# Effects of Thermal Radiation in a Rotating Fluid on an Oscillating Vertical Plate with Variable Temperature and Mass Diffusion

A. Antony Gnana Aravind and J. Ravikumar*

*Abstract*—**A vertically oscillating plate with a temperature difference and mass diffusion is used to investigate the role of thermal radiation and chemical reaction in the unexpected convection flow of a viscous unstable rotating fluid. The results of the calculations indicate that the fluid is a grey medium that absorbs and emits radiation. With time, both the plate's and the surrounding environment's temperatures and concentrations rise. The Laplace transform method finds solutions to the dimensionless governing equations for a plate, which is oscillating in its own plane. In relation to one another, the phase angle, radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number, and time are all investigated.**

*Index Terms*—**Oscillating, Phase angle, Vertical plate, Variable temperature, mass diffusion.**

## I. Introduction

In natural systems, there are fluid movements that arise from not just temperature gradients but also variations in concentration. The variations in mass transfer have an impact on the rate of heat transmission. In several industrial sectors, numerous transportation processes occur whereby the combined buoyancy effect with the influence of thermal radiation facilitates the simultaneous occurrence of heat and mass transfer. Therefore, the design of fins, steel rolling, nuclear power plants, gas turbines, and other propulsion devices for aircraft, missiles, and satellites, as well as the processing of materials, energy utilisation, temperature measurements, food processing, cryogenic engineering, remote sensing for astronomy and space exploration, and many other agricultural, medical, and military applications, depend heavily on radiative heat and mass transfer. When the temperature of the surrounding fluid is high, radiation effects become essential, which occurs in space technology. In these cases, the effects of heat radiation and mass diffusion must be considered.

England and Emery [1] examined how heat radiation affected the gas boundary layer with laminar-free convection. Singh [2] investigated hydromagnetic convection flow in an abruptly rotating fluid through an infinite vertical plate. The effects of mass transfer and convection flow currents flowing across an oscillating vertical plate were studied by Soundalgekar, Lahurikar, Pohanerkar, and Birajdar [3]. Soundalgekar and Akolkar [4] studied free convection currents and mass transfer when they changed the flow across a vertically oscillating plate. Mansour [5] examined a vertical plate to determine how radiation and free convection impact oscillatory flow. Using computational methods, Muthucumaraswamy and Saravanan [6] investigated the radiative flow of an oscillating, vertical, semi-infinite plate exposed to a constant mass flux. Radiation and chemical reactions were studied by Manivannan, Muthucumaraswamy, and Thangaraj [7] on an isothermal, vertically oscillating plate with varying mass diffusion.

The precise solution of a vertical plate immersed in a rotating fluid with temperature, mass diffusion, and variations in thermal radiation is the subject of the research conducted by Ravikumar and Vijayalakshmi [8]. Radiation effects on MHD were studied by Ravikumar and Vijayalakshmi [9] in a rotating fluid passing a vertical plate of varying temperature and mass. Heat and mass transfer in porous media under the influence of radiation and slip conditions were studied by Giulio Lorenzini, Halima Usman, and Fazle Mabood [10]. Govind Pathak and Rakesh Kumar [11] looked into the impact of thermal radiation and heat generation on a plate moving vertically at different temperatures and with different levels of mass diffusion. Nagarajan and Sundar Raj [12] investigated the effects of radiation on a vertically oscillating plate subject to first-order chemical processes at different temperatures. Heat and mass transfer effects on flow across a vertically oscillating plate of varying temperatures were studied by R. Muthucumaraswamy and A. Vijayalakshmi [13]. The authors of this study, Titilayo Morenike Agbaje, Sandile Sydney Motsa, Peter Leach, and Precious Sibanda [14], researched on an efficient large spectral collocation technique for analyzing the behavior of MHD laminar natural convection flow from a vertical permeable flat plate. The study included many factors, including uniform surface temperature, Soret and Dufour effects, chemical reactions, and thermal radiation.

The effects of heat and mass transfer in a rotating fluid with a temperature difference and mass diffusion have not been studied, but a vertical plate that constantly oscillates while being exposed to thermal radiation is an exciting research topic. Using a vertically oscillating plate with temperature and mass diffusion, we will examine the impact of heat radiation on a rotating, unstable fluid flow. The Laplace transform is used for solving the governing equations since they are dimensionless.

## II. Mathematical Analysis

The flow of a rotating fluid is examined in three dimensions across an infinite vertical oscillating plate while being

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viscous, incompressible, and electrically conducting. On this plate, the $x'$ and $y'$ axes are normal to the plate's plane, whereas the $z'$ axis is perpendicular to the plate's surface. We now set the plate, which is at a fixed vertical angle on the $x'$ axis. The fluid and the plate spin at the same constant angular velocity $\Omega'$ around the $z'$ axis. This fluid is a non-scattering grey absorbing-emitting substance. The plate and fluid are initially at rest, with uniform temperatures and concentrations $T'_\infty$ and $C'_\infty$ respectively. The plate is given an oscillating motion with the velocity $u_0 \cos \omega t$ and $u_0 \sin \omega t$ in a fluid, together with thermal radiation at time $t'$ against the gravitational field in the vertical direction. The concentration $C'_w$ is then held constant while the temperature $T'_w$ of the plate is increased for $t'$. All physical quantities are proportional to $z'$ because the plate occupying the plane $z' = 0$ has an infinite extent. This transient flow is governed by the following equations, which are based on the conventional Boussinesq's approximation:

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + v\frac{\partial^2 u'}{\partial z'^2} \qquad (1)$$

$$\frac{\partial v'}{\partial t'} - 2\Omega' u' = v\frac{\partial^2 u'}{\partial z'^2} \qquad (2)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z'} \qquad (3)$$

$$\frac{\partial C'}{\partial t'} = \frac{\partial^2 c'}{\partial z'^2} - K_1(C' - C'_\infty) \qquad (4)$$

To begin, suppose we have the required beginning and boundary conditions:

$$t' \le 0 : u' = 0, \qquad v' = 0,$$
$$T' = T'_\infty, \qquad C' = C'_\infty \quad \text{for all } z'$$

For cosine oscillation,

$$t' > 0 : u' = u_0 \cos \omega t', \qquad v' = 0,$$
$$T' = T'_w + (T'_w - T'_\infty)At',$$
$$C' = C'_w + (C'_w - C'_\infty)At' \quad \text{at } z' = 0$$

For sine oscillation,

$$t' > 0 : u' = u_0 \sin \omega t', \qquad v' = 0,$$
$$T' = T'_w + (T'_w - T'_\infty)At',$$
$$C' = C'_w + (C'_w - C'_\infty)At' \quad \text{at } z' = 0$$

$$u' = 0, \qquad v' = 0,$$
$$T' \to T'_\infty, \qquad C' \to C'_\infty \quad \text{at } z' \to \infty \qquad (5)$$

where $A = \frac{u_0^2}{v}$.

The local radiant is stated in the case of an optically thin grey gas by

$$\frac{\partial q_r}{\partial z'} = -4a^*\sigma(T'^4_\infty - T'^4) \qquad (6)$$

Temperature differences within the flow are thought to be small enough in practice that $T'^4$ may be characterized as a linear function of temperature. This is performed in a Taylor series based on $T'_\infty$ by eliminating higher-order components and expanding $T'^4$, so

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \qquad (7)$$

By using equations (6) and (7), equation (3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial z'^2} + 16a^*\sigma T'^3_\infty(T'_\infty - T') \qquad (8)$$

The following dimensionless quantities are introduced:

$$(u, v) = \frac{(u', v')}{u_0}, \quad t = \frac{t' u'^2_0}{v}, \quad z = \frac{z' u_0}{v}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},$$

$$Gr = \frac{g\beta v(T'_w - T'_\infty)}{u'^3_0}, \qquad C = \frac{C' - C'_\infty}{C'_w - C'_\infty},$$

$$Gc = \frac{g\beta^* v(C'_w - C'_\infty)}{u'^3_0}, \qquad K = K_1\left[\frac{v}{u_0^2}\right]^{\frac{1}{3}} \qquad (9)$$

$$Pr = \frac{\mu c_p}{k}, \quad \Omega = \frac{\Omega' v}{u'^2_0}, \quad R = \frac{16a^* v^2 \sigma T'^3_\infty}{k u'^2_0}$$

and the problem's equations are simplified from (1) to (5) by the statement $q = u + iv$, where $q$ is a complex velocity with sign $i = \sqrt{-1}$, as

$$\frac{\partial q}{\partial t} + 2i\Omega = Gr\theta + GcC + \frac{\partial^2 q}{\partial z^2} \qquad (10)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial z^2} - \frac{R}{Pr}\theta \qquad (11)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial z^2} - KC \qquad (12)$$

Dimensionless initial and boundary variables are as follows:

$$q = 0, \qquad \theta = 0, \qquad C = 0 \quad \text{for all } z \le 0 \text{ and } t \le 0$$

For cosine oscillation,

$$t > 0 : \; q = \cos \omega t, \qquad \theta = t, \qquad C = t \quad \text{at } z = 0$$

For sine oscillation,

$$t > 0 : \; q = \sin \omega t, \qquad \theta = t, \qquad C = t \quad \text{at } z = 0$$

$$q = 0, \qquad \theta \to 0, \qquad C \to 0 \quad \text{as } z \to \infty \qquad (13)$$

The solutions to equations (10), (11), and (12) with boundary conditions (13), are given below which is obtained using the standard Laplace-transform approach,

$$\theta = \frac{t}{2}\Big[\exp(-2\eta\sqrt{Prat})\; erfc(\eta\sqrt{Pr} - \sqrt{at}) + \exp(2\eta\sqrt{Prat})\; erfc(\eta\sqrt{Pr} + \sqrt{at})\Big] - \frac{\eta\sqrt{Prt}}{2\sqrt{a}}\Big[\exp(-2\eta\sqrt{Prat})\; erfc(\eta\sqrt{Pr} - \sqrt{at}) - \exp(2\eta\sqrt{Prat})\; erfc(\eta\sqrt{Pr} + \sqrt{at})\Big]$$

$$C = \frac{t}{2}\Big[\exp(-2\eta\sqrt{ScKt})\; erfc(\eta\sqrt{Sc} - \sqrt{Kt}) + \exp(2\eta\sqrt{ScKt})\; erfc(\eta\sqrt{Sc} + \sqrt{Kt})\Big] - \frac{\eta\sqrt{Sct}}{2\sqrt{K}}\Big[\exp(-2\eta\sqrt{ScKt})\; erfc(\eta\sqrt{Sc} - \sqrt{Kt}) - \exp(2\eta\sqrt{ScKt})\; erfc(\eta\sqrt{Sc} + \sqrt{Kt})\Big]$$

We obtain velocity for cosine oscillations,

$$
\begin{aligned}
q_c = {} & \frac{\exp(i\omega t)}{2} \\
& \Bigg\{ \frac{1}{2} \Bigg[ \exp(2\eta\sqrt{bt+i\omega t})\ erfc(\eta+\sqrt{bt+i\omega t}) \\
& + \exp(-2\eta\sqrt{bt+i\omega t})\ erfc(\eta-\sqrt{bt+i\omega t}) \Bigg] \Bigg\} \\
& + \frac{\exp(-i\omega t)}{2} \\
& \Bigg\{ \frac{1}{2} \Bigg[ \exp(-2\eta\sqrt{bt-i\omega t})\ erfc(\eta-\sqrt{bt-i\omega t}) \\
& + \exp(2\eta\sqrt{bt-i\omega t})\ erfc(\eta+\sqrt{bt-i\omega t}) \Bigg] \Bigg\} \\
& + \frac{Gr}{(1-Pr)} \Bigg\{ \frac{1}{2c^2} \Bigg[ (\exp -2\eta\sqrt{bt})(\eta-\sqrt{bt}) \\
& + \exp 2\eta\sqrt{bt}(\eta+\sqrt{bt}) \Bigg] \\
& + \frac{t}{2c} \Bigg[ (\exp -2\eta\sqrt{bt})\ erfc(\eta-\sqrt{bt}) \\
& + \exp 2\eta\sqrt{bt}\ erfc(\eta+\sqrt{bt}) \Bigg] \\
& - \frac{\exp(ct)}{2c^2} \Bigg[ (\exp(-2\eta\sqrt{(c+b)t}))\ erfc(\eta-\sqrt{(c+b)t}) \\
& + \exp(2\eta\sqrt{(c+b)t})\ erfc(\eta+\sqrt{(c+b)t}) \Bigg] \\
& - \frac{1}{2c^2} \Bigg[ (\exp(-2\eta\sqrt{Prat}))\ erfc(\eta\sqrt{Pr}-\sqrt{at}) \\
& + \exp(2\eta\sqrt{Prat})\ erfc(\eta\sqrt{Pr}+\sqrt{at}) \Bigg] \\
& - \Bigg[ \frac{t}{2c} \Big[ \exp(-2\eta\sqrt{Prat})\ erfc(\eta\sqrt{Pr}-\sqrt{at}) \\
& + \exp(2\eta\sqrt{Prat})\ erfc(\eta\sqrt{Pr}+\sqrt{at}) \Big] \\
& - \frac{\eta\sqrt{Prt}}{2c\sqrt{a}} \Big[ \exp(-2\eta\sqrt{Prat})\ erfc(\eta\sqrt{Pr}-\sqrt{at}) \\
& - \exp(2\eta\sqrt{Prat})\ erfc(\eta\sqrt{Pr}+\sqrt{at}) \Big] \Bigg] \\
& + \frac{\exp(ct)}{2c^2} \\
& \Bigg[ (\exp(-2\eta\sqrt{Pr(a+c)t}))\ erfc(\eta\sqrt{Pr}-\sqrt{(a+c)t}) \\
& + \exp(2\eta\sqrt{Pr(a+c)t})\ erfc(\eta\sqrt{Pr}+\sqrt{(a+c)t}) \Bigg] \Bigg\} \\
& + \frac{GC}{(1-Sc)} \Bigg\{ \frac{1}{2d^2} \Bigg[ (\exp -2\eta\sqrt{bt})(\eta-\sqrt{bt}) \\
& + \exp 2\eta\sqrt{bt}(\eta+\sqrt{bt}) \Bigg] \\
& + \frac{t}{2d} \Bigg[ (\exp -2\eta\sqrt{bt})\ erfc(\eta-\sqrt{bt}) \\
& + \exp 2\eta\sqrt{bt}\ erfc(\eta+\sqrt{bt}) \Bigg] \\
& - \frac{\exp(dt)}{2d^2} \\
& \Bigg[ (\exp(-2\eta\sqrt{(d+b)t}))\ erfc(\eta-\sqrt{(d+b)t}) \\
& + \exp(2\eta\sqrt{(d+b)t})\ erfc(\eta+\sqrt{(d+b)t}) \Bigg] \\
& - \frac{1}{2d^2} \Bigg[ (\exp(-2\eta\sqrt{ScKt}))\ erfc(\eta\sqrt{Sc}-\sqrt{Kt}) \\
& + \exp(2\eta\sqrt{ScKt})\ erfc(\eta\sqrt{Sc}+\sqrt{Kt}) \Bigg] \\
& - \Bigg[ \frac{t}{2d} \Big[ \exp(-2\eta\sqrt{ScKt})\ erfc(\eta\sqrt{Sc}-\sqrt{Kt}) \\
& + \exp(2\eta\sqrt{ScKt})\ erfc(\eta\sqrt{Sc}+\sqrt{Kt}) \Big] \\
& - \frac{\eta\sqrt{Sct}}{2d\sqrt{K}} \\
& \Big[ \exp(-2\eta\sqrt{ScKt})\ erfc(\eta\sqrt{Sc}-\sqrt{Kt}) \\
& - \exp(2\eta\sqrt{ScKt})\ erfc(\eta\sqrt{Sc}+\sqrt{Kt}) \Big] \Bigg] \\
& + \frac{\exp(dt)}{2d^2} \\
& \Bigg[ \exp(-2\eta\sqrt{Sc(K+d)t})\ erfc(\eta\sqrt{Sc}-\sqrt{(K+d)t}) \\
& + \exp(2\eta\sqrt{Sc(K+d)t})\ erfc(\eta\sqrt{Sc}+\sqrt{(K+d)t}) \Bigg] \Bigg\}
\end{aligned}
$$

Similarly, velocity corresponding to sine oscillation is,

$$
\begin{aligned}
q_s = {} & \frac{\exp(i\omega t)}{2i} \\
& \Bigg\{ \frac{1}{2} \Bigg[ \exp(2\eta\sqrt{bt+i\omega t})\ erfc(\eta+\sqrt{bt+i\omega t}) \\
& + \exp(-2\eta\sqrt{bt+i\omega t})\ erfc(\eta-\sqrt{bt+i\omega t}) \Bigg] \Bigg\} \\
& - \frac{\exp(-i\omega t)}{2i} \\
& \Bigg\{ \frac{1}{2} \Bigg[ \exp(-2\eta\sqrt{bt-i\omega t})\ erfc(\eta-\sqrt{bt-i\omega t}) \\
& + \exp(2\eta\sqrt{bt-i\omega t})\ erfc(\eta+\sqrt{bt-i\omega t}) \Bigg] \Bigg\} \\
& + \frac{Gr}{(1-Pr)} \Bigg\{ \frac{1}{2c^2} \Bigg[ (\exp -2\eta\sqrt{bt})\ erfc(\eta-\sqrt{bt})
\end{aligned}
$$

$$+ \exp 2\eta\sqrt{bt}\ erfc(\eta + \sqrt{bt})\Bigg]$$

$$+ \frac{t}{2c}\Bigg[\exp\left(-2\eta\sqrt{bt}\right)\ erfc(\eta - \sqrt{bt})$$

$$+ \exp 2\eta\sqrt{bt}\ erfc(\eta + \sqrt{bt})\Bigg]$$

$$- \frac{\exp\left(ct\right)}{2c^2}\Bigg[\exp\left(-2\eta\sqrt{(c+b)t}\right)\ erfc(\eta - \sqrt{(c+b)t})$$

$$+ \exp\left(2\eta\sqrt{(c+b)t}\right)\ erfc(\eta + \sqrt{(c+b)t})\Big)\Bigg]$$

$$- \frac{1}{2c^2}\Bigg[(\exp\left(-2\eta\sqrt{Prat}\right))\ erfc(\eta\sqrt{Pr} - \sqrt{at})$$

$$+ \exp\left(2\eta\sqrt{Prat}\right)\ erfc(\eta\sqrt{Pr} + \sqrt{at})\Bigg]$$

$$- \Bigg[\frac{t}{2c}\Big(\exp\left(-2\eta\sqrt{Prat}\right)\ erfc(\eta\sqrt{Pr} - \sqrt{at})$$

$$+ \exp\left(2\eta\sqrt{Prat}\right)\ erfc(\eta\sqrt{Pr} + \sqrt{at})\Big)$$

$$- \frac{\eta\sqrt{Prt}}{2c\sqrt{a}}\Big(\exp\left(-2\eta\sqrt{Prat}\right)\ erfc(\eta\sqrt{Pr} - \sqrt{at})$$

$$- \exp\left(2\eta\sqrt{Prat}\right)\ erfc(\eta\sqrt{Pr} + \sqrt{at})\Big)\Bigg]$$

$$+ \frac{\exp(ct)}{2c^2}$$

$$\Bigg[\exp\left(-2\eta\sqrt{Pr(a+c)t}\right))\ erfc(\eta\sqrt{Pr} - \sqrt{(a+c)t})$$

$$+ \exp\left(2\eta\sqrt{Pr(a+c)t}\right)\ erfc(\eta\sqrt{Pr} + \sqrt{(a+c)t})\Bigg]\Bigg\}$$

$$+ \frac{GC}{(1-Sc)}\Bigg\{\frac{1}{2d^2}\Bigg[\exp\left(-2\eta\sqrt{bt}\right)\ erfc(\eta - \sqrt{bt})$$

$$+ \exp\left(2\eta\sqrt{bt}\right)\ erfc(\eta + \sqrt{bt})\Bigg]$$

$$+ \frac{t}{2d}\Bigg[\exp\left(-2\eta\sqrt{bt}\right)\ erfc(\eta - \sqrt{bt})$$

$$+ \exp\left(2\eta\sqrt{bt}\right)\ erfc(\eta + \sqrt{bt})\Bigg]$$

$$- \frac{\exp\left(dt\right)}{2d^2}$$

$$\Bigg[\exp\left(-2\eta\sqrt{(d+b)t}\right)\ erfc(\eta - \sqrt{(d+b)t})$$

$$+ \exp\left(2\eta\sqrt{(d+b)t}\right)\ erfc(\eta + \sqrt{(d+b)t})\Bigg]$$

$$- \frac{1}{2d^2}\Bigg[\exp\left(-2\eta\sqrt{ScKt}\right) erfc(\eta\sqrt{Sc} - \sqrt{Kt})$$

$$+ \exp\left(2\eta\sqrt{ScKt}\right)\ erfc(\eta\sqrt{Sc} + \sqrt{Kt})\Bigg]$$

$$- \Bigg[\frac{t}{2d}\Big[\exp\left(-2\eta\sqrt{ScKt}\right)\ erfc(\eta\sqrt{Sc} - \sqrt{Kt})$$

$$+ \exp\left(2\eta\sqrt{ScKt}\right)\ erfc(\eta\sqrt{Sc} + \sqrt{Kt})\Big]$$

$$- \frac{\eta\sqrt{Sct}}{2d\sqrt{K}}\Big[\exp\left(-2\eta\sqrt{ScKt}\right)\ erfc(\eta\sqrt{Sc} - \sqrt{Kt})$$

$$- \exp\left(2\eta\sqrt{ScKt}\right)\ erfc(\eta\sqrt{Sc} + \sqrt{Kt})\Big]\Bigg]$$

$$+ \frac{\exp(dt)}{2d^2}$$

$$\Bigg[\exp\left(-2\eta\sqrt{Sc(K+d)t}\right)\ erfc(\eta\sqrt{Sc} - \sqrt{(K+d)t})$$

$$+ \exp\left(2\eta\sqrt{Sc(K+d)t}\right)\ erfc(\eta\sqrt{Sc} + \sqrt{(K+d)t})\Bigg]\Bigg\}$$

Where $a = \frac{R}{Pr}$, $b = 2i\Omega$, $c = \frac{R-b}{1-Pr}$, $d = \frac{ScK-b}{1-Sc}$, $\eta = \frac{z}{2\sqrt{t}}$, $erfc$ is the error complimentary function.

Complex arguments are used in the error function and error complimenting function. The term $q$ represents a complex velocity and is decomposed according to its real and imaginary components using the formula below.

$$erfc(x+iy) = erf(x) + \frac{\exp(-x^2)}{2x\pi}[1-\cos(2xy)] + \frac{\exp(-x^2)}{2x\pi}[i\sin(2xy)] + \frac{2\exp(-x^2)}{\pi}\sum_{n=1}^{\infty}\frac{\exp(-n^2/4)}{n^2+4x^2}[f_n(x,y) + ig_n(x,y)] + \epsilon(x,y)$$

Where,

$$f_n = 2x - 2x\cosh(ny)\cos(2xy) + n\sinh(ny)\sin(2xy)$$

$$g_n = 2x\cosh(ny)\sin(2xy) + n\sinh(ny)cos(2xy)$$

$$\mid \epsilon(x,y) \mid \approx 10^{-16} \mid erf(x+iy) \mid$$

## III. Results and Discussion

To understand the physical context completely, one must know the physical values of velocity, temperature, and concentration at different times, different phase angle values, Schmidt number, radiation parameter, chemical reaction parameter, thermal Grashof number, and mass Grashof number. The purpose of the analyses depicted here is to evaluate how different $R, K, Gr, Gc, Pr, Sc, t$ and $\omega t$ for cosine and sine oscillations affect the characteristics of the flow and transport. The complementary and exponential error functions can be used to express solutions by the Laplace transform.
Figure 1 shows profiles of temperatures at different times ($t = 0.3, 0.5, 0.7$ and $0.9$), $Pr = 7$ when thermal radiation $R = 5$ is present.

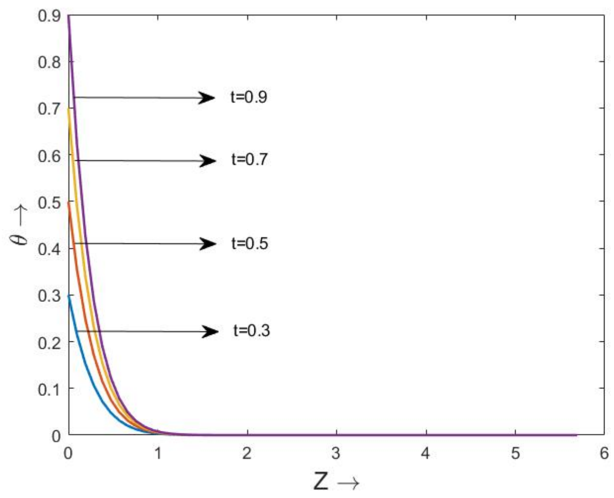


Figure 1. Temperature Profile for various t.

It is obvious that the temperature rises as the value of time $t$ increases.

Figure 2 depicts temperature profiles computed for several thermal radiation parameter values $(R = 0.5, 3, 7$ and $10)$ at $t = 0.8, (Pr = 7)$.

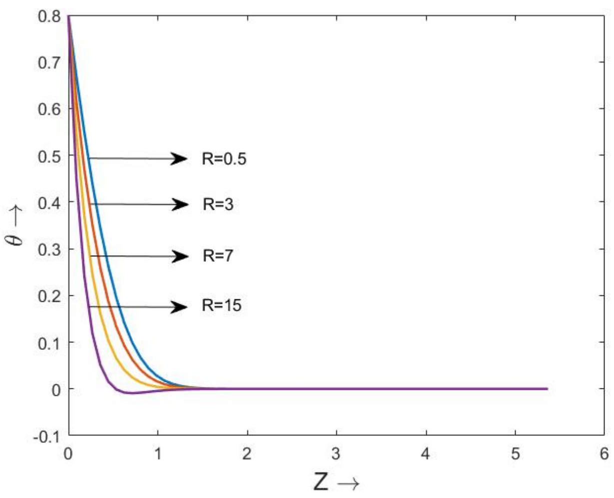


Figure 2. Temperature Profile for various R.

The temperature drops when the radiation parameter rises.

The concentration patterns are shown graphically in Figure 3 for various time values $(t = 0.3, 0.5, 0.7, 0.9), K = 0.2, Sc = 2.01$.

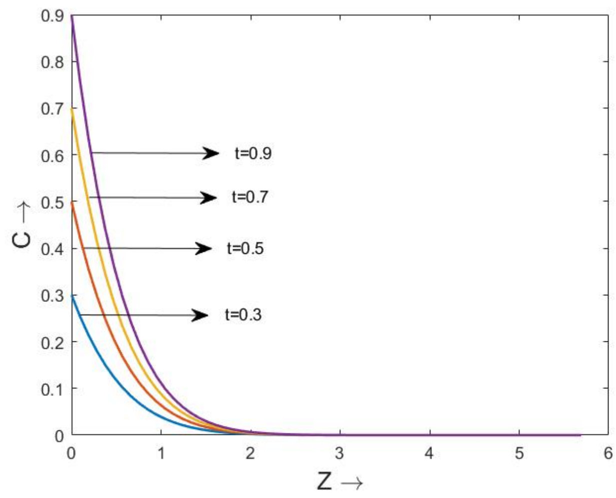


Figure 3. Concentration Profile for various t.

The trend indicates that the concentration rises as the value of time $t$ increases.

Figure 4 depicts Concentration profiles computed for several chemical reaction parameter values $(K = 0.5, 2, 5$ and $8)$ at $t = 0.8, (Sc = 2.01)$.

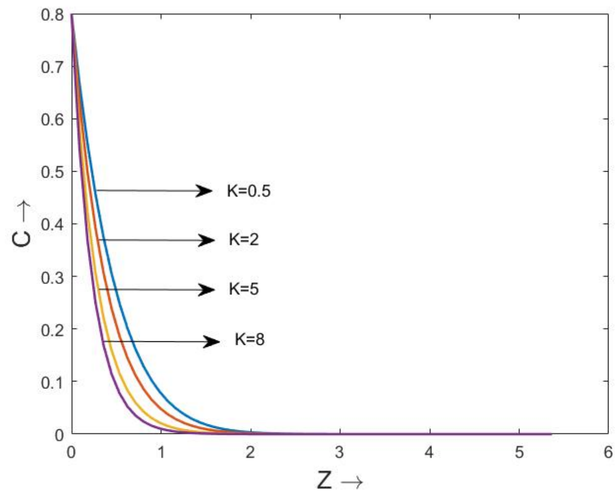


Figure 4. Concentration Profile for various K.

The concentration drops when the chemical reaction parameter rises.

Figure 5 depicts the primary velocity profiles for cosine oscillation at different phase angles $(\omega t = \pi/6, \pi/3, \pi/2), R = 15, K = 0.5, Gr = 5, Gc = 5, Sc = 2.01, Pr = 7, \Omega = 0.5$ and $t = 0.2$.

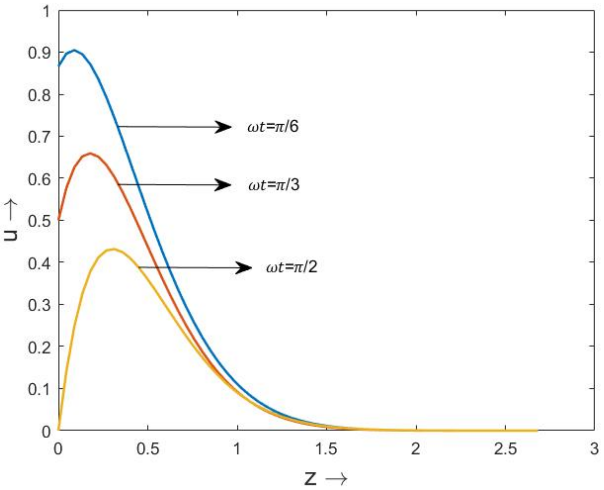


Figure 5. Primary velocity in cosine oscillation for various $\omega t$.

It is evident that when the phase angle$(\omega t)$ rises, the velocity drops.

Figure 6 shows secondary velocity profiles for several phase angles $\omega t = \pi/6, \pi/3, \pi/2), R = 15, K = 0.5, Gr = 5, Gc = 5, Sc = 2.01, Pr = 7, \Omega = 0.5$ and $t = 0.2$ for cosine oscillation.

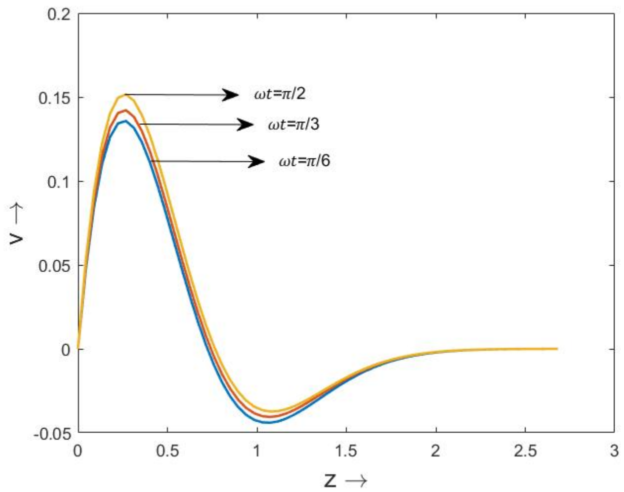


Figure 6. Secondary velocity in cosine oscillation for various $\omega t$.

It is evident that when the phase angle$(\omega t)$ rises, the velocity drops.

Figure 7 shows the effects of primary velocity for various radiation parameter values $(R = 1, 1.5, 2, 2.5), \omega t = \pi/4, K = 0.2, Gr = 10, Gc = 10, Pr = 7, Sc = 2.01, \Omega = 0.5$ and $t = 0.8$ for cosine oscillation.

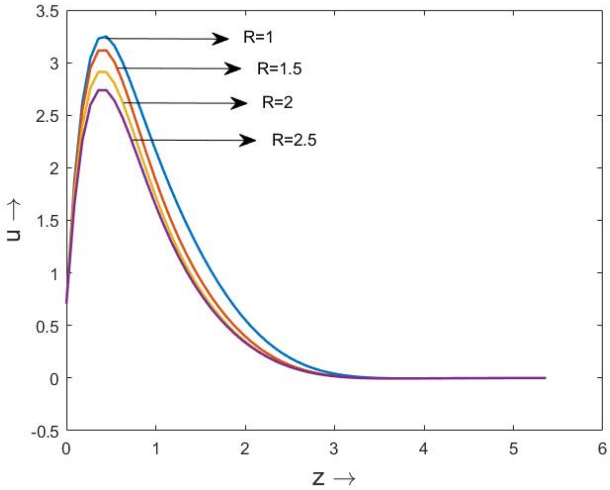


Figure 7. Primary velocity in cosine oscillation for various $R$.

It seems that the velocity rises when the radiation parameter's value decreases.

Figure 8 depicts the effect of secondary velocity for several radiation parameter values, such as $(R = 1, 1.5, 2, 2.5), \omega t = \pi/4, K = 0.2, Gr = 10, Gc = 10, Pr = 7, Sc = 2.01, \Omega = 0.5$ and $t = 0.8$ for cosine oscillation.

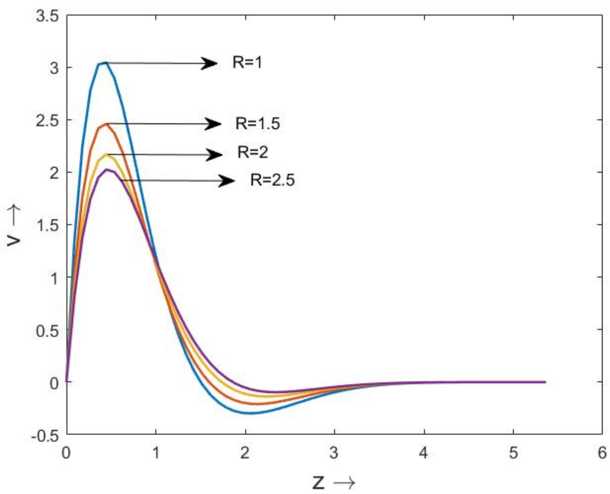


Figure 8. Secondary velocity in cosine oscillation for various $R$.

Based on the pattern, the radiation parameter seems to decrease when the velocity increases.

Figure 9 displays the rotational parameter's primary velocity profiles for various values. The values of the rotational parameters are $\Omega = 1, 1.5, 2, 2.5$ with $R = 10, K = 0.5, Gr = 10, Gc = 10, Sc = 2.01, t = 0.8, \omega t = \pi/6$ and $Pr = 7$ for cosine oscillation.

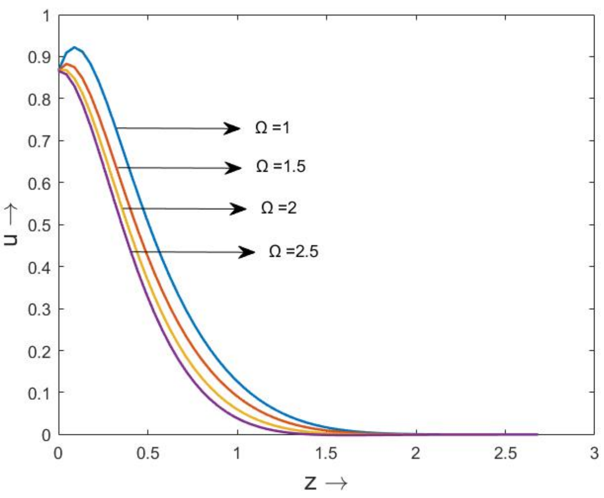


Figure 9. Primary velocity in cosine oscillation for various $\Omega$.

The primary velocity increases as the rotation parameter $\Omega$ decreases.

Figure 10 depicts the secondary velocity patterns for different values of rotational parameters. The values of the rotational parameters are $\Omega = 1, 1.5, 2, 2.5$ with $R = 10, K = 0.5, Gr = 10, Gc = 10, Sc = 2.01, t = 0.8, \omega t = \pi/6$ and $Pr = 7$ for cosine oscillation.

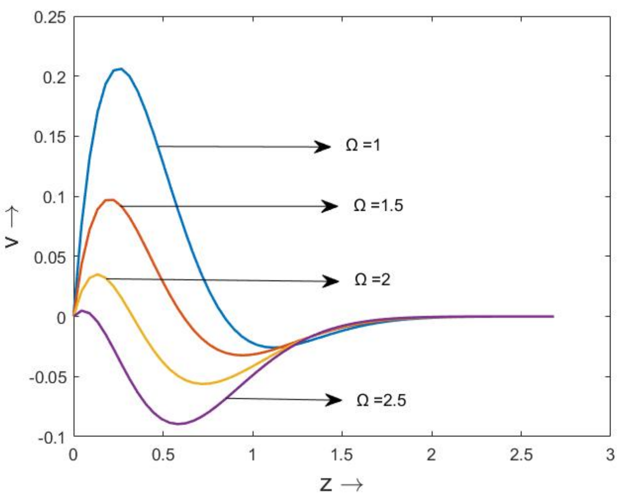


Figure 10. Secondary velocity in cosine oscillation for various $\Omega$.

As a result of the rotation parameter's influence, the secondary velocity decreases.

Figure 11 displays the rotational parameter's primary velocity profiles for various values. The values of the rotational parameters are $(K = 1, 1.5, 2, 2.5)$ with $R = 5, \Omega = 0.5, Gr = 10, Gc = 10, Sc = 2.01, t = 0.8, \omega t = \pi/4$ and $Pr = 7$ for cosine oscillation.

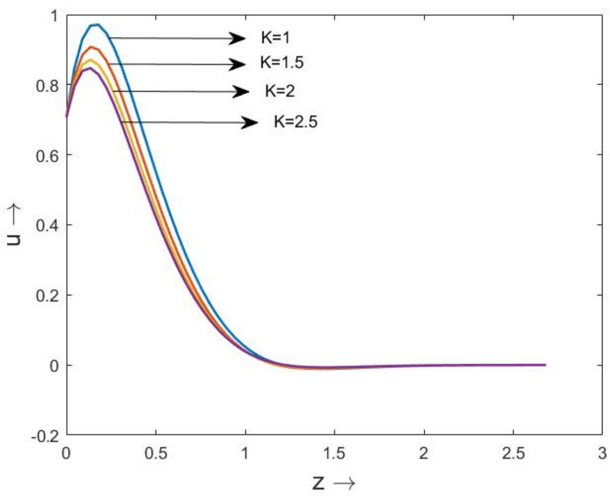


Figure 11. Primary velocity in cosine oscillation for various $K$.

As a result of the chemical reaction parameter's influence, the secondary velocity decreases.

Figure 12 depicts the effect of secondary velocity for several radiation parameter values, such as $(K = 1, 1.5, 2, 2.5), \omega t = \pi/4, R = 5, Gr = 10, Gc = 10, Pr = 7, Sc = 2.01, \Omega = 0.5$ and $t = 0.8$ for cosine oscillation.

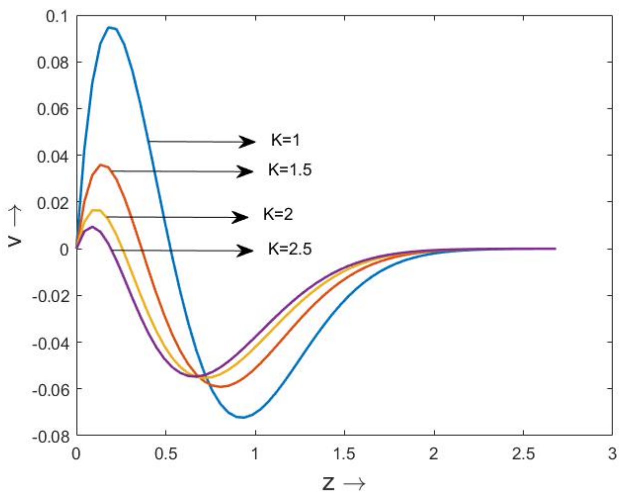


Figure 12. Secondary velocity in cosine oscillation for various $K$.

As a result of the chemical reaction parameter's influence, the secondary velocity decreases.

Figure 13 shows the effects of primary velocity for various thermal Grasshof number $(Gr = 10, 20, 30, 40), \omega t = \pi/2, K = 0.5, R = 5, Gc = 5, Pr = 7, Sc = 2.01, \Omega = 0.5$ and $t = 0.2$ for cosine oscillation.

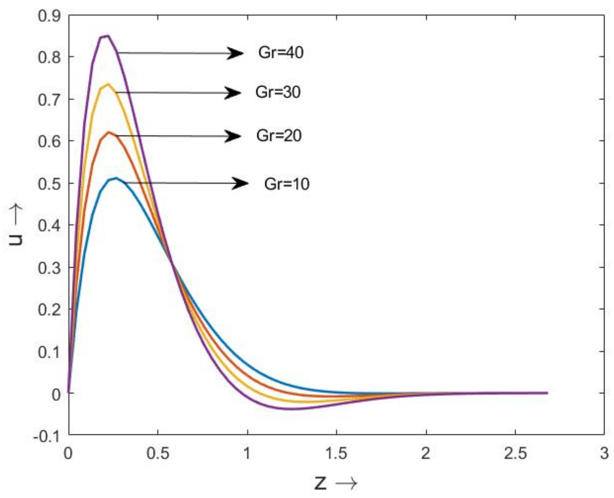


Figure 13. Primary velocity in cosine oscillation for various $Gr$.

The primary velocity increases as the thermal Grasshof number increases.

Figure 14 depicts the effect of secondary velocity for several thermal Grasshof number $(Gr = 10, 20, 30, 40), \omega t = \pi/2, K = 0.5, R = 5, Gc = 5, Pr = 7, Sc = 2.01, \Omega = 0.5$ and $t = 0.2$ for cosine oscillation.

