Integral Solutions of the Ternary Diophantine Equation $945(x^\wedge 2+y^\wedge 2)-1889xy+10(x+y)+100=pz^\wedge q$

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***Abstract*—This article introduces a new equation of the form $945(x^2+y^2)-1889xy+10(x+y)+100=pz^q$ and explores its solution by gradually changing the parameter q from 1 to 10 with generalized value $q=m$. A transformation $x=u+v$ and $y=u-v$ simplifies the equation which results $(u+10)^2+3779v^2=pz^q$. We have employed multiple techniques to discover integral solutions for the equation and conducted a thorough analysis to examine the properties and characteristics of these solutions.**

***Index Terms*—Figurate numbers, Integral solutions, Linear equation, quadratic equation, cubic equation, biquadratic equation, pentic equation.**

I. Introduction

DIOPHANTINE equations are polynomial equations that require integer solutions. They are named after the ancient Greek mathematician Diophantus and have been studied extensively due to their diverse applications in cryptography, number theory, and other mathematical fields. This research article examines various methods for finding non-zero distinct integer solutions in non-homogeneous Diophantine equations. Our discussion includes multiple studies by different researchers, each focusing on specific equations and presenting unique techniques for determining integral solutions. In this article we studied the characteristics of the respective diophantine equation and discuss about some special numbers like figurate numbers etc [1]- [4].

Several researchers have delved into the study of various Diophantine equations. P. Jayakumar, V. Pandian, and A. Nirmala have investigated the homogeneous Diophantine equation $5(x^2+y^2)-9xy=23z^2$ and introduced a linear transformation to simplify it [22]. They discovered relations between special numbers and solutions to the equation, found non-negative integer solutions, and made noteworthy observations. M.A. Gopalan, S. Mallika, and S. Vidhyalakshmi focused on finding non-zero distinct integral solutions for a non-homogeneous cubic equation with three unknowns, presenting three methods for determining integral solutions [17]. M.A. Gopalan, S. Vidhyalakshmi, and R. Maheshwari explored five distinct patterns of non-zero integer solutions for the homogeneous cone defined by the diophantine equation $4x^2+y^2-7xy=16z^2$, identifying and describing these patterns [16]. M.A. Gopalan and V. Geetha obtained an infinite number of non-zero distinct integral points on the homogeneous cone, the equation $7(x^2+y^2)-13xy=28z^2$, revealing exciting relations between the solutions obtained and particular number patterns [18]. S. Vidhyalakshmi, T.R. Usharani, and K. Hemalatha successfully obtained an infinite number of non-zero distinct integer solutions for the ternary quadratic Diophantine equation $(x^2+y^2)-xy+x+y+1=16z^2$ [32]. K. Dhivya and T.R. Usha Rani comprehensively investigated a non-homogeneous quadratic equation with five unknowns. They obtained a set of integer solutions, discovering intriguing relationships between these solutions and unique numbers [13]. S. Vidhyalakshmi and M.A. Gopalan attempted to find non-zero distinct integer solutions to the non-homogeneous cubic equation with three unknowns represented by the equation $x^2+xy+y^2=(m^2+3n^2)z^3$ [33]. S. Thenmozhi and S. Vidhyalakshmi presented an infinite number of non-zero distinct integer solutions to the ternary quadratic equation represented by $3(x^2+y^2)-5xy=75z^2$, exploring four different integer solutions for the given equation [30]. The purpose of this study is to identify essential points on mathematical surfaces and understand their properties to gain a better understanding of these objects. A. Priya and S. Vidhyalakshmi have found infinite non-zero distinct integer solutions for the ternary quadratic Diophantine equation represented by $2(x^2+y^2)-3xy+(x+y)+1=z^2$. They also discovered four sets of distinct integer solutions and their relationship with polygonal numbers [5].

Researchers can explore other quadratic equations with variables greater than or equal to 3 to determine their unique properties. N. Bharathi and S. Vidhyalakshmi have searched for all integer solutions for the ternary quadratic equation represented by the equation $x^2-xy+y^2+2(x+y)+4=12z^2$ [20]. Similarly, researchers can explore other ternary quadratic equations to identify integer solutions and their properties. Researchers can investigate other general integer solutions for cubic equations with three unknowns to gain a deeper understanding of the properties and characteristics of these equations. P. Jayakumar and G. Shankarakalidoss have identified six distinct patterns of non-zero integer solutions for homogeneous equations represented by $4(x^2+y^2)-3xy=19z^2$ [21]. Further research is encouraged to explore other patterns of solutions and their related properties. G. Janaki and C. Saranya have found infinite non-zero distinct integer solutions for the ternary quadratic Diophantine equation represented by $5(x^2+y^2)-6xy=4z^2$ [10]. Further exploration can uncover new configurations of integer so-

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lutions and expand the understanding of the rich structure and properties inherent in the equation. G. Janaki and C. Saranya have presented four distinct patterns of non-zero integer solutions for the non-homogeneous cone associated with the non-homogeneous cubic equation involving three unknowns $3(x^2+y^2)-4xy+2(x+y+1)=972z^3$ [11]. Further investigations are needed to discover other patterns of non-zero integer solutions and explore the corresponding properties. C. Saranya and P. Gayathri have presented three different patterns of non-zero distinct integer solutions for the non-homogeneous cone represented by the ternary cubic Diophantine equation $6(x^2+y^2)-11xy=288z^3$ [6]. The study also discusses exciting aspects related to the solutions. Researchers have examined non-zero unique integer solutions to several quadratic and cubic Diophantine equations, revealing exciting relationships between the solutions and unique polygonal and pyramidal numbers.

Researchers have studied the non-zero unique integer solutions to the ternary homogeneous quadratic equation $3(x^2+y^2)-4xy=42z^2$, as well as the non-zero unique integer points on the Ternary Quadratic Diophantine Equation that represents a non-homogeneous cone [14]. The equation is given by $6(x^2+y^2)-11xy+3x+3y+9=72z^2$ [9].Researchers have also examined the Homogeneous Ternary Quadratic Diophantine Equation $2x^2-3xy+2y^2=56z^2$, as well as the ternary cubic equation $5(x^2+y^2)-9xy+x+y+1=28z^3$ and $5(x^2+y^2)-9xy+x+y+1=35z^3$ [7], [23], [31] . In each case, researchers have discovered various patterns of non-zero distinct integer solutions, uncovering intriguing connections between the solutions of the equations and unique polygonal, pyramidal, and central pyramidal numbers. Additionally, researchers have derived formulas for generating sequences of solutions based on given solutions. By exploring these equations and their properties, researchers can gain valuable insights into the nature of their integer solutions, expanding our understanding of ternary quadratic Diophantine equations.

Thangamalar has discovered three unique patterns of non-zero integer solutions for a specific non-homogeneous cone equation [15]. Their study found various integer points that satisfy the given equation and identified exciting relationships between these solutions and specific number patterns. Their research suggests conducting further investigations to explore patterns of non-zero integer solutions and related properties for other ternary quadratic Diophantine equations. This research contributes to understanding the complex relationships between equations, solutions, and unique numbers in these equations. Similarly, Manju Somanath, Radhika Das, and Bindu V.A. analyzed a homogeneous cubic equation with four unknowns and discovered four distinct patterns of non-zero integral solutions [19]. They observed interesting relations between these solutions and unique numbers, such as Polygonal, Three-Dimensional Figurate, Star, Rhombic Dodecahedral, and more. This study highlights the diverse nature of Diophantine equations. Researchers should continue investigating different solution patterns and related properties to further understand the relationships between equations, solutions, and unique numbers in Diophantine equations. The equations $x^3+y^3=13(z+w)p^2$ $2(l^2+m^2)-3lm=56t^3$, $x^2+xy+y^2=12z^2$, $7(x^2+y^2)-13xy+x+y+1=31z^2$, $(x^2-y^2)(5x^2+5y^2-8xy)=13(x^2-y^2)z^3$, $21(x^2+y^2)-41xy=99z^7$, $x^2+y^2+2(x+y)+2=10z^2$ are also studied [8], [12], [24]- [29].

This article delves into different equations, such as linear, quadratic, cubic and more. We showcase various techniques and transformations to obtain multiple non-zero distinct integer solutions patterns. This research provides insights into the behaviour and structure of Diophantine equations, which has implications for cryptography and number theory. Additionally, we suggest future research avenues that urge an investigation into other forms of Diophantine equations and their properties.

II. Notations

$T_{m,n}-n^{th}$ m-gonal number
$P(n)-n^{th}$ pronic number
$C(n)-n^{th}$ cubic number
O_n-n^{th} octahedral number
SO_n-n^{th} Stella octangular number
TO_n-n^{th} truncated octahedral number
Gno_n-n^{th} Gnomonic number
${P_n}^3-(n-1)^{th}$ Tetrahedral number
${P_n}^5-n^{th}$ Pentagonal Pyramidal number
${P_n}^6-n^{th}$ Hexagonal Pyramidal Number
${P_n}^7-n^{th}$ Heptagonal Pyramidal Number
RD_n-n^{th} Rhombic dodecagonal Number
HO_n-n^{th} Hauy Octahedral Number
$C^k(n)-n^{th}$ k-dimensional hypercube number.
Nex_n-n^{th} Nexus number.
PT_n-n^{th} Pentatope number.
$4DF_n-n^{th}$ Four dimensional figurate number whose generating polygon is a square.

III. Section-1

If $q=1$ then $945(x^2+y^2)-1889xy+10(x+y)+100=pz$

A. Technique for Analysis

For obtaining non zero integral solution the linear equation can be solved as follows:

$$945(x^2+y^2)-1889xy+10(x+y)+100=pz \tag{1}$$

Note that $p=l+k^2$, where some fixed integer $l=3779$ and k is a perfect number.
When the transformation is substituted,

$$y=u-v,\ x=u+v \tag{2}$$

Hence Equation (1) becomes

$$(u+10)^2+3779v^2=pz \tag{3}$$

We put different values of k in equation (1) to obtain different types of equations and by solving these equations, we have

Example − 1
If $k=6\Rightarrow p=3815$

Technique − 1
Assume

$$Z=a^2+3779b^2 \tag{4}$$

Note that a and b are non-zero integers in the above equation. And write

$$3815 = \frac{(6n + in\sqrt{3779})(6n - in\sqrt{3779})}{n^2} \quad (5)$$

Replacing equations (4) and (5) in equation (3) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(6n+in\sqrt{3779})(6n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)(a-i\sqrt{3779}b)) \quad (6)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = u(a,b) = 6a - 3779b - 10$
$v = v(a,b) = a + 6b$
the integral solutions of (1) is obtained as follows by substituting the values of u, v in equation (2)
$x = x(a,b) = 7a - 3773b - 10$
$y = y(a,b) = 5a - 3785b - 10$
$z = z(a,b) = a^2 + 3779b^2$

Properties

- $x(1,n) - x(n,1) + 1890Gno_n$ is a perfect number.
- $y(1,n) - y(n,1) + 1895Gno_n \equiv 0(mod\ 379)$
- $x(1,n) - y(n,1) + 1889Gno_n$ is a smith number.
- $x(n,1) - y(1,n) - 1896Gno_n \equiv 0(mod\ 941)$

TABLE I
NUMERICAL EXAMPLES OF TECHNIQUE - 1

$b = 20a$

x	y	z
-75463	-75705	1511601
-150916	-151400	6046404
-226369	-227095	13604409
-301822	-302790	24185616
-377275	-378485	37790025

Example – 2
If $k = 28 \Rightarrow p = 4563$

Technique – 2
write

$$4563 = \frac{(28n + in\sqrt{3779})(28n - in\sqrt{3779})}{n^2} \quad (7)$$

Replacing equations (4) and (7) in equation (3) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(28n+in\sqrt{3779})(28n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)(a-i\sqrt{3779}b)) \quad (8)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = u(a,b) = 28a - 3779b - 10$
$v = v(a,b) = a + 28b$
the integral solutions of (1) is obtained as follows by substituting the values of u, v in equation (2)
$x = x(a,b) = 29a - 3751b - 10$
$y = y(a,b) = 27a - 3807b - 10$
$z = z(a,b) = a^2 + 3779b^2$

Properties

- $x(1,n) - y(1,n) - 28Gno_n$ is a super niven number.
- $y(n,1) - x(1,n) - 1889Gno_n \equiv 0(mod\ 59)$
- $x(n,1) - y(n,1) - Gno_n$ is a leyland number.
- $x(n,1) - y(1,n) - 1918Gno_n \equiv 0(mod\ 31)$

TABLE II
NUMERICAL EXAMPLES OF TECHNIQUE-2

$a = 20b$

x	y	z
-3181	-3277	4179
-6352	-6544	16716
-9523	-9811	37611
-12694	-13078	66864
-15865	-16345	104475

Example – 3
If $k = 496 \Rightarrow p = 249795$

Technique – 3
write

$$249795 = \frac{(496n + in\sqrt{3779})(496n - in\sqrt{3779})}{n^2} \quad (9)$$

Replacing equations (4) and (9) in equation (3) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(496n+in\sqrt{3779})(496n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)(a-i\sqrt{3779}b)) \quad (10)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = u(a,b) = 496a - 3779b - 10$
$v = v(a,b) = a + 496b$
the integral solutions of (1) is obtained as follows by substituting the values of u, v in equation (2)
$x = x(a,b) = 497a - 3283b - 10$
$y = y(a,b) = 495a - 4275b - 10$
$z = z(a,b) = a^2 + 3779b^2$

Properties

- $x(1,n) - y(1,n) - 496Gno_n$ is a primitive abundant number.
- $y(n,1) - y(1,n) - 2385Gno_n \equiv 0(mod\ 53)$
- $x(n,1) - y(n,1) - Gno_n \equiv 0(mod\ 331)$
- $x(n,1) - x(1,n) - 1918Gno_n \equiv 0(mod\ 7)$

TABLE III
NUMERICAL EXAMPLES OF TECHNIQUE - 3

$a = 30b$

x	y	z
11617	10565	4679
23244	21140	18716
34871	31715	42111
46498	42290	74864
58125	52865	116975

Example – 4
If $k = 8128 \Rightarrow p = 66068163$

Technique – 4
write

$$66068163 = \frac{(8128n + in\sqrt{3779})(8128n - in\sqrt{3779})}{n^2} \quad (11)$$

Replacing equations (4) and (11) in equation (3) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(8128n+in\sqrt{3779})(8128n-in\sqrt{3779})}{n^2} ((a + i\sqrt{3779}b)(a - i\sqrt{3779})b) \quad (12)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = u(a, b) = 8128a - 3779b - 10$
$v = v(a, b) = a + 8128b$
the integral solutions of (1) is obtained as follows by substituting the values of u, v in equation (2)
$x = x(a, b) = 8129a + 4349b - 10$
$y = y(a, b) = 8127a - 11907b - 10$
$z = z(a, b) = a^2 + 3779b^2$

Properties

- $x(1, n) + y(n, 1) - 6238Gno_n$ is a gapful number.
- $x(n, 1) - y(1, n) - 10018Gno_n$ is a saint exupery number.
- $x(1, n) - y(1, n) - 8128Gno_n \equiv 0(mod\ 271)$
- $x(n, 1) - y(n, 1) - Gno_n \equiv 0(mod\ 5419)$

TABLE IV
NUMERICAL EXAMPLES OF TECHNIQUE - 4

$a = 40b$

x	y	z
329499	313163	5379
659008	626336	21516
988517	939509	48411
1318026	1252682	86064
1647535	1565855	134475

IV. SECTION - 2

If $q = 2$ then $945(x^2+y^2)-1889xy+10(x+y)+100 = pz^2$

A. Technique for Analysis

For obtaining non zero integral solution the quadratic equation can be solved as follows:

$$945(x^2 + y^2) - 1889xy + 10(x + y) + 100 = pz^2 \quad (13)$$

Note that $p = l + k^2$, where some fixed integer $l = 3779$ and k is an abundant number.
When the transformation is substituted,

$$y = u - v,\ x = u + v, \quad (14)$$

Hence Equation (13) becomes

$$(u + 10)^2 + 3779v^2 = pz^2 \quad (15)$$

We put different values of k in equation (13) to obtain different types of equations and by solving these equations, we have

Example – 1
If $k = 12 \Rightarrow p = 3923$,

Technique – 5
write

$$3923 = \frac{(12n + in\sqrt{3779})(12n - in\sqrt{3779})}{n^2} \quad (16)$$

Replacing equations (4) and (16) in equation (15) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(12n+in\sqrt{3779})(12n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^2(a-i\sqrt{3779}b)^2) \quad (17)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = u(a, b) = 12a^2 - 7558ab - 45348b^2 - 10$
$v = v(a, b) = a^2 + 24ab - 3779b^2$
the integral solutions of (1) is obtained as follows by substituting the values of u, v in equation (2)
$x = x(a, b) = 13a^2 - 7534ab - 49127b^2 - 10$
$y = y(a, b) = 11a^2 - 7582ab - 41569b^2 - 10$
$z = z(a, b) = a^2 + 3779b^2$

Properties

- $-[x(n, n + 1) + 12433T_{4,n} + 8679Gno_n]$ is a sphenic number.
- $x(n, 2n-1)+34703T_{4,n}-13591Gno_n$ is a weak prime number.
- $-[y(n+1, n+2)+6T_{16382,n}+119067Gno_n]$ is a happy number.
- $y(n, 2n - 1) + 397T_{916,n} + 3587Gno_n \equiv 0(mod\ 2053)$

TABLE V
NUMERICAL EXAMPLES OF TECHNIQUE - 5

$a = 85b$

x	y	z
-595602	-606574	11004
-2382378	-2426266	44016
-5360338	-5459086	99036
-9529482	-9705034	176064
-14889810	-15164110	275100

Technique – 6
Equation (15) can be written as

$$(u+10)^2 = pz^2 - 3779v^2, where\ p = 3923 \qquad (18)$$

introducing the linear transformation

$$z = X + 3779T\ and\ v = X + 3923T \qquad (19)$$

in (18) we get

$$X^2 = 14825017T^2 + \left(\frac{u+10}{12}\right)^2 \qquad (20)$$

which is satisfied by

$$\left.\begin{array}{l} T(c,d) = 2cd \\ u(c,d) = 177900204c^2 - 12d^2 - 10 \\ X(c,d) = 14825017c^2 + d^2 \end{array}\right\} \qquad (21)$$

substituting the values of (21) in (19) and using (14) the corresponding integer solutions of (13) are given by,

$$\left.\begin{array}{l} x = 192725221c^2 - 11d^2 + 7846cd - 10 \\ y = 163075187c^2 - 13d^2 - 7846cd - 10 \\ z = 14825017c^2 + d^2 + 7558cd \end{array}\right\} \qquad (22)$$

Thus the equation (22) represent a nonzero distinct integral solutions of (13) in two parameters.

Properties

- $x(1,n) - 11T_{4,n} - 3923Gno_n \equiv 0(mod\ 887)$
- $x(n,1) - 14825017T_{28,n} - 88957948Gno_n$ is a super - 2 number.
- $y(1,n) + 13T_{4,n} + 3923Gno_n \equiv 0(mod\ 7333)$
- $y(n,1) - 14825017T_{24,n} - 74121162Gno_n$ is an emirpimes number.

TABLE VI
NUMERICAL EXAMPLES OF TECHNIQUE - 6

$d = 95c$

x	y	z
193371306	162212482	15552052
773485254	648849958	62208208
1740341834	1459912418	139968468
3093941046	2595399862	248832832
4834282890	4055312290	388801300

Note
In addition to (14) one may also consider the linear transformations $z = X - 3779T$ and $v = X - 3923T$ following the method presented above different set of solutions are obtained.

Technique – 7
Consider (15) as

$$(u+10)^2 - 144v^2 = p(z^2 - v^2), where\ p = 3923 \qquad (23)$$

Write (23) in the form of ratio as

$$\frac{(u+10)+12v}{z+v} = \frac{3923(z-v)}{(u+10)-12v} = \frac{A}{B}, B \neq 0$$

Which is equivalent to the following two equations
$B(u+10) + (12B - A)v - Az = 0$
$A(u+10) + (3923B - 12A)v - 3923BZ = 0$
on employing the method of cross multiplication, we get

$$\left.\begin{array}{l} u = -12A^2 - 47076B^2 + 7846AB - 10 \\ v = A^2 - 3923B^2 \end{array}\right\} \qquad (24)$$

$$z = A^2 - 24AB + 3923B^2 \qquad (25)$$

Substituting the values of u and v from (24) in (14), the non-zero distinct integral values of x and y are given by

$$\left.\begin{array}{l} x = -11A^2 - 50999B^2 + 7846AB - 10 \\ y = -13A^2 + 7846AB - 43153B^2 - 10 \\ z = A^2 - 24AB + 3923B^2 \end{array}\right\} \qquad (26)$$

Thus the equation (26) represent non-zero distinct integral solutions of (13) in two parameters.

Properties

- $-z(1,n) + 3923T_{4,n} - 12Gno_n$ is a sophie germain number.
- $x(1,n) - 3923T_{28,n} - 27461Gno_n$ is a tau number.
- $y(1,n) + 3923T_{24,n} + 15692Gno_n \equiv 0(mod\ 449)$

TABLE VII
NUMERICAL EXAMPLES OF TECHNIQUE - 7

$A = 75B$

x	y	z
475566	472162	7748
1902294	1888678	30992
4280174	4249538	69732
7609206	7554742	123968
11889390	11804290	193700

Technique − 8
Write (23) in the form of ratio as

$$\frac{(u+10)+12v}{z-v} = \frac{3923(z+v)}{(u+10)-12v} = \frac{A}{B}, B \neq 0$$

Solutions of technique − 8

$$\left.\begin{aligned} x &= A^2 + 43153B^2 + 7846AB - 10 \\ y &= 11A^2 + 7846AB + 50999B^2 - 10 \\ z &= A^2 - 24AB + 3923B^2 \end{aligned}\right\} \tag{27}$$

Thus the equation (27) represent non-zero distinct integral solutions of (13) in two parameters.

Properties

- $z(1,n) - 3923T_{4,n} - 12Gno_n$ is a 2nd star number.
- $x(1,n) - 11T_{7848,n} - 25494Gno_n$ is an unprimeable number.
- $y(1,n) - 13T_{7848,n} - 29416Gno_n$ is an interprime number.

TABLE VIII
NUMERICAL EXAMPLES OF TECHNIQUE - 8

$A = 65B$

x	y	z
557358	607454	6588
2229462	2429846	26352
5016302	5467166	59292
8917878	9719414	105408
13934190	15186590	164700

Technique − 9
Write (23) in the form of ratio as

$$\frac{(u+10)+12v}{3923(z-v)} = \frac{z+v}{(u+10)-12v} = \frac{A}{B}, B \neq 0$$

Solutions of technique − 9

$$\left.\begin{aligned} x &= 50999A^2 + 11B^2 + 7846AB - 10 \\ y &= 43153A^2 + 7846AB + 13B^2 - 10 \\ z &= 3923A^2 + 24AB + B^2 \end{aligned}\right\} \tag{28}$$

Thus the equation (28) represent non-zero distinct integral solutions of (13) in two parameters.

Properties

- $z(1,n) - T_{4,n} - 12Gno_n$ is a semiprime number.
- $x(1,n) - 11T_{4,n} - 3923Gno_n$ is a pseudo perfect number.
- $y(1,n) - 13T_{4,n} - 3923Gno_n \equiv 0(mod\ 233)$

TABLE IX
NUMERICAL EXAMPLES OF TECHNIQUE - 9

$B = 55A$

x	y	z
515794	513998	8268
2063206	2056022	33072
4642226	4626062	74412
8252854	8224118	132288
12895090	12850190	206700

Example − 2
If $k = 18 \Rightarrow p = 4103$

Technique − 10
write

$$4103 = \frac{(18n + in\sqrt{3779})(18n - in\sqrt{3779})}{n^2} \tag{29}$$

Replacing equations (4) and (29) in equation (15) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(18n+in\sqrt{3779})(18n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^2(a-i\sqrt{3779}b)^2) \tag{30}$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = u(a,b) = 18a^2 - 7558ab - 68088b^2 - 10$
$v = v(a,b) = a^2 + 36ab - 3779b^2$
the integral solutions of (13) is obtained as follows by substituting the values of u, v in equation (14)
$x = x(a,b) = 19a^2 - 7522ab - 71801b^2 - 10$
$y = y(a,b) = 17a^2 - 7594ab - 64243b^2 - 10$
$z = z(a,b) = a^2 + 3779b^2$

Properties

- $-[x(n, 2n+1) + 9T_{67164,n} + 298473Gno_n]$ is an untouchable number.
- $x(n+1, n-1) + 14683T_{4,n} - 7199Gno_n$ is a curzon number.
- $-[y(2n-1, n) + 19T_{8356,n} + 35909Gno_n]$ is a self number.
- $-y(n+1, n) - 3T_{9336,n} - 10779Gno_n$ is a polite number.

TABLE X
NUMERICAL EXAMPLES OF TECHNIQUE - 10

$a = 100b$

x	y	z
-634011	-653653	13779
-2536014	-2614582	55116
-5706019	-5882797	124011
-10144026	-10458298	220464
-15850035	-16341085	344475

Example – 3
If $k = 20 \Rightarrow p = 4179$

Technique – 11
write

$$4179 = \frac{(20n + in\sqrt{3779})(20n - in\sqrt{3779})}{n^2} \quad (31)$$

Replacing equations (4) and (31) in equation (15) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(20n+in\sqrt{3779})(20n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^2(a-i\sqrt{3779}b)^2) \quad (32)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = u(a,b) = 20a^2 - 7558ab - 75580b^2 - 10$
$v = v(a,b) = a^2 + 40ab - 3779b^2$
the integral solutions of (13) is obtained as follows by substituting the values of u, v in equation (14)
$x = x(a,b) = 21a^2 - 7518ab - 79359b^2 - 10$
$y = y(a,b) = 19a^2 - 7598ab - 71801b^2 - 10$
$z = z(a,b) = a^2 + 3779b^2$

Properties
- $-[x(n-1, n-1) + 2T_{86858,n} - 43429Gno_n]$ is a strong prime number.
- $x(3n-1, 3n) - 2T_{781706,n} - 402065Gno_n$ is a wasteful number.
- $-[y(3n, 3n-1) + 2T_{714422,n} + 130409Gno_n]$ is a psudo perfect number.

TABLE XI
NUMERICAL EXAMPLES OF TECHNIQUE - 11

$a = 90b$

x	y	z
-585889	-601731	11879
-2343526	-2406894	47516
-5272921	-5415499	106911
-9374074	-9627546	190064
-14646985	-15043035	296975

Example – 4
If $k = 24 \Rightarrow p = 4355$

Technique – 12
write

$$4355 = \frac{(24n + in\sqrt{3779})(24n - in\sqrt{3779})}{n^2} \quad (33)$$

Replacing equations (4) and (33) in equation (15) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(24n+in\sqrt{3779})(24n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^2(a-i\sqrt{3779}b)^2) \quad (34)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = u(a,b) = 24a^2 - 7558ab - 90696b^2 - 10$
$v = v(a,b) = a^2 + 48ab - 3779b^2$
the integral solutions of (13) is obtained as follows by substituting the values of u, v in equation (14)
$x = x(a,b) = 25a^2 - 7510ab - 94475b^2 - 10$
$y = y(a,b) = 23a^2 - 7606ab - 86917b^2 - 10$
$z = z(a,b) = a^2 + 3779b^2$

Properties
- $-[x(1,n) + 5T_{37792,n} + 50990Gno_n]$ is a deficient number.
- $-x(n,1) + T_{52,n} - 3743Gno_n$ is an amenable number.
- $y(1,n) - 23T_{7560,n} - 47250Gno_n]$ is a chen prime number.
- $y(n,1) + 23T_{4,n} - 3803Gno_n \equiv 0(mod\ 211)$

TABLE XII
NUMERICAL EXAMPLES OF TECHNIQUE - 12

$a = 80b$

x	y	z
-535285	-548207	10179
-2141110	-2192798	40716
-4817485	-4933783	91611
-8564410	-8771162	162864
-13381885	13704935	254475

V. SECTION - 3

If $q = 3$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^3$

A. Technique for Analysis

For obtaining non zero integral solution the cubic equation can be solved as follows:

$$945(x^2 + y^2) - 1889xy + 10(x + y) + 100 = pz^3 \quad (35)$$

Note that $p = l + k^2$,where some fixed integer $l = 3779$ and k is an abundant number.
When the transformation is substituted,

$$y = u - v,\ x = u + v, \quad (36)$$

Hence Equation (35) becomes

$$(u + 10)^2 + 3779v^2 = pz^3 \quad (37)$$

We put different values of k in equation (35) to obtain different types of equations and by solving these equations, we have

Example − 1
If $k = 30 \Rightarrow p = 4679$

Technique − 13
write

$$4679 = \frac{(30n + in\sqrt{3779})(30n - in\sqrt{3779})}{n^2} \tag{38}$$

Replacing equations (4) and (38) in equation (37) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(30n+in\sqrt{3779})(30n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^3(a-i\sqrt{3779}b)^3) \tag{39}$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = 30a^3 + 142808841b^3 - 11337a^2b - 340110ab^2 - 10$
$v = a^3 - 113370b^3 + 90a^2b - 11337ab^2$
the integral solutions of (35) is obtained as follows by substituting the values of u, v in equation (36)
$x = 31a^3 + 141167471b^3 - 11247a^2b - 351447ab^2 - 10$
$y = 29a^3 + 143942111b^3 - 11427a^2b - 328773ab^2 - 10$
$z = a^2 + 3779b^2$

Properties

- $x(1,n) - 28334942{P_n}^5 + 854054T_{36,n} - 13676111T_{4,n} + 13676111p(n)$ is an alternating number.
- $y(1,n) - 86365266{P_n}^3 - 11337T_{60,n} - 7350110Gno_n \equiv 0(mod\ 272227)$
- $31y(n,1) - 29x(n,1) + 28074T_{4,n}$ is a wasteful number.

TABLE XIII
NUMERICAL EXAMPLES OF TECHNIQUE - 13

$a = 70b$

x	y	z
-54911129	-54665209	8679
-439288962	-437321602	34716
-1482600223	-1475960383	78111
-3514311626	-3498572746	138864
-6863889885	-6833149885	216975

Example − 2
If $k = 36 \Rightarrow p = 5075$

Technique − 14
write

$$5075 = \frac{(36n + in\sqrt{3779})(36n - in\sqrt{3779})}{n^2} \tag{40}$$

Replacing equations (4) and (40) in equation (37) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(36n+in\sqrt{3779})(36n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^3(a-i\sqrt{3779}b)^3) \tag{41}$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = 36a^3 + 142808841b^3 - 11337a^2b - 408132ab^2 - 10$
$v = a^3 - 136044b^3 + 108a^2b - 11337ab^2$
the integral solutions of (35) is obtained as follows by substituting the values of u, v in equation (36)
$x = 37a^3 + 14144797b^3 - 11229a^2b - 419469ab^2 - 10$
$y = 35a^3 + 144168885b^3 - 11445a^2b - 396795ab^2 - 10$
$z = a^2 + 3779b^2$

Properties

- $x(1,n) - 14144797c(n) - 3779T_{224,n} + 404461T_{4,n} - 404461p(n)$ is a powerful number.
- $y(1,n) - 14416885c(n) - 18895T_{44,n} + 366455T_{4,n} - 366455p(n)$ is a proth number.
- $37y(n,1) - 35x(n,1) + 30450T_{4,n}$ is a pernicious number.

TABLE XIV
NUMERICAL EXAMPLES OF TECHNIQUE - 14

$a = 60b$

x	y	z
-43455753	-43032825	7379
-347645954	-344262530	29516
-1173305071	-1161886015	66411
-2781167562	-2754100170	118064
-5431967885	-5379101885	184475

Example − 3
If $k = 40 \Rightarrow p = 5399$

Technique − 15
write

$$5399 = \frac{(40n + in\sqrt{3779})(40n - in\sqrt{3779})}{n^2} \tag{42}$$

Replacing equations (4) and (42) in equation (37) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(40n+in\sqrt{3779})(40n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^3(a-i\sqrt{3779}b)^3) \tag{43}$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = 40a^3 + 142808841b^3 - 11337a^2b - 453480ab^2 - 10$
$v = a^3 - 151160b^3 + 120a^2b - 11337ab^2$
the integral solutions of (35) is obtained as follows by substituting the values of u, v in equation (36)
$x = 41a^3 + 141129681b^3 - 11217a^2b - 464817ab^2 - 10$
$y = 39a^3 + 144432001b^3 - 11457a^2b - 442143ab^2 - 10$
$z = a^2 + 3779b^2$

Properties

- $x(1,n) - 28259362{P_n}^5 + 7558T_{3864,n} - 14598157T_{4,n} + 14598157p(n)$ is a 6th hogben number.
- $y(1,n) - 86592006{P_n}^3 + 11337T_{80,n} - 6994869Gno_n \equiv 0(mod\ 152063)$
- $41y(n,1) - 39x(n,1) + 32274T_{4,n} \equiv 0(mod\ 6203)$

TABLE XV
NUMERICAL EXAMPLES OF TECHNIQUE - 15

$a = 50b$

x	y	z
-32028679	-31442659	6279
-256229362	-251541202	25116
-864774073	-848951533	56511
-2049834826	-2012329546	100464
-4003583635	-3930331135	156975

Example – 4
If $k = 42 \Rightarrow p = 5543$

Technique – 16
write

$$5543 = \frac{(42n + in\sqrt{3779})(42n - in\sqrt{3779})}{n^2} \quad (44)$$

Replacing equations (4) and (44) in equation (37) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(42n+in\sqrt{3779})(42n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^3(a-i\sqrt{3779}b)^3) \quad (45)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = 42a^3 + 142808841b^3 - 11337a^2b - 476154ab^2 - 10$
$v = a^3 - 158718b^3 + 126a^2b - 11337ab^2$
the integral solutions of (35) is obtained as follows by substituting the values of u, v in equation (36)
$x = 43a^3 + 141122123b^3 - 11211a^2b - 487491ab^2 - 10$
$y = 41a^3 + 144439559b^3 - 11463a^2b - 464817ab^2 - 10$
$z = a^2 + 3779b^2$

Properties

- $x(1, n) - 14122123c(n) + 11337T_{88,n} - 487365T_{4,n} + 487365p(n)$ is a cunningham number.
- $y(1, n) - 28879118{P_n}^5 + 128486T_{234,n} - 14787353T_{4,n} + 14787353p(n)$ is a equidigital number.
- $43y(n, 1) - 41x(n, 1) + 33258T_{4,n}$ is an arithmetic number.

TABLE XVI
NUMERICAL EXAMPLES OF TECHNIQUE - 16

$a = 40b$

x	y	z
-20563127	-19869931	5379
-164504946	-158959378	21516
-555204169	-536487877	48411
-1316039498	-1271674954	86064
-2570389635	-2483740135	134475

Example – 5
If $k = 48 \Rightarrow p = 6083$

Technique – 17
write

$$6083 = \frac{(48n + in\sqrt{3779})(48n - in\sqrt{3779})}{n^2} \quad (46)$$

Replacing equations (4) and (46) in equation (37) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(48n+in\sqrt{3779})(48n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^3(a-i\sqrt{3779}b)^3) \quad (47)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = 48a^3 + 142808841b^3 - 11337a^2b - 544176ab^2 - 10$
$v = a^3 - 181392b^3 + 144a^2b - 11337ab^2$
the integral solutions of (35) is obtained as follows by substituting the values of u, v in equation (36)
$x = 49a^3 + 140999449b^3 - 11193a^2b - 555513ab^2 - 10$
$y = 47a^3 + 144622233b^3 - 11481a^2b - 532839ab^2 - 10$
$z = a^2 + 3779b^2$

Properties

- $x(1, n) - 14099449c(n) + 11337T_{100,n} - 555369T_{4,n} + 555369p(n)$ is a deficient number.
- $y(1, n) - 86773398{P_n}^3 - 733126T_{40,n} + 13184787T_{4,n} - 13184787p(n)$ is a pancake number.
- $49y(n, 1) - 47x(n, 1) + 36498T_{4,n} \equiv 0 (mod\ 82393)$

TABLE XVII
NUMERICAL EXAMPLES OF TECHNIQUE - 17

$a = 30b$

x	y	z
-11316651	-10586847	4679
-90533138	-84694706	18716
-305549317	-285844609	42111
-724265034	-677557578	74864
-1414580135	-1323354635	116975

VI. SECTION - 4

If $q = 4$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^4$

A. Technique for Analysis

For obtaining non zero integral solution the biquadratic equation can be solved as follows:

$$945(x^2 + y^2) - 1889xy + 10(x + y) + 100 = pz^4 \quad (48)$$

Note that $p = l + k^2$,where some fixed integer $l = 3779$ and k is a deficient number.
When the transformation is substituted,

$$y = u - v,\ x = u + v \quad (49)$$

Hence Equation (48) becomes

$$(u + 10)^2 + 3779v^2 = pz^4 \quad (50)$$

We put different values of k in equation (48) to obtain different types of equations and by solving these equations, we have

Example − 1
If $k = 3 \Rightarrow p = 3782$

Technique − 18
write

$$3782 = \frac{(3n + in\sqrt{3779})(3n - in\sqrt{3779})}{n^2} \tag{51}$$

Replacing equations (4) and (51) in equation (50) and technique of factorizing is applied,

$$((u+10)+i\sqrt{3779}v)((u+10)-i\sqrt{3779}v) = \frac{(3n+in\sqrt{3779})(3n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^4(a-i\sqrt{3779}b)^4) \tag{52}$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = 3a^4 - 68022a^2b^2 + 428425236^4 - 15116a^3b - 57123364ab^3 - 10$
$v = a^4 - 22674a^2b^2 + 14280841b^4 + 12a^3b - 45348ab^3$
the integral solutions of (48) is obtained as follows by substituting the values of u, v in equation (49)
$x = 4a^4 - 90696a^2b^2 + 571233646^4 - 15104a^3b - 57168712ab^3 - 10$
$y = 2a^4 - 45348a^2b^2 + 285616826^4 - 15128a^3b - 57078016ab^3 - 10$
$z = a^2 + 3779b^2$

Properties

- $2x(n,1) - 4y(n,1) - 1894TO_n - 62502T_{4,n} - 56964592Gno_n$ is a hidden beast number.
- $14280841x(n,1) - x(1,n) + 53910163438RD_n + 2T_{1618676045270,n} + 408910160811601Gno_n \equiv 0(mod\ 16272218595427)$

TABLE XVIII
NUMERICAL EXAMPLES OF TECHNIQUE - 18

$a = 10b$

x	y	z
-538697366	-561861288	3879
-4309578858	-4494890234	15516
-14544828622	-15170254516	34911
-34476630794	-35959121802	62064
-67337169510	-70232659760	96975

Example − 2
If $k = 7 \Rightarrow p = 3828$

Technique − 19
write

$$3828 = \frac{(7n + in\sqrt{3779})(7n - in\sqrt{3779})}{n^2} \tag{53}$$

Replacing equations (4) and (53) in equation (50) and technique of factorizing is applied,

$$((u+10)+i\sqrt{3779}v)((u+10)-i\sqrt{3779}v) = \frac{(7n+in\sqrt{3779})(7n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^4(a-i\sqrt{3779}b)^4) \tag{54}$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = 7a^4 - 158718a^2b^2 + 99965887b^4 - 15116a^3b - 57123364ab^3 - 10$
$v = a^4 - 22674a^2b^2 + 142808416^4 + 28a^3b - 105812ab^3$
the integral solutions of (48) is obtained as follows by substituting the values of u, v in equation (49)
$x = 8a^4 - 181392a^2b^2 + 1142467286^4 - 15088a^3b - 57229176ab^3 - 10$
$y = 6a^4 - 136044a^2b^2 + 85685046b^4 - 15144a^3b - 57017552ab^3 - 10$
$z = a^2 + 3779b^2$

Properties

- $6x(n,1) - 8y(n,1) - 15312SO_n - 56390336Gno_n \equiv 0(mod\ 31234589)$
- $14280841x(n,1) - x(1,n) + 161559074874HO_n - 2T_{2913548279030,n} + 407399019461283Gno_n \equiv 0(mod\ 3)$

TABLE XIX
NUMERICAL EXAMPLES OF TECHNIQUE - 19

$a = 5b$

x	y	z
-178314962	-204693074	3804
-1426519626	-1637544522	15216
-4814503714	-5526712738	34236
-11412156938	-13100356106	60864
-22289369010	-25586633010	95100

VII. SECTION - 5

If $q = 5$ then $945(x^2+y^2)-1889xy+10(x+y)+100 = pz^5$

A. Technique for Analysis

For obtaining non zero integral solution the pentic equation can be solved as follows:

$$945(x^2 + y^2) - 1889xy + 10(x + y) + 100 = pz^5 \tag{55}$$

Note that $p = l + k^2$,where some fixed integer $l = 3779$ and k is a semiprime number.
When the transformation is substituted,

$$y = u - v, x = u + v, \tag{56}$$

Hence Equation (55) becomes

$$(u + 10)^2 + 3779v^2 = pz^5 \tag{57}$$

We put different values of k in equation (55) to obtain different types of equations and by solving these equations, we have

Example − 1
If $k = 6 \Rightarrow p = 3815$

Technique – 20
write

$$3815=\frac{(6n+in\sqrt{3779})(6n-in\sqrt{3779})}{n^2} \quad (58)$$

Replacing equations (4) and (58) in equation (57) and technique of factorizing is applied,

$$((u+10)+i\sqrt{3779}v)((u+10)-i\sqrt{3779}v)$$
$$\frac{(6n+in\sqrt{3779})(6n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^5(a-i\sqrt{3779}b)^5) \quad (59)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u=6a^5-18895a^4b-226740a^3b^2+142808410a^2b^3+428425230ab^4-53967298139b^5-10$
$v=a^5+30a^4b-37790a^3b^2-226740a^2b^3+71404205ab^4+856850466b^5$
the integral solutions of (55) is obtained as follows by substituting the values of u, v in equation (56)
$x=7a^5-18865a^4b-264530a^3b^2+142581670a^2b^3+499829435ab^4-538816130093b^5-10$
$y=5a^5-18925a^4b-188950a^3b^2+143035150a^2b^3+357021025ab^4-540529831185b^5-10$
$z=a^2+3779b^2$

Properties

- $y(1,n)+54052983185c^5(n)-357021025c^4(n)-71517575SO_n+7552T_{52,n}-35658629Gno_n$ is an equidigital number.
- $5x(n,1)-7y(n,1)-38150c^4(n)+288337700T_{4,n}$ is a pseudo perfect number.
- $x(n,1)-7c^5(n)+3773Nex_n+16200TO_n-7104240T_{42,n}-317608830Gno_n\equiv 0(mod\ 3207431)$

TABLE XX
NUMERICAL EXAMPLES OF TECHNIQUE - 20

$a=2b$

x	y	z
-52314045409	-52768614785	3783
-1674049452778	-1688595672810	15132
-12712313031967	-12822773390335	34047
-53569582488586	-54035061529610	60528
-163481391871885	-164901921171885	94575

VIII. SECTION - 6

If $q=6$ then $945(x^2+y^2)-1889xy+10(x+y)+100=pz^6$

A. Technique for Analysis

For obtaining non zero integral solution the hextic equation can be solved as follows:

$$945(x^2+y^2)-1889xy+10(x+y)+100=pz^6 \quad (60)$$

Note that $p=l+k^2$,where some fixed integer $l=3779$ and k is an interprime number.
When the transformation is substituted,

$$y=u-v,\ x=u+v, \quad (61)$$

Hence Equation (60) becomes

$$(u+10)^2+3779v^2=pz^6 \quad (62)$$

We put different values of k in equation (60) to obtain different types of equations and by solving these equations, we have

Example – 1
If $k=4\Rightarrow p=3795$,

Technique – 21
write

$$3795=\frac{(4n+in\sqrt{3779})(4n-in\sqrt{3779})}{n^2} \quad (63)$$

Replacing equations (4) and (63) in equation (62) and technique of factorizing is applied,

$$((u+10)+i\sqrt{3779}v)((u+10)-i\sqrt{3779}v)=$$
$$\frac{(4n+in\sqrt{3779})(4n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^6(a-i\sqrt{3779}b)^6) \quad (64)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u=4a^6-22650a^5b-283425a^4b^2+285314500a^3b^3+1071063075a^2b^4-323461048650ab^5-269836490695b^6-10$
$v=a^6+24a^5b-56685a^4b^2-302320a^3b^3+214212615a^2b^4+342740184ab^5-53967298139b^6$
the integral solutions of (60) is obtained as follows by substituting the values of u, v in equation (61)
$x=5a^6-22674a^5b-226740a^4b^2+285616820a^3b^3+856850460a^2b^4-323803788834ab^5-215869192556b^6-10$
$y=3a^6-22698a^5b-170055a^4b^2+285919140a^3b^3+642637845a^2b^4+324146529018ab^5-161901894417b^6-10$
$z=a^2+3779b^2$

Properties

- $-y(n,1)+3c^6(n)-22698c^5(n)-4081320PT_n+143469735SO_n+35806025T_{38,n}-161696668264Gno_n$ is a strong prime number.
- $3x(n,1)-5y(n,1)-45540c^5(n)+3441913200{P_n}^3-324887923470Gno_n\equiv 0(mod\ 5771681)$
- $y(1,n)+161901894417c^6(n)+324146529018c^5(n)-128527569Nex_n+1499034825O_n+285665461T_{92,n}+699931276Gno_n\equiv 0(mod\ 15931901)$

TABLE XXI
NUMERICAL EXAMPLES OF TECHNIQUE - 21

$b=a$

x	y	z
-538530763529	163172998826	3780
-34465968865226	10443071925494	15120
-392588926605361	118953116151434	34020
-2205822007373834	668356603232246	60480
-8414543179984386	2549578106812480	94500

IX. SECTION - 7

If $q=7$ then $945(x^2+y^2)-1889xy+10(x+y)+100=pz^7$

A. Technique for Analysis

For obtaining non zero integral solution the heptic equation can be solved as follows:

$$945(x^2+y^2)-1889xy+10(x+y)+100=pz^7 \quad (65)$$

Note that $p=l+k^2$, where some fixed integer $l=3779$ and k is a chenprime number.
When the transformation is substituted,

$$y=u-v,\ x=u+v, \quad (66)$$

Hence Equation (65) becomes

$$(u+10)^2+3779v^2=pz^7 \quad (67)$$

We put different values of k in equation (65) to obtain different types of equations and by solving these equations, we have

Example – 1
If $k=3 \Rightarrow p=3788$

Technique – 22
write

$$3788=\frac{(3n+in\sqrt{3779})(3n-in\sqrt{3779})}{n^2} \quad (68)$$

Replacing equations (4) and (68) in equation (67) and technique of factorizing is applied,

$$((u+10)+i\sqrt{3779}v)((u+10)-i\sqrt{3779}v)= \frac{(3n+in\sqrt{3779})(3n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^7(a-i\sqrt{3779}b)^7) \quad (69)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u=3a^7-26453a^6b-238077a^5b^2+499829435a^4b^3+1499488305a^3b^4-1133313260919a^2b^5-1133313260919ab^6+2039424196672815b^7-10$
$v=a^7+21a^6b-79359a^5b^2-396795a^4b^3+499829435a^3b^4+899692983a^2b^5-377771086973ab^6-161901894417b^7$
the integral solutions of (65) is obtained as follows by substituting the values of u, v in equation (66)
$x=4a^7-26432a^6b-317436a^5b^2+499432640a^4b^3+1999317740a^3b^4-1132413567936a^2b^5-1511084347892ab^6+2037805177728864b^7-10$
$y=2a^7-26474a^6b-158718a^5b^2+500226230a^4b^3+999658870a^3b^4-1134212953902a^2b^5-755542173946ab^6+2041043215616698b^7-10$
$z=a^2+3779b^2$

Properties

- $-y(1,n)+2041043215616698c^7(n)-755542173946c^6(n)-1134212953902c^5(n)+23991812880PT_n-6597272388{P_n}^7-45525613T_{630,n}+18048405767T_{4,n}-18048405767p(n)$ is an ABA number.
- $2x(n,1)-4y(n,1)+48097902720DF_n-226200428333T_{4,n}\equiv 0(mod\ 127919963)$
- $x(1,n)-2037805177728864c^7(n)+1511084347892c^6(n)+1132413567936c^5(n)-399863548Nex_n+5248804260{P_n}^6+363734T_{7560,n}+2124165967Gno_n\equiv 0(mod\ 13499)$

TABLE XXII
NUMERICAL EXAMPLES OF TECHNIQUE - 22

$a=-b$

x	y	z
204157688958710	203725151481334	3780
26132184186716150	26076819389612022	15120
446492865752720630	445546906289699318	34020
3344919575899668470	3337832881870340086	60480
15949819449899999990	15916027459479999990	94500

X. SECTION - 8

If $q=8$ then $945(x^2+y^2)-1889xy+10(x+y)+100=pz^8$

A. Technique for Analysis

For obtaining non zero integral solution the octic equation can be solved as follows:

$$945(x^2+y^2)-1889xy+10(x+y)+100=pz^8 \quad (70)$$

Note that $p=l+k^2$, where some fixed integer $l=3779$ and k is a prime number.
When the transformation is substituted,

$$y=u-v,\ x=u+v, \quad (71)$$

Hence Equation (70) becomes

$$(u+10)^2+3779v^2=pz^8 \quad (72)$$

We put different values of k in equation (70) to obtain different types of equations and by solving these equations, we have

Example – 1
If $k=5 \Rightarrow p=3804$

Technique – 23
write

$$3804=\frac{(5n+in\sqrt{3779})(5n-in\sqrt{3779})}{n^2} \quad (73)$$

Replacing equations (4) and (73) in equation (72) and technique of factorizing is applied,

$$((u+10)+i\sqrt{3779}v)((u+10)-i\sqrt{3779}v)= \frac{(5n+in\sqrt{3779})(5n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^8(a-i\sqrt{3779}b)^8) \quad (74)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u=5a^8-30232a^7b-529060a^6b^2+799727096a^5b^3+4998294350a^4b^4-3022168695784a^3b^5-7555421739460a^2b^6+16315393573382480ab^7+10197120983364050b^8-10$
$v=a^8+40a^7b-105812a^6b^2-1058120a^5b^3+999658870a^4b^4+3998635480a^3b^5-1511084347892a^2b^6-$

$2158691925560ab^7 + 2039424196672816b^8$
the integral solutions of (70) is obtained as follows by substituting the values of u, v in equation (71)
$x = 6a^8 - 30192a^7b - 634872a^6b^2 + 798668976a^5b^3 + 5997953220a^4b^4 - 3018170060304a^3b^5 - 9066506087352a^2b^6 + 1629380665412688ab^7 + 1223654518003686b^8 - 10$
$y = 4a^8 - 30272a^7b - 423248a^6b^2 + 800785216a^5b^3 + 3998635480a^4b^4 - 3026167331264a^3b^5 - 6044337391568a^2b^6 + 1633698049263808ab^7 + 815769678669124b^8 - 10$
$z = a^2 + 3779b^2$

Properties

- $-x(1,n) + 1223654518003686c^8(n) + 1629380665412688c^7(n) - 9066506087352c^6(n) - 3018170060304c^5(n) + 1199590644Nex_n - 5598618732So_n - 999711776T_{26,n} - 11296715840Gno_n$ is a Psuedo perfect number.
- $y(1,n) - 815769678669124c^8(n) - 1633698049263808c^7(n) + 6044337391568c^6(n) + 3026167331264c^5(n) - 3998635480c^4(n) - 50049076To_n - 165119626T_{22,n} - 142434269Gno_n \equiv 0(mod\ 16931)$

TABLE XXIII
NUMERICAL EXAMPLES OF TECHNIQUE - 23

$a = 3b$

x	y	z
2840957303225846	5581276377547254	3788
727285069625819126	1428806752652099574	15152
18639520866464841206	36618754313087599094	34092
186184977824209698806	365774528678937493494	60608
1109748946572599999990	2180186084979399999990	94700

XI. SECTION - 9

If $q = 9$ then $945(x^2+y^2)-1889xy+10(x+y)+100 = pz^9$

A. Technique for Analysis

For obtaining non zero integral solution the nonic equation can be solved as follows:

$$945(x^2 + y^2) - 1889xy + 10(x + y) + 100 = pz^9 \quad (75)$$

Note that $p = l + k^2$, where some fixed integer $l = 3779$ and k is a prime number.
When the transformation is substituted,

$$y = u - v,\ x = u + v, \quad (76)$$

Hence Equation (75) becomes

$$(u + 10)^2 + 3779v^2 = pz^9 \quad (77)$$

We put different values of k in equation (75) to obtain different types of equations and by solving these equations, we have

Example – 1
If $k = 2 \Rightarrow p = 3783$

Technique – 24
write

$$3783 = \frac{(2n + in\sqrt{3779})(2n - in\sqrt{3779})}{n^2} \quad (78)$$

Replacing equations (4) and (78) in equation (77) and technique of factorizing is applied,

$$((u + 10) + i\sqrt{3779}v)((u + 10) - i\sqrt{3779}v) = \frac{(2n+in\sqrt{3779})(2n-in\sqrt{3779})}{n^2}((a + i\sqrt{3779}b)^9(a - i\sqrt{3779}b)^9) \quad (79)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = 2a^9 - 34011a^8b - 272088a^7b^2 + 1199590644a^6b^3 + 3598771932a^5b^4 - 6799879565514a^4b^5 - 9066506087352a^3b^6 + 7341927108022116a^2b^7 + 3670963554011058ab^8 - 7706984039226548996b^9 - 10$
$v = a^9 + 18a^8b - 136044a^7b^2 - 634872a^6b^3 + 1799385966a^5b^4 + 3598771932a^4b^5 - 4533253043676a^3b^6 - 3885645466008a^2b^7 + 1835481777005529ab^8 + 4078848393345626b^9$
the integral solutions of (75) is obtained as follows by substituting the values of u, v in equation (76)
$x = 3a^9 - 33993a^8b - 408132a^7b^2 + 1198955772a^6b^3 + 5398157898a^5b^4 - 6796280793582a^4b^5 - 13599759131028a^3b^6 + 7338041462556108a^2b^7 + 5506445331016587ab^8 - 7702905190833203376b^9 - 10$
$y = a^9 - 34029a^8b - 136044a^7b^2 + 1200225516a^6b^3 + 1799385966a^5b^4 - 6803478337446a^4b^5 - 4533253043676a^3b^6 + 7345812753488124a^2b^7 + 1835481777005529ab^8 - 7711062887619894616b^9 - 10$
$z = a^2 + 3779b^2$

Properties

- $-x(1,n) - 7702905190833320337c^9(n) + 5506445331016587c^8(n) + 7338041462556108c^7(n) - 13599759131028c^6(n) - 6796280793582c^5(n) + 129555789552PT_n - 7797497904RD_n - 399924012T_{20,n} - 66778365093p(n) + 66778365093T_{4,n}$ is a semiprime number.
- $-y(1,n) - 7711062887619894616c^9(n) + 1835481777005529c^8(n) + 7345812753488124c^7(n) - 4533253043676c^6(n) - 6803478337446c^5(n) + 1799385966c^4(n) + 1800338274P_n{}^6 - 300101727T_{8,n} + 1200100791p(n) - 1200100791T_{4,n}$ is a cullen number.

TABLE XXIV
NUMERICAL EXAMPLES OF TECHNIQUE - 24

$a = -3b$

x	y	z
-720951224205221410	-710928665436911110	3788
-369127026793073356810	-363995476703698483210	15152
-14190482946031372816210	-13993208921794721181310	34092
-188993037718053558681610	-186365684072293623398410	60608
-1408107859775823046875010	-1388532549681466992187510	94700

XII. SECTION - 10

If $q = 10$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^{10}$

A. Technique for Analysis

For obtaining non zero integral solution the decic equation can be solved as follows:

$$945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^{10} \quad (80)$$

Note that $p = l + k^2$, where some fixed integer $l = 3779$ and k is a non zero integer.
When the transformation is substituted,

$$y = u - v,\ x = u + v, \quad (81)$$

Hence Equation (80) becomes

$$(u+10)^2 + 3779v^2 = pz^{10} \quad (82)$$

We put different values of k in equation (80) to obtain different types of equations and by solving these equations, we have

Example − 1
If $k = 1 \Rightarrow p = 3780$

Technique − 25
write

$$3780 = \frac{(n + in\sqrt{3779})(n - in\sqrt{3779})}{n^2} \quad (83)$$

Replacing equations (4) and (83) in equation (82) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(n+in\sqrt{3779})(n-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^{10}(a-i\sqrt{3779}b)^{10}) \quad (84)$$

Now compare real and imaginary parts and equating like expressions, we arrive
$u = a^{10} - 37790a^9b - 170055a^8b^2 + 1713700920a^7b^3 + 2998976610a^6b^4 - 13599759131028a^5b^5 - 11333132609190a^4b^6 + 24473090360073720a^3b^7 + 91774088850276 45a^2b^8 - 7706984039226548990ab^9 - 770698403922654899b^{10} - 10$
$v = a^{10} + 10a^9b - 170055a^8b^2 - 453480a^7b^3 + 2998976610a^6b^4 + 3598771932a^5b^5 - 11333132609190a^4b^6 - 6476075776680a^3b^7 + 9177408885027645a^2b^8 + 2039424196672810ab^9 - 770698403922654899b^{10}$
the integral solutions of (80) is obtained as follows by substituting the values of u, v in equation (81)
$x = 2a^{10} - 37780a^9b - 340110a^8b^2 + 1713247440a^7b^3 + 5997953220a^6b^4 - 13596160359096a^5b^5 - 22666265218380a^4b^6 + 24466614284297040a^3b^7 + 18354817770055290a^2b^8 - 7704944615029876180ab^9 - 1541396807845309798b^{10} - 10$
$y = -37800a^9b + 1714154400a^7b^3 - 13603357902960a^5b^5 + 24479566435850400a^3b^7 - 7709023463423221800ab^9 - 10$
$z = a^2 + 3779b^2$

Properties

- $y(1,n) + 7709023463423221800c^9(n) - 24479566435850400c^7(n) + 13603357902960c^5(n) - 2571231600{P_n}^6 + 1285615800T_{4,n} - 214250400Gno_n$ is a congruent number.
- $-y(n,1) - 37800c^9(n) + 1714154400c^7(n) - 13603357902960c^5(n) + 18359674826887800HO_n + 36719349653775600T_{4,n} - 38667515149295361 00Gno_n \equiv 0(mod\ 30248131393)$

TABLE XXV
NUMERICAL EXAMPLES OF TECHNIQUE - 25

$a = 4b$

x	y	z
-30521307296281244960	-29283303365494446250	3815
-3125381867139199482 8810	-2998610264626631294 9770	15260
-180225267453811123305 2560	-172914978042908155602 5770	34335
-32003910319505402704691210	-30705769109776704460554250	61040
-298059641565246532714843760	-285969759428656701562500010	95375

Result :

- If $q = 1$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz$
- If $q = 2$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^2$
- If $q = 3$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^3$
- If $q = 4$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^4$
- If $q = 5$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^5$
- If $q = 6$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^6$
- If $q = 7$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^7$
- If $q = 8$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^8$
- If $q = 9$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^9$
- If $q = 10$ then $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^{10}$

In general, If $q = m$ then

$$945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^m$$

XIII. GENERAL CASE

$945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^m$

A. Technique for Analysis

For obtaining non zero integral solution the m^{th} degree equation can be solved as follows:

$$945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^m \quad (85)$$

Note that $p = l + k^2$, where some fixed integer $l = 3779$ and $k \in \mathbf{Z}\backslash\{0\}$
When the transformation is substituted,

$$y = u - v,\ x = u + v, \quad (86)$$

Hence Equation (85) becomes

$$(u+10)^2 + 3779v^2 = pz^m \tag{87}$$

We put different values of k in equation (85) to obtain different types of equations and by solving these equations, we have

Technique − 26
write

$$p = \frac{(kn + in\sqrt{3779})(kn - in\sqrt{3779})}{n^2} \tag{88}$$

Replacing equations (4) and (88) in equation (87) and technique of factorizing is applied,

$$((u+10) + i\sqrt{3779}v)((u+10) - i\sqrt{3779}v) = \frac{(kn+in\sqrt{3779})(kn-in\sqrt{3779})}{n^2}((a+i\sqrt{3779}b)^m(a-i\sqrt{3779}b)^m) \tag{89}$$

Now compare real and imaginary parts and equating like expressions, we arrive

$$\begin{aligned} u &= ka^m - \left(\frac{km(m-1)}{2}\right)3779a^{m-2}b^2 - 3779ma^{m-1}b \\ &\quad + 14280841\left(\frac{m(m-1)(m-2)}{3}\right)a^{m-3}b^3 + \ldots - 10 \\ v &= a^m + kma^{m-1}b + \left(\frac{m(m-1)}{2}\right)3779a^{m-2}b^2 \\ &\quad - 3779\left(\frac{km(m-1)(m-2)}{3}\right)a^{m-3}b^3 + \ldots \end{aligned}$$

the integral solutions of (81) is obtained as follows by substituting the values of u, v in equation (82)

$$\begin{aligned} x &= (k+1)a^m - [3779m - km]\,a^{m-1}b \\ &- \left[\left(\frac{km(m-1)}{2}\right)3779 - \left(\frac{m(m-1)}{2}\right)3779\right]a^{m-2}b^2 \\ &+ \left[14280841\left(\frac{m(m-1)(m-2)}{3}\right)\right. \\ &\left. -3779\left(\frac{km(m-1)(m-2)}{3}\right)\right]a^{m-3}b^3 + \ldots - 10 \\ y &= (k-1)a^m - [3779m + km]\,a^{m-1}b \\ &- \left[\left(\frac{km(m-1)}{2}\right)3779 + \left(\frac{m(m-1)}{2}\right)3779\right]a^{m-2}b^2 \\ &+ \left[14280841\left(\frac{m(m-1)(m-2)}{3}\right)\right. \\ &\left. +3779\left(\frac{km(m-1)(m-2)}{3}\right)\right]a^{m-3}b^3 + \ldots - 10 \end{aligned}$$

XIV. Remarks

- For different values of k, the Diophantine equation $945(x^2+y^2) - 1889xy + 10(x+y) + 100 = pz^q$, where $p = l + k^2$ furnishes numerous techniques.
- Similarly, if we continue the above process for the Diophantine equation $945(x^2+y^2) - 1889xy - 10(x+y) + 100 = pz^q$, then it also yields the integer solutions.

XV. Conclusion

In this study, we thoroughly investigate non-zero integral solutions to an equation and explore novel methods for solving it. We have successfully obtained 26 techniques, by solving the equation for q values upto 10 including generalized value $q = m$. Our findings suggests that there is potential for further exploration by varying q up to n and using diverse techniques to solve the diophantine equation. Future research opportunities include exploring alternative equation forms with more than three variables, which can be solved using various techniques.

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