A Hybrid MADM Approach for the Analysis of the Most Polluted Region

A. Mohamed Nusaf, R. Kumaravel and J. Jayapragash

***Abstract*—In India, there are myriad causes for air pollution, and celebrations of a festival like Bhogi also significantly pollute the air. This study aims to analyse the air quality in the regions of Chennai, India, during the Bhogi festival from 2019 to 2021, based on multiple air pollutants. The study introduces a hybrid approach, namely the Analytical Hierarchy Process – Entropy - Technique for Order Preference by Similarity to an Ideal Solution (AHP-Entropy-TOPSIS), to analyse and rank the areas based on the quality of air. A combined approach of AHP and Entropy is employed to determine the weights of multiple air pollutants. The TOPSIS approach ranks the city areas and identifies the area with the worst air quality during the festival. The proposed model is validated by performing the Spearman's rank correlation with three other existing Multi-Attribute Decision-Making (MADM) methods, namely Combinative Distance Based Assessment (CODAS), Weighted Aggregated Sum Product Assessment (WASPAS) and Multi-Objective Optimization on Basis of Ratio Analysis (MOORA). Sensitivity analysis is carried out to assess the effects of the priority weights and the dependency of the pollutants in ranking the regions. The highest air pollution level during the festival was seen in Ambattur (2019), Royapuram (2020), and Tondiarpet (2021) in their respective year. The results demonstrate the consistency and efficiency of the proposed approach.**

***Index Terms*—AHP, air pollution, decision analysis, entropy, MADM, TOPSIS.**

I. Introduction

One of the most severe global challenges is air pollution, primarily driven by the mass production of automobiles and the rapid expansion of industries. It has led to a worldwide surge in car usage, depleting fossil fuels and disrupting ecosystems [1]. Besides vehicle carbon emissions and industrial pollutants, our environment suffers due to the incineration of waste materials like plastics and tires, which exacerbates their harmful effects on the environment and human health [2], [3], [4], [5], [6]. In 2017, air pollution caused 1.24 million deaths worldwide. This issue is even more critical in India, with ambient air pollution ranking as the third leading risk factor and home to 11 of Central and South Asia's 15 most polluted cities in 2021 [7], [8].

In India, festivals like Diwali and Bhogi contribute significantly to environmental pollution. This study specifically focuses on the Bhogi festival, which marks the start of the harvest season and is a day of gratitude to the Sun God. On the second week of January each year, people celebrate Bhogi by lighting bonfires in the morning. During this ritual, unwanted items, including plastics and rubber tires due to the lack of agricultural waste in cities, are burned, emitting toxic gases like sulphur dioxide, carbon monoxide, and nitrogen oxide. Unfortunately, this has led to a notable increase in air pollution. It is crucial to assess air quality based on these various pollutants. [9]. These pollutants cause various effects, and assessing air quality based on these multiple pollutants is essential. Numerous studies have been conducted using mathematical methods that convert the concentrations of various air pollutants into a single value across a given area [10], [11], [12], [13].

The sources of these pollutants are diverse and interconnected. Therefore, the current study employs a hybrid MADM model that provides rankings and analysis to assist decision-makers in addressing real-time issues involving independent attributes. MADM is a field of Operations Research that aids in evaluating decision alternatives in various domains using performance attributes by a decision-maker or a group of decision-makers [14], [15]. Various decision-making methodologies, including AHP, ELECTRE, PROMETHEE, TOPSIS, TODIM, and VIKOR, exist to facilitate efficient decision-making [16]. Many methods consider attribute weights during aggregation, which can be categorised into subjective and objective weighting methods. Subjective methods determine weights based on decision-makers preferences, with the most popular options being AHP and the Best-Worst Method (BWM). Objective methods calculate attribute weights using mathematical models and do not rely on the decision maker's subjective judgment information, with the most popular objective weighting methods being Entropy and Standard Deviation [17]. The applications of the methods are enormous [18], [19], [20], [21], [22], [23], [24].

Researchers in different regions have used various methods in air quality assessment. Wang et al. [25] applied Entropy and TOPSIS to assess air pollution's impact on Wuhan city's economic development in China. Grecu [26] used the TOPSIS method to rank different periods of air quality in Mehedinti city. Chen et al. [27] employed VIKOR and DANP to analyse air quality improvement strategies in Kaohsiung, Taiwan. Ozkaya and Erdin [28] used TOPSIS and VIKOR to evaluate sustainable forest and air quality management in European countries. Lin et al. [29] proposed an air quality assessment method using Entropy and TOPSIS for multiple pollutants. Xu and Chernikov [30] applied a combined Entropy-TOPSIS-PROMETHEE method to assess air quality in various Chinese cities. Torkayesh et al. [31] analysed air pollutants in 22 European countries using the BWM

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and MARCOS methods to provide insights for improving environmental sustainability on regional and national levels.

The literature indicates that many studies have viewed air quality assessment as a significant problem within the MADM framework, given its reliance on multiple quality indicators. Several studies have employed the Entropy approach for air quality assessment, but none have presented the combined AHP-Entropy weighting approach for regional air pollutant evaluation. Therefore, our study introduces the AHP-Entropy-TOPSIS method to evaluate air quality and rank areas. The paper is structured as follows: the study area and data collection encompass the considered alternatives, attributes, and statistical data. The methodology section details the algorithmic aspects of the methods. Research findings are summarized and validated using Spearman's rank correlation, and sensitivity analysis is conducted in the results and discussion section. Finally, the article concludes in the conclusion section.

II. Study Area and Data Collection

This research intends to estimate the air quality and determination of the most polluted area during the Bhogi festival, specifically in Thiruvotriyur (TVT), Manali (MNI), Madhavaram (MDM), Tondiarpet (TNP), Royapuram (RPM), Thiru. Vi. Ka. Nagar (TVK), Ambattur (ABU), Anna Nagar (ANN), Teynampet (TYT), Kodambakkam (KMM), Valasaravakkam (VSM), Alandur (ANR), Adyar (ADR), Perungudi (PGD), Shozhinganallur (SHU). These areas of Chennai city are considered alternatives in this study and are represented in Fig. 1.

