A Role of Triangular Fuzzy Neutrosophic Numbers in Solving Neutrosophic Transportation Problem

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Abstract—**The transportation problem (TP) is a prominent kind of linear programming problem (LPP) in which it is necessary to transfer goods from several sources to several destinations while minimizing the overall cost of transportation is its objective. Because of its many practical uses, it's a popular tool in operation research. The utilization of neutrosophic sets (NS) to analyze and resolve diverse decision-making challenges has quickly gained popularity. As a consequence, the neutrosophic theory is progressively being the subject of several current research investigations. In this study, we assess the TP in a neutrosophic setting, where the neutrosophic transportation problem (NTP) is expressed as a table called the neutrosophic transportation table, with triangular fuzzy neutrosophic numbers as its core elements. We make use of a score function to convert the triangular fuzzy neutrosophic values to their equivalent crisp numbers, followed by a stepwise methodology of the proposed approach to obtain the optimal solution. The outcomes are then compared with the previously acquired solutions to demonstrate the method's effectiveness.**

Index Terms—**Neutrosophic set, neutrosophic number, triangular fuzzy neutrosophic number, transportation problem, balanced transportation problem, unbalanced transportation problem, neutrosophic transportation problem, score function, range, optimal solution.**

I. Introduction

We constantly come into a variety of imprecise, ambiguous, and insufficient circumstances in our everyday routines. Consequently, in 1965, Zadeh [1] established the notion of fuzzy sets as an extension of classical sets that allows for partial membership, i.e., assigns a membership grade for each element. Due to its ability to deal with inconsistency, the fuzzy set theory has gained tremendous success across a broad spectrum of fields. As an extension of fuzzy sets, Atanassov [2] introduced the aspect of intuitionistic fuzzy sets in 1983. These sets include both the membership grade and the non-membership grade of each element as a result of specific restrictions. The truth (T), indeterminacy (I), and falsity (F) membership grades for each element are included in neutrosophic sets, an extension of intuitionistic fuzzy sets. Smarandache [3] initially articulated the idea in 1995.

Neutrosophic sets (NS) have been widely used in decision-making, pattern recognition, and medical diagnosis, among several other areas. Quite enough has been accomplished in the research of NS-based decision-making problems. In artificial intelligence, multiple-attribute decision-making, visualisation, medical examination, defect identification, optimization design, and other fields, a variety of unique neutrosophic ideas have been put forth and utilized. The NS can also be employed to resolve the TP in addition to the numerous places mentioned above. The French mathematician Gaspard Monge initially defined the transportation issue, commonly referred to as the study of optimum transportation and resource allocation, in 1781. The basic objective of the transportation issue, a distribution-type problem, is to determine the least or most profitable way to move items from different sending locations (also known as origins) to numerous receiving locations (also known as destinations). It is one of the most intensively studied problems in optimization. It serves as a benchmark for numerous optimization techniques. The TP has a significant impact on a variety of research and application fields like inventory control, production planning, scheduling, personal allocation, and so forth. This draws interest and serves as a motivation to explore and try solving the fuzzy neutrosophic transportation problem.

Scholars have conducted extensive studies on the fuzzy numbers, triangular fuzzy, trapezoidal fuzzy, and pentagonal fuzzy neutrosophic numbers, as well as devised score functions for the same [4]-[13]. Chakraborty et al [14] created a score function to defuzzify triangular fuzzy neutrosophic numbers. Dhouib [15], [16] presented the Dhouib-matrix-TP1 heuristic to tackle transportation and the trapezoidal fuzzy transportation problems. The fuzzy TP has been answered using G.M, H.M, Q.M, and S.D [17]-[20]. The intuitionistic fuzzy transportation problem and a few other neutrosophic optimization problems were overcome using a few different methods [21]-[29]. Sikkannanl and Shanmugavel [30] unraveled the neutrosophic transportation problem making use of the cost mean and complete contingency cost table. Pratihar et al [31] resolved the transportation problem in a neutrosophic environment. Dhouib [32] attacked the single-valued trapezoidal neutrosophic transportation problems employing the novel Dhouib-matrix-TP1 heuristic.

In this paper, we investigate the transportation problem in a neutrosophic environment, talk about the characteristics of triangular fuzzy neutrosophic numbers and types of neutrosophic transportation problem, employ a score function, and provide a step-by-step method of the proposed algorithm to solve the neutrosophic transportation problem. The neutrosophic transportation problem is expressed as a table called the neutrosophic transportation table, whose elements are triangular fuzzy neutrosophic numbers to arrive at the optimal solution and compare the outcomes with the earlier results found. The following is how the paper is set up: The abstract and introduction are included in section 1. We provide some fundamental definitions in the preliminaries section of section 2. Neutrosophic numbers, triangular fuzzy neutrosophic numbers and both their characteristics are talked about in

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section 3. Section 4 explains the transportation problem (TP), NTP, their features, types, and solutions, along with a score function. The proposed methodology for solving the neutrosophic transportation issue is presented in section 5, along with the defuzzification of the neutrosophic data before using the suggested algorithm stepwise to attain the best answer. The proposed approach to tackle the transportation problem in a neutrosophic environment is illustrated using a few real-world problems in section 6. Section 7 discusses about the important outcomes of the work done and concludes the article.

II. Preliminaries

Definition 2.1 : Let X be a non-empty set. A fuzzy set H in X is characterized by its membership function $\mu_H : X \longrightarrow [0,1]$ and $\mu_H(x)$ is interpreted as the degree of membership of element x in fuzzy set H, for each $x \in X$, given by,

$$H = \{(x, \mu_H(x)) : x \in X\}$$

Definition 2.2 : The fuzzy set H defined on the set of real numbers is said to be a fuzzy number if H and its membership function $\mu_H(X)$ has the following properties:

1) H is normal and convex
2) H is bounded
3) $\mu_H(X)$ is piecewise continuous

Definition 2.3 : Let X be a non-empty set. An intuitionistic fuzzy set H in X is of the form $H = \{(x, \mu_H(x), \nu_H(x)) : x \in X\}$, where the functions μ_H, $\nu_H : X \longrightarrow [0,1]$ define respectively the degree of membership and the degree of non-membership for every element $x \in X$ to the set H, which is a subset of X.

$$0 \le \mu_H(x) + \nu_H(x) \le 1$$

Furthermore, we have $\pi_H(x) = 1 - \mu_H(x) - \nu_H(x)$ is called the intuitionistic fuzzy set index or hesitation margin is the degree of indeterminacy of x in H where $\pi_H(x) \in [0,1]$ i.e., $\pi_H : X \longrightarrow [0,1]$ and $0 \le \pi_H(x) \le 1$ for every x∈X.

Definition 2.4 : An intuitionistic fuzzy set H of the real line R is called an intuitionistic fuzzy number if the following conditions hold :

1) There exists $x_0 \in R$, such that, $\mu_H(x_0) = 1$ and $\nu_H(x_0) = 0$, where x_0 stands for the mean value of H.
2) $\mu_H(x_0)$ is a continuous mapping from R to the closed interval $[0,1]$ and for all $x \in R$, the relation $0 \le \mu_H(x) + \nu_H(x) \le 1$ holds.

