Distributed Sliding Mode Control for Average Formation Tracking of Multi-agent Systems with Multiple Active Leaders

Xin Cai, Xiaozhou Zhu, Wen Yao

Abstract—This article adopts sliding mode technology to investigate average formation tracking issue of multi-agent systems, followers can track reference trajectory derived from average state of multiple leaders, while achieving expected timevarying formation. Firstly, multiple active leaders with bounded but unknown control inputs are introduced that can manipulate the motion trajectory of the swarm system in response to changes or threats in the environment. Secondly, utilizing sliding mode theory method and states of adjacent agents, the distributed protocol is proposed, and the stableness is demonstrated through Lyapunov criterion. Finally, the performance of the constructed scheme is validated via the numerical case of multi-unmanned aerial vehicle(UAV) formation.

Index Terms—average formation, active leaders, multi-agent systems, sliding mode, Lyapunov criterion.

I. INTRODUCTION

R ECENTLY, cooperative algorithm has been investigated in some industries that is logistics transportation, search and rescue, localization and monitoring [1], [2]. Cooperative control involves multiple aspects, formation is a significant issue and has been widely used in autonomous underwater vehicle, unmanned aerial vehicle, spacecraft and so on. With more complex scenarios, it is very important and challenging to design efficient, stable, and robust formation protocol.

Many methods to study the formation problem have been proposed. [3] adopted the leader-follower strategy to complete formation control task. [4] solved the formation problem through swarm behavior. The scheme based on virtual structure was used in [5]. [6] studied a protocol using the information exchanged between agents, and proposed that classical formation methods can be unified under the consensus control framework. Due to the consensus theory has made great progress, consensus-based methods have been adopted to study formation control. Based on consensus theory, [7] proposed an event-triggering function to generate a series of control quantity, avoiding frequent controller updates and saving communication resources. [8] studied the formation strategy of the swarm systems under varying

Manuscript received May 12, 2023; revised September 21, 2023. This work was supported by the National Natural Science Foundation of China under Grant 11725211.

Xin Cai is a PhD student at the Defense Innovation Institute, Chinese Academy of Military Science, Beijing, 100071, China (phone:+86 15189828030;e-mail:caixin94@outlook.com).

Xiaozhou Zhu is an associate professor at the Defense Innovation Institute, Chinese Academy of Military Science, Beijing, 100071, China(email:zhuxiazhou.niidt@gmail.com).

Wen Yao is a professor at the Defense Innovation Institute, Chinese Academy of Military Science, Beijing, 100071, China(email:wendyo782@126.com). topologies, proposed a robust control method consisting of position and attitude controllers, and carried out UAVs flight experiment. [9] discussed the distributed formation control problem with external disturbance, and proposed the finitetime observer to compensate external disturbance.

Significantly, the formation formed by above methods is time-invariant. In many scenarios, the formation needs to change in real-time to adapt to dynamically changing environments, so it promotes the development of time-varying formation [10]. Utilizing the commonly used Lyapunov method and riccati equation technology, a formation control law was designed in [11]. In [12], the formation control of fractional-order system was discussed, the formation problem was convert to the asymptotic stability problem through linear transformations.

Considering the tracking leader in the swarm systems, it translates to formation tracking control. The swarm systems consist of two types of agents: followers and leaders. The goals of leaders are to generate a desired trajectory, while followers are responsible for forming the expected formation and tracking the reference trajectory. In [13], a continuous repulsive vector was incorporated into the speed of the agent to ensure that the swarm systems complete formation tracking and avoids obstacles. [14] explored the formation tracking control using a broad learning system, taking into account input saturation and actuator fault. [15] solved the autonomous formation tracking problem by utilizing model predictive control. However, it is worth noting that in [13], [14], [15], the assumption is made that there is only one leader, it has certain limitations. In many practical scenarios, in scenarios such as coordinated flight of multiple manned/unmanned aerial vehicles, it becomes necessary for the formation to track trajectories generated by multiple leaders [16]. This is particularly crucial for ensuring the safety of manned aerial vehicles. Multiple UAVs maintain a formation centered around the average position of the manned aerial vehicles, effectively encircling them. This configuration allows for the performance of dangerous and dirty tasks by the unmanned aerial vehicles. Dealing with multiple leaders adds an extra layer of complexity to the problem compared to the case of a single leader.