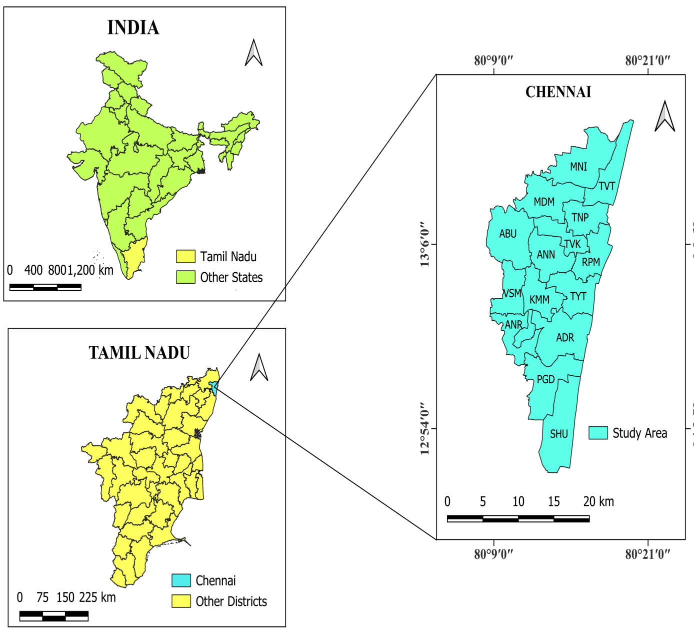
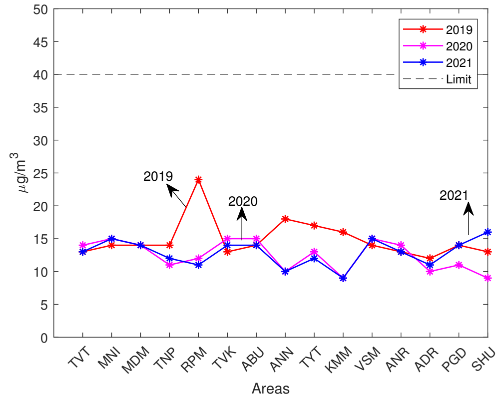
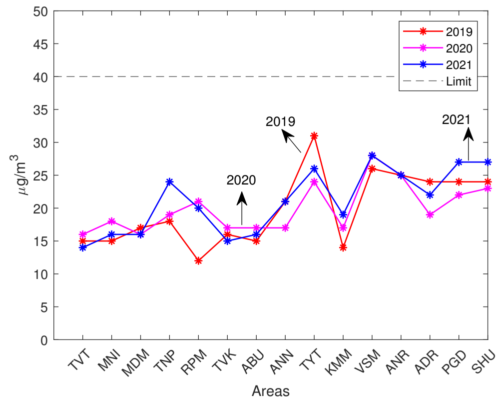


Fig. 1: The locations of considered areas in Chennai on the map of India

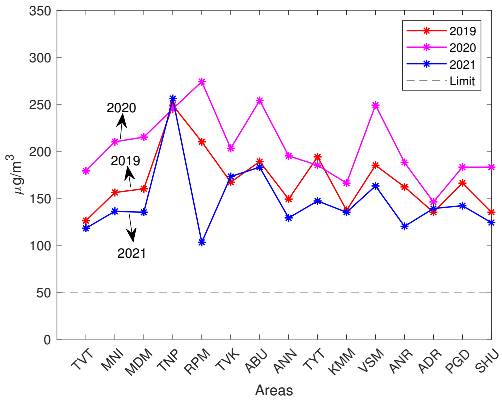
The air pollutants sulphur dioxide (SO_2), nitrogen dioxide (NO_2), and particulate matter (PM_{10} and $PM_{2.5}$) are selected as the attributes to evaluate the air quality using the MADM method. The concentration of the pollutants during the Bhogi festival in 2019-2021 was collected from Tamil Nadu Pollution Control Board (TNPCB), Chennai [32] and depicted in Fig. 2.



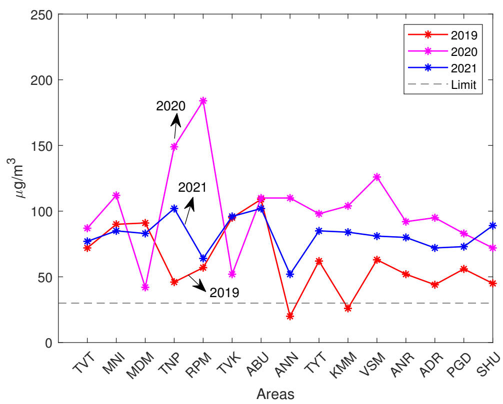
(a) SO_2



(b) NO_2



(c) PM_{10}



(d) $PM_{2.5}$

Fig. 2: Data distribution of each pollutant

Descriptive statistics of the pollutant studied in the community of Chennai have been carried out. Table I shows a statistical summary of these data, including measures of central tendency and variability. The maximum concentration values of SO_2 and NO_2 in 2019, 2020, and 2021 were within the permissible limits, with values ranging from 15 $\mu g \backslash m^3$ to 31 $\mu g \backslash m^3$. However, all three years exceeded the average daily limit of 100 $\mu g \backslash m^3$ for PM_{10}, with maximum values ranging from 249 $\mu g \backslash m^3$ to 274 $\mu g \backslash m^3$. Similarly, all three years exceeded the average limit of 60 $\mu g \backslash m^3$ for $PM_{2.5}$, with maximum values ranging from 102 $\mu g \backslash m^3$ to 184 $\mu g \backslash m^3$. It is noticed that large standard deviations were found for PM_{10} in all years, indicating significant variations in the concentration levels within the studied regions. Additionally, the coefficient of variation for $PM_{2.5}$ was higher in 2019 and 2020, suggesting more variability in the concentration levels of this pollutant during those years. Furthermore, the skewness values indicate the distribution characteristics of the contaminants. Each pollutant, SO_2, NO_2, and PM_{10}, was more skewed in 2019, 2020, and 2021 respectively. These observations highlight the elevated levels of particulate matter, specifically PM_{10} and $PM_{2.5}$, during the Bhogi festival period. The results emphasise the need for effective measures to reduce and control particulate matter pollution, as it poses a significant health risk to the population.

III. Methodology

This study proposes a hybrid approach that includes the AHP and Entropy weighting techniques to assign relative importance to each attribute (parameter). The TOPSIS approach is used to classify the alternatives (areas) to identify the most polluted region. The flowchart for the AHP-Entropy-TOPSIS approach is shown in Fig. 3.

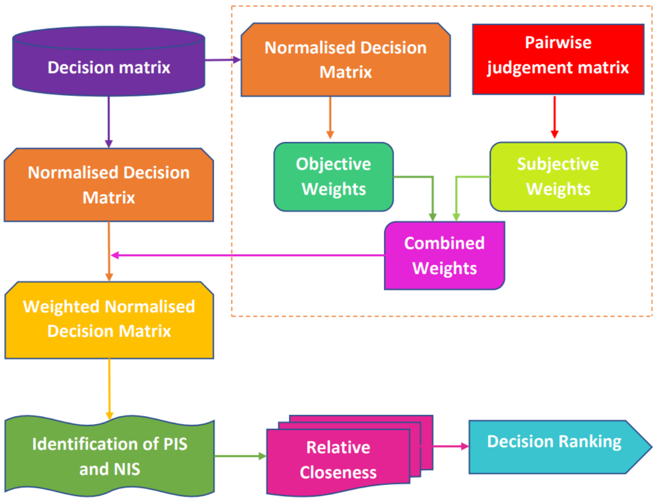


Fig. 3: AHP-Entropy-TOPSIS Model

A. Initial Decision Matrix

A MADM decision matrix $A = [a_{ij}]_{p \times k}$ consists of p alternatives and k attributes as presented in equation 1

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{12} & a_{22} & \dots & a_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ a_{p1} & a_{p2} & \dots & a_{pk} \end{bmatrix} \tag{1}$$

Here, a_{ij} $(1 \le i \le p, \ \ 1 \le j \le k)$ indicates the performance ratings of i^{th} alternative to the j^{th} attribute for the initial data shown in Fig 2.