Definition 2.5 : Let X be a non-empty set. A neutrosophic set $H \in X$ is of the form $H = \{(x, T_H(x), I_H(x), F_H(x)) : x \in X\}$, where the functions $T_H, I_H, F_H : X \longrightarrow ^-]0,1[^+$ define respectively the degree of truth membership, the degree of indeterminacy and the degree of falsity membership for every element $x \in X$ to the set H, which is a subset of X.

$$^-0 \le T_H(x) + I_H(x) + F_H(x) \le 3^+.$$

III. Neutrosophic numbers and its properties

A. Neutrosophic numbers

Definition 3.1 : A neutrosophic set H defined on the universal set of real numbers R is called a neutrosophic number if it has the following properties :

1) H is normal if there exists $x_0 \in R$, such that $T_H(x_0) = 1$, $I_H(x_0) = F_H(x_0) = 0$.
2) H is a convex set for the truth function $T_H(x)$, i.e., $T_H(\mu x_1 + (1-\mu)x_2) \ge min(T_H(x_1), T_H(x_2))$, $\forall$x$_1, x_2 \in R$ and $\mu \in [0,1]$.
3) H is a concave set for the indeterministic function and false function $I_H(x)$ and $F_H(x)$, i.e., $I_H(\mu x_1 + (1-\mu)x_2) \ge max(I_H(x_1), I_H(x_2))$, $\forall$x$_1, x_2 \in R$ and $\mu \in [0,1]$ and $F_H(\mu x_1 + (1-\mu)x_2) \ge max(F_H(x_1), F_H(x_2))$, $\forall$x$_1, x_2 \in R$ and $\mu \in [0,1]$.

It is also possible to define the neutrosophic number in an alternative way as follows:
Let X be a non-empty set. A neutrosophic set $H \in X$ is of the form $H = \{(x, T_H(x), I_H(x), F_H(x)) : x \in X\}$, where the functions $T_H, I_H, F_H : X \longrightarrow ^-]0,1[^+$ define respectively the degree of truth membership, the degree of indeterminacy and the degree of falsity membership for every element $x \in X$ to the set H, which is a subset of X.

$$^-0 \le T_H(x) + I_H(x) + F_H(x) \le 3^+.$$

Example : Neutrosophic Number for a Student's Performance Grade - Representing a neutrosophic fuzzy number for a student's performance grade in a subject : (5.2, 5.5, 5.8). This means that the student's grade is most likely around 5.5 (truth-membership degree of 5.5), with a small level of indeterminacy (0.3) and a very low level of falsity (0.6).

Remark 1 : If H is a neutrosophic set in a non-empty set X, then for convenience, we denote a neutrosophic number by, $H = (T_H(x), I_H(x), F_H(x))$.

1) Properties of neutrosophic numbers: Let $G, H \in X$. Then their operations are defined as,

- $(T_G(x), I_G(x), F_G(x)) + (T_H(x), I_H(x), F_H(x))$ $= (T_G(x) + T_H(x) - T_G(x)T_H(x),$ $I_G(x)I_H(x), F_G(x)F_H(x))$
- $(T_G(x), I_G(x), F_G(x)).(T_H(x), I_H(x), F_H(x))$ $= (T_G(x)T_H(x), I_G(x) + I_H(x) - I_G(x)I_H(x),$ $F_G(x) + F_H(x) - F_G(x)F_H(x))$
- $k(T_G(x), I_G(x), F_G(x))$ $= (1 - (1 - T_G(x))k, I_G(x)k, F_G(x)k), (k \in R)$
- $(T_G(x), I_G(x), F_G(x))k$ $= (T_G(x)k, 1-(1-I_G(x))k, 1-(1-F_G(x))k), (k \in R)$

B. Triangular fuzzy neutrosophic number

Let X be the universal set and let the set of all triangular fuzzy numbers on $[0,1]$ be denoted by $F[0,1]$. A triangular fuzzy neutrosophic set, H in X is written as,

$$H = \{x : (T_H(x), I_H(x), F_H(x)), x \in X\},$$

where, $T_H(x), I_H(x), F_H(x) : X \longrightarrow F[0,1]$.

The triangular fuzzy numbers, $T_H(x) = (T_H^1(x), T_H^2(x), T_H^3(x))$, $I_H(x) = (I_H^1(x), I_H^2(x), I_H^3(x))$ and $F_H(x) = (F_H^1(x), F_H^2(x), F_H^3(x))$

respectively denote the truth-membership, indeterminacy-membership and falsity-membership of x in H and for every $x \in X$, $0 \leq T_H^3(x) + I_H^3(x) + F_H^3(x) \leq 3$.

For convenience, we indicate the triangular fuzzy neutrosophic number H as, $H = ((h_1, h_2, h_3), (h_4, h_5, h_6), (h_7, h_8, h_9))$, where $(T_H^1(x), T_H^2(x), T_H^3(x)) = (h_1, h_2, h_3)$, $(I_H^1(x), I_H^2(x), I_H^3(x)) = (h_4, h_5, h_6)$ and $(F_H^1(x), F_H^2(x), F_H^3(x)) = (h_7, h_8, h_9)$. The triangular fuzzy neutrosophic number can be expressed as mentioned above.

1) Properties of triangular fuzzy neutrosophic numbers: Let $G = ((g_1, g_2, g_3), (g_4, g_5, g_6), (g_7, g_8, g_9))$ and $H = ((h_1, h_2, h_3), (h_4, h_5, h_6), (h_7, h_8, h_9))$ be two triangular fuzzy neutrosophic numbers in the set of real numbers and $\lambda > 0$. Then, the operations involving them are listed as follows :

- $G \oplus H = ((g_1 + h_1 - g_1h_1, g_2 + h_2 - g_2h_2, g_3+h_3-g_3h_3), (g_4h_4, g_5h_5, g_6h_6), (g_7h_7, g_8h_8, g_9h_9))$.
- $G \otimes H = ((g_1h_1, g_2h_2, g_3h_3), (g_4 + h_4 - g_4h_4, g_5 + h_5 - g_5h_5, g_6 + h_6 - g_6h_6), (g_7 + h_7 - g_7h_7, g_8 + h_8 - g_8h_8, g_9 + h_9 - g_9h_9))$.
- $\lambda G = (((1 - (1 - g_1)^\lambda, 1 - (1 - g_2)^\lambda, 1 - (1 - g_3)^\lambda)), (g_4^\lambda, g_5^\lambda, g_6^\lambda), (g_7^\lambda, g_8^\lambda, g_9^\lambda))$.
- $G^\lambda = ((g_1^\lambda, g_2^\lambda, g_3^\lambda), ((1 - (1 - g_4)^\lambda, 1 - (1 - g_5)^\lambda, 1 - (1 - g_6)^\lambda)), ((1 - (1 - g_7)^\lambda, 1 - (1 - g_8)^\lambda, 1 - (1 - g_9)^\lambda)))$, where $\lambda > 0$.

IV. Transportation problem(TP) - Features, types and solutions

A. Features of the transportation problem(TP)

1) Description of TP : The transportation problem is a particular type of linear programming problem (LPP), where goods must be shuttled from a collection of sources to a collection of destinations whilst also taking into account the supply and demand at every location (i.e., without compromising on either), intending to minimize the overall transportation cost. It comprises things like the distance between two locations, the route used, the way of getting there, how many components are transferred, how swiftly they are delivered, etc. Columns indicate the destinations, whereas rows represent the sources. A transportation issue typically involves m rows and n columns. The quantity of supply from each source and the volume of demand at each destination are regarded as known quantities. The unit transportation costs of goods between sources and destinations are known. The goal is to figure out the quantity that must be transferred from each source to each destination so that the overall transportation expense is as low as possible. If there are precisely $(m + n - 1)$ fundamental variables, the problem is fixable. It is sometimes referred to as the Hitchcock issue.

2) Some basic notations :

- m - number of sources.
- n - number of destinations.
- c_{ij} - delivery fee per item from source i to destination j.
- x_{ij} - quantity of goods carried from source i to destination j.
- a_i - supply available at source i.
- b_j - demand available at destination j.

