Congestion Propagation of Double-layer Urban Rail Transit Considering Passengers' Bounded Rationality

Changfeng Zhu, Xuejiao Ma, Xuegui Wang, Jinhao Fang, Jie Wang, and Linna Cheng

Abstract—Based on the multi-layer network perspective, a method for constructing the double-layer urban rail transit topology network was proposed. Then, the choice behavior of passengers at congested stations was analyzed, and the passenger flow loss rate was quantified. Moreover, a congestion propagation model for the double-layer urban rail transit network was proposed, and the congestion propagation process was analyzed using a real network as an illustration. The results indicate that once the congestion disturbance reaches an influence threshold, the congestion would propagate to other stations in the same or different layers, ultimately spreading widely in the network. The preferences and risk attitudes of passengers at congested stations affect the passenger flow loss rate. Furthermore, the spontaneous travel choice behavior of passengers at congested stations can regulate the congestion propagation process of urban rail transit networks. The stations with high topological importance are more capable of propagating congestion than stations with high functional importance, and the passenger flow distribution structure of the network is more fragile than the network topological structure. The increase in coupling coefficient leads to a rise in station-to-station interaction, making congestion more likely to propagate. The research results can provide a reference for urban rail transit passenger flow operation organization.

Index Terms—Multi-layer network, Congestion propagation, Prospect theory (PT), Coupled map lattices (CML), Influence threshold

I. INTRODUCTION

Multi-layer integrated urban rail transit networks lead the development of metropolitan areas. However, the rapidly growing passenger flow and tightening connected

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line networks increase the risk of congestion propagation on multi-layer rail networks [1][2]. Congestion has become a severe issue that rail operation management should address to prevent accidents and enhance operational service.

A great deal of research has been conducted on the congestion propagation process of urban rail transit networks. Zhou et al. [3] were the first to propose the concept of passenger flow propagation during peak hours and analyzed the influencing factors in the congestion propagation process. Subsequently, numerous scholars have studied the passenger flow propagation process. Zhang et al. [4] established the interaction rules for passenger flow and analyzed the propagation process of commuter passenger flow using the meta-cellular automata model. Jiang et al. [5] defined a set of state parameters to reflect the dissipation propagation rate and simulated the dissipation process using the meta-cellular automata model. Zhao et al. [6] analyzed the delay propagation of urban rail transit by developing a meta-cellular automata model for single-station failure and a propagation model for multi-station failure. Li et al. [7] analyzed the propagation dynamics of passenger flow in train delay scenarios and proposed measures for passenger flow control. Xu et al. [8] developed a comprehensive model for large passenger flow propagation using AFC data, train operating data, and network topology data.

The theory of cascade failure in networks provides a novel approach for analyzing the process of congestion propagation. The cascade failure model is particularly effective in describing the successive failure of nodes[9][10], eventually leading to failure propagation throughout the network. And the cascade failure model is particularly effective in describing the occurrence of station congestion in urban rail transit networks, which leads to subsequent congestion at other stations. The CML cascade failure model can reveal complicated phenomena such as chaotic traffic flow characteristics in the network, so it was commonly used to analyze the cascading failure of multi-layer traffic networks. Huang et al. [12] developed a weighted CML model to analyze the cascading failure process of rail transit networks by considering the network topology and the characteristics of passenger flow. Zhu et al. [13] analyzed the vulnerability of rail transit networks by using a CML model that accounts for the dynamic nature of passenger flow propagation. Zhang et al. [17] proposed an ICML model to investigate the impact of different attack strategies on the vulnerability of urban rail transit networks.

The CML model mainly simulates the cascade failure dynamics process through the evolution of node states, and

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inappropriate parameter settings may result in inaccurate results. The parameter settings of the established literature on the CML model are shown in Table.1.

TABLE I THE PARAMETER SETTINGS OF ESTABLISHED LITERATURE

Authors	Subjects	Considerations	The initial state of the station
Zhang [11]	Road Network	Topological factors	section saturation
Huang [12]	Rail network	Topological factors, passenger flow factors	Random numbers
Zhu [13]	Rail network	Topological factors	Random numbers
Ma [14]	Bus - metro network	Topological factors, passenger flow factors	Random numbers
Xiong [15]	Rail network	Topological factors	Cross-sectional passenger flow
Zhang [16]	Multimodal Transport Network	Topological factors, passenger flow factors, directional factors	Random numbers
Zhang [17]	Rail network	Topological factors, passenger flow factors, directional factors	Random numbers
Wu [18]	Rail network	Topological factors	Random numbers
Gao [19]	Double-layer rail network	Topological factors, passenger flow factors	Station load
Huang [20]	Rail network	Topological factors, passenger flow factors	Random numbers

As shown in Table.1, The initial state of the CML model was assigned randomly in the majority of studies, and the relationship between passenger flow and model parameters was not quantified, making it challenging to ensure the accuracy of the iterative process. Consequently, further investigation into the quantification of model parameters is warranted.

Furthermore, the majority of studies that analyze the congestion propagation process by cascade failure models commonly abstracted the urban rail transit network as a single-layer network. However, scholars have gradually realized that single-layer networks fail to adequately capture the differences between lines and stations in the urban rail transit network [21]. Consequently, an increasing number of scholars have recently conducted studies on the dynamics of multi-layer networks [22][23][24]. These researches are beneficial to characterize the process of congestion propagation in urban rail transit networks.

The travel choice behavior of passengers is a bounded rational decision-making process influenced by subjective and objective factors, which subsequently affects the process of congestion propagation. Probabilistic choice models serve as the prevalent approach for analyzing travel choice behavior. In 1959, Luce developed the logit model as an extension of the disaggregate model. The majority of traditional logit models are based on the random utility theory, which assumes that passengers are perfectly rational and choose the option that maximizes their expectations.