B. Weighting Methods

The weights of the attributes can be evaluated using alternative data or expert opinion. In this study, we consider the standard weighting approaches, such as AHP and Entropy.

C. AHP

T. L. Saaty developed the AHP in 1977 to assign relative importance to choices based on how they compare on a ratio scale [33]. The decision hierarchy of the AHP method for identifying the most polluted area is shown in Fig. 4.

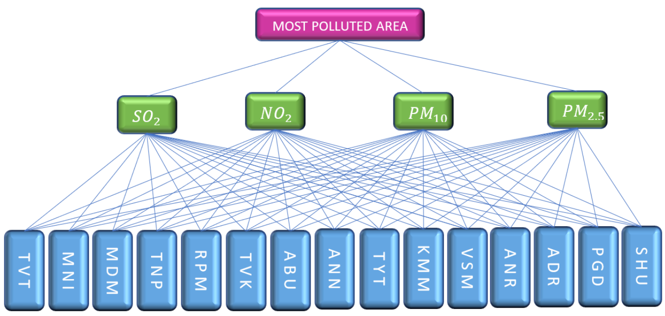


Fig. 4: Decision hierarchy of parameters and areas

The relative importance corresponding to each attribute is ranked using the ratio scale represented in Table II.

TABLE II: Ratio Scale

Numerical Rating	Importance
1	Equal Importance
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme Importance
2,4,6,8	Intermediate Values

The paired comparison matrix $P = [p_{ij}]_{p \times k}$ to determine the weight of each attribute used in this analysis is shown in equation 2.

$$P = \begin{bmatrix} 1 & 3 & 0.33 & 0.20 \\ 0.33 & 1 & 0.20 & 0.14 \\ 3 & 5 & 1 & 0.33 \\ 5 & 7 & 3 & 1 \end{bmatrix} \tag{2}$$

We require the vector $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_k]$ that represents the weight of each attribute in the matrix P. To get the vector α from P, divide each P column by its sum to get a new P column. The resulting matrix is denoted as P_{norm} and take the mean of the values in the i^{th} row of P_{norm} to estimate α_i.

The Consistency Index (CI) and Consistency Ratio (CR) are determined using equations 3 and 4, and the Random Index (RI) value for a 4×4 matrix is 0.9.

$$CI = \frac{\lambda_{max} \ - \ n}{n - 1} \tag{3}$$

TABLE I: Summary statistics of the air pollutants concentrations

Year	2019				2020				2021			
Pollutants	SO_2	NO_2	PM_{10}	$PM_{2.5}$	SO_2	NO_2	PM_{10}	$PM_{2.5}$	SO_2	NO_2	PM_{10}	$PM_{2.5}$
Count	15	15	15	15	15	15	15	15	15	15	15	15
Average ($\mu g/m^3$)	14.9	19.8	168	61.9	12.5	19.9	205	101.1	12.9	21.1	146.9	81.7
Standard dev. ($\mu g/m^3$)	2.99	5.57	32.99	25.51	2.29	3.71	36.12	35.33	1.99	4.91	36.81	13.37
Coeff. of variation %	20	28	20	41	18	19	18	35	16	23	25	16
Minimum ($\mu g/m^3$)	12	12	126	20	9	16	146	42	9	14	103	52
Maximum ($\mu g/m^3$)	24	31	249	109	15	28	274	184	16	28	256	102
Range ($\mu g/m^3$)	12	19	123	89	6	12	128	142	7	14	153	50
Stnd. skewness	2.30	0.44	1.04	0.27	-0.30	0.85	0.49	0.64	-0.41	-0.01	2.01	-0.45
Stnd. kurtosis	6.04	-0.85	1.19	-0.53	-1.58	-0.26	-0.52	1.30	-0.57	-1.58	5.18	0.62

Here, $\lambda_{max} = \frac{1}{n}\sum_{i=1}^{p}\frac{P\alpha_i^T}{\alpha_i}$

$$CR = \frac{CI}{RI} \tag{4}$$

For results to be considered, the C.R. value must be lower than 0.1.

D. Entropy

An objective weighting approach, Entropy, proposed by Shannon in 1948 [34], has chosen to assign relative importance to the various criteria. The algorithm for Entropy is as follows:

Step 1. Using the following formula, normalise the decision matrix by substituting each a_{ij} by n_{ij}.

$$n_{ij} = \frac{a_{ij}}{\sum\limits_{i=1}^{p} a_{ij}} \qquad (1 \le i \le p, 1 \le j \le k) \tag{5}$$

Step 2. Calculate the entropy value e_j of j^{th} attribute by

$$e_j = -h\sum_{i=1}^{p} n_{ij}\ln n_{ij} \qquad (h = \frac{1}{\ln p}, 1 \le j \le k) \tag{6}$$

Step 3. Determine the degree of diversification of the j^{th} attribute

$$v_j = 1 - e_j \qquad (1 \le j \le k) \tag{7}$$

Step 4. Calculate the weight of the attributes using

$$\beta_j = \frac{v_j}{\sum\limits_{j=1}^{k} v_j} \qquad (1 \le i \le p, 1 \le j \le k) \tag{8}$$

E. Combined Weight

To maximise the benefits of the AHP and the Entropy methods, this research utilises a hybrid approach by combining the relative importance of each attribute. The final weight of the attribute is calculated as

$$w_j = \frac{\alpha_j\beta_j}{\sum_{j=1}^{k}\alpha_j\beta_j} \tag{9}$$

where α_j is the weight assigned to the j^{th} attribute determined using the AHP approach and β_j is the weight assigned to the j^{th} attribute determined using the Entropy method.

F. TOPSIS

TOPSIS was proposed by Hwang and Yoon in 1981 [35] based on the assumption that the selected alternative should be closest to the Positive Ideal Solution (PIS) and farthest from the Negative Ideal Solution (NIS). The TOPSIS approach can be described this way:

Step 1. Using the following formula, normalise the decision matrix by substituting each a_{ij} by z_{ij}.

$$z_{ij} = \frac{a_{ij}}{\sqrt{\sum\limits_{i=1}^{p} a_{ij}^2}} \qquad (1 \le i \le p, 1 \le j \le k) \tag{10}$$

Step 2. Determine the weighted normalized values as

$$s_{ij} = w_j * z_{ij} \qquad (1 \le i \le p, 1 \le j \le k) \tag{11}$$

where w_j correspond to the weight of the j^{th} attribute.