3) Mathematical formulation of TP : The TP can be formulated mathematically as follows:

$$MinZ = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\ x_{ij},$$

subject to,

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1, 2, ...n$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1, 2, ...n$$

where, $x_{ij} \in R$ and $x_{ij} \geq 0$.

4) The structure of the problem : Let the given number of origins (O) and destinations (D) be m and n respectively. Let a_i represent the supply level at the i^{th} origin and b_j represent the demand level at the j^{th} destination. It is known for all combinations (i, j) that, the cost of carrying one unit of a product from its source to its destination is c_{ij}. Let x_{ij} be the quantity transported from origin i to destination j. The goal is to reduce the overall cost of transportation by identifying the amount x_{ij} to be transferred along all routes (i, j). Supply constraints at sources must be met, as must demand needs at destinations. The aforementioned transportation problem may be stated in tabular format as follows :

TABLE I
THE TRANSPORTATION TABLE

O/D	G_1	...	G_n	Supply
S_1	c_{11} x_{11}	...	c_{1n} x_{1n}	a_1
...	...	...	...	...
S_m	c_{m1} x_{m1}	...	c_{mn} x_{mn}	a_m
Demand	b_1	...	b_n	

5) Mathematical formulation of NTP :

- Type 1 NTP : If the component c_{ij} is substituted by neutrosophic cost parameters, i.e., c_{ij}^N, the model looks like this:
  $$MinZ = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^N\ x_{ij},$$
  subject to,
  $$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1, 2, ...n$$
  $$\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1, 2, ...n$$
  where, $x_{ij} \in R$ and $x_{ij} \geq 0, \forall i, j$.

- Type 2 NTP : If the components a_i and b_j are substituted by neutrosophic cost parameters, i.e., a_i^N and b_j^N, then the model looks like this :
  $$MinZ = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\ x_{ij},$$
  subject to,
  $$\sum_{j=1}^{n} x_{ij} = a_i^N,\ i = 1, 2, ...n$$
  $$\sum_{i=1}^{m} x_{ij} = b_j^N,\ j = 1, 2, ...n$$

where, $x_{ij} \in R$ and $x_{ij} \geq 0, \forall i, j$.

- Type 3 NTP : If the components a_i, b_j and c_{ij} are substituted by neutrosophic cost parameters, i.e., a_i^N, b_j^N and c_{ij}^N, then the model looks like this :
$MinZ = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^N \; x_{ij}$,
subject to,
$\sum_{j=1}^{n} x_{ij} = a_i^N$, $i = 1, 2, ...n$
$\sum_{i=1}^{m} x_{ij} = b_j^N$, $j = 1, 2, ...n$
where, $x_{ij} \in R$ and $x_{ij} \geq 0, \forall i, j$.

B. Types of transportation problem

Transportation problems are broadly classified into balanced and unbalanced, depending on the source's supply and the requirement at the destination.
1) Balanced transportation problem: If the overall supply equals the whole demand, the situation is regarded as a balanced transportation problem., i.e., if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.
2) Unbalanced transportation problem: An unbalanced transportation problem refers to a scenario where the overall supply does not match the whole demand., i.e., if $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$.

- When the overall supply exceeds the whole demand., i.e., if $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$, to make it equal to the supply, a dummy destination with zero cost components is put into the transportation table.
- On the flip hand, when the overall demand exceeds the entire supply., i.e., if $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$, to make it equal to the demand, a dummy source with zero cost components is put into the transportation table.

Remark 2 : A pre-requisite and adequate requirement for a transportation issue to be dealt with is that the overall demand equals the overall supply., i.e., it should be a balanced transportation problem($\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$). If the issue is an unbalanced one, then a dummy row or a dummy column is included in the transportation table to make it a balanced problem, depending on the need. The issue can then be treated similarly to the balanced problem.

C. Solutions of the TP

1) Different types of solutions of the TP : The following are the possible types of solutions for a given transportation problem:

a) Feasible solution: A feasible solution is a collection of non-negative numbers x_{ij}, $i = 1, 2, ...m$ and $j = 1, 2, ...n$ that meet the requirements.

b) Basic feasible solution: A basic feasible solution is a feasible solution to a $m \times n$ transportation issue that comprises not more than $m+n-1$ non-negative independent allocations.

c) Non-degenerate basic feasible solution: A non-degenerate basic feasible solution to a $m \times n$ transportation issue includes precisely $m + n - 1$ non-negative allocations in independent places.

d) Degenerate basic feasible solution: A basic feasible solution with fewer than $m+n-1$ non-negative allocations is considered to as a degenerate basic feasible solution.

e) Optimal solution: A feasible (but not necessarily basic) solution is referred to as the optimal solution if it reduces the overall transportation costs.

Remark 3 : An $m \times n$ balanced transportation issue has a maximum of $m + n - 1$ fundamental variables.

2) Some common methods for solving a transportation problem : There are numerous approaches for determining the initial basic feasible solution to a transportation challenge. Among them, a few most frequently employed techniques are :

- North-West Corner Rule (NWCR)
- Least Cost method (LCM) and
- Vogel's Approximation method (VAM).

After computing the initial basic feasible solution of a given transportation problem, the following step (or) the major goal is to determine whether it constitutes an optimal solution (i.e., optimization of the acquired initial basic feasible solution) using the MODI (Modified Distribution) approach.

Remark 4: The derived optimal solution might or might not be equivalent to the initial basic feasible solution estimated before it.

3) Score function : The score function as in [14], used here for converting the neutrosophic data of the TSP into crisp data is as follows:

$$S(H) = \frac{1}{12}[(h_1 + 2h_2 + h_3) + (h_4 + 2h_5 + h_6) + (h_7 + 2h_8 + h_9)] \quad (1)$$

V. Methodology for solving neutrosophic transportation problem

A. Defuzzification of the neutrosophic data

As an initial task of solving a neutrosophic transportation problem, each entry (being a triangular fuzzy neutrosophic number) of the given neutrosophic transportation problem, which is represented by means of a neutrosophic transportation table, is defuzzified using the above score function (1) $S(H) = \frac{1}{12}[(h_1+2h_2+h_3)+(h_4+2h_5+h_6)+(h_7+2h_8+h_9)]$, hence being converted into their respective crisp numbers.

B. The proposed methodology for solving neutrosophic transportation problem

The stages included in the proposed methodology for tackling the neutrosophic transportation problem are :

Step 1: From the provided neutrosophic transportation table, the first step is to transform all the triangular fuzzy neutrosophic data into their corresponding crisp data using the score function (1) mentioned above.

Step 2: The next step is to verify if the considered neutrosophic transportation problem is balanced.

- If yes, go to step 3.
- Orelse, if it is unbalanced, make it a balanced one by either including a dummy row or a dummy column appropriate to the situation's requirement and then go to step 3.

Step 3: Now, after making the problem balanced., i.e., after making the overall supply meet the whole demand, the next step is to include a row underneath called the Range Demand Column (RDC) and a column at the right called the Range Supply Row (RSR) in the neutrosophic transportation table.

Step 4: Now, after introducing the above-mentioned row (RDC) and column (RSR), find out the corresponding values for the newly added row and column by calculating the range value for every row and column, utilizing the formula, Range = Highest value - Least value.

Step 5: On having obtained all the values of the RSR and the RDC, now spot and select the highest value among all the found new entries of the RSR and the RDC.

- If this selected value is in the RSR, then choose the least element (c_{ij}) of its corresponding row.
- Else, if this highest value lies in the RDC, then choose the least element (c_{ij}) of its corresponding column.

Step 6 : The next task after spotting the cell with the minimal element is to compare the corresponding supply and demand availability (a_i and b_j).,i.e., to check whether, $a_i \leq b_j$ (or) $a_i > b_j$.