Inspired by the above problem discussed, formation tracking control is studied. The major **contributions** of this article are

 The formation is time-varying, with both position and velocity components varying, rendering it more applicable to real-world situations. The second-order system is adopted to describe dynamics of each individual agent, it can be applied to actual robot systems.

- Multiple active leaders with unknown control inputs are introduced to provide reference signals for multiagent systems. It allows team behavior to be modified to deal with environmental threats, thus avoiding unexpected situations.
- 3) Based on information interaction between neighbors, the distributed control law is proposed via sliding mode strategy, the Lyapunov criterion is utilized to evaluate the stableness.

The rest of this article are arranged as. The problem formulation is provided in Section II. The distributed sliding mode control protocol is constructed in Section III. An application example is given in Section IV. The article is concluded in Section V.

Notations: $\|\cdot\|$ is the 2-norm. \mathbb{R}^n represents $n \times 1$ real vectors. \otimes , I_N denote the Kronecker product and $n \times n$ identity matrices. The vector $\mathbf{1}_n$ is a column vector where elements are 1.

II. PRELIMINARIES AND PROBLEM DESCRIPTION

A. Graph theory

The multi-agent systems includes M leaders and N followers. The information topology among agents is described by the weighted directed graph $G = (V, \mathcal{E}, A)$, where Vindicates the set of nodes, \mathcal{E} indicates the set of edges, and the adjacency matrix is expressed by A. Each edge e_{ij} connects nodes V_i and V_j , and the weight of the edge is denoted by a_{ij} . If e_{ij} exists in \mathcal{E} , then a_{ij} is positive; otherwise, it is zero. Denote L as the Laplacian matrix.

The weight among leaders and follower *i* is represented by a_{i0} . If followers can get the status of one or more leaders, then $a_{i0} = 1$, and if not $a_{i0} = 0$. Moreover, define $Q = \text{diag}(a_{10}, a_{20}, \ldots, a_{N0})$, $L_Q = L + Q$. If a follower has at least one leader in its neighbor set, it is referred to as an informed follower. Conversely, if its neighbor set does not include any leaders, it is considered an uninformed follower. Furthermore, an informed one is classified as a well-informed agent if its neighbor set includes all leaders.

Assumption 1: Each informed one is well-informed, while each uninformed follower is connected to at least one wellinformed agent through the directed path.

Assumption 2: The communication is unidirectional between leaders and followers, meaning that followers can get the information of leaders, but they cannot send information to leaders.

Lemma 1: [17] Let equation $\dot{x} = f(x), x \in \mathbb{R}^n$, if V is a positive continuously function and satisfies $\dot{V}(x) \leq -c(V(x))^\vartheta$, c > 0, $0 < \vartheta < 1$. In this case, the system is stabilized within a finite time T, and T meets $T \leq (V(x_0))^{1-\vartheta}/c(1-\eta)$.

B. Problem description

The mathematical model of follower i is

$$\begin{cases} \dot{p}_i = v_i \\ \dot{v}_i = u_i \end{cases}$$
(1)

where the control input $u_i \in \mathbb{R}^n$, $p_i \in \mathbb{R}^n$, $v_i \in \mathbb{R}^n$ are the states. Define $x_i = \begin{bmatrix} p_i^T, v_i^T \end{bmatrix}^T$, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$, the dynamics model (1) can be rewritten as

$$\dot{x}_i = Ax_i + Bu_i \tag{2}$$

Define $U = [u_1, u_2, ..., u_N]^T$, $X = [x_1^T, x_2^T, ..., x_N^T]^T$, then the multi-agent systems is described

$$\dot{X} = (I_N \otimes A) X + (I_N \otimes B) U \tag{3}$$

Similarly, the dynamics of each leader is written

$$\dot{\tau}_k = A \tau_k + B r_k \tag{4}$$

where $\tau_k = \left[p_k^T, v_k^T\right]^T \in \mathbb{R}^{2n}$. The control input $|r_k| \leq \varsigma$, $\varsigma > 0$.

Remark 1: In [18], leaders are passive and have no control input. However, in this paper, leaders are active and have unknown but bounded inputs, which means that their motion trajectory can be manipulated in real time to deal with complex environments.