Step 3. Calculate the PIS (S^+) and the NIS (S^-) as follows:

$$\begin{aligned} PIS = S^+ &= (s_1^+, s_2^+, ..., s_k^+) \\ NIS = S^- &= (s_1^-, s_2^-, ..., s_k^-) \end{aligned} \tag{12}$$

where $s_k^+ = max_i s_{ij}$ and $s_k^- = min_i s_{ij}$ if the j^{th} attribute is benifit attribute; $s_k^+ = min_i s_{ij}$ and $s_k^- = max_i s_{ij}$ if the j^{th} attribute is non-benifit attribute.

Step 4. Determine the distances between each alternative and the ideal solutions, where d_i^+ is the distance to S^+ and d_i^- is the distance to S^-, as shown below.

$$d_i{}^+ = \sqrt{\sum_{j=1}^{k}\left(s_{ij} - s_k^+\right)^2} \tag{13}$$

$$d_i{}^- = \sqrt{\sum_{j=1}^{k}(s_{ij} - s_k^-)^2} \tag{14}$$

Step 5. Evaluate the relative closeness using

$$R_i = \frac{d_i{}^-}{d_i{}^+ + d_i{}^-} \qquad (i = 1, 2, ...p) \tag{15}$$

Step 6. Rank the alternatives according to the highest value of R_i.

G. CODAS

The MADM method CODAS was proposed by Ghorabaee et al. in 2016 [36] and the desirability of alternatives is determined by using two measures Euclidean distance and Taxicab distance. The alternative which has greater distances from the negative-ideal solution is more desirable. The CODAS method is presented as follows:

Step 1. Calculate the normalised decision matrix as follows:

$$f_{ij} = \frac{a_{ij}}{max_i x_{ij}} \tag{16}$$

Step 2. Calculate the weighted normalized decision matrix as

$$g_{ij} = w_j * f_{ij} \tag{17}$$

Step 3. Determine the NIS as follows:

$$l_j = min_i g_{ij} \tag{18}$$

Step 4. Calculate the Euclidean and Taxicab distances of alternatives from the NIS, shown as follows:

$$E_i = \sqrt{\sum_{j=1}^{k} (g_{ij} - l_j)^2} \tag{19}$$

$$T_i = \sum_{j=1}^{k} |g_{ij} - l_j| \tag{20}$$

Step 5. Construct the relative assessment matrix

$$o_{ij} = (E_i - E_k) + (\psi(E_i - E_k) \times (T_i - T_k)) \tag{21}$$

where k ϵ 1,2, ..., n and ψ denotes a threshold function to recognise the equality of the Euclidean distances of two alternatives and is defined as follows:

$$\psi(x) = \begin{cases} 1 & \text{if } |x| \geq \tau \\ 0 & \text{if } |x| < \tau \end{cases}$$

In this calculation τ is taken as 0.02.

Step 6. Calculate the assessment score of each alternative, shown as follows:

$$O_i = \sum_{k=1}^{n} o_{ik} \tag{22}$$

Step 7. Rank the alternatives according to the decreasing values of O_i.

H. WASPAS

The WASPAS method is a MADM technique developed by Zavadskas et al. in 2012 [37] used to evaluate and rank alternatives based on multiple criteria. It is a combination of two well-known methods, the weighted sum model (WSM) and the weighted product method (WPM). The WASPAS method is presented as follows:

Step 1. Calculate the normalised decision matrix as follows:

$$f_{ij} = \frac{a_{ij}}{max_i x_{ij}} \tag{23}$$

Step 2. A joint generalized attribute of weighted aggregation of additive and multiplicative methods is as follows:

$$q_i = 0.5 \sum_{j=1}^{k} f_{ij} * w_j + 0.5 \prod_{j=1}^{k} f_{ij}^{w_j} \tag{24}$$

Step 3. In order to have increased ranking accuracy and effectiveness of the decision-making process. The total relative importance of the i^{th} alternative is defined as follows:

$$Q_i = \lambda(q_i^{(1)} + q_i^{(2)}), \lambda = 0.5 \tag{25}$$

Step 4. Rank the alternatives based on the highest values of Q_i.

I. MOORA

The MOORA method is a decision making technique introduced by Brauers in 2004 [38] used to rank and select alternatives based on multiple attributes. MOORA uses a ratio-based approach to assess the relative performance of each alternative concerning different alternatives. The algorithm for the MOORA method is as follows:

Step 1. Using the following formula, normalise the decision matrix by substituting each a_{ij} by z_{ij}.

$$z_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{p} a_{ij}^2}} \qquad (1 \leq i \leq p, 1 \leq j \leq k) \tag{26}$$

Step 2. For multi-objective optimization, these normalized performances are added in case of maximization (for beneficial attributes) and subtracted in case of minimization (for nonbeneficial attributes). Then the optimization problem becomes:

$$y_i = \sum_{i=1}^{g} z_{ij} - \sum_{g+1}^{n} z_{ij} \tag{27}$$

where g is the number of attributes to be maximised, (n-g) is the number of attributes to be minimised.

Step 3. To give more importance to an attribute, it is multiplied with its corresponding weight then equation 27 becomes as follows:

$$y_i = \sum_{i=1}^{g} w_j * z_{ij} - \sum_{g+1}^{n} w_j * z_{ij} \tag{28}$$

Step 4. Rank the alternatives based on the highest values of y_i.

IV. Results and Discussion

An attempt has been made to determine the most polluted area using the proposed TOPSIS model, based on the concentration of air pollutants in different areas of Chennai city. The AHP calculates the subjective weights of the attributes using the pair-wise comparison matrix. The weights calculated are $\alpha_j = [0.1219\ 0.0569\ 0.2633\ 0.5579]$. After constructing the pair-wise comparison matrix, it is required to verify its consistency. The Eigenvalue, λ_{max} obtained is 4.1185, and the consistency ratio is 0.04, less than the allowed value of 0.1. Thus, there is good consistency in the judgments made. The Entropy method calculates the objective weights of the attributes using the initial data shown in Fig. 2. The obtained weight β_j using equation 8 is represented in Table III.

TABLE III: Entropy weights of the attributes

Year	β_1	β_2	β_3	β_4
2019	0.112	0.239	0.113	0.537
2020	0.157	0.152	0.139	0.552
2021	0.151	0.336	0.344	0.169

A combined weighting method is used to obtain more reasonable weights using AHP and Entropy methods. The final weights w_j of the attributes evaluated using equation 9 is given in Table IV.

