Currency Exchange Portfolio Risk Estimation Using Copula-Based Value at Risk and Conditional Value at Risk

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***Abstract*—The recent currency fluctuations, which have created uncertainty in currency markets about the movement of exchange rates, have spurred renewed interest in portfolio dependence modelling and risk modelling. Currency exchange portfolio risks can be measured using Value at Risk (VaR) and Conditional Value at Risk (CVaR) based on Monte Carlo simulation. However, this method of risk estimation involves considerable challenges owing to the complexity of modelling the joint multivariate distribution of the assets in the portfolio. Therefore, copula functions, such as the *t*-Student and Clayton copulas, have been proposed to measure the dependence structure of the return of a currency exchange portfolio. This study proposes the use of the copula-VaR and copula-CVaR approaches to make strategic choices in currency management when evaluating the risks of equal- and mixed-weighted portfolios of the returns of investment in five foreign currencies in Malaysia: the United States dollar, United Kingdom pound sterling, European Union euro, Japanese yen, and Singapore dollar. The generalised autoregressive conditional heteroscedasticity (GARCH)-copula models are also evaluated. We find that the marginal distribution of the returns series of the currency exchange rates can be modelled using the Glosten–Jagannathan–Runkle (GJR)-GARCH model with the *t*-Student distribution, and the dependence structure of the currency exchange portfolio can be depicted by the *t*-Student copula. The best investment performance tends to the Singapore dollar.**

***Index Terms*—GARCH-copula model, Exchange rate, Portfolio risk, VaR and CVaR.**

I. Introduction

CURRENCY fluctuations are changes in the value of a currency relative to another currency. Financial data usually fluctuate rapidly due to factors such as supply and demand, economic growth of countries, and inflation, giving rise to heteroscedasticity. Currency fluctuations significantly influence risk estimation in financial markets. The outbreak and spread of COVID-19 in 2019 caused economic instability. The resulting subdued market sentiment slowed down economic activity and triggered currency fluctuations. Such fluctuations may lead to uncertainties about exchange rate movements in currency exchange portfolio, increasing currency exchange portfolio risks and complicating investment decision-making.

Currency exchange portfolio risks need to be studied because investment in currency exchange rates affects the amount of money investors see at the end of the day, and this in turn determines the ultimate rate of return for investors. Previous studies have used methods such as the standard deviation, value at risk (VaR), and conditional value at risk (CVaR) to measure portfolio risk [1], [2], [3]. VaR and CVaR are applied to estimate currency exchange portfolio risks in this study because they are a quantile risk measure and a coherent risk measure, respectively, with properties such as translation invariance, positive homogeneity, monotonicity, and sub-additivity. VaR is a statistical risk assessment measure that gives the probability that a portfolio will experience losses within a fixed time frame with a specified probability. The greatest advantage of using VaR is that risks can be represented by a single number [3]. CVaR is a risk assessment measure that quantifies the tail risk of a portfolio at a certain confidence level. CVaR is similar to VaR except that it provides a relatively conservative measure of loss [4]. This study uses VaR and CVaR to estimate currency exchange portfolio risks.

The autoregressive conditional heteroscedasticity (ARCH) and generalised autoregressive conditional heteroscedasticity (GARCH) models are used to capture the heteroscedasticity effect of marginal returns. It is an appropriate method to measure VaR because uncertainties about exchange rate movements in currency exchange portfolios increase currency exchange portfolio risks and complicate investment decision-making. The positive and negative errors assumed by the GARCH model will have the same volatility effect (symmetry). However, because most data show that the positive and negative errors of market volatility are asymmetric, the symmetry assumption is often violated in financial markets. A drawback of the GARCH model is the problems stemming from the asymmetric effect, as the model assumes that all coefficients are greater than zero. This drawback makes the model difficult to apply. The GJR-GARCH model, which was introduced by Glosten et al. [5] to deal with the asymmetry problem, analyses volatility effects from asymmetric conditional heteroscedasticity by adding seasonal terms to distinguish the positive and negative shocks. In the present study, the ARCH/GARCH family of

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models and the GJR-GARCH model are used to measure portfolio risk.

Further, financial data are not always independently and identically distributed; they exhibit a heteroscedasticity effect. To improve the accuracy of the predicted results, it is crucial to understand the factors that contribute to currency exchange portfolio risks. Typically, the risks faced by investors may increase if a high dependence structure exists. For several reasons, copula-VaR is the best fit to apply. The assumption of joint normality is not required in the copula. Copula-VaR allows high-dimensional joint distributions to be decomposed into marginal distributions and links them together to estimate currency exchange portfolio risks. In addition, the nonlinear, asymmetrical structure of multiple risk factors can be accurately described using the copula. Many applications of copula-VaR have been described in [6], [7], and other works. In the present study, an elliptical copula model (*t*-Student) and an Archimedean copula (the Clayton copula) are used to depict the dependence structure of the residuals and obtain the copula parameters.

II. The model

Financial data are statistically analysed to summarise the data series and observe the nature of data before fitting mathematical and statistical models. However, financial data are not always independently and identically distributed; they often exhibit a heteroscedasticity effect. Therefore, the time series model is used to fit the data. The ARCH model, which was introduced by Engle [8], is applied to estimate financial time series because it considers the relationship between the past conditional variance and the current conditional variance. Let ε_t be the error term of the time series; the return R_t can then be represented as

$$R_t = \mu_t + \varepsilon_t. \tag{1}$$

μ_t is the independent variable's vector multiplied by the slope's vector. In the ARCH model, we model the residual term ε_t, which can be represented as

$$\varepsilon_t = \sigma_t z_t. \tag{2}$$

z_t is a stochastic piece that follows a normal distribution with a mean of zero and a variance of one. That is, $z_t \sim N(0,1)$. By modelling σ_t^2 with an autoregressive model, σ_t^2 can be represented as

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2. \tag{3}$$

Engle [8] showed that for a positive σ_t^2, the parameters must be constrained as follows: $\alpha_0 > 0$ and $\alpha_j \geq 0$ for $j = 1,2,...,q$, where $q > 0$ and q is the length of the ARCH lags. Unfortunately, when the ARCH model is used to describe volatility, the length of the lag q may become very large. Therefore, the GARCH(p,q) model, which uses the autoregressive moving average model instead of the autoregressive model to describe conditional variance, is much more suitable for modelling the conditional variance:

$$R_t = \mu_t + \varepsilon_t \tag{4}$$

$$\varepsilon_t = \sigma_t z_t \tag{5}$$

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2, \tag{6}$$

where β_i is the parameter of the GARCH(p,q) component model, and σ_{t-i}^2 is the conditional variance in the previous i time step. Bollerslev [9] showed that the parameters must be constrained as follows for a positive σ_t^2: $\alpha_0 > 0$, $\alpha_j \geq 0$ for $j = 1,2,...,q$, where $q > 0$, and $\beta_i \geq 0$ for $i = 1,2,...,p$, where $p \geq 0$. $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$. However, the positive and negative return rates of assets have different effects on volatility, and a negative effect on volatility is often greater than a positive effect (the leverage effect) of the same magnitude. The GJR-GARCH model is used to consider this asymmetry, and its general form is as follows:

$$R_t = \mu_t + \varepsilon_t \tag{7}$$

$$\varepsilon_t = \sigma_t z_t \tag{8}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{k=1}^{r} \gamma_k I_{t-k} \varepsilon_{t-k}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{9}$$

where $z_t \sim N(0,1)$, and γ_k is the GJR's threshold value. [5] showed that for a positive σ_t^2, the parameters must be constrained as follows: $\alpha_0 > 0$, $\alpha_i > 0$, $\beta_j > 0$, and $\beta_j + \gamma_k \geq 0$, $\sum_{i=1}^{q} \alpha_i + \frac{1}{2}\sum_{k=1}^{r} \gamma_k + \sum_{j=1}^{p} \beta_j < 1$. I_{t-k} is an indicator variable that takes the value of 1 if an error (defined as $\varepsilon_{t-k} < 0$) occurs and zero otherwise. $\sum_{k=1}^{r} \gamma_k I_{t-k} \varepsilon_{t-k}^2$ is the leverage when $\varepsilon_{t-k} < 0$. If the residual $\varepsilon_{t-i} > 0$, the positive effect will be $(\alpha_k + 0) \times \varepsilon_{t-i}$. If $\varepsilon_{t-k} < 0$, the negative effect will be $(\alpha_k + \gamma_k) \times \varepsilon_{t-k}$. When $\gamma_k > 0$, the model shows a larger negative effect than the positive effect. Therefore, the GJR-GARCH model can be considered a GARCH model that includes leverage by capturing the asymmetrical nature of a time series. To select the best-fit model for capturing the volatility of returns, the log-likelihood function (LLF) is used. Some tests are presented in the next section to validate this model. Let R_i be the return of asset i, and L_i be the loss of asset i; the return of n assets at time t in vector R and the loss of n assets at time t in vector L can be represented as

$$R_t = [R_{1t}, R_{2t}, ..., R_{nt}] \tag{10}$$

and

$$L_t = [L_{1t}, L_{2t}, ..., L_{nt}]. \tag{11}$$

The weight of an optimal portfolio of n assets can be expressed as $W = [w_1, w_2, ..., w_n]$, where $0 \leq w_i \leq 1$ and $w_1 + w_2 + w_3 + ... + w_n = 1$. The total returns (TR) of the portfolio are obtained by the above simulation as follows:

$$\begin{aligned} \text{TR} &= R_t \times W^T \\ &= [R_{1t}, R_{2t}, ..., R_{nt}] \times [w_1, w_2, ..., w_n]^T. \end{aligned} \tag{12}$$

The risks of equal-weighted and mixed-weighted portfolios are computed by using VaR and CVaR, which are defined as below:

$$\text{VaR}_\alpha(L) = inf\{l, F_L(l) \geq \alpha\} \tag{13}$$

CVaR is the mean of the losses that exceed VaR threshold:

$$\mathrm{CVaR}_\alpha(L) = E\left[L|L > VaR_\alpha\right], \tag{14}$$

Assuming an equal-weighted portfolio with n assets, the weight in each asset should be the same, that is, $W = \left[\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right]$. The TR can be represented as

$$\begin{aligned} \mathrm{TR} &= [R_{1t}, R_{2t}, ..., R_{nt}] \times \left[\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right]^T \\ &= E[\bar{R}_{it}]. \end{aligned} \tag{15}$$

The total portfolio return is therefore the average return of the assets. The probability that the loss exceeds VaR is α, which can be expressed as

$$\mathrm{VaR}_\alpha(L) = \mathrm{VaR}_\alpha\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) = \mathrm{VaR}_\alpha\left(\bar{L}_i\right), \tag{16}$$

where L is a random variable that represents the total loss with continuous F_L. In the case of multiple assets, copulas can be used as dependence functions to characterise independence and perfect dependence in a straightforward way [10]. According to Sklar's theorem (1959) [11], if F is an n-dimensional cumulative distribution function with continuous margins $F_1, F_2, F_3, ..., F_n$, a unique copula, C, exists such that

$$\begin{aligned} &F(l_1, l_2, l_3, ..., l_n) \\ &= C\left(F_1(l_1), F_2(l_2), F_3(l_3), ..., F_n(l_n)\right), \end{aligned} \tag{17}$$

where $F(l_1, l_2, l_3, ..., l_n)$ is the joint cumulative density function. After determining the best marginal distributions, multivariate *t*-Student and Clayton copulas are used to describe the dependence structure. The cumulative density function of the multivariate *t*-Student and Clayton copulas can be written as follows:

$$\begin{aligned} &C^t_{v,\rho}(u_{1t}, u_{2t}, ..., u_{nt}) \\ &= t_{v,\rho}\left(t_v^{-1}(u_{1t}), t_v^{-1}(u_{2t}), ..., t_v^{-1}(u_{nt})\right), \end{aligned} \tag{18}$$

$$\begin{aligned} &C^{\mathrm{Clayton}}(u_{1t}, u_{2t}, u_{3t}, ..., u_{nt}) \\ &= \left(u_{1t}^{-\theta}, u_{2t}^{-\theta}, u_{3t}^{-\theta}, ..., u_{nt}^{-\theta} - n + 1\right)^{-\frac{1}{\theta}}, \end{aligned} \tag{19}$$

where v is the degrees of freedom, ρ is the correlation matrix, $t_{v,\rho}$ is the standardised multivariate *t*-Student distribution, t_v^{-1} is the inverse of the *t*-Student cumulative density function, and $\theta \geq 0$ is a parameter of the Clayton copula. The parameters of each copula are obtained from its cumulative density function. The tail dependence of each copula model is computed to measure the co-movements and assess the strength of the dependence on extreme negative or positive returns. The formulas for the tail dependence of each copula model are listed in Table I.

To select an appropriate copula function for dependence modelling, the LLF, Akaike information criterion (AIC), and Bayesian information criterion (BIC) are used for data processing. After modelling the marginal distribution and dependence structure separately, a large number of M's are generated by Monte Carlo simulation based on the best-fit GARCH-copula model. Because the independent uniform random variables, which are $X = (X_1, X_2, ..., X_n), X_i \sim U[0,1]$, are simulated, the numbers are transformed into the standardised residuals:

TABLE I: Tail Dependence of Copula Models

Copula	*t*-student	Clayton
Lower Tail Dependence, λ_l	$2t_{v+1}\left(-\sqrt{\frac{(v+1)(1-\rho)}{1+\rho}}\right)$	$2^{-\frac{1}{\theta}}$
Upper Tail Dependence, λ_u	$2t_{v+1}\left(-\sqrt{\frac{(v+1)(1-\rho)}{1+\rho}}\right)$	0

$$R = \left(F_1^{-1}(X_1), F_2^{-1}(X_2), ..., F_i^{-1}(X_n)\right), \tag{20}$$

where F_i^{-1}, $i = 1, 2, ..., n$, is the inverse of the distribution that represents the simulated returns of each corresponding margin. The returns of each margin are computed using standardised residuals and the conditional mean and variance terms observed in the original data at time t.

III. Results and Discussion

The currency exchange rates of the United States dollar (USD), United Kingdom pound sterling (GBP), European Union euro (EUR), Japanese yen (JPY), and Singapore dollar (SGD) relative to the Malaysian ringgit (MYR) are used to demonstrate the application of VaR and CVaR with the copula. The data were obtained from Bank Negara Malaysia [12] for the sample period October 2017 to October 2020. For each currency exchange rate, 732 observations were obtained from the original data by using the daily logarithmic return $r_t = \ln P_t - \ln P_{t-1}$, where P_t is the middle price of the exchange rate at time t. Daily logarithmic returns and histograms of daily logarithmic returns of the five selected currency exchange rates are illustrated in Figures 1 and 2, respectively. The figures show that all of the returns exhibit a pronounced heteroscedasticity effect, which means that the currency exchange rates changed dramatically in both directions during our sample period.

Table II summarises descriptive statistics for each return series. The results in the table show that, in general, all of the returns are typically skewed, with heavy tails, and are not normally distributed. These characteristics occur because all of the returns have nonzero skewness with a kurtosis coefficient greater than three and have a large Jarque–Bera coefficient that is far from zero. The augmented Dickey–Fuller test results show that there is no unit root and follows a stationary stochastic process but heteroscedasticity . The normal quantile-quantile (QQ) plot for each return series is presented in Figure 3.

Figure 3 clearly shows that all of the returns have fat tails, which indicates the strong influence of extreme observations on the expected future risk. To prove the aforesaid performance statistically, the Ljung–Box test was performed to check whether autocorrelations are present in the time series. The results of the test are shown in Table III.

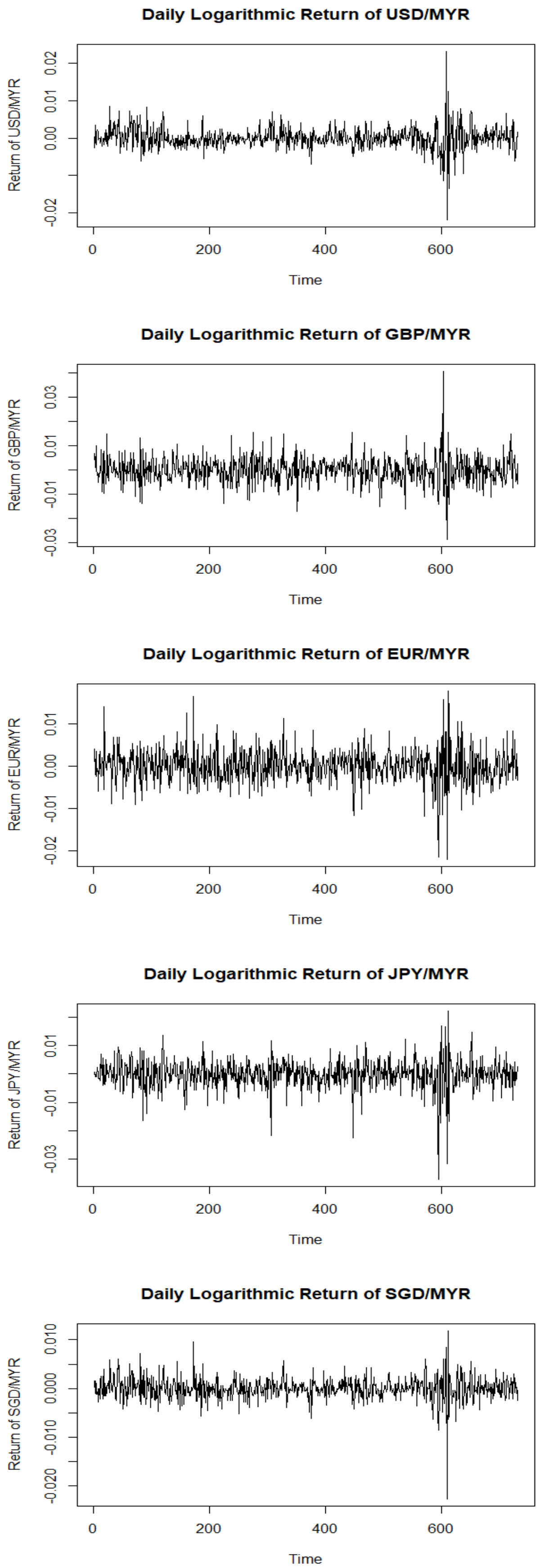

Fig. 1: Daily Logarithmic Returns of Five Selected Currency Exchange Rates of the Malaysian Ringgit

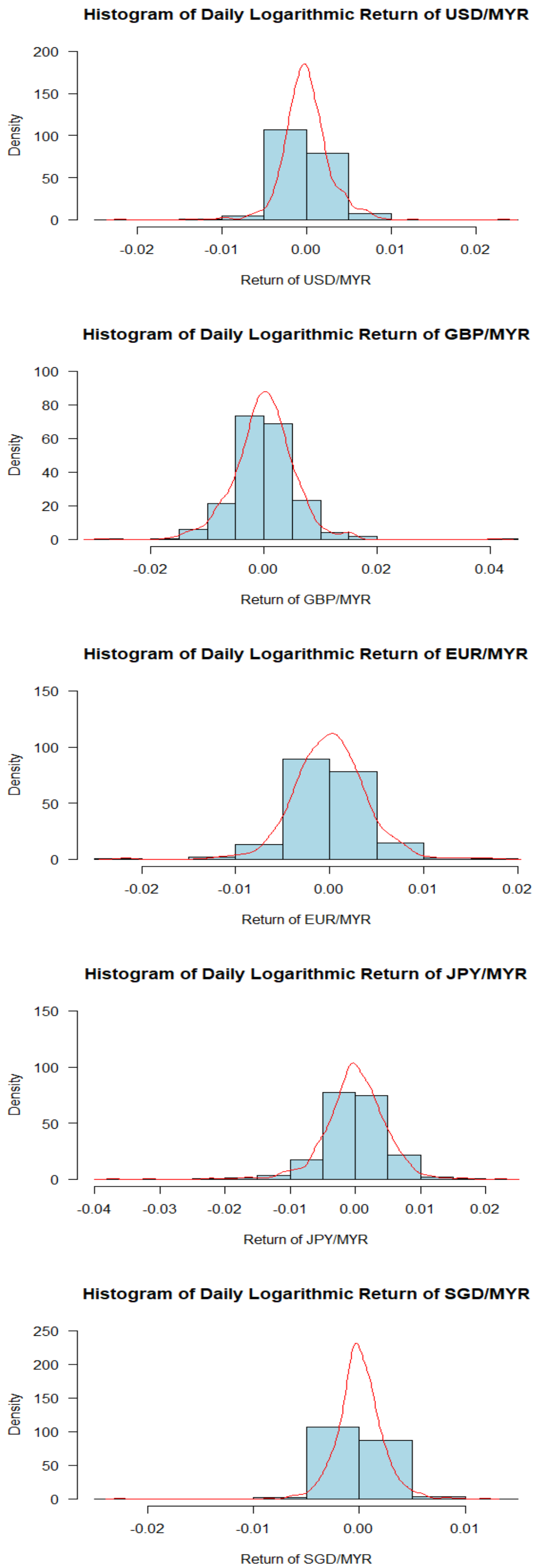

Fig. 2: Histogram of Daily Logarithmic Returns of Five Selected Currency Exchange Rates of the Malaysian Ringgit

TABLE II: Descriptive Statistics of Daily Logarithmic Returns of Five Selected Currency Exchange Rates of the Malaysian Ringgit

Statistics	USD/MYR	GBP/MYR	EUR/MYR	JPY/MYR	SGD/MYR
Mean	2.349×10^{-5}	7.882×10^{-5}	2.896×10^{-5}	-6.671×10^{-5}	3.148×10^{-5}
Maximum	0.0233	0.0409	0.0177	0.0223	0.0118
Minimum	-0.0220	-0.0287	-0.0220	-0.0372	-0.0227
Standard deviation	0.0029	0.0054	0.0040	0.0051	0.0023
Skewness	0.1328	0.3205	-0.0795	-1.1089	-0.9459
Kurtosis	13.4320	8.2909	6.3660	10.3266	17.1957
Jarque-Bera	3321.4000*	866.3300*	346.3400*	1787.2000*	6255.4000*

*Rejection of the null hypothesis at the 5% significance level.

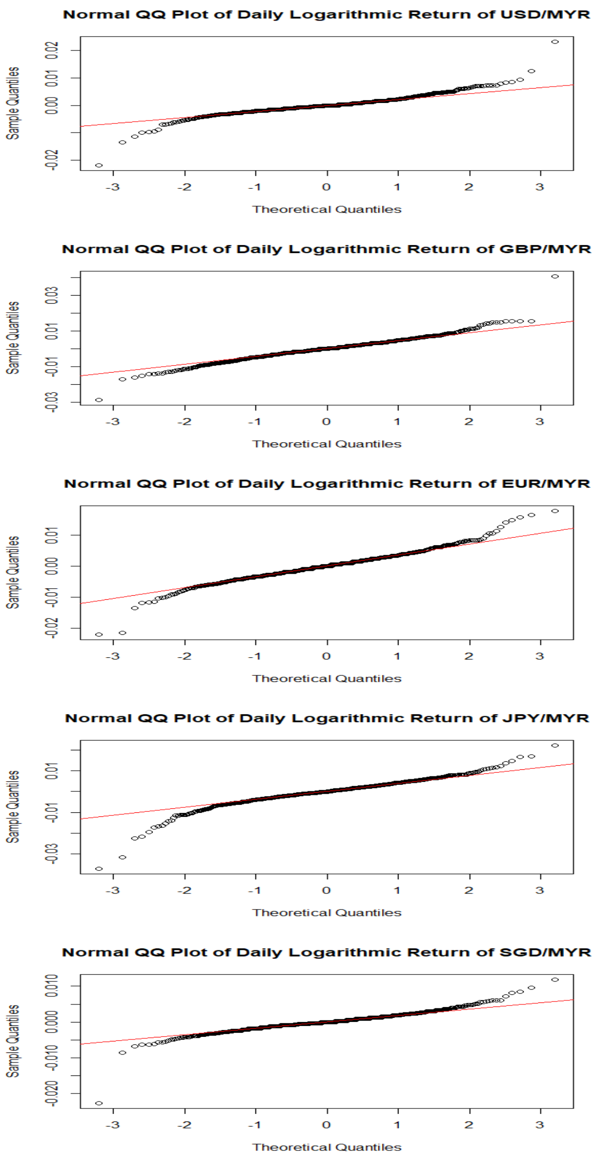

Fig. 3: Normal Q-Q Plot of Daily Logarithmic Returns of Five Selected Currency Exchange Rates of the Malaysian Ringgit

TABLE III: The Ljung–Box Test

Return Series	χ^2	p-value
USD/MYR	18.6390*	0.00008964
GBP/MYR	15.3400*	0.03188
EUR/MYR	62.2460*	0.01775
JPY/MYR	8.5278*	0.03627
SGD/MYR	5.3061*	0.02125

Notes: χ^2 is the chi-square distribution.
*Rejection of the null hypothesis at the 5% significance level.

The results of the Ljung–Box test in Table III show that all of the returns series have autocorrelations at the 95% confidence level. This can improve the accuracy of all return estimations, which can help investors estimate the future price in recent days and develop investment strategies with less risk.

Moreover, because all of the returns are not independently and identically distributed and exhibit a heteroscedasticity effect, the marginal distribution is determined using an innovative approach that combines the AR(1)-GARCH(1,1) and AR(1)-GJR-GARCH(1,1) models with the normal and t-Student distributions. The parameters estimated by the LLF are shown in Table IV.

Table IV shows that the AR(1)-GJR-GARCH(1,1) model with the t-Student distribution is ideal for estimation in marginal distribution modelling because most of the LLF coefficients are larger than those of other models, except for a special case (SGD/MYR). Therefore, we chose the AR(1)-GJR-GARCH(1,1) model and tested for the existence of an ARCH effect. The Ljung–Box test on the standardised residual model, which is used to check whether the model exhibits lack of fit, and the ARCH Lagrange Multiplier, which is used to detect the ARCH effect, are presented in Tables V and VI, respectively. These tables show that the AR(1)-GJR-GARCH(1,1) model with the t-Student distribution exhibits neither lack of fit nor the ARCH effect up to lag 7 in the residuals. Thus, we can conclude that the AR(1)-GJR(1,1) model with the t-Student distribution describes the heteroscedasticity in each marginal return sufficiently well.

Further, the t-Student and Clayton copulas are used to describe the dependence structure. The results of the estimated parameters are summarised in Table VII. According to Table VII we can see that the lower and upper tail dependences for the t-Student copula are positive and equal, which indicates that there is no difference in the dependence between any of the returns during bull and bear periods.

TABLE IV: Parameter Estimation of AR(1)-GARCH(1,1) and AR(1)-GJR-GARCH(1,1) Models with Normal and t-Student Distributions

Currency	Parameter	AR(1)-GARCH(1,1)		AR(1)-GJR-GARCH(1,1)	
		Normal	t-Student	Normal	t-Student
USD/MYR	α	0.13984	0.14095	0.16789	0.16540
	β	0.80996	0.82221	0.79670	0.80768
	γ			-0.04269	-0.03765
	LLF	3355.1	3365.6	3355.7	3366.0
GBP/MYR	α	0.09605	0.07738	0.10498	0.04348
	β	0.79923	0.80239	0.79133	0.81977
	γ			-0.01415	0.06033
	LLF	2807.9	2825.3	2808.0	2826.1
EUR/MYR	α	0.05467	0.04020	0.00000	0.00115
	β	0.91035	0.93784	0.91942	0.93898
	γ			0.08315	0.06726
	LLF	3034.3	3048.3	3040.0	3052.0
JPY/MYR	α	0.14097	0.14234	0.08151	0.11760
	β	0.69633	0.68829	0.74967	0.69128
	γ			0.06541	0.03965
	LLF	2886.4	2922.4	2887.2	2922.5
SGD/MYR	α		0.07181	0.08929	0.09216
	β		0.89869	0.90285	0.90518
	γ			-0.05100	-0.05633
	LLF		3506.0	3479.2	3479.2

Notes: α and β are the parameters of the volatility and the variance during the previous period, respectively. γ is the GJR's threshold value.

TABLE V: The Ljung–Box Test on the Standardised Residual Model

Series	χ^2	p-value
USD/MYR	2.3170	0.1280
GBP/MYR	0.0247	0.8750
EUR/MYR	0.0014	0.9699
JPY/MYR	0.8027	0.3703
SGD/MYR	0.2518	0.6158

Notes: χ^2 is the chi-square distribution.
*Rejection of the null hypothesis at the 5% significance level.

TABLE VI: Autoregressive Conditional Heteroscedasticity Lagrange Multiplier Test

Series	Lag	p-value
USD/MYR	3	0.4597
	5	0.7308
	7	0.5991
GBP/MYR	3	0.2678
	5	0.4394
	7	0.1337
EUR/MYR	3	0.1819
	5	0.3490
	7	0.1883
JPY/MYR	3	0.8090
	5	0.8547
	7	0.9426
SGD/MYR	3	0.3746
	5	0.5488
	7	0.7496

*Rejection of the null hypothesis at the 5% significance level.

The lower and upper tail dependences for the Clayton copula are positive and different, which indicates that the dependence structure during a recession is stronger than that during a boom. The results in Table VII show that the t-Student copula performs better than the Clayton copula as it has the largest LLF coefficient and the lowest AIC and BIC values. Therefore, it is the best-fit copula to describe the dependence structure. After modelling the marginal distribution and the dependence structure separately, VaR and CVaR are estimated based on the best-fit GJR-GARCH-copula model. The results for the portfolio risk under an equal-weighted portfolio and optimal investment proportion with minimum risk are shown in Tables VIII and IX, respectively.

TABLE VII: Parameter Estimation for Copula Families and Model Selection Statistics

Copula	t-Student	Clayton
ρ	0.3748	
θ		0.4714
v	4.3466	
λ_u	0.1759	0.0000
λ_l	0.1759	0.2298
LLF	459.1138	305.6843
AIC	-914.2275	-609.3686
BIC	-905.0360	-604.7728

The results in Table VIII show that the VaRs obtained by analysis with 90%, 95%, and 99% confidence levels are 0.003274, 0.004216, and 0.008207, respectively. Therefore, we expect that our worst daily loss will not exceed 0.003274, 0.004216, and 0.008207 with 90%, 95%, and 99% confidence levels, respectively. Because the CVaR's obtained by analysis with 90%, 95%, and 99% confidence levels are 0.004920, 0.006111, and 0.009225, respectively, the averages of the losses that exceed the VaR threshold are 0.004920, 0.006111, and 0.009225, respectively. The best currency exchange portfolio is at the 90% confidence level because it has the lowest risk associated with obtaining a profit. As CVaR is always greater than VaR, the conclusion is that CVaR

TABLE VIII: Portfolio Risk under Equal-Weighted Portfolio

Copula	Confidence Levels	VaR	Corresponding CVaR	Weights W_1	W_2	W_3	W_4	W_5
t-student	90%	0.003274	0.004920	0.2	0.2	0.2	0.2	0.2
	95%	0.004216	0.006111	0.2	0.2	0.2	0.2	0.2
	99%	0.008207	0.009225	0.2	0.2	0.2	0.2	0.2

Notes: W_1, W_2, W_3, W_4, and W_5 are the weights of USD, GBP, EUR, JPY, and SGD, respectively.

TABLE IX: Optimal Investment Proportion of Portfolio with Minimum Risk

Copula	Confidence Levels	VaR	Corresponding CVaR	Weights W_1	W_2	W_3	W_4	W_5
t-student	90%	0.0026	0.0043	0.3051	0.0068	-0.0538	-0.1112	0.8531
	95%	0.0034	0.0055	0.2986	-0.0002	-0.0544	-0.0945	-0.8506
	99%	0.0058	0.0096	0.2921	-0.0073	-0.0550	-0.0778	0.8480

Notes: W_1, W_2, W_3, W_4, and W_5 are the weights of USD, GBP, EUR, JPY, and SGD, respectively.

performs better than VaR. Table IX presents the optimal portfolio weights under the minimum portfolio risk, VaR, and CVaR across various confidence levels by using the t-Student copula. The table shows that the best investment tends to focus on investment in the SGD, because it has the largest optimal weight. As the confidence levels increase, the SGD weight decreases, whereas the weights of the other currencies increase, showing that investors are willing to take more risks to achieve a higher expected return.

IV. Conclusion

The copula-VaR and copula-CVaR approaches were introduced and applied to measure currency exchange portfolio risks. The copula was applied to capture the dependence structure between currency exchanges in a portfolio. We found that the GJR-GARCH model with the t-Student distribution and the t-Student copula model are the best fits for describing the marginal distribution and dependence structure in currency exchange portfolios, respectively. We also found that copula-CVaR is always greater than copula-VaR for equal-weighted currency exchange portfolios because the former performs better than the latter. In addition, the optimal portfolio weights are similar across different confidence levels when minimising the portfolio risk. The best investment tends to focus on investment in the SGD. At higher confidence levels, the weights of the SGD decrease, whereas the weights of the other currencies increase. This behaviour can help currency exchange investors better manage currency exchange risks. Having said that, we also found that our solutions can be further improved in how VaR is used to estimate currency exchange portfolio risks. Therefore, a backtesting method needs to be applied to determine the accuracy of the VaR model. The GJR-GARCH-Extreme Value Theory model and dynamic copulas could be considered in future studies for the backtesting.

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