Definition 1: Let $H = \begin{bmatrix} h_1^T, h_2^T, \dots, h_N^T \end{bmatrix}^T$ is the timevarying formation, where $h_i = \begin{bmatrix} h_{ip}^T, h_{iv}^T \end{bmatrix}^T \in \mathbb{R}^{2n}$ is differentiable and continuous. For the multi-agent systems (3):

$$\lim_{t \to \infty} \|x_i - h_i - x_0\| = 0$$
 (5)

where $x_0 = \frac{1}{M} \sum_{k=1}^{M} \tau_k$, then the multi-agent systems that include multiple leaders are capable of completing average formation tracking.

Remark 2: Definition 1 includes two tasks to be completed simultaneously. As shown in Fig. 1, one is that followers' states keep offset signal h_i , followers are responsible for completing formation task. Additionally, the followers' states converge to a consensus on the reference signal, it corresponds to the average state of the leaders.



Fig. 1. Demonstration of time-varying formation tracking.

Hence, the major purpose addressed is to propose a control method for followers to accomplish average formation tracking task defined in Definition 1.

III. CONTROL PROTOCOL DESIGN

The formation error system for follower i is denoted as

$$\epsilon_{i} = \sum_{j=1}^{N} a_{ij} \left(x_{i} - h_{i} - x_{j} + h_{j} \right) + a_{i0} \left(x_{i} - h_{i} - x_{0} \right)$$
(6)

The following switching function s_i is denoted by using sliding mode control method

$$s_i = K\epsilon_i \tag{7}$$

where $K = [k_1, k_2] \in \mathbb{R}^{1 \times 2}$.

Volume 53, Issue 4: December 2023

The switching function $S = [s_1, s_2, \dots, s_N]^T$ is defined as

$$S = (L_Q \otimes K) (X - H) - (Q \otimes K) (\mathbf{1}_N \otimes x_0), \quad (8)$$

Lemma 2: The multi-agent systems (3) successfully accomplish the desired tracking task, as expressed by the vector H, when the switching function S achieves and maintains a state of $S = \mathbf{0}$.

Proof: From (6), we can get the state error equation

$$\epsilon_{ip} = \sum_{j=1}^{N} a_{ij} \left(p_i - h_{ip} - p_j + h_{jp} \right) + a_{i0} \left(p_i - h_{ip} - p_0 \right),$$

$$\epsilon_{iv} = \sum_{j=1}^{N} a_{ij} \left(v_i - h_{iv} - v_j + f_{jv} \right) + a_{i0} \left(v_i - h_{iv} - v_0 \right).$$
(9)

one has $\epsilon_i = [\epsilon_{ip}, \epsilon_{iv}]^{\mathrm{T}}$, $\dot{\epsilon}_{ip} = \epsilon_{iv}$. The switching function (7) is

$$s_i = k_1 \epsilon_{ip} + k_2 \epsilon_{iv}. \tag{10}$$

Let $P = [p_1, p_2, \dots, p_N]^T$, $V = [v_1, v_2, \dots, v_N]^T$, $H_p = [h_{1p}, h_{2p}, \dots, h_{Np}]^T$, and $H_v = [h_{1v}, h_{2v}, \dots, h_{Nv}(t)]^T$. Moreover, define

$$\tilde{P} = P - H_p - \mathbf{1}_N \otimes p_0$$

$$\tilde{V} = V - H_v - \mathbf{1}_N \otimes v_0$$
(11)

then it gets $\dot{\tilde{P}} = \tilde{V}$. The equation (8) is rewritten

$$S = k_1 L_Q \tilde{P} + k_2 L_Q \tilde{V} \tag{12}$$

Take Lyapunov function as

$$V_{\epsilon} = \frac{1}{2} \tilde{P}^{\mathrm{T}} \tilde{P} \tag{13}$$

and it gets

$$\dot{V}_{\epsilon} = \tilde{P}^{\mathrm{T}} \dot{\tilde{P}} = \tilde{P}^{\mathrm{T}} \tilde{V}$$
(14)