- If $a_i \leq b_j$, then allot the a_i amount of units to c_{ij}, which gives, $b_j = b_j - a_i$ and delete row i.
- If $a_i > b_j$, then allot the b_j amount of units to c_{ij}, which gives, $a_i = a_i - b_j$ and delete column j.

Step 7: Re-perform Steps 3-6 until all columns disappear. Then, verify if the overall number of allotments precisely equals $m+n-1$, which guarantees the prevalence of a non-degenerate basic feasible solution to the given neutrosophic transportation problem.

- If yes, move forward to step 8.
- Or else, make it non-degenerate by making the overall number of allotments equal to $m + n - 1$ and then proceed to step 8.

Step 8: Finally, compute the initial basic feasible solution and the total minimal transportation cost, which itself will serve as the optimal solution and the optimal transportation cost for the chosen neutrosophic transportation problem respectively.

VI. Illustrations for the Suggested Methodology

A. Illustration 1 - Type 1 NTP

In this type of problem [31], the cost of transportation in each cell is considered a neutrosophic expense, and supply and demand entries are crisp values, which are expressed through the following Table II:

TABLE II
THE NEUTROSOPHIC TRANSPORTATION TABLE OF TYPE 1 NTP

	G_1	G_2	G_3	**Supply**
S_1	c_{11}	c_{12}	c_{13}	15
S_2	c_{21}	c_{22}	c_{23}	25
S_3	c_{31}	c_{32}	c_{33}	20
Demand	18	20	22	

where each cell's cost of transportation (c_{ij}^N.,i.e., triangular fuzzy neutrosophic numbers) are as follows:
c_{11} = $((1, 4, 7), (1, 3, 5), (3.5, 6, 7.5))$
c_{12} = $((0.5, 2.5, 4.5), (1, 2, 3), (1.5, 3.5, 5.5))$
c_{13} = $((1, 3, 5), (0.5, 1.5, 3.5), (2, 4, 6))$
c_{21} = $((1, 2, 3), (0.5, 1.5, 2.5), (1.5, 2.5, 3.5))$
c_{22} = $((1, 1.5, 4), (0.5, 1, 2.5), (1.25, 3, 4.25))$
c_{23} = $((1.5, 2.5, 3.5), (1, 1.5, 3), (2, 3, 4))$
c_{31} = $((2, 4, 6), (1.5, 2.5, 4.5), (3, 5, 7))$
c_{32} = $((1, 5, 8), (1.5, 4.5, 7.5), (4, 6.5, 9))$
c_{33} = $((1, 5, 8), (1.5, 3, 6.5), (4, 7, 9))$

Step 1: Using the above score function (1),
$S(H) = \frac{1}{12}[(h_1+2h_2+h_3)+(h_4+2h_5+h_6)+(h_7+2h_8+h_9)]$,
converting the given neutrosophic data into their corresponding crisp data, we obtain,
$S(c_{11})$ = 4.25 ; $S(c_{12})$ = 2.67 ; $S(c_{13})$ = 2.92
$S(c_{21})$ = 2 ; $S(c_{22})$ = 1.71 ; $S(c_{23})$ = 2.75
$S(c_{31})$ = 3.92 ; $S(c_{32})$ = 5.25 ; $S(c_{33})$ = 5.08

The finally obtained crisp equivalent transportation table is given by the following Table III:

TABLE III
THE CRISP EQUIVALENT TRANSPORTATION TABLE OF TYPE 1 NTP

	G_1	G_2	G_3	**Supply**
S_1	4.25	2.67	2.92	15
S_2	2	1.71	2.75	25
S_3	3.92	5.25	5.08	20
Demand	18	20	22	60

Step 2: The next step is to verify if the considered neutrosophic transportation problem is balanced. Here, since $\sum_{i=1}^{m} a_i$ = $\sum_{j=1}^{n} b_j$ ($\sum_{i=1}^{3} a_i$ = $\sum_{j=1}^{3} b_j$), i.e., 15+25+20 = 18+20+22 = 60, the given transportation problem is balanced. Hence, we can move forward to step 3.

Step 3: Now, after knowing that the problem is balanced, the next step is to include a row underneath called the Range Demand Column (RDC) and a column at the right called the Range Supply Row (RSR) in the transportation table as follows:

	G_1	G_2	G_3	**Supply**	**RSR**
S_1	4.25	2.67	2.92	15	
S_2	2	1.71	2.75	25	
S_3	3.92	5.25	5.08	20	
Demand	18	20	22	60	
RDC					

Step 4: Now, after having introduced the above-mentioned row (RDC) and column (RSR), the next task is to find out the corresponding values for the newly added row and column by calculating the range entry for every row and column utilizing the formula, Range = Highest entry - Least entry, as follows:

	G_1	G_2	G_3	**Supply**	**RSR**
S_1	4.25	2.67	2.92	15	1.58
S_2	2	1.71	2.75	25	1.04
S_3	3.92	5.25	5.08	20	1.33
Demand	18	20	22	60	
RDC	2.25	3.54	2.33		

Step 5: Now, after obtaining all the values of the RSR and the RDC, we have to spot and select the highest value among all the found new entries of the RSR and the RDC. Here, the highest value is 3.54, which lies in the RDC. Hence, we have to choose the least element (c_{ij}) of its corresponding column. The corresponding column is G_2 and its least element is 1.71 (c_{22}) as shown below:

	G_1	G_2	G_3	**Supply**	**RSR**
S_1	4.25	2.67	2.92	15	1.58
S_2	2	1.71	2.75	25	1.04
S_3	3.92	5.25	5.08	20	1.33
Demand	18	20	22	60	
RDC	2.25	3.54	2.33		

Step 6: The next task after spotting the cell with the minimal element is to compare the corresponding supply and demand availability (a_i and b_j).,i.e., to check whether $a_i \leq b_j$ (or) $a_i > b_j$. Here, the corresponding supply and demand availability are 25 and 20 respectively, of which the least value is 20. Hence, since $a_i > b_j$, we allocate the b_j amount of units to c_{ij}, which changes the new supply value as, $a_i = a_i - b_j$., i.e., since $a_2 > b_2$ (25 > 20), we allocate the b_2 (20) amount of units to c_{22}, which changes the new supply value as, $a_2 = a_2 - b_2$ (25 - 20 = 5), which is seen in the following transportation table:

	G_1	G_2	G_3	**Supply**	**RSR**
S_1	4.25	2.67	2.92	15	1.58
S_2	2	20 1.71	2.75	~~25~~ 5	1.04
S_3	3.92	5.25	5.08	20	1.33
Demand	18	20	22	60	
RDC	2.25	3.54	2.33		

Thus, column j is discarded., i.e., column 2 (G_2) is discarded, since the demand availability for this column is allocated completely to c_{22} (i.e., fully exhausted), as shown below in the following table:

	G_1	G_3	**Supply**
S_1	4.2	2.92	15
S_2	2	2.75	5
S_3	3.92	5.08	20
Demand	18	22	60

Step 7: The next step is to keep repeating steps 3-6 until all columns disappear. Hence, here since not all columns are discarded, we again start the same procedure and continue repeating steps 3-6. We add a row below (RDC) and a column at the right (RSR) in the transportation table, with their corresponding values as follows:

	G_1	G_3	**Supply**	**RSR**
S_1	4.25	2.92	15	1.33
S_2	2	2.75	5	0.75
S_3	3.92	5.08	20	1.16
Demand	18	22	60	
RDC	2.25	2.33		

Now, we have to spot and select the highest value among all the found out new entries of the RSR and the RDC. The highest value is 2.33, which lies in the RDC. Hence, we have to choose the least element (c_{ij}) of its corresponding column. The corresponding column is G_3 and its least element is 2.75 (c_{23}) as shown below:

	G_1	G_3	**Supply**	**RSR**
S_1	4.25	2.92	15	1.33
S_2	2	2.75	5	0.75
S_3	3.92	5.08	20	1.16
Demand	18	22	60	
RDC	2.25	2.33		

Now, the next task is to compare the corresponding supply and demand availability (a_i and b_j).,i.e., to check whether, $a_i \leq b_j$ (or) $a_i > b_j$. Here, the corresponding supply and demand availability are 5 and 22 respectively, of which the least value is 5.
Hence, here since $a_i < b_j$, we allocate the a_i amount of units to c_{ij}, which changes the new demand value as, $b_j = b_j - a_i$., i.e., since $a_2 < b_2$ ($5 < 22$), we allocate the a_2 (5) amount of units to c_{23}, which changes the new demand value as, $b_2 = b_2 - a_2$ (22 - 5 = 17), which is seen in the following transportation table:

	G_1	G_3	**Supply**	**RSR**
S_1	4.25	2.92	15	1.33
S_2	2	5 2.75	5	0.75
S_3	3.92	5.08	20	1.16
Demand	18	~~22~~ 17	60	
RDC	2.25	2.33		

Hence, row j is discarded., row 2 (S_2) is discarded since the supply available for this row is allocated completely to c_{23} (i.e., fully exhausted), which is shown in the following table:

	G_1	G_3	**Supply**
S_1	4.25	2.92	15
S_3	3.92	5.08	20
Demand	18	17	60

Now, still since not all columns are discarded, we have to keep repeating steps 3-6 till all the columns are discarded. Hence, following the same, we obtain the final resulting transportation table as,

	G_3	**Supply**
S_3	5.08 [2]	2
Demand	2	

Thus, the resulting final transportation table with all the allocations is given below in the Table IV:

TABLE IV
THE RESULTING ALLOCATED TRANSPORTATION TABLE OF TYPE 1 NTP

	G_1	G_2	G_3	**Supply**
S_1	4.25	2.67	2.92 [15]	15
S_2	2	1.71 [20]	2.75 [5]	25
S_3	3.92 [18]	5.25	5.08 [2]	20
Demand	18	20	22	

Our next task is to verify if the overall number of allotments precisely equals $m+n-1$. Here, since we have 5 allocations, which is precisely $m+n-1$ non-negative allotments in independent places, it is guaranteed that there prevails a basic feasible solution to the given $m \times n$ transportation issue. Thus, we can go to step 8.

Step 8: Finally, we have to compute the initial basic feasible solution and the total minimal transportation cost, which itself will serve as the optimal solution and the optimal transportation cost respectively, for the considered neutrosophic transportation problem. Hence, the optimal solution of the considered neutrosophic transportation problem is, x_{22} = 20, x_{23} = 5, x_{13} = 15, $x_{31} = 18$ and x_{33} = 2, with the optimal transportation cost (OTC), Z = Rs. (1.71 $\times 20) + (2.75 \times 5) + (2.92 \times 15) + (3.92 \times 18) + (5.08 \times 2) = Rs.172.47$

B. 6.2 Illustration 2 - Type 2 NTP

In this type of problem [31], the cost of transportation in each cell is represented as crisp entries and supply and demand entries are taken as triangular fuzzy neutrosophic numbers, expressed in the following Table V, where the supply and demand entries (a_i^N and b_j^N .,i.e., triangular fuzzy neutrosophic numbers) are as follows:

s_1 = ((10, 15, 20), (14, 16, 22), (12, 15, 19))
s_2 = ((20, 25, 30), (24, 26, 32), (22, 25, 29))
s_3 = ((15, 20, 25), (19, 21, 27), (17, 20, 24))
d_1 = ((13, 18, 23), (17, 19, 25), (15, 18, 22))
d_2 = ((15, 20, 25), (19, 21, 27), (17, 20, 24))
d_3 = ((17, 22, 27), (21, 23, 29), (19, 22, 26))

TABLE V
THE NEUTROSOPHIC TRANSPORTATION TABLE OF TYPE 2 NTP

	G_1	G_2	G_3	**Supply**
S_1	4	2	2	s_1
S_2	2	1	2	s_2
S_3	3	5	5	s_3
Demand	d_1	d_2	d_3	

Step 1: Using the above score function (1),

$S(H) = \frac{1}{12}[(h_1+2h_2+h_3)+(h_4+2h_5+h_6)+(h_7+2h_8+h_9)]$,

converting the given neutrosophic data into their corresponding crisp data, we obtain,
$S(s_1)$ = 15.75 ; $S(s_2)$ = 25.4167 ; $S(s_3)$ = 20.75
$S(d_1)$ = 18.75 ; $S(d_2)$ = 20.75 ; $S(d_3)$ = 22.75

The finally obtained crisp equivalent transportation table is given by the following Table VI:

TABLE VI
THE CRISP EQUIVALENT TRANSPORTATION TABLE OF TYPE 2 NTP

	G_1	G_2	G_3	**Supply**
S_1	4	2	2	15.75
S_2	2	1	2	25.4167
S_3	3	5	5	20.75
Demand	18.75	20.75	22.75	

Step 2: The next step is to verify if the considered neutrosophic transportation problem is balanced. Here, since $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$ ($\sum_{i=1}^{3} a_i \neq \sum_{j=1}^{3} b_j$), i.e., 15.75 + 25.4167 + 20.75 = 61.9167 $\neq$ 62.25 = 18.75 + 20.75 + 22.75, the given transportation problem is not balanced. Hence, we have to make it balanced, before we move forward to step 3. Hence, here since $\sum_{i=1}^{3} a_i < \sum_{j=1}^{3} b_j$, we add a dummy row with the required supply value, which now makes the problem balanced, shown in the following table :

	G_1	G_2	G_3	**Supply**
S_1	4	2	2	15.75
S_2	2	1	2	25.4167
S_3	3	5	5	20.75
S_4	0	0	0	0.3333
Demand	18.75	20.75	22.75	

Step 3: Now, after making the problem balanced, the next step is to add a row below called the Range Demand Column (RDC) and a column at the right called the Range Supply Row (RSR) in the transportation table as follows:

	G_1	G_2	G_3	**Supply**	**RSR**
S_1	4	2	2	15.75	
S_2	2	1	2	25.4167	
S_3	3	5	5	20.75	
S_4	0	0	0	0.3333	
Demand	18.75	20.75	22.75		
RDC					

Step 4: Now, after having introduced the above-mentioned row (RDC) and column (RSR), the next task is to find out the corresponding values for the newly added row and column by calculating the range for every row and column as follows:

	G_1	G_2	G_3	**Supply**	**RSR**
S_1	4	2	2	15.75	2
S_2	2	1	2	25.4167	1
S_3	3	5	5	20.75	2
S_4	0	0	0	0.3333	0
Demand	18.75	20.75	22.75		
RDC	4	5	5		

Step 5: Now, after obtaining all the values of the RSR and the RDC, we have to spot and select the highest value among all the found new entries of the RSR and the RDC. Here, the highest value is 5, which lies in the RDC. Hence, we have to choose the least element (c_{ij}) of its corresponding column. The corresponding column is G_2 and its least element is 0 (c_{42}) as shown below :

	G_1	G_2	G_3	**Supply**	**RSR**
S_1	4	2	2	15.75	2
S_2	2	1	2	25.4167	1
S_3	3	5	5	20.75	2
S_4	0	0	0	0.3333	0
Demand	18.75	20.75	22.75		
RDC	4	5	5		