However, in the actual decision-making process, passengers exhibit bounded rationality due to variations in individuals' preferences for risk, and an increasing number of scholars in the field of human decision-making behavior have embraced the notion that passengers are bounded rational. The concept of bounded rationality was initially introduced by Simon [25], and one of the subsequent highly influential theories developed in the field was prospect theory (PT) [26][27]. Prospect theory directly integrates psychological perceptions into the decision-making process, and the application in passengers' choice behavior focuses on route and departure time choice[28][29], etc. In recent years, the application of prospect theory in the field of transportation has become increasingly widespread, Gao et al. [30] proposed a travel mode choice model for commuters based on cumulative prospect theory and a multi-attribute decision-making method. Cheng et al. [31] developed a multi-objective optimization model to optimize urban rail transit-stopping schemes by employing prospect theory to maximize travel time savings and minimize congestion costs. Zhu et al. [32] and Ma et al. [33] proposed a route choice model for emergency logistics based on prospect theory that considered both route attributes and the risk attitudes of decision-makers.

Some researchers attempt to incorporate the concept of bounded rationality into the traditional probabilistic choice model. Zhang et al. [34] proposed a hierarchical Logit model that considered the perceived cost difference, incorporating bounded rationality. Yao et al. [35] developed a travel mode choice model that took into account the psychological reference value of alternatives, combining rational decision-making and the choice preferences of passengers. Ma et al. [36] created an NL-cumulative prospect theory travel mode prediction model, which subjectivized the objective utility obtained from the NL model through prospect theory. The majority of the previous studies have focused on commuting choice behavior, and the choice behavior of passengers at congested stations deserves further investigation.

In conclusion, while the existing literature offers valuable insights, further research is still needed. Specifically: (1) The majority of the existing studies regarded the entire urban rail transit network as a single-layer network, with few studies proposing a method to construct a double-layer urban rail transit network systematically. (2) Secondly, the primary cause of congestion propagation is passenger travel choice behavior, however, previous studies have not adequately explored the impact of this behavior on the propagation of congestion. (3) The initial state of the CML model was assigned randomly in the majority of existing studies and failed to quantify the relationship between passenger flow and model parameters, making it challenging to ensure the accuracy of the iterative process.

In this paper, a congestion propagation model for a double-layer urban rail transit network based on CML was proposed, and the choice behavior of passengers at congested stations was analyzed using the prospect theory model. Then the congestion propagation of the double-layer urban rail transit network considering passengers' bounded rationality was studied.

II. DOUBLE-LAYER URBAN RAIL TRANSIT TOPOLOGY NETWORK CONSTRUCTION

A. Model Assumptions

The double-layer urban rail transit topology network is constructed with the following assumptions:

Hypothesis 1: Two systems comprise the double-layer urban rail transit network: a subway network and a regional rapid rail network.

Hypothesis 2: Typically, the direction of urban rail transit lines is symmetrical, abstracting it as an undirected network.

Hypothesis 3: Define stations as network nodes and the line connection relationship between stations as intra-layer connected edges. In order to establish relationships between the different layers of the network, the transfer of passengers between stations of different layers is defined as inter-layer connected edges.

B. Composite network construction

Based on these assumptions, a double-layer urban rail transit network (*DRTN*) is defined as

$$DRTN = \langle G_R^1, G_S^2 \rangle \tag{1}$$

Where, $G_R^1 = (V_R^1, E_R^1)$ is the regional rapid rail layer of the *DRTN*, $G_S^2 = (V_S^2, E_S^2)$ is the subway layer of the *DRTN*. $V \in \{V^{(n)}, n = 1.2\}$ is the node set of the *DRTN*, and $V^{(n)} = \{1^{(n)}, ..., i^{(n)}, ..., N_T^{(n)}\}$ are the stations in each layer of the *DRTN*. $E \in \{E^{(n)}, E^{(nm)}, n \neq m = 1.2\}$ is the edge set of the *DRTN*, $E^{(n)} = \{e_{ij}^{(n)} = (v_i^{(n)}, v_j^{(n)})\}$ is the intra-layer connected link of station *i* and station *j*, $E^{(nm)} = \{e_{ij}^{(nm)} = (v_i^{(n)}, v_j^{(m)})\}$ is the inter-layer connected link of station *i* and station *i* and station *j*.

The schematic diagram of the double-layer urban rail transit network is shown in Fig.1.



Fig. 1. Schematic diagram of double-layer urban rail transit network

C. Basic characteristics of the stations

The basic characteristics of the stations in a double-layer urban rail transit network are analyzed as follows:

(1) The station degree

The degree reflects the importance of the station in the network. In a double-layer urban rail transit network, stations may exist in more than one layer of the network, then the degree of station i in layer n is:

$$k_i^{(n)} = \sum_{j^{(n)}=1, j^{(n)} \neq i^{(n)}}^N e_{ij}^{(n)} + \sum_{j^{(m)}=1, n \neq m}^M e_{ij}^{(nm)}$$
(2)

(2) The station strength

In order to more realistically reflect the impact of the passenger flow on the station, the passenger flow is defined by the in-flow and out-flow:

$$F_i = F_{i \to j}^{out} + F_{j \to i}^{in} \tag{3}$$

The strength of station i is determined by the total passenger flow. The larger the strength, the more critical the passenger flow transport capacity of the station in the network. The strength of station i in layer n is:

$$s_{i}^{(n)} = \sum_{j^{(n)}=1, j^{(n)} \neq i^{(n)}}^{N} \left(F_{i \to j}^{out} + F_{j \to i}^{in} \right)$$
(4)

III. BOUNDED RATIONALITY BEHAVIOR ANALYSIS OF PASSENGERS AT CONGESTED STATIONS

When an emergency event causes a large amount of passenger collecting volume and exceeds the allowable threshold, the station will become congested. Congestion can result in passengers experiencing irritability, anger, and anxiety, influencing travel choice as a bounded rational decision-making process.

A. Analysis of factors influencing passengers' travel choice behavior

Define the available travel modes for passengers traveling between origin and destination (OD) pairs, including urban rail transit, buses, and taxis. The route of each mode is determined by the shortest route principle. When the station is congested, passengers at the congested station have the option to switch from urban rail transit to buses or taxis as an alternative mode of transportation. Typically, the travel mode choice of passengers is determined by the travel time and travel cost.