When S = 0, it gets

$$k_1 L_Q \tilde{P} + k_2 L_Q \tilde{V} = \mathbf{0} \tag{15}$$

Due to the invertibility of L_Q , \dot{V}_{ϵ} is expressed

$$\dot{V}_{\epsilon} = -\frac{k_1}{k_2} \tilde{P}^{\mathrm{T}} \tilde{P}$$

$$\leq -\frac{2k_1}{k_2} V_{\epsilon}$$
(16)

From the Lyapunov stability theory, we can get $\lim_{t\to\infty} V_{\epsilon} = \mathbf{0}$, and it has $\lim_{t\to\infty} \tilde{P} = \mathbf{0}$. In addition, it obtains that $\lim_{t\to\infty} \tilde{V} = 0$. Thus, it gets

$$\lim_{t \to \infty} \|x_i - h_i - x_0\| = 0$$
 (17)

From Definition 1, it gets that the multi-agent systems (3) complete tracking task. This completes the proof.

The goal of the sliding mode method is to guarantee that follower agent i reaches and maintains $s_i = 0$. Thus, by utilizing Lemma 2, the formation tracking control can

be converted to the sliding mode problem. Consequently, a distributed control protocol is developed as follows:

$$u_{i} = (KB (d_{i} + a_{i0}))^{-1} \left(KB \sum_{j=1, j \neq i}^{N} a_{ij} u_{j} - [KA ((d_{i} + a_{i0}) x_{i} - \sum_{j=1, j \neq i}^{N} a_{ij} x_{j}) - a_{i0} KA x_{0} - a_{i0} KB \varsigma + \rho \operatorname{sgn}(s_{i}) - K \left((d_{i} + a_{i0}) \dot{h}_{i} - \sum_{j=1, j \neq i}^{N} a_{ij} \dot{h}_{j} \right) \right] \right)$$
(18)

where $\rho > 0$, $d_i = \sum_{j=1, j \neq i}^N a_{ij}$. *Theorem 1:* If the matrix $K = [k_1, k_2]$ satisfies the conditions $k_1 > 0$, $k_2 > 0$, and $\rho > 0$, Assumption 1 and Assumption 2 hold, then the swarm systems (3) are capable of achieving the average formation tracking through the control law (18).

Proof: From (8), one has

$$\dot{S} = (L_Q \otimes K) \left(\dot{X} - \dot{H} \right) - (Q \otimes K) \left(\mathbf{1}_N \otimes \dot{x}_0 \right)$$
(19)

Let $u_0 = \frac{1}{M} \sum_{k=1}^{M} r_k$, and substituting \dot{X} , \dot{x}_0 into the system (19), it gets

$$\dot{S} = (L_Q \otimes K) \left((I_N \otimes A) X + (I_N \otimes B) U \right) - (L_Q \otimes K) \dot{H} - (Q \otimes K) \left(1_N \otimes (Ax_0 + Bu_0) \right)$$
(20)

Define $U = [u_1, u_2, \dots, u_N]^T$, the protocol is rewritten

$$U = -(L_Q \otimes (KB))^{-1} ((L_Q \otimes KA) X - Q_1 \otimes (KAx_0 + KB\varsigma) + \rho \operatorname{sgn}(S) - (L_Q \otimes K) \dot{H})$$
(21)

Combine (20) and (21), we can get

$$\dot{S} = -\rho \operatorname{sgn}(S) - Q_1 \otimes (KB \left(u_0(t) - \varsigma \right))$$
(22)

For each agent i, it obtains

$$\dot{s}_i = -\rho \operatorname{sgn}\left(s_i\right) - a_{i0} KB\left(u_0 - \varsigma\right) \tag{23}$$

Take the Lyapunov function

$$V_{is} = \frac{1}{2}s_i^2 \tag{24}$$

It is further concluded that

$$\dot{V}_{is} = s_i \dot{s}_i \tag{25}$$

Due to $-(u_0(t) - \varsigma) \le 0$, it has $\dot{s}_i \le -\rho \operatorname{sgn}(s_i)$. Hence,

$$\dot{V}_{is} = s_i \dot{s}_i
\leq s_i \left(-\rho \operatorname{sgn}(s_i)\right)
\leq -\sqrt{2}\rho\left(V_{is}\right)^{\frac{1}{2}}$$
(26)

From the Lemma 1, each agent can reach the sliding surface $s_i = 0$. Further, according to Lemma 2, the swarm systems (3) has achieved average formation tracking. The proof is completed.