TABLE IV: Final weights of the attributes

Year	w_1	w_2	w_3	w_4
2019	0.038	0.038	0.083	0.840
2020	0.052	0.023	0.098	0.827
2021	0.083	0.086	0.407	0.425

Using TOPSIS approach, the initial data is normalised using equation 26, and then the weighted normalised matrix is calculated using equation 11 and represented in Table V. The positive and negative ideal solutions of attributes are obtained by equation 12. The distance between each alternative and the ideal solution is determined using equations 13 and 14. The relative closeness of each alternative to the ideal solution is calculated using equation 15 and represented in Table VI.

It can be seen from Table VI that the maximum closeness attained in the year 2019 was 0.9585, which corresponds to ABU. In 2021, the maximum value of 0.9869 corresponds to RPM and in 2021, 0.9421 corresponds to TNP. Finally, the areas are ranked in descending order based on relative closeness and represented in Fig. 5.

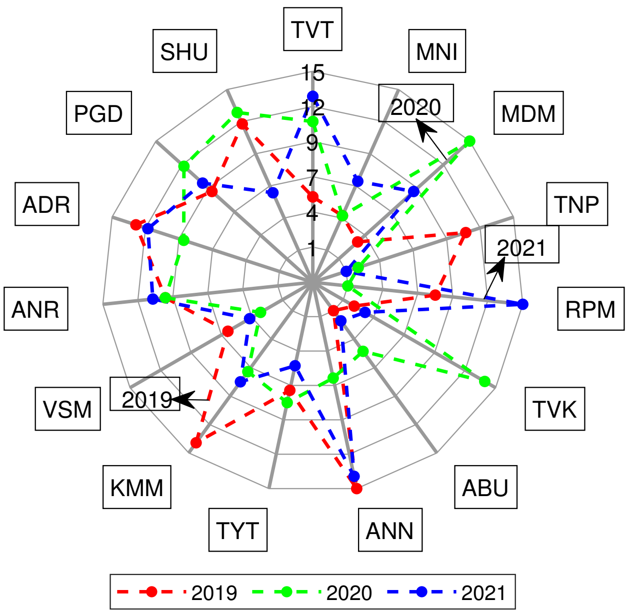


Fig. 5: Ranks obtained from TOPSIS method

From Fig. 5 it can be seen that Ambattur, Royapuram and Tondiarpet are ranked as one among the fifteen areas of Chennai cities, and Anna Nagar, Madhavaram and Royapuram are ranked as fifteen in the year 2019, 2020 and 2021 respectively. However, since the TNPCB result [32] was based on the breakpoint concentration of these pollutants and obtained through the maximum aggregation operator, the ranking of these fifteen areas are slightly different and the result is represented in Fig. 6.

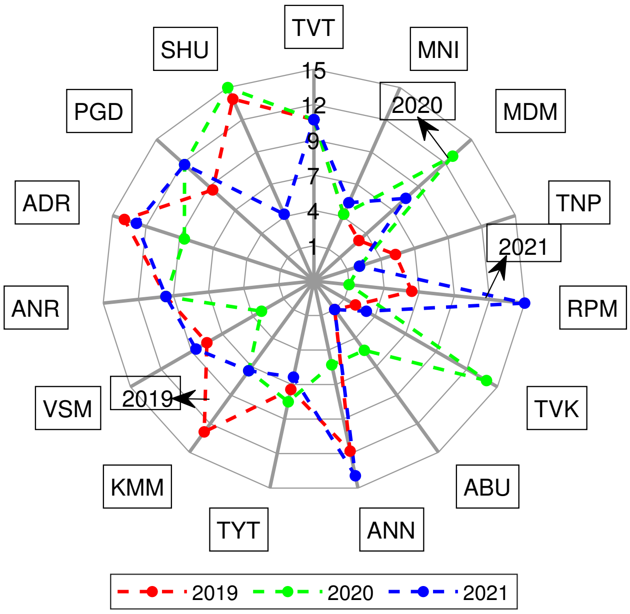


Fig. 6: Ranks obtained from TNPCB

Fig. 6 represents that Ambattur, Royapuram, and Ambattur were the worst polluted areas in 2019, 2020 and 2021, respectively, whereas Shozhinganallur was considered the least contaminated area in 2019 and 2020. Royapuram was regarded as the least polluted area in 2021. For the most polluted area, the result obtained from AHP-Entropy-TOPSIS is similar to TNPCB result.

A. *Comparison with MADM methods:*

The current work presents the methodology validation by comparing the proposed TOPSIS approach with the three well-known MADM approaches: CODAS, WASPAS and MOORA. All techniques are applied to the same dataset for ranking the areas to obtain the most polluted area of Chennai city. The ranking results of these methods are represented in Table VII.

TABLE VII: Ranking of CODAS, WASPAS and MOORA

Year	2019			2020			2021		
Methods	CODAS	WASPAS	MOORA	CODAS	WASPAS	MOORA	CODAS	WASPAS	MOORA
TVT	5	5	5	11	11	11	13	13	13
MNI	4	4	4	4	5	4	7	7	7
MDM	3	3	3	15	15	15	8	9	9
TNP	11	11	11	2	2	2	1	1	1
RPM	8	8	8	1	1	1	14	14	15
TVK	2	2	2	14	14	14	3	3	3
ABU	1	1	1	5	4	5	2	2	2
ANN	15	15	15	6	6	6	15	15	14
TYT	7	6	7	8	8	8	6	5	5
KMM	14	14	14	7	7	7	9	10	8
VSM	6	7	6	3	3	3	4	4	4
ANR	10	10	10	9	9	10	11	11	11
ADR	13	13	13	10	10	9	12	12	12
PGD	9	9	9	12	12	12	10	8	10
SHU	12	12	12	13	13	13	5	6	6

From Table VII, the CODAS, WASPAS and MOORA results indicate that Ambattur, Royapuram and Tondiarpet are ranked as one of the fifteen areas of Chennai cities, and