(1) Travel time

When the station becomes congested, the travel time of passengers includes in-vehicle travel time, transfer time, waiting time, and additional time resulting from the congestion.

The total travel time of mode k is

$$t_k = t_k^{vehicle} + t_k^{trans} + t_k^{wait} + t'$$
(5)

Where, $t_k^{vehicle}$ is the in-vehicle travel time, t_k^{trans} is the transfer time, t_k^{wait} is the waiting time, and t' is the additional time resulting from the congestion.

(2) Travel cost

Different travel modes lead to differences in costs. Urban rail transit and taxis usually adopt mileage-based fare pricing and buses mostly adopt universal fare pricing. The total travel cost of mode k is

$$E_k = E_k^{vehicle} + E' \tag{6}$$

Where, $E^{vehicle}$ is the actual costs for choosing travel mode k, E' is the additional costs incurred by abandoning urban rail transit resulting from the congestion.

B. A travel mode choice behavior model based on prospect theory

Due to a lack of information, passengers at congested stations usually set their mental expectations by referring to the current circumstances as their reference points. The disparity between mental expectation and actual perception is used to calculate gain or loss while traveling.

The average travel time for each travel mode between OD is defined as the time reference point: $T_0^* = \frac{1}{k} \sum_k t_k^{(OD)}$, while the average travel cost for each travel mode between OD is defined as the cost reference point: $E_0^* = \frac{1}{k} \sum_k E_k^{(OD)}$.

When passengers make their travel mode choices at congested stations, two possible outcomes arise:

(1) If $T_k > T_0^*$ or $E_k > E_0^*$, it indicates that the passengers' actual perception of the travel mode falls short of their mental expectation, resulting in relative losses. In this case, passengers tend to choose travel modes with shorter travel times or lower costs, manifested as risk-appetite.

(2) If $T_k \leq T_0^*$ or $E_k \leq E_0^*$, it indicates that the passengers' actual perception of the travel mode exceeds their mental expectation, resulting in a relative gain. In this case, passengers will avoid changing the travel mode, manifested as risk-averse.

Based on these observations, the travel choice value functions for passengers at congested stations are built as follows:

$$v(T) = \begin{cases} \left(T_{k} - T_{0}^{*}\right)^{\alpha}, & T_{k} \ge T_{0}^{*} \\ -b\left(T_{0}^{*} - T_{k}\right)^{\beta}, & T_{k} < T_{0}^{*} \end{cases}$$
(7)

$$v(E) = \begin{cases} \left(E_{k} - E_{0}^{*}\right)^{\alpha}, & E_{k} \ge E_{0}^{*} \\ -d\left(E_{0}^{*} - E_{k}\right)^{\beta}, & E_{k} < E_{0}^{*} \end{cases}$$
(8)

Where, v(T) is the travel time value function, v(E) is the travel cost value function. α , β are risk sensitivity coefficients, and b, d ($b \ge 1$, $d \ge 1$) are loss aversion coefficients.

The value function curves of travel time and travel cost are shown in Fig.2.



Fig. 2. The value function curve of travel time and travel cost. (a) travel time. (b) travel cost.

At the same time, passengers usually make subjective judgments and travel decisions based on their perceived probability and usually overestimate low-probability events and underestimate large-probability events when making decisions. The subjective probability function based on the perceived probability of passengers is:

$$\omega^{+}(p) = \frac{(p_{k})^{\gamma}}{[(p_{k})^{\gamma} + (1 - p_{k})^{\gamma}]^{\frac{1}{\gamma}}}$$
(9)

$$v^{-}(p) = \frac{(p_{k})^{\delta}}{[(p_{k})^{\delta} + (1 - p_{k})^{\delta}]^{\frac{1}{\delta}}}$$
(10)

Where, p_k is the actual probability of choosing travel mode k, γ is the gain-perceived probability coefficient, and δ is the loss-perceived probability coefficient. Typically, $\gamma = 0.61$ and $\delta = 0.69$. The subjective probability function curve is shown in Fig.3.



Fig. 3. Subjective probability function curve

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The prospect value can be expressed as a weighted combination of the value function and the subjective probability function. Therefore, the prospect values for travel time and travel cost are:

$$V_T^{(OD)} = v(T) \cdot \omega(p) \tag{11}$$

$$V_{W}^{(OD)} = v(W) \cdot \omega(p) \tag{12}$$

To mitigate the impact of the reference point dimension on the results, the prospect values $V_T^{(OD)}$ and $V_W^{(OD)}$ are normalized:

$$\begin{cases} \overline{V}_{T}^{(OD)} = \frac{V_{T}^{(OD)}}{|V_{T}^{(OD)}|_{\max}}, |V_{T}^{(OD)}|_{\max} = \max\left\{V_{T}^{(OD)}\right\} \\ \overline{V}_{W}^{(OD)} = \frac{V_{W}^{(OD)}}{|V_{W}^{(OD)}|_{\max}}, |V_{W}^{(OD)}|_{\max} = \max\left\{V_{W}^{(OD)}\right\} \end{cases}$$
(13)

The preference coefficient η is introduced to reflect the preferences of various types of passengers for travel time and travel costs, and the combined prospect value of travel mode *k* is:

$$V_k^{(OD)} = \eta \overline{V}_T^{(OD)} + (1 - \eta) \overline{V}_W^{(OD)}$$
(14)

Calculate the probability of passengers choosing mode k from the congested station using the MNL (Multinomial Logit) model:

$$p_{k}^{(OD)} = \frac{e^{V_{k}^{(OD)}}}{\sum_{k \in K} e^{V_{k}^{(OD)}}}$$
(15)

Where, $p_k^{(OD)}$ is the probability that the passengers at the congested station will choose travel mode *k*, and *K* is the total number of travel modes between that OD.

The macroscopic results of travel mode choice are presented as passenger flow loss at congested stations, which affects the network congestion propagation process.

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The principle of passenger flow loss at congested stations under bounded rationality is shown in Fig.4.