Volume 53, Issue 4: December 2023



Fig. 2. Formation tracking process of multiple UAVs.



Fig. 3. Dual closed-loop control framework of UAV.



Fig. 4. Communication graph.

IV. NUMERICAL SIMULATION

To illustrate the performance of designed control framework, a numerical example involving multiple unmanned aerial vehicles (UAVs) is provided. Fig. 2 depicts the tracking process of UAVs.

The altitude channel of the UAV can be independently controlled to enable the UAV to fly at the specified altitude. Hence, the main focus is on the control of formation tracking for UAVs on the horizontal plane, once the desired altitude has been achieved. For each UAV in the swarm systems, Fig. 3 illustrates the dual closed-loop control structure of the UAV. Within this framework, the outer loop is responsible for guiding the agent towards the expected position, a inner loop controller is tasked with tracking the attitude. Notably, the position controller exhibits a larger time constant compared to the attitude controller. This article applies the designed control protocol to the outer loop system to achieve multi-UAV formation tracking. Denote $p_i = [p_{iX}, p_{iY}]^T$, $v_i = [v_{iX}, v_{iY}]^T$, where p_{iX}, p_{iY} indicate the positions, v_{iX}, v_{iY} are the velocities, the outer loop dynamics of UAV *i* in the

swarm systems are depicted as follows

$$\begin{cases} \dot{p}_i = v_i \\ m_i \dot{v}_i = -T_{\tau_i} R_i e_3 + m_i g e_3 i = 1, 2, \dots, N \end{cases}$$
(27)

where m_i, g represent the the mass and gravitational acceleration, respectively. $R_i \in \mathbb{R}^{3\times3}$ is the rotation matrix, T_{τ_i} is lift, $e_3 = [0, 0, 1]^T$. Furthermore, let control input $u_i = -\frac{T_{\tau_i}}{m_i}R_ie_3 + ge_3$, then the UAV dynamics (27) can be transformed into the model described in (2). The multi-UAV system comprises of three leaders and six followers. The interaction topology is depicted in Fig. 4.

The initial state of UAVs are randomly generated. The expected formation is defined:

$$h_{i} = 2 \begin{bmatrix} \sin\left(t + \frac{i}{3}\pi\right) \\ \cos\left(t + \frac{i}{3}\pi\right) \\ \cos\left(t + \frac{i}{3}\pi\right) \\ -\sin\left(t + \frac{i}{3}\pi\right) \end{bmatrix}, i = 1, 2, \cdots, 6$$
(28)

The unknown control input $r_k = [r_{kX}, r_{kY}]^T$ (k = 1, 2, 3)of leaders are given by $r_1 = [6.5 \cos 0.1t, 5.4 \sin 0.1t]^T$, $r_2 = [6.2 \cos 0.1t, 5.8 \sin 0.1t]^T$, $r_3 = [5.3 \cos 0.1t, 6.8 \sin 0.1t]^T$, and ς is selected as 4. The gain matrix K is chosen as $K = I_2 \otimes [3, 0.5]$, and the parameter ρ is 5.

The simulation results are presented in Figs. 5-7. Fig. 5 gives state snapshots of the UAVs at different time instances, specifically t = 0.2 s, 1.5 s, and 3 s. In the simulation, the leaders are depicted by diamonds in various colors, while the followers are represented by pentagrams. At the initial moment, UAVs are randomly distributed in space. To complete formation tracking tasks, UAVs begin to move. At t = 3s, the snapshot shows the followers form the hexagonal formation around the leaders. Fig. 6 shows the trajectories of the UAVs within 50 s. Define $\mathcal{L} = [e_1^T, e_2^T, \cdots, e_6^T]^T$, where $e_i = x_i - h_i - x_0$, it can be observed that the combined formation errors $\mathcal{L}^T \mathcal{L}$ of the whole multi-UAV systems converge to 0 in the finite time in Fig. 7.

V. CONCLUSION

This article discussed average formation tracking issue, where the leaders have unknown control inputs. By employing the sliding mode technique, a tracking control protocol was developed. Additionally, a sufficient condition for



Fig. 5. State snapshots of the UAVs at different times.