TABLE V: Weighted normalised matrix of the initial data

Parameter	SO_2			NO_2			PM_{10}			$PM_{2.5}$		
Area\Year	2019	2020	2021	2019	2020	2021	2019	2020	2021	2019	2020	2021
TVT	0.008	0.015	0.021	0.007	0.005	0.014	0.016	0.022	0.082	0.235	0.174	0.102
MNI	0.009	0.016	0.025	0.007	0.005	0.016	0.020	0.026	0.095	0.293	0.224	0.113
MDM	0.009	0.015	0.023	0.008	0.005	0.016	0.020	0.026	0.094	0.297	0.084	0.110
TNP	0.009	0.012	0.020	0.009	0.006	0.025	0.031	0.030	0.178	0.150	0.298	0.135
RPM	0.016	0.013	0.018	0.006	0.006	0.021	0.026	0.033	0.072	0.186	0.368	0.085
TVK	0.008	0.016	0.023	0.008	0.005	0.015	0.021	0.025	0.120	0.310	0.104	0.127
ABU	0.009	0.016	0.023	0.007	0.005	0.016	0.024	0.031	0.127	0.355	0.220	0.135
ANN	0.012	0.011	0.016	0.010	0.005	0.022	0.019	0.024	0.090	0.065	0.220	0.069
TYT	0.011	0.014	0.020	0.015	0.007	0.027	0.024	0.023	0.102	0.202	0.196	0.113
KMM	0.010	0.010	0.015	0.007	0.005	0.020	0.017	0.020	0.094	0.085	0.208	0.111
VSM	0.009	0.016	0.025	0.013	0.008	0.029	0.023	0.030	0.113	0.205	0.252	0.107
ANR	0.008	0.015	0.021	0.012	0.007	0.026	0.020	0.023	0.084	0.169	0.184	0.106
ADR	0.008	0.011	0.018	0.012	0.006	0.023	0.017	0.018	0.097	0.143	0.190	0.095
PGD	0.009	0.012	0.023	0.012	0.007	0.028	0.021	0.022	0.099	0.183	0.166	0.097
SHU	0.008	0.010	0.026	0.012	0.007	0.028	0.017	0.022	0.086	0.147	0.144	0.118

TABLE VI: Relative closeness of alternatives to the ideal solution

Year	2019			2020			2021		
Area	d_i^+	d_i^-	R_i	d_i^+	d_i^-	R_i	d_i^+	d_i^-	R_i
TVT	0.122	0.169	0.581	0.195	0.090	0.317	0.103	0.035	0.256
MNI	0.064	0.228	0.781	0.144	0.141	0.493	0.087	0.050	0.366
MDM	0.060	0.231	0.793	0.284	0.010	0.034	0.089	0.047	0.348
TNP	0.205	0.086	0.295	0.070	0.215	0.753	0.008	0.126	0.942
RPM	0.170	0.121	0.417	0.004	0.285	0.987	0.118	0.017	0.128
TVK	0.048	0.244	0.836	0.264	0.022	0.077	0.060	0.076	0.561
ABU	0.013	0.290	0.958	0.148	0.137	0.480	0.052	0.087	0.624
ANN	0.290	0.006	0.022	0.149	0.136	0.478	0.111	0.020	0.149
TYT	0.153	0.137	0.473	0.173	0.112	0.394	0.079	0.055	0.409
KMM	0.271	0.020	0.068	0.161	0.124	0.436	0.089	0.048	0.352
VSM	0.150	0.140	0.483	0.116	0.169	0.592	0.070	0.059	0.457
ANR	0.186	0.105	0.360	0.185	0.100	0.352	0.099	0.041	0.293
ADR	0.212	0.078	0.270	0.179	0.106	0.372	0.091	0.038	0.292
PGD	0.173	0.118	0.404	0.203	0.082	0.289	0.088	0.042	0.322
SHU	0.209	0.082	0.281	0.225	0.060	0.212	0.093	0.054	0.367

Anna Nagar, Madhavaram and Anna Nagar are ranked as fifteen in 2019, 2020 and 2021 respectively. The comparison of these ranks is presented in Figure 7.

Figure 7 presents a comparison of the ranks obtained by the TOPSIS method against three alternative approaches, CODAS, WASPAS and MOORA, to assess their performance and effectiveness in ranking various areas. The results for the years 2019-2021 show that the TOPSIS method precisely matched rankings in all areas compared to the MOORA approach. Moreover, in 2019 and 2020, the TOPSIS method achieved identical rankings in all specified areas when compared to CODAS except for few areas in 2021. Similarly, concerning the WASPAS method, the TOPSIS rankings coincided with WASPAS in 13 areas for 2019-2020 and 11 areas for 2021, indicating considerable consistency across different periods. This substantial consistency in rankings highlights the potential of the TOPSIS method. It is important to note slight variations in rankings when comparing the TOPSIS method to the other three approaches. To verify the accuracy of the TOPSIS method, it is essential to perform an evaluation using appropriate statistical tools. Spearman's rank correlation test, is applied to test the interrelationship among the rankings obtained from the AHP-Entropy-TOPSIS with CODAS, WASPAS and MOORA results.

Spearman's coefficient determines the significance of correlation among two or more rankings. Suppose there are two ranking datasets (A_1 and A_2). Spearman's rank (ρ) over these datasets is calculated using the equation.

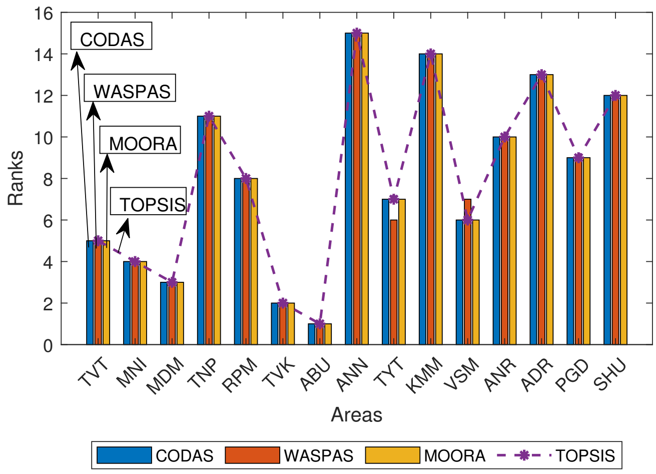
$$\rho = 1 - \frac{6\sum d_i^2}{n(n^2-1)} \tag{29}$$

Here, n is the number of alternatives in the data set, and d^2 is the difference between the two rankings (A_1 and A_2). The resultant ρ value specifies the relationship between the two ranking sets. The value closer to +1 signifies a strong positive relationship, whereas the value nearer to -1 indicates a strong negative relationship.

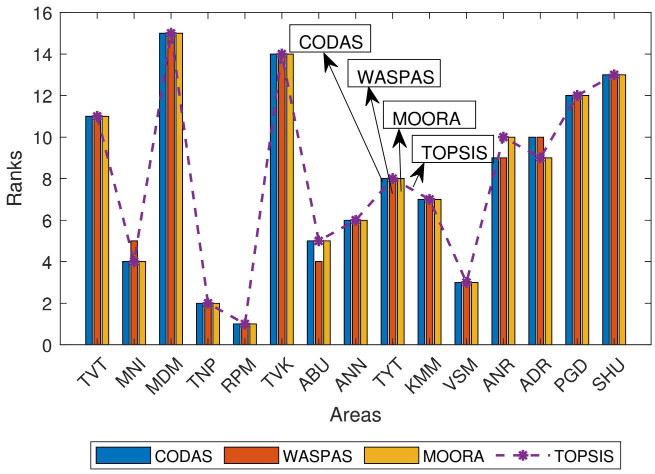
From the ranks obtained from AHP-Entropy-TOPSIS, CODAS, WASPAS and MOORA the correlation is determined using the equation 29. The spearman's rank correlation among the AHP-Entropy-TOPSIS approach with CODAS, WASPAS and MOORA methods in 2019 is 0.999, 0.995 and 0.999; in 2020, it is 0.994, 0.991 and 0.999; in 2021, it is 0.987, 0.980 and 0.999 respectively. It is also observed that the correlation between the AHP-Entropy-TOPSIS and the other methods indicate a strong positive correlation between the methods.