Fig. 4. The principle of passenger flow loss at the congested station

As shown in Fig. 4, the passenger flow volume between a specific OD starting at station *j* is $q_j^{(OD)}$, the probability of passengers continuing to choose urban rail transit is $p_{RTN}^{(OD)}$, and the passenger flow loss between this OD is $(p_{Taxi}^{(OD)} + p_{Bus}^{(OD)}) \cdot q_j^{(OD)}$. Then the passenger flow loss rate at congested station *j* is:

$$p_{j} = p_{j,Taxi} + p_{j,Bus} = \frac{\sum_{J} (p_{Taxi}^{(OD)} + p_{Bus}^{(OD)}) \cdot q_{j}^{(OD)}}{\sum_{J} q_{j}^{(OD)}}$$
(16)

Where, p_j is the passenger flow loss rate at congested station *j*, $q_i^{(OD)}$ is the passenger flow volume between OD.

IV. A DOUBLE-LAYER URBAN RAIL TRANSIT NETWORK CONGESTION PROPAGATION MODEL

The propagation of congestion is a dynamic process. When the passenger collecting volume at a station exceeds capacity at a station, some passengers may opt to exit the urban rail transit system and choose alternative travel modes. Others will be transported to adjacent stations with the train operation, leading to congestion propagation within the network. The principle of congestion propagation in urban rail transit networks is shown in Fig.5.



Fig. 5. The principle of congestion propagation

Based on the analysis of the congestion propagation mechanism, a cascade failure model is proposed to describe the congestion propagation process in the double-layer urban rail transit network:

$$\chi_{i}^{(n)}(t+1) = \begin{cases} (1 - \varepsilon_{1} - \varepsilon_{2})f\left[x_{i}^{(n)}(t)\right] + \\ \sum_{j=1, j \neq i}^{N} e_{ij}f\left[x_{j}^{(n)}(t)\right] \\ k_{i}^{(n)} + \\ \sum_{i}^{N} \sum_{j=1, j \neq i}^{N} (1 - p_{j}) \cdot e_{ij}F_{i \rightarrow j}^{in}(t)f\left[x_{j}^{(n)}(t)\right] \\ k_{i}^{(n)} + \\ \sum_{i}^{N} \sum_{j=1, j \neq i}^{N} e_{ij}F_{j \rightarrow i}^{out}(t)f\left[x_{j}^{(n)}(t)\right] \\ k_{i}^{(n)} + \\ k_{i}^{(n)} + k_{i}^{(n)} +$$

Where, ε_1 is the topological network coupling coefficient, ε_2 is the passenger flow distribution coupling coefficient, $\varepsilon_1 \in (0,1)$, $\varepsilon_2 \in (0,1)$, e_{ij} is the connection information between station *i* and station *j*, p_j is the passenger flow loss rate at congested station *j*, $k_i^{(n)}$ is the degree of station *i*, $s_i^{(n)}$ is the strength of station *i*, i.e., the total passenger flow at the station.

The mapping function $f[x_i^{(n)}(t)] = 4[x_i^{(n)}(t) \cdot (1-x_i^{(n)}(t))]$ is chosen to describe the dynamic behavior of congestion at station *i* in layer *n* of the double-layer urban rail transit network, $x_i^{(n)}(t)$ is the state of station *i* at time step *t*. Initially, $x_i^{(n)}(t-1) \in (0,1)$ is defined as the maximum section load factor at station *i* in layer *n* of the double-layer urban rail transit network, which represents the state of station *i* before congestion:

$$x_{i}^{(n)}(t-1) = \max\left\{\frac{V_{i,1}^{(n)}(l)}{C_{i,1}^{(n)}(l)}, \frac{V_{i,2}^{(n)}(l)}{C_{i,2}^{(n)}(l)}, ..., \frac{V_{i,k}^{(n)}(l)}{C_{i,k}^{(n)}(l)}\right\}$$
(18)

Where, $V_{i,k}^{(n)}(l)$ is the passenger flow volume of adjacent section k at station i in unit period l, $C_{i,k}^{(n)}(l)$ is the train capacity passing through adjacent section k at station i in unit period l.

An emergency event may cause a large amount of passenger collecting volume in a short period. To quantify the passenger collecting volume, a congestion disturbance $R \ge 1$ is defined. For stations unaffected by emergency events, R = 0. If all stations always maintain $0 \le x_i^{(n)}(t) < 1$, then there is no congested station in the network, if $x_i^{(n)}(t) \ge 1$, station *i* becomes congested, after that $x_i^{(n)}(t) \equiv 0$ within $(t+1,+\infty)$, and neighboring stations will be affected by the state of station *i*, which causes the congestion propagation in the double-layer urban rail transit network.

Based on the above analysis, the process of the simulation algorithm for congestion propagation in double-layer urban rail transit network is shown in Fig.6.



Fig. 6. The process of the simulation algorithm

V. EXAMPLE ANALYSIS

The simulation analysis focuses on a partial topology of the Shanghai urban rail transit network. The topology of the network is shown in Fig.7, and the in-flow and out-flow of passenger flow at the stations are shown in Fig. 8.





Fig. 7. Urban rail transit topology network





Fig. 9. The congestion propagation process in the double-layer rail transit network.

As shown in Fig.9, when the passenger collecting volume at the station reaches a significant scale, the trains are unable to meet the transportation demand, leading to passenger detention. At the same time, operating trains close to full load will cause congestion to spread to adjacent stations, manifested as point-to-line propagation. When the train reaches an interchange station, congestion extends to other stations in adjacent layers of the double-layer urban rail

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transit network, which is manifested as line-to-surface propagation.

The process of congestion propagation can be influenced by various parameter values, making it essential to analyze the impact of each parameter on congestion propagation of the network.

A. Effect of choice behavior on passenger flow loss rate

The choice behavior of passengers plays a crucial role in passenger flow loss at congested stations, consequently affecting the congestion propagation process. Taking the choice behavior of passengers at station 27 as an example, the effects of choice behavior on the passenger flow loss rate are shown in Fig.10-17.



Fig. 10. The relationship between the risk appetite coefficient α and passenger flow loss rate.



Fig. 11. The relationship between the risk aversion coefficient β and passenger flow loss rate.