Step 6: The next task after spotting the cell with the minimal element, is to compare the corresponding supply and demand availability (a_i and b_j).,i.e., to check whether $a_i \leq b_j$ (or) $a_i > b_j$. Here, the corresponding supply and demand availability are 0.3333 and 20.75 respectively, of which the least value is 0.3333. Hence, since $a_i < b_j$, we allocate the a_i amount of units to c_{ij}, which changes the new demand value as, $b_j = b_j - a_i$., i.e., since $a_4 < b_2$ (0.3333 $<$ 20.75), we allocate the a_4 (0.3333) amount of units to c_{42}, which changes the new demand value as, $b_2 = b_2 - a_4$ (20.75 - 0.3333 = 20.417), as shown below:

	G_1	G_2	G_3	**Supply**	**RSR**
S_1	4	2	2	15.75	2
S_2	2	1	2	25.4167	1
S_3	3	5	5	20.75	2
S_4	0	0.3333 0	0	0.3333	0
Demand	18.75	~~20.75~~ 20.4167	22.75		
RDC	4	5	5		

Thus, row i is discarded., row 4 (S_4) is discarded, since the supply availability for this row is allocated completely to c_{42} (i.e., fully exhausted), as shown below in the following table:

	G_1	G_2	G_3	**Supply**
S_1	4	2	2	15.75
S_2	2	1	2	25.4167
S_3	3	5	5	20.75
Demand	18.75	20.4167	22.75	

Step 7: The next step is to keep repeating steps 3-6 until all columns disappear. Hence, here since not all columns are discarded, we again start the same procedure and continue repeating steps 3-6. After repeating the same above procedure a few times, we obtain the following table:

	O_3	**Supply**
S_3	2 5	2
Demand	2	

Thus, the resulting final transportation table with all the allocations is given below in Table VII:

TABLE VII
THE RESULTING ALLOCATED TRANSPORTATION TABLE OF TYPE 2 NTP

	G_1	G_2	G_3	**Supply**
S_1	4	2	15.75 2	15.75
S_2	2	20.4167 1	5 2	25.4167
S_3	18.75 3	5	2 5	20.75
S_4	0	0.3333 0	0	0.3333
Demand	18.75	20.75	22.75	

Our next task is to verify if the overall number of allotments precisely equals $m+n-1$. Here, since we have 6 allotments, which is precisely $m+n-1$ non-negative allotments in independent places, it is guaranteed that there prevails a basic feasible solution to the given $m \times n$ transportation issue. Thus, we can go to step 8.

Step 8: Finally, we have to compute the initial basic feasible solution and the total minimal transportation cost, which itself will serve as the optimal solution and the optimal transportation cost respectively, for the considered neutrosophic transportation problem. Thus, the optimal solution of the considered transportation problem is, x_{42} = 0.3333, x_{22} = 20.4167, x_{13} = 15.75, x_{23} = 5, x_{31} = 18.75 and x_{33} = 2, with the optimal transportation cost (OTC), Z = Rs. (0 $\times$0.3333) + (1 $\times$ 20.4167) + (2 $\times$ 15.75) + (2 $\times$ 5) + (3 $\times$ 18.75) + (5 $\times$ 2) $= Rs.128.1667$

C. Illustration 3 - Type 3 NTP

In this type of problem [31], the cost of transportation in each cell is considered a neutrosophic cost, and supply and demand entries are also represented as triangular fuzzy neutrosophic numbers, which are expressed through the following Table VIII:

TABLE VIII
THE NEUTROSOPHIC TRANSPORTATION TABLE OF TYPE 3 NTP

	G_1	G_2	G_3	**Supply**
S_1	c_{11}	c_{12}	c_{13}	s_1
S_2	c_{21}	c_{22}	c_{23}	s_2
S_3	c_{31}	c_{32}	c_{33}	s_3
Demand	d_1	d_2	d_3	

where each cell's cost of transportation, supply, and demand entries (c_{ij}^N, a_i^N and b_j^N.,i.e., triangular fuzzy neutrosophic numbers) are as follows:
$c_{11} = ((1,4,7),(1,3,5),(3.5,6,7.5))$
$c_{12} = ((0.5,2.5,4.5),(1,2,3),(1.5,3.5,5.5))$
$c_{13} = ((1,3,5),(0.5,1.5,3.5),(2,4,6))$
$c_{21} = ((1,2,3),(0.5,1.5,2.5),(1.5,2.5,3.5))$
$c_{22} = ((1,1.5,4),(0.5,1,2.5),(1.25,3,4.25))$
$c_{23} = ((1.5,2.5,3.5),(1,1.5,3),(2,3,4))$
$c_{31} = ((2,4,6),(1.5,2.5,4.5),(3,5,7))$
$c_{32} = ((1,5,8),(1.5,4.5,7.5),(4,6.5,9))$
$c_{33} = ((1,5,8),(1.5,3,6.5),(4,7,9))$
$s_1 = ((10,15,20),(14,16,22),(12,15,19))$
$s_2 = ((20,25,30),(24,26,32),(22,25,29))$
$s_3 = ((15,20,25),(19,21,27),(17,20,24))$
$d_1 = ((13,18,23),(17,19,25),(15,18,22))$
$d_2 = ((15,20,25),(19,21,27),(17,20,24))$
$d_3 = ((17,22,27),(21,23,29),(19,22,26))$

Step 1: Using the above score function (1),
$S(H) = \frac{1}{12}[(h_1+2h_2+h_3)+(h_4+2h_5+h_6)+(h_7+2h_8+h_9)]$,
converting the given neutrosophic data into their corresponding crisp data, we obtain,
$S(c_{11})$ = 4.25 ; $S(c_{12})$ = 2.67 ; $S(c_{13})$ = 2.92
$S(c_{21})$ = 2 ; $S(c_{22})$ = 1.71 ; $S(c_{23})$ = 2.75
$S(c_{31})$ = 3.92 ; $S(c_{32})$ = 5.25 ; $S(c_{33})$ = 5.08
$S(s_1)$ = 15.75 ; $S(s_2)$ = 25.4167 ; $S(s_3)$ = 20.75
$S(d_1)$ = 18.75 ; $S(d_2)$ = 20.75 ; $S(d_3)$ = 22.75

The finally obtained crisp equivalent transportation table is given by the following Table IX:

TABLE IX
THE CRISP EQUIVALENT TRANSPORTATION TABLE OF TYPE 3 NTP

	G_1	G_2	G_3	**Supply**
S_1	4.25	2.67	2.92	15.75
S_2	2	1.71	2.75	25.4167
S_3	3.92	5.25	5.08	20.75
Demand	18.75	20.75	22.75	

Steps 2-7: Now, following the same above step-by-step procedure similar to the previous two illustrations, we obtain the final resulting transportation table with all the allocations as follows in the Table X:

TABLE X
THE RESULTING ALLOCATED TRANSPORTATION TABLE OF TYPE 3 NTP

	G_1	G_2	G_3	**Supply**
S_1	4.25	2.67	15.75 2.92	15.75
S_2	2	20.4167 1.71	5 2.75	25.4167
S_3	18.75 3.92	5.25	2 5.08	20.75
S_4	0	0.3333 0	0	0.3333
Demand	18.75	20.75	22.75	

Our next task is to verify if the overall number of allotments precisely equals $m+n-1$. Here, since we have 6 allotments, which is precisely $m+n-1$ non-negative allotments in independent places, it is guaranteed that there prevails a basic feasible solution to the given $m \times n$ transportation issue. Thus, we can go to step 8.

