 2)$-KF	-970.69	0.09090624

TABLE VIII
ARCH-LM AND LJUNG–BOX TESTS FOR THE RESIDUAL SERIES

Model	Residual test	
	ARCH-LM test	Ljung–Box test
ARFIMA $(2, d_1, 2)$	9.66×10^{-6}	0.02449
ARFIMA $(2, d_1, 2)$-KF	0.2667	0.1208

TABLE IX
SUMMARY OF MODELS

Model	Training set	Test set
	AIC	RMSE
ARFIMA $(1, d_1, 0)$	-962.91	0.08981462
ARFIMA $(2, d_1, 1)$	-966.25	0.08886256
ARFIMA $(2, d_1, 2)$	-966.07	0.08800826
ARFIMA $(1, d_1, 0)$-AHW	629.5825	0.08080500
ARFIMA $(2, d_1, 1)$-AHW	630.2137	0.07599763
ARFIMA $(2, d_1, 2)$-AHW	629.3643	0.07768752

TABLE X
ARCH-LM AND LJUNG–BOX TESTS OF THE RESIDUALS

Model	ARCH-LM test	Ljung–Box test
ARFIMA $(2, d_1, 2)$	9.66×10^{-6}	0.02449
ARFIMA $(2, d_1, 1)$-AHW	2.585×10^{-5}	0.97540
ARFIMA $(2, d_1, 2)$-AHW	5.815×10^{-6}	0.19970

types in Table X. That is, both models (i.e., individual ARFIMA and hybrid AHW) do not yield an acceptable result in the residual test. In such case, the AHW model exhibits practically no improvement in hybrid modeling fit. Moreover, the ARCH-LM test restricts the quality and efficiency of the listed models, explaining why the performance of these models is not too good.

Lastly, from the results presented in Tables VIII and X, the ARFIMA individual and ARFIMA-AHW hybrid models are inappropriate for modeling and forecasting Brent crude oil prices, but the ARFIMA$(2, d_1, 2)$-KF model satisfies all model measurement validation criteria, although the ARFIMA-AHW hybrid models outperformed in terms of the RMSE criterion for the test set compared to the other proposed hybrid KF models. Notably here, the ARFIMA$(2, d_1, 1)$-AHW model has a good advantage, it takes 633 steps during processing to model.

From the aforementioned results, the ARFIMA model produces good results with the KF procedure. Thus, the proposed model (i.e., ARFIMA-KF) is an efficient forecasting method for obtaining an accurate forecast model. Moreover, the KF approach improves the ARFIMA method and enhances the accuracy and efficiency of the appropriate hybrid model compared with the AHW or MHW methods. However, the best model for the dataset does not achieve the best AIC value, and thus, the hypothesis of this study can be accepted. Another significant result is the incapability of individual ARFIMA models to model accurately the dataset used in this study. Consequently, the attained experimental results are stimulating, in the sense that realizing precision in the modeling of Brent prices is hard and it requires caution and accuracy. Therefore, the hybrid proposed model can be popularized for other commodities, not just oil, as mentioned in [51]. This is confirmed by [2] that the hybridization method with the KF has major significance for statistical applications in econometrics. For that, this study discussed how to choose an effective modeling technique and modify it to improve the efficiency and quality of the chosen model for any real-time series dataset. Accordingly, the empirical comparison results present that this technique has two main advantages over other techniques proposed in this study. First, its performance is highly dependent on the number and values of the initial modeling parameters which must be carefully and accurately estimated. Second, we find that the KF hybridized technique is more accurate than others when tested and evaluated through necessary statistical tests. In addition, (Fig.3) shows the ARFIMA(2,0.3589648,2)-KF model with the actual data series set used in our study to ensure the accuracy of its efficiency. These findings show that increasing the number of parameters in the base model allows the filtering technique to approximate the resulting data to the actual values more accurately, especially in necessary forecast times. This is due to the fact that the filtering system is continuously updated after new observations are received, which reduces the error percentage of the resulting values, thus the hybrid model becomes able to forecast accurately. In other words, this process can be likened to a cycle, where the previous step is the starting value of the next step. Thus, the results highlight the significance of the proposed filtering

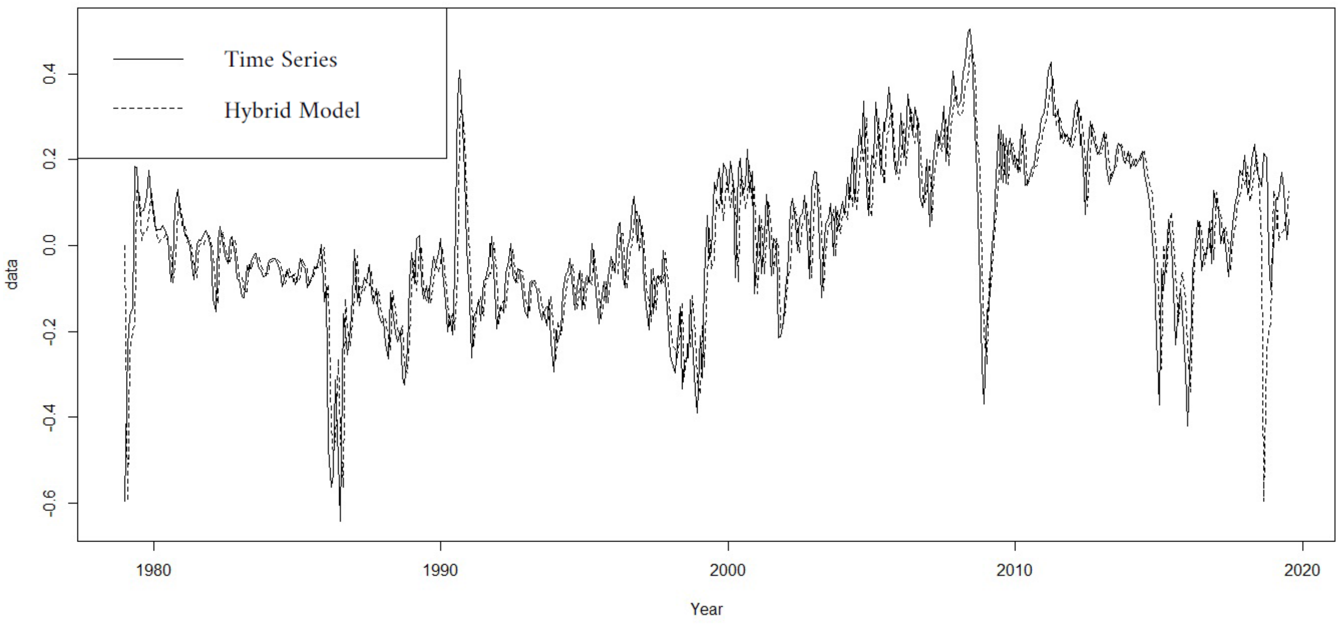

Fig. 3. The time series and Hybrid ARFIMA(2,0.3589648,2)-Kalman filter

methodology. However, the question remains open, to what extent the KF improves the accuracy and suitability of linear and nonlinear hybrid models when modeled?

For future research, we recommend replacing the hybrid method for individual models by using KF because it saves time and steps required for modeling in the case of non-normality in residuals. In addition to focusing on the KF procedure and the hybrid HW model based on ARFIMA and their use of actual time series because both may be used as a tool by modelers and forecasters, we realize that choosing between them is a difficult task, as confirmed by our study. The ARCH-LM test of the models' residuals signifies the need to practice caution when selecting a fit model for forecasting, and thus, we also recommend the importance of performing this model residual test when modeling. Thus, by furthering our understanding of modeling optimization, researchers' choice of filtering techniques can be stimulated (e.g., the hybridization with the KF) to increase the impact of improving model accuracy when the empirical analysis and the modeling are used. Consequently, this paper focuses the value of the extent to which this proposed method deserves to be disseminated and popularized.

V. Conclusion

In this study, we improve the ARFIMA model to avoid two problems, namely, non-normality residuals and inaccuracy in selecting an appropriate model for forecasting, by using the KF approach. The simulation outcomes exhibit that the performance of the ARFIMA-KF model is better in terms of accuracy compared with those of other models, such as ARFIMA-AHW and ARFIMA-MHW, by using the ARCH-LM and Ljung-Box tests. Moreover, the primary feature of the proposed ARFIMA$(2, d_1, 2)$-KF model, is its ability to capture many of the features found in the series of Brent prices through the unique characteristics of the individual models that compose it. The most important among them, is the new observations processing cycle through the use of filtering technology (this means, the KF controls feedback processing over time). This leads to the fact that, the KF has the ability and the force to control the reduction of the error value. Moreover, the simulation results confirmed that using different methods in one hybrid model integrates the power of the individual models and produces a more accurate and efficient model.

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