Fig. 12. The relationship between the α , β , and passenger flow loss rate.

As shown in Fig.10-12, the passenger flow loss rate increases with the increase of risk appetite coefficient α , while it decreases with the increase of risk aversion coefficient β . Taking Fig.10 as an example, when risk appetite coefficient α is small, the majority of passengers will conservatively choose urban rail transit because the travel time and cost of urban rail transit are more balanced. In contrast, a minority of passengers will take the risk of choosing other modes. As the α increases, the number of passengers willing to explore other modes also increases, resulting in a higher passenger flow loss rate. The impact of the risk aversion coefficient β on the passenger flow loss rate follows a similar pattern, as the β increases, passengers are more willing to choose urban rail transit to avoid potential risks, resulting in a lower passenger flow loss rate.







Fig. 14. The relationship between loss-perceived probability coefficient δ and passenger flow loss rate.



Fig. 15. The relationship between γ , δ , and passenger flow loss rate.

As shown in Fig.13-15, the passenger flow loss rate increases with the increase of the gain-perceived probability coefficient γ and decreases with the increase of the loss-perceived probability coefficient δ .

Taking Fig.13 as an example, when passengers at congested rail transit stations choose their travel modes, they tend to exhibit the characteristic of overestimating lowprobability events and underestimating large-probability events. Urban rail transit offers a higher prospect value than other travel modes, if passengers are perfectly rational, choosing urban rail transit becomes a probabilistic event. However, if passengers are bounded rational, they tend to underestimate the benefits of choosing urban rail transit. Therefore, with the increase of gain-perceived probability coefficient γ , the perceived value of choosing alternative travel modes increases rapidly, leading to a higher passenger flow loss rate in the urban rail transit network. The impact of the loss-perceived probability coefficient δ on the passenger flow loss rate follows a similar pattern. with the increase of the loss-perceived probability coefficient δ , passengers tend to overestimate the loss of choosing other travel modes, leading to a decrease in passenger flow loss rate.

Comparing Fig.12 and Fig.15, it can be seen that the passenger flow loss rate is more sensitive to β and δ . It is evident that passengers are more sensitive to the perception of loss and are more tend to avoid potential risks in their travels.



Fig. 16. The relationship between prospective preference coefficient η and the passenger flow loss rate

As shown in Fig.16, the passenger flow loss rate decreases as the prospective preference coefficient η increases initially and then increases. When $\eta < 0.5$, it indicates that passengers prioritize saving travel costs over saving travel time, resulting in more passengers opting for urban rail transit and buses, and urban rail transit is a superior option. Similarly, when $\eta > 0.5$, it indicates that passengers prioritize saving travel time over saving travel costs, leading to an increase in the choice of urban rail transit and taxis, urban rail transit also is a superior option.

At $\eta = 0.4$, the passenger loss rate reaches its minimum, indicating passengers who value travel time are more likely to choose other travel modes. Therefore, ensuring passenger travel time is a crucial strategy for reducing the passenger flow loss rate.



Fig. 17. The relationship between additional congestion time t' and passenger flow loss rate.

As shown in Fig.17, it is observed that a longer duration of congestion in urban rail transit results in a higher passenger flow loss rate, there is a linear correlation between the passenger flow loss rate and the additional congestion time. The longer the duration of congestion, the lower the willingness of passengers to choose urban rail transit. Therefore, it is essential to implement timely measures to alleviate congestion, minimize the duration of congestion, and reduce the passenger flow loss rate as much as possible when congestion occurs.

B. Effect of the passenger flow loss rate on congestion propagation

From the analysis in part *A*, it is evident that choice behavior has a significant impact on the passenger flow loss rate. It is necessary to further analyze the effect of the passenger flow loss rate on the congestion propagation process. Assuming R=3, the passenger flow loss rate at each station is equal. The results are shown in Fig 18- 20.



Fig. 18. Congestion propagation process with the different passenger flow loss rates when the station with the largest degree is congested. (a) the proportion of congested stations. (b) number of congested stations.



Fig. 19. Congestion propagation process with the different passenger flow loss rates when the station with the largest passenger flow is congested. (a) the proportion of congested stations. (b)number of congested stations.



Fig. 20. Congestion propagation process with the different passenger flow loss rates when the station with the largest initial state is congested. (a) the proportion of congested stations. (b) number of congested stations.

As shown in Fig.18-20, it can be observed that the congestion propagation rate decreases with the increase of the passenger flow loss rate. This implies that as the value of the passenger flow loss rate p_i increases, more passengers

choose to leave the urban rail transit system. The loss of passenger flow reduces the burden on transportation within the system, leading to the speed of congestion propagation slowing down. It is demonstrated that the spontaneous travel choice behavior of passengers can effectively regulate the passenger flow in the urban rail transit system during congested periods. However, the loss of passenger flow negatively impacts the revenue and passenger service quality of urban rail transit, highlighting the need for timely passenger flow control measures.

When comparing Fig.18-20, it is evident that the process of congestion propagation varies when the initial congested station is different, even with the same passenger flow loss rate. Therefore, it is necessary to further analyze the effect of the initial congested station on congestion propagation.

C. Effect of the different initial congested stations and congestion disturbance on congestion propagation

Considering the significant impact of the initial congested station and the passenger collecting volume on the process of congestion propagation, the stations with the largest degree, the largest passenger flow, and the largest initial state are subjected to different congestion disturbances, respectively. And the congestion propagation threshold R_c is introduced, when $R \ge R_c$, congestion spreads widely in the network. Taking $\varepsilon_1 = \varepsilon_2 = 0.25$, the congestion propagation processes at the stations with the largest degree, passenger flow, and initial state are depicted in Figs. 21-22, 23-24, and 25-26, respectively.



Fig. 21. The proportion of congested stations with time when the station with the largest degree is congested. (a) without considering passenger travel behavior at congested station. (b) considering passenger travel behavior at congested station.



Fig. 22. The number of congested stations with time when the station with the largest degree is congested. (a) without considering passenger travel behavior at congested station. (b) considering passenger travel behavior at congested station.