Step 8: Finally, we have to compute the initial basic feasible solution and the total minimal transportation cost, which itself will serve as the optimal solution and the optimal transportation cost respectively, for the considered neutrosophic transportation problem. Hence, the optimal solution of the considered transportation problem is, x_{42} = 0.3333, x_{22} = 20.4167, x_{23} = 5, x_{13} = 15.75, x_{31} = 18.75 and x_{33} = 2, with the optimal transportation cost (OTC), Z = Rs. $(0 \times 0.3333) + (1.71 \times 20.4167) + (2.75 \times 5) + (2.92 \times 15.75) + (3.92 \times 18.75) + (5.08 \times 2) = Rs.178.3126$

Remark 5: We find that the initial basic feasible solution and the total transportation costs obtained here for illustrations 1, 2, and 3 using the proposed method differ from that of the corresponding illustrations using the existing methods in [31]. Further discussions and inferences about the neutrosophic transportation problem considered in this research article are given in the following section.

VII. RESULTS AND CONCLUSION

A. Results

Some of the important results are discussed in this section through the following Tables XI-XIV and Fig. 1-5 which provide a comparison of the solutions of the proposed approach here to solve the NTP, with the other existing methods in [31] :

TABLE XI
SOLUTIONS (OTC) OF THE NTP OBTAINED USING THE PROPOSED APPROACH

Illustrations	**Proposed method**
Illustration 1	Rs.172.47
Illustration 2	Rs.128.1667
Illustration 3	Rs.178.3126

- The Tables XI and XII display the solutions (OTC) of the NTP obtained using the proposed approach and a

TABLE XII
COMPARISON OF THE SOLUTIONS (ITC/OTC) OF THE NTP OBTAINED USING THE PROPOSED APPROACH WITH SOME METHODS IN EXISTENCE

Illustrations	NWCR	VAM	MODI method	Proposed method
Illustration 1	Rs.211.05	Rs.190.12	Rs.172.47	Rs.172.47
Illustration 2	Rs.197.5	Rs.128.5	Rs.128.5	Rs.128.1667
Illustration 3	Rs.219.33	Rs.197.43	Rs.178.88	Rs.178.3126

comparison of the total transportation costs (ITC/OTC) of the NTP taken into consideration, calculated using the proposed method with that of all the three illustrations considered above, using the other three existing methods in [31] respectively, namely:

1) North-West Corner Rule (NWCR)
2) Vogel's approximation method (VAM) and
3) Modified Distribution method (MODI).

TABLE XIII
SOLUTIONS (IBFS/OVERALL ALLOTMENTS/OPTIMAL SOLUTION) OF THE NTP OBTAINED USING THE PROPOSED APPROACH

Illustrations	Illustration 1	Illustration 2	Illustration 3
Proposed method	x_{13} = 15; x_{22} = 20; x_{23} = 5; x_{31} = 18; x_{33} = 2	x_{13} = 15.75; x_{22} = 20.4167; x_{23} = 5; x_{31} = 18.75; x_{33} = 2; x_{42} = 0.3333	x_{13} = 15.75; x_{22} = 20.4167; x_{23} = 5; x_{31} = 18.75; x_{33} = 2; x_{42} = 0.3333

TABLE XIV
SOLUTIONS (IBFS/OVERALL ALLOTMENTS) OF THE NTP OBTAINED USING THE PROPOSED APPROACH COMPARED WITH SOME METHODS IN EXISTENCE

Illustrations	Illustration 1	Illustration 2	Illustration 3
NWCR	x_{11} = 15; x_{21} = 3; x_{22} = 20; x_{23} = 2; x_{33} = 20	x_{11} = 15.75; x_{21} = 3; x_{22} = 20.75; x_{23} = 2; x_{33} = 20.75	x_{11} = 15.75; x_{21} = 3; x_{22} = 20.75; x_{23} = 2; x_{33} = 20.75
VAM	x_{12} = 13; x_{13} = 2; x_{21} = 18; x_{22} = 7; x_{33} = 20	x_{13} = 15.75; x_{22} = 20.75; x_{23} = 5; x_{31} = 18.75; x_{33} = 2	x_{12} = 13.75; x_{13} = 2; x_{21} = 18.75; x_{22} = 7; x_{33} = 20.75
MODI method	x_{13} = 15; x_{22} = 20; x_{23} = 5; x_{31} = 18; x_{33} = 2	x_{13} = 15.75; x_{22} = 20.75; x_{23} = 5; x_{31} = 18.75; x_{33} = 2	x_{13} = 15.75; x_{22} = 20.75; x_{23} = 5; x_{31} = 18.75; x_{33} = 2
Proposed method	x_{13} = 15; x_{22} = 20; x_{23} = 5; x_{31} = 18; x_{33} = 2	x_{13} = 15.75; x_{22} = 20.4167; x_{23} = 5; x_{31} = 18.75; x_{33} = 2; x_{42} = 0.3333	x_{13} = 15.75; x_{22} = 20.4167; x_{23} = 5; x_{31} = 18.75; x_{33} = 2; x_{42} = 0.3333

- The above Tables XIII and XIV display the solutions (IBFS/overall allotments/optimal solution) of the NTP obtained using the proposed approach and a comparison of the initial basic feasible solution (overall allotments/optimal solution) of the NTP taken into consideration, calculated using the proposed method with that of all the three illustrations considered above, using the other three existing methods as used in [31].
- Thus the above mentioned comparisons guarantee that the initial basic feasible solution and the total minimal transportation cost (ITC) found by utilizing the proposed method itself serve as the optimal solution and the optimal transportation cost respectively (OTC), for the given neutrosophic transportation problem.

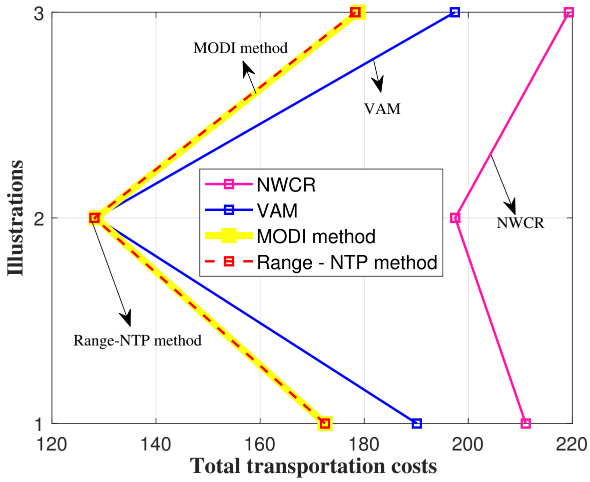

Fig. 1. An overview of the solutions of the NTP using the proposed approach and a few other classical methods in existence

- The same above inferences can be drawn from Fig. 1, which provides an overview of the solutions to all three neutrosophic transportation problems that were previously taken into consideration and solved using the four methods mentioned earlier by showing the difference in the values of the solutions.
- Fig. 1 also shows that the solutions to the above three neutrosophic transportation problems obtained using the proposed method (the red dotted line) almost coincide with those calculated using the MODI method (the most common way for computing the optimal solution, along with the optimal transportation cost) (the yellow line) and better than those obtained using the NWCR (the pink line) and VAM methods (in terms of total transportation costs) (the blue line).
- Having drawn the previously mentioned inferences from Fig. 1, Fig. 2 presents an even more clear picture of how the solution of the NTPs under consideration using the proposed method (the red dotted line) is better than that of the same NTPs using the other methods in [31], by providing just a glimpse of an enlarged version of the sample with a considerable amount of variation in the total transportation costs (ITC/OTC).
- Knowing that the MODI method which is used to check the optimality of the given transportation problem gives better solutions than both the NWCR and VAM methods, we observe from Fig. 2 that the proposed method also appears to serve the same purpose as the MODI method (thereby giving the best possible solution (or) making the total transportation costs as minimal as possible), by providing either the same or lesser values for the total transportation costs of all the considered three NTPs in this article, when compared to the MODI method.
- Fig. 3 presents a picture of the IBFS of illustration 1 using the proposed method (x_{13} = 15; x_{22} = 20; x_{23} = 5; x_{31} = 18; x_{33} = 2), where the violet arrow line represents the allotment of the row 1, the green arrow line represents the allotments of the row 2 and the maroon arrow line represents the allotments of the