Fig. 6. Agents trajectories within 50s.



Fig. 7. Time-varying formation error

achieving formation tracking task was provided. In the future, the research can be extended to the group formation tracking problem, and several subgroups can complete different tasks. Furthermore, it is of great significance to take into account additional practical constraints, such as actuator failure, collision avoidance and input saturation, which obviously makes the problem more complex and challenging.

REFERENCES

- M. Akafan, B. Minaei-Bidgoli, and A. Bagheri, "Determining the membership percentage of each node in overlapping communities using multi-agent collective intelligence." *Engineering Letters*, vol. 28, no. 4, pp. 996–1010, 2020.
- [2] H. Miao and N. Yu, "Targeted multi-agent communication with deep metric learning." *Engineering Letters*, vol. 31, no. 2, pp. 712–723, 2023.
- [3] M. H. Trinh, Q. Van Tran, D. Van Vu, P. D. Nguyen, and H.-S. Ahn, "Robust tracking control of bearing-constrained leader-follower formation," *Automatica*, vol. 131, p. 109733, 2021.
- [4] S. Li and X. Fang, "A modified adaptive formation of uav swarm by pigeon flock behavior within local visual field," *Aerospace Science* and *Technology*, vol. 114, p. 106736, 2021.
- [5] V. S. M. Rosa and E. M. Belo, "Virtual structure formation flight control based on nonlinear mpc," in 2021 International Conference on Unmanned Aircraft Systems (ICUAS). IEEE, 2021, pp. 1383–1390.
- [6] W. Ren, "Consensus strategies for cooperative control of vehicle formations," *IET Control Theory & Applications*, vol. 1, no. 2, pp. 505–512, 2007.
- [7] P. Yang, A. Zhang, and D. Zhou, "Event-triggered finite-time formation control for multiple unmanned aerial vehicles with input saturation," *International Journal of Control, Automation and Systems*, vol. 19, pp. 1760–1773, 2021.
- [8] H. Liu, Y. Wang, F. L. Lewis, and K. P. Valavanis, "Robust formation tracking control for multiple quadrotors subject to switching topologies," *IEEE Transactions on Control of Network Systems*, vol. 7, no. 3, pp. 1319–1329, 2020.
- [9] M. Fu and L. Yu, "Finite-time extended state observer-based distributed formation control for marine surface vehicles with input saturation and disturbances," *Ocean Engineering*, vol. 159, pp. 219– 227, 2018.
- [10] L. Brinón-Arranz, A. Seuret, and C. Canudas-de Wit, "Cooperative control design for time-varying formations of multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 59, no. 8, pp. 2283– 2288, 2014.
- [11] X. Dong, Y. Zhou, Z. Ren, and Y. Zhong, "Time-varying formation control for unmanned aerial vehicles with switching interaction topologies," *Control Engineering Practice*, vol. 46, pp. 26–36, 2016.

- [12] Y. Gong, G. Wen, Z. Peng, T. Huang, and Y. Chen, "Observer-based time-varying formation control of fractional-order multi-agent systems with general linear dynamics," IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 67, no. 1, pp. 82–86, 2019. [13] Y. Guo, J. Zhou, G. Li, and J. Zhang, "Robust formation tracking
- and collision avoidance for uncertain nonlinear multi-agent systems subjected to heterogeneous communication delays," Neurocomputing, vol. 395, pp. 107-116, 2020.
- [14] Z. Yang, S. Li, D. Yu, and C. Chen, "Bls-based formation control for nonlinear multi-agent systems with actuator fault and input saturation," Nonlinear Dynamics, vol. 109, no. 4, pp. 2657–2673, 2022. [15] A. T. Hafez, S. N. Givigi, and S. Yousefi, "Unmanned aerial vehi-
- cles formation using learning based model predictive control," Asian Journal of Control, vol. 20, no. 3, pp. 1014-1026, 2018.
- [16] X. Dong and G. Hu, "Time-varying formation tracking for linear
- vol. 38, no. 3, pp. 751-766, 2000.
- [18] Q. Li, J. Wei, Q. Gou, and Z. Niu, "Distributed adaptive fixed-time formation control for second-order multi-agent systems with collision avoidance," Information Sciences, vol. 564, pp. 27-44, 2021.