As shown in Fig.21, the rate of congestion propagation increases with the increase of the congestion disturbance. When congestion occurs at the station with the largest degree, $R_c = 3.2$ regardless of whether the choice behavior of passengers is considered. When $R \ge 3.2$, the congestion spreads widely in the urban rail transit network. It can be seen that the choice behavior of passengers has less impact on the congestion propagation threshold at stations with high topological importance. Moreover, when congestion disturbances R are identical, considering passenger travel choice behavior results in a slower congestion propagation speed, indicating that spontaneous travel choice behavior of passengers can slow down the congestion propagation process.

As shown in Fig.22, the number of congested stations and congestion time approximately follow a normal distribution. Additionally, the peak periods of the number of congested stations vary when different types of stations experience congestion. When the congestion disturbance is small, the congestion will only spread in a small area of the network. As the congestion disturbance increases, the congestion gradually spreads within the urban rail transit network, and the larger the congestion disturbance, the earlier the peak period of the number of congested stations appears. It proved that passenger flow control measures should be implemented as early as possible before the peak period of the number of congested stations appears to minimize the speed of congestion propagation and potential for hazardous accidents.



Fig. 23. The proportion of congested stations with time when the station with the largest passenger flow is congested. (a) without considering passenger travel behavior at congested station. (b) considering passenger travel behavior at congested station.



Fig. 24. The number of congested stations with time when the station with the largest passenger flow is congested. (a) without considering passenger travel behavior at congested station. (b) considering passenger travel behavior at congested station.

As shown in Fig.23, $R_c = 2.8$ is the congestion threshold at the station with the largest passenger flow, without considering the travel choice behavior of passengers. When $R \ge 2.8$, the congestion spreads widely in the urban rail transit network. $R_c = 3.2$ is the congestion propagation threshold considering the choice behavior of passengers. When $R \ge 3.2$, the congestion spreads widely in the urban rail transit network. It can be seen that the travel choice behavior of passengers at stations with high functional importance has an essential influence on the congestion propagation process, significantly increasing the congestion threshold.

Comparing Fig.21 and Fig.23, it can be seen that the station with the largest passenger flow exhibits a faster congestion propagation speed compared to the station with the largest degree. Moreover, the congestion propagation threshold R_c at the station with the largest passenger flow is lower than that at the station with the largest degree. It is found that the stations with high topological importance are more capable of propagating congestion than stations with high functional importance, and the passenger flow distribution structure of the network is more fragile than the network topological structure.

Comparing Fig.22 and Fig.24, it can be seen that after reaching the congestion propagation threshold, the peak period of the number of congested stations at the station with the largest degree appears later than that at the station with the largest passenger flow, which further proves that the station with high topological importance has a stronger ability to propagate congestion.



Fig. 25. The proportion of congested stations with time when the station with the largest initial state is congested. (a) without considering passenger travel behavior at congested station. (b) considering passenger travel behavior at congested station.



Fig. 26. The number of congested stations with time when the station with the largest initial state is congested. (a) without considering passenger travel behavior at congested station. (b) considering passenger travel behavior at congested station.

As shown in Fig.25, When congestion occurs at the station with the largest initial state, $R_c = 2.4$ is the congestion propagation threshold regardless of whether the travel choice behavior of passengers is considered. Notably, the congestion propagation threshold is the lowest at the station with the largest initial state, indicating that the initial state has a critical effect on the occurrence of congestion propagation.

As shown in Fig.26, when the congestion propagation disturbance at stations with the largest initial state is small $(R \le 2.4)$, the number of congested stations converges at T=22. In contrast, when the congestion disturbance is large (R>2.4), the peak period of the number of congested stations appears earlier, and the number of congested stations converges at $T\le 16$. It can be seen that the size of the initial aggregated passenger flow has a significant influence on the congestion propagation process at the station with the largest initial state. When the aggregated passenger flow is low, the congestion propagation period is prolonged, whereas it converges rapidly when the aggregated passenger flow is high.

D. Effect of coupling coefficient on congestion propagation threshold

Since the topological network coupling coefficient ε_1 and the passenger flow distribution coupling coefficient ε_2 describe the interaction between network topology and passenger flow, it is necessary to explore their effect on the congestion propagation threshold. The thresholds for different coupling coefficients are shown in Fig.27.



Fig. 27. The relationship between influence thresholds and coupling coefficient. (a)station with the largest degree. (b)station with the largest passenger flow. (c)The station with the largest initial state.

As shown in Fig.27, both ε_1 and ε_2 affect the congestion propagation threshold R_c . When ε_1 is held constant, there is a gradual decrease in R_c as ε_2 increases. Similarly, when ε_2 is held constant, there is a gradual decrease in R_c with an increase in ε_1 . The primary factor is the increase in the coupling coefficient, which results in higher station-tostation interaction and an increased likelihood of congestion propagation.

Meanwhile, the congestion propagation threshold R_c of the station with the largest degree is more sensitive to ε_1 , demonstrating that the tightness of inter-station connectivity has a greater impact on the congestion propagation process at stations with high topological importance, an increase in ε_1 is more likely to cause congestion spreading widely in the network. Similarly, the congestion propagation threshold R_c of the station with the largest passenger flow is more sensitive to ε_2 , demonstrating that the closeness of passenger flow interactions between stations has a greater impact on the congestion propagation process at stations with high functional importance, an increase in ε_2 is more likely to cause congestion spreading widely in the network. Moreover, the congestion propagation threshold R_c of the station with the largest initial state is equally sensitive to both ε_1 and ε_2 .

E. Effect of network coupling on congestion propagation

The process of congestion propagation at the station may differ in single-layer and double-layer networks. For example, station 27, a transfer point between the subway and regional rapid rail, is compared in terms of the process of congestion propagation in the single-layer regional rail network, single-layer subway network, and double-layer urban rail transit network is compared. The results are shown in Fig.28-29.