B. Sensitivity Analysis

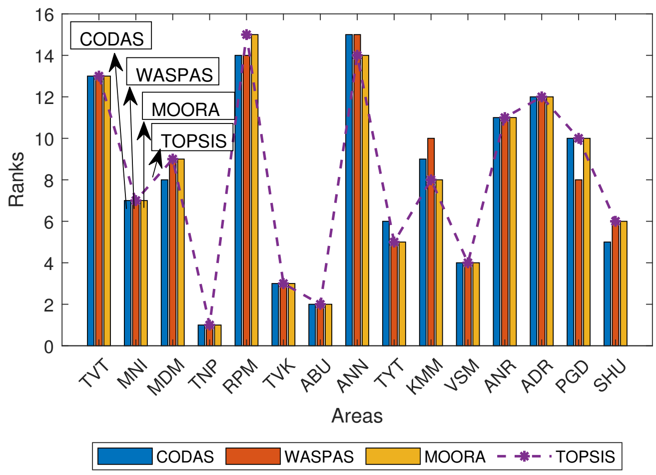
A sensitivity analysis is carried out to examine the effect of priority weights on the ranking of regions and to identify



(a) Comparion of the ranks for the year 2019



(b) Comparion of the ranks for the year 2020



(c) Comparion of the ranks for the year 2021

Fig. 7: Comparative ranks of CODAS, WASPAS and MOORA methods with TOPSIS

the impact of a specific air pollutant on air quality. The sensitivity analysis in the present study considers two scenarios:

- The priority weights have changed.
- Reduction of pollutants

1) Scenario 1: This scenario presents the changes in the weights of the attributes and the comparison between the 2 cases:

Case 1: By applying a combined AHP-Entropy (AE) approach.

Case 2: Equal weights (EW) are considered for all attributes.

Fig. 8 depicts the ranking of the areas varies while applying the AE approach and the EW approach. It can be seen that the ranking of the areas changes in many places when using equal weights across the years. In 2019, the area ranks were not disturbed in ANR, ADR and SHU, whereas in 2020, MDM is the only area where the rank is unchanged. In 2021 TVT, TNP, RPM and ANN attained the same rank, and the ranks of other areas are changed when applying the EW approach.

When evaluating any decision-making problem, the weights of the attributes are vital. Air has different concentrations of multiple pollutants. So, this scenario 1 indicates that computing the pollutant's weights is essential while finding the most polluted region.

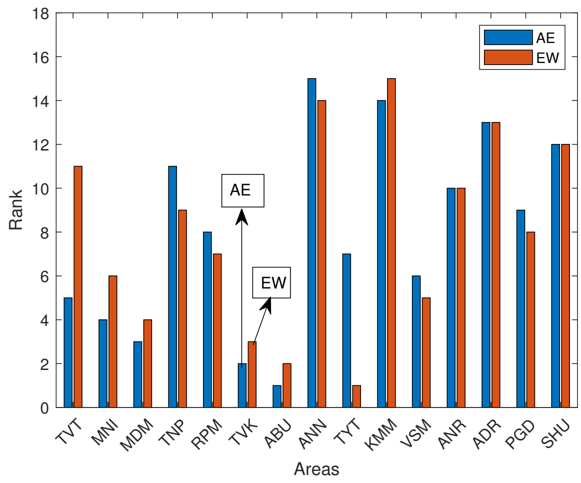
2) Scenario 2: In this case, sensitivity analysis removes air pollutants one at a time to observe how they affect the ranking of contaminated places. This kind of sensitivity analysis gives information on the pollutant that significantly impacts air quality and affects the region's rank. For the analysis SO_2 is eliminated first, NO_2 second, then PM_{10} finally $PM_{2.5}$ is eliminated, and Fig. 9 illustrates the areas ranking with all pollutants and after removing the SO_2, NO_2, PM_{10} and $PM_{2.5}$ pollutants.

This scenario represents the impact of a particular pollutant and changes in ranking while eliminating each pollutant. When eliminating the air pollutant SO_2, NO_2, and PM_{10} the ranks of the areas are not affected, but while eliminating the pollutant $PM_{2.5}$ it causes a significant change in rank across the years. This sensitivity case concludes the effect of the pollutant $PM_{2.5}$ causes more impact while ranking the most polluted areas. Since $PM_{2.5}$ is identified as the prominent pollutant in ranking the most polluted region, a correlation between the concentration of different pollutants with $PM_{2.5}$ allows the extrapolation of the concentration of another.

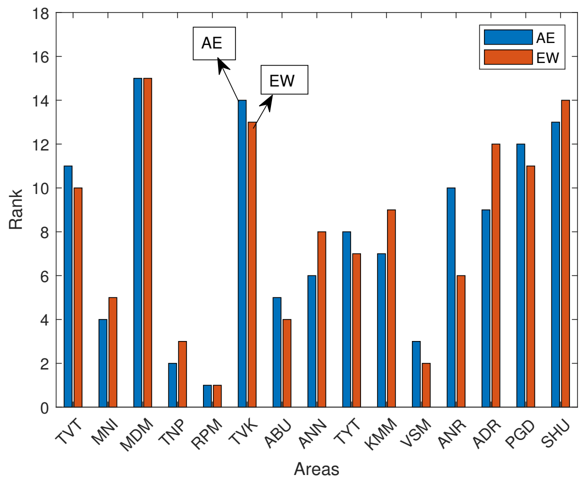
The correlation coefficient r measures the degree of linear dependency between two variables. Its value can lie in a range between -1 and 1, depending on whether the correlation is completely negative or positive, respectively. If it equals zero, there is a lack of linear dependency.