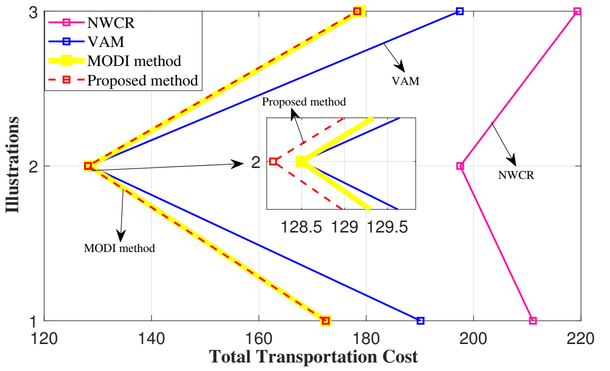

Fig. 2. A glimpse of an enlarged version of the sample with a considerable amount of variation in the total transportation costs

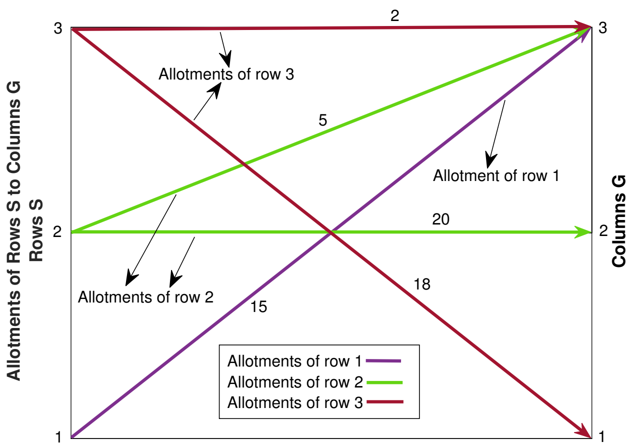

Fig. 3. A picture of the IBFS (overall allotments/OS) of illustration 1 using the proposed approach

row 3 of the NTP taken into consideration.

- Since we have already established that the proposed method gives us the OS along with the OTC, Fig. 3 can be viewed as the OS of Illustration 1 using the proposed method.
- Fig. 4 presents a picture of the IBFS of illustration 2 using the proposed method (x_{13} = 15.75; x_{22} = 20.4167; x_{23} = 5; x_{31} = 18.75; x_{33} = 2; x_{42} = 0.3333), where the violet arrow line represents the allotment of the row 1, the green arrow line represents the allotments of the row 2 and the maroon arrow line represents the allotments of the row 3 of the NTP taken into consideration.
- As we have already established that the proposed method gives us the OS along with the OTC, Fig. 4 can be viewed as the OS of Illustration 2 using the proposed method.
- Fig. 5 presents a picture of the IBFS of illustration 3 using the proposed method (x_{13} = 15.75; x_{22} = 20.4167; x_{23} = 5; x_{31} = 18.75; x_{33} = 2; x_{42} = 0.3333), where the violet arrow line represents the allotment of the row 1, the green arrow line represents the allotments of the row 2 and the maroon arrow line represents the allotments of the row 3 of the NTP taken into consideration.
- Since we have already established that the proposed method gives us the OS along with the OTC, Fig. 5 can

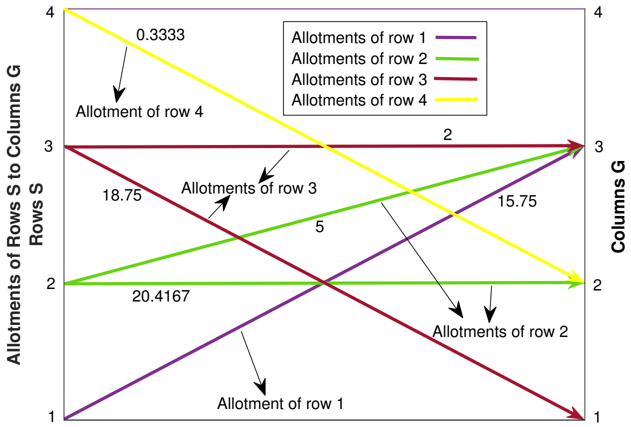

Fig. 4. A picture of the IBFS (overall allotments/OS) of illustration 2 using the proposed approach

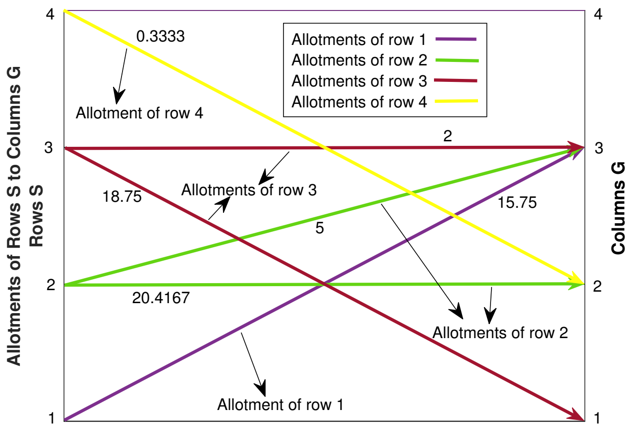

Fig. 5. A picture of the IBFS (overall allotments/OS) of illustration 3 using the proposed approach

be viewed as the OS of Illustration 3 using the proposed method.

B. Conclusion

This research piece investigates the transportation problem in a neutrosophic environment using triangular fuzzy neutrosophic numbers. Dealing with uncertainty was simplified since the neutrosophic data also takes into account an indeterminacy factor, apart from the truth and falsity factors, and the computations were also performed using these values. The neutrosophic transportation problem has been resolved by employing the stepwise procedure of the proposed method, according to the above-discussed instances. From the above Tables XI-XIV and Fig. 1-5, we can conclude that the IBFS and the total minimal transportation cost found by applying the proposed method itself serve as the optimal solution and the optimal transportation cost respectively, for the neutrosophic transportation problems taken into consideration of all the three different types, since the obtained values of the above considered three illustrations using the proposed method here, are either the same or a little lesser than that of the corresponding three illustrations obtained using the MODI method (used to verify the optimality of the given transportation problem) in [31]. Therefore, unlike other existing classical approaches like NWCR and VAM, the proposed method does not require the utilization of the MODI method to check the optimality of the solution after getting the initial basic feasible solution and the initial transportation cost. This, in turn, makes this proposed approach to solve the NTP, a little more efficient compared to the other existing classical methods taken into account in this paper since it gives us fruitful answers by reducing the transportation costs and the computation time to solve the NTP as much as possible. The use of triangular fuzzy neutrosophic numbers in solving transportation problems presents a groundbreaking approach. It enhances our ability to grapple with uncertainty in logistics and transportation planning. The proposed methodology equips decision-makers with a versatile tool to navigate the complexities of real-world data. Its practical applications extend to supply chain optimization and logistics management, offering businesses a competitive edge. Though this field is not without its challenges, this study has illuminated its potential and the road ahead is rich with research opportunities, from refining solution algorithms to exploring diverse industry applications. Knowing that there is a considerable amount of study being carried out in this field, it may be possible to investigate if the presented approach could be used to tackle the multi-objective transportation problem in addition to this one. Hence, this would enhance the findings and benefit the neutrosophic research environment. The future of transportation optimization lies in the dynamic world of neutrosophic numbers, promising innovation and efficiency in an unpredictable landscape.

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