Fig. 28. The proportion of congested stations in different networks



Fig. 29. The number of congested stations with time in different networks

As shown in Fig.28-29, when congestion occurs at station 27 in the regional rapid rail network, congestion will only spread in a small area due to poor network connectivity. However, for station 27 in the subway network and double-layer urban rail transit network, the congestion will spread extensively, potentially reaching stations unaffected by congestion in the single-layer regional rapid rail network. This is primarily due to enhanced station-to-station interaction resulting from network coupling. Furthermore, the final extent of congestion propagation in the double-layer urban rail transit network is smaller compared to the subway network because the regional rapid rail can distribute the passenger flow among subways, proving that the network coupling can improve the stability of the network.

VI. CONCLUSIONS

The main conclusions were obtained by constructing a double-layer urban rail transit topological network (*DRTN*) and developing a congestion propagation model to analyze the congestion propagation process in urban rail transit networks:

(1) When the passenger collecting volume at the station reaches a significant scale, the trains are unable to meet the transportation demand. At the same time, operating trains close to full load will cause congestion to propagate to adjacent stations, manifested as point-to-line propagation. When the train reaches an interchange station, congestion extends to other stations in adjacent layers of the urban rail transit network, which is manifested as line-to-surface propagation.

(2) Travel preferences and risk attitudes influence the travel choice behavior of passengers at congested stations, resulting in a loss of passenger flow. Specifically, passengers who are more risk- appetite are more likely to choose other travel modes, resulting in a higher passenger flow loss rate. Conversely, passengers who are more risk-averse are more likely to continue choosing urban rail transit. Furthermore, a longer duration of congestion in the urban rail transit system correlates with a higher passenger flow loss rate, and the passenger flow loss rate exhibits a linear correlation with the amount of additional congestion time.

(3) The spontaneous travel choice behavior of passengers can effectively regulate the passenger flow in the urban rail transit system during congested periods and reduce the rate of congestion propagation. However, the loss of passenger flow negatively impacts the revenue and passenger service quality of urban rail transit, highlighting the need for timely passenger flow control measures.

(4) The initial congested station and the passenger collecting volume significantly affect the process of congestion propagation. The rate of congestion propagation increases with the increase of the congestion disturbance, while the number of congested stations and congestion time approximately follow a normal distribution.

(5) The stations with high topological importance are more capable of propagating congestion than stations with high functional importance, and the passenger flow distribution structure of the network is more fragile than the network topological structure.

(6) The increase in coupling coefficient leads to a rise in station-to-station interaction, making congestion more likely to propagate. Specifically, the topological distribution coupling coefficient has a more significant influence on the threshold for stations with high topological importance, while the passenger flow distribution coupling coefficient has a more substantial impact on the threshold for stations with high functional importance.

(7) The coupling of networks increases the risk of congestion at previously uncongested stations in the singlelayer regional rapid rail network. Moreover, the final extent of congestion propagation in the double-layer urban rail transit network is smaller compared to the subway network, proving that the network coupling can improve the stability of the network.

REFERENCES

- D. Hörcher, D. J. Graham, and R. J. Anderson, "Crowding cost estimation with large scale smart card and vehicle location data," *Transportation Research Part B: Methodological*, vol. 95, pp. 105-125, 2017.
- [2] C. L. Du, X. L. Li, R. R. Sun, P. Zhang, and G. Y. Zhu, "Classification of urban rail stations based on passenger flow congestion propagation," *Journal of Beijing Jiaotong University*, vol. 45, no. 1, pp. 39-46. 2021.
- [3] Y. F. Zhou, L. S. Zhou, and Y. X. Yue, "Synchronized and coordinated train connecting optimization for transfer stations of urban rail networks," *Journal of the China Railway Society*, vol. 33, no. 3, pp. 9-16, 2011.
- [4] Q. Zhang, W. J. Xiao, and G. Pan, "A CA-based simulation model of urban railway large passenger flow congestion transmission," *Journal* of *Transportation Systems Engineering and Information Technology*, vol. 17, no. 4, pp. 83-89, 2017.
- [5] Y. S. Jiang, W. T. Liu, and Z. H. Yao, "Evolution mechanism of congestion and dissipation of sudden passenger flow in urban rail transit based on cellular automaton," *Journal of Transportation Systems Engineering and Information Technology*, vol. 20, no. 5, pp. 121-127, 2020.
- [6] Y. Zhao, and X. Ding, "The research on delay propagation of urban rail transit operation under sudden failure," *Journal of Advanced Transportation*, vol. 2021, Article ID. 8984474, 2021.
- [7] M. Li, X. Zhou, Y. Wang, L. Jia and M. An, "Modelling cascade dynamics of passenger flow congestion in urban rail transit network induced by train delay," *Alexandria Engineering Journal*, vol. 61, no.11, pp. 8797-8807, 2022.
- [8] R. Xu, X. Song, F. Wang, and F. Zhou, "Assessment method for dynamic impact of large passenger flow on urban rail transit network: a case study on Chengdu east railway station," *Transportation Research Record*, vol. 2676, no. 4, pp. 296-307, 2022.
- [9] Y. Zhang, C. Zhu, B. Ma, X. Wang, and E. Xu, "Analysis of connectivity reliability of multi-level rail transit network considering passengers' bounded rationality," *IAENG International Journal of Applied Mathematics*, vol. 52, no.3, pp. 667-677, 2022.
- [10] Z. Yang, X. Dong, and L. Guo, "Scenario inference model of urban metro system cascading failure under extreme rainfall conditions," *Reliability Engineering & System Safety*, vol. 229, Article ID. 108888, 2023.
- [11] Y. Zhang, Y. Lu, G. Lu, P. Chen, and C. Ding, "Analysis of road traffic network cascade failures with coupled map lattice method," *Mathematical Problems in Engineering*, vol. 2015, Article ID. 101059, 2015.
- [12] A. Huang, H. M. Zhang, W. Guan, Y. Yang, and G. Zong, "Cascading failures in weighted complex networks of transit systems based on coupled map lattices," *Mathematical Problems in Engineering*, vol. 2015, Article ID. 940795, 2015.
- [13] Y. Zhu, J. Sun, and J. Wang, "Reliability analysis of urban rail transit system network based on coupled map lattice model," *Journal of University of Shanghai for Science and Technology*, vol. 43, no. 1, pp. 93-101, 2021.
- [14] F. Ma, F. Liu, K. F. Yuen, P. Lai, Q. Sun, and X. Li, "Cascading failures and vulnerability evolution in bus-metro complex bilayer networks under rainstorm weather conditions," *International Journal* of Environmental Research and Public Health, vol. 16, no. 3, Article ID. 329, 2019.
- [15] Z.H. Xiong, and Z. S. Yao, "Influence scope of cascading failure on rail transit system," *Journal of Transportation Systems Engineering* and Information Technology, vol. 20, no. 1, pp. 13-18, 2020.
- [16] L. Zhang, M. Xu, and S. Wang, "Quantifying bus route service disruptions under interdependent cascading failures of a multimodal public transit system based on an improved coupled map lattice model," *Reliability Engineering & System Safety*, vol. 235, Article ID. 109250, 2023.
- [17] J. Zhang, Z. Wang, S. Wang, W. Shao, X. Zhao, and W. Liu, "Vulnerability assessments of weighted urban rail transit networks with integrated coupled map lattices," *Reliability Engineering and System Safety*, vol. 214, Article ID. 107707, 2021.
- [18] S. Wu, Y. Zhu, N. Li, Y. Wang, X. Wang, and D. J. Sun, "Urban rail transit system network reliability analysis based on a coupled map lattice model," *Journal of Advanced Transportation*, vol. 2021, Article ID. 5548956, 2021.
- [19] C. Gao, Y. Fan, S. Jiang, Y. Deng, J. Liu, and X. Li, "Dynamic robustness analysis of a two-layer rail transit network model," *IEEE Transactions on Intelligent Transportation Systems*. Vol. 23, pp. 6509-6524, 2021.

- [20] A. L. Huang, X. H. Xu, W. Guan, and M. Y. Duan. "Evolution of metro network stability based on weighted coupled map lattice," *Journal of Transportation Systems Engineering and Information Technology*, vol. 21, no. 3, 2021.
- [21] J. Zhou, D. Coit, F. Felder, and S. Tsianikas, "Combined optimization of system reliability improvement and resilience with mixed cascading failures in dependent network systems," *Reliability Engineering & System Safety*, Article ID. 109376, 2023.
- [22] L. Lu, X. Wang, Y. Ouyang, J. Roningen, N. Myers, and G. Calfas, "Vulnerability of interdependent urban infrastructure networks: Equilibrium after failure propagation and cascading impacts," *Computer-Aided Civil and Infrastructure Engineering*, vol. 33, no. 4, pp. 300-315, 2018.
- [23] J. Zhu, T. Xue, P. Pei, X. Chen, H. Gao, and Q. Liu, "The synchronization ability of a class of multi-layer coupled networks," *IAENG International Journal of Applied Mathematics*, vol. 53, no.1, pp. 209-216, 2023.
- [24] W. Du, Y. Li, J. Zhang, and J. Yu, "Stochastic synchronization between different networks and its application in bilayer coupled public traffic network," *IAENG International Journal of Computer Science*, vol. 46, no.1, pp. 102-108, 2019.
- [25] H. A. Simon, "A behavioral model of rational choice," *The Quarterly Journal of Economics*, vol. 69, no.1, pp. 99-118, 1955.
- [26] A. Tversky, and D. Kahneman, "Advances in prospect theory: Cumulative representation of uncertainty," *Journal of Risk and Uncertainty*, vol. 5, no. 4, pp. 297-323, 1992.
- [27] S.N. Liu, C.F. Zhu, Y.H. Zheng, Q.R. Wang, "Research on bi-level game scheduling for emergency rescue workers under the condition of limited rationality," *Engineering Letters*, vol. 28, no. 4, pp. 1338-1347, 2020.
- [28] W. Sun, C. Zhang, and H. Li, "Evolutionary game of emergency logistics path selection under bounded rationality," *Socio-Economic Planning Sciences*, vol. 82, Article ID. 101311, 2022.
- [29] K. Geng, Y. Wang, E. Cherchi and P. Guarda, "Commuter departure time choice behavior under congestion charge: analysis based on cumulative prospect theory," *Transportation Research Part A: Policy* and Practice, vol. 168, Article ID. 103564, 2023.
- [30] K. Gao, L. Sun, Y. Yang, F. Meng, and X. Qu, "Cumulative prospect theory coupled with multi-attribute decision making for modeling travel behavior," *Transportation Research Part A: policy and practice*, vol. 148, pp.1-21, 2021.
- [31] L. Cheng, C. Zhu, Q. Wang, W. Wang, Z. Zhang, and W. Sun, "Skip-stop operation plan for urban rail transit considering bounded rationality of passengers," *IET Intelligent Transport Systems*, vol. 16, no. 1, pp. 24-40, 2022.
- [32] C. F. Zhu, Z. K. Zhang, and Q. R. Wang, "Path choice of emergency logistics based on cumulative prospect theory," *Journal of Advanced Transportation*, vol. 2019, Article ID.8512423, 2019.
- [33] B. Ma, C. Zhu, Y. Zhang, and Q. Wang, "Research on evolutionary game of emergency material allocation under bounded rationality," *Engineering Letters*, vol. 29, no.4, pp. 1524-1534, 2021
- [34] X. J. Zhang, H. Z. Guan, L. Zhao, and F. Bian, "Nested logit model on travel mode choice under boundebly rational view," *Journal of Transportation Systems Engineering and Information Technology*, vol. 18, no. 6, pp. 110-116, 2018.
- [35] E. J. Yao, W. D. Chen, T. W. Lu, and Y. Yang, "Transportation mode selection model considering traveler's personal preferences," *Journal* of *Beijing Jiaotong University*, vol. 44, no. 1, pp. 42-48, 2020.
- [36] S. H. Ma, Y. C. Zhou, and Y. Zhang. "Travel mode choice forecasting based on Nested Logit-cumulative prospect theory model," *Journal of Transportation Systems Engineering and Information Technology*, vol. 19, no. 4, pp. 135-142, 2019.



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