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}} \tag{30}$$

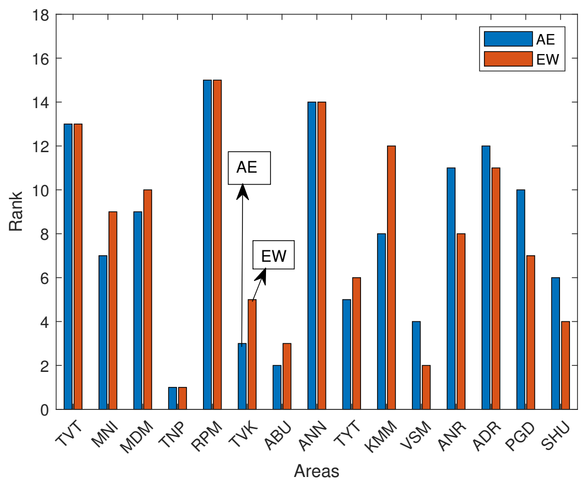
r = correlation coefficient
x_i = values of the x-variable in a sample
$\bar{x}$ = mean of the values of the x-variable



(a) AHP-Entropy and equal weights ranking for the year 2019

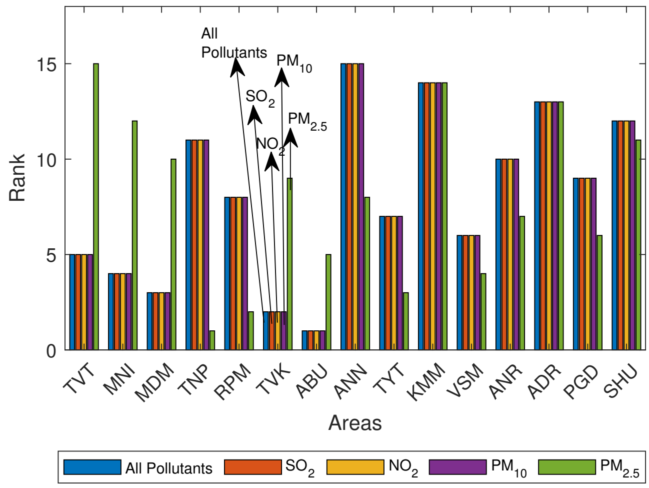


(b) AHP-Entropy and equal weights ranking for the year 2020

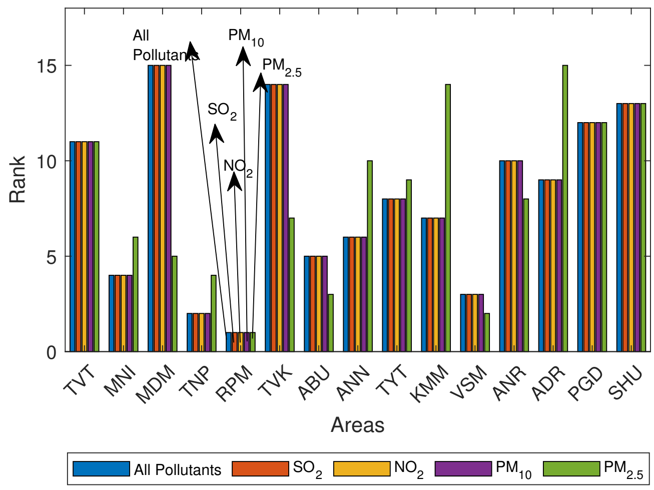


(c) AHP-Entropy and equal weights ranking for the year 2021

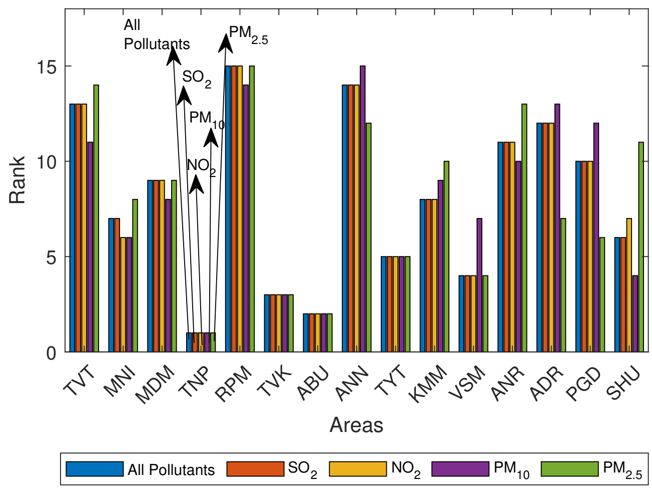
Fig. 8: Sensitivity analysis for scenario 1



(a) Changes in ranking for the year 2019



(b) Changes in ranking for the year 2020



(c) Changes in ranking for the year 2021

Fig. 9: Sensitivity analysis for scenario 2

y_i = values of the y-variable in a sample
$\bar{y}$ = mean of the values of the y-variable

The correlation between PM2.5 and other pollutants for the year 2019-2021 was calculated using equation 17 and the obtained results are represented in Table VIII.

TABLE VIII: Correlation between the concentration of other pollutants with $PM_{2.5}$

Pollutants	$PM_{2.5}$ (2019)	$PM_{2.5}$ (2020)	$PM_{2.5}$ (2021)
SO_2	-0.2506	-0.1211	0.4428
NO_2	-0.3047	0.2260	-0.1271
PM_{10}	0.1372	0.5882	0.6785

The results in the Table VIII show a statistically significant relationship between the levels of $PM_{2.5}$ and other pollutants in the air. The pollutant $PM_{2.5}$ is highly correlated with the PM_{10} pollutant every year rather than other pollutants. These pollutants' correlations were 0.1372 in 2019, 0.5882 in 2020 and 0.6785 in 2021. Since PM_{10} and $PM_{2.5}$ are both particles that are less than 10 or 2.5 micrometres in diameter, they make up a large proportion of dust that can be drawn deep into the lungs. The air quality of these areas can be improved by reducing the sources which emit the air pollutants $PM_{2.5}$ and PM_{10}.

V. Conclusion

In this study, the estimation of the most polluted area in Chennai city during the Bhogi festival in the year 2019-2021 was carried out by the AHP-Entropy-TOPSIS approach. The weights of the four different pollutants are derived using the AHP and Entropy methods to attain the advantage from both the subjective and objective weight approach. Fifteen different areas of Chennai city were considered as alternatives. The proposed model is implemented to rank these areas based on the priority weights attained from the combined AHP and Entropy methods. It is clear from the results that Ambattur (ABU), Royapuram (RPM), and Tondiarpet (TNP) were the most polluted areas in the years 2019, 2020, and 2021 respectively. The proposed model has many advantages, like less complexity and computation of air pollutants weights using subjective and objective weight concepts.

Furthermore, the validation of the proposed decision support system is checked through spearman's rank correlation with the other three existing MADM approaches namely, CODAS, WASPAS and MOORA. The result proved the consistency and strong correlation in the ranking of the proposed approach. Sensitivity analysis is also carried out to identify the importance of specific air pollutants on overall air quality and to determine whether the pollutants' weight affects the ranks of the city. The first analysis provided the ranks of the areas to illustrate that the priority weights of the pollutants are vital in ranking the areas, and the second analysis indicated that the pollutant $PM_{2.5}$ has more impact on the areas' rank. These results might help government agencies in making the right decisions. In future, the present work can be enhanced by incorporating more pollutants and factors like temperature, humidity, and wind to analyse the air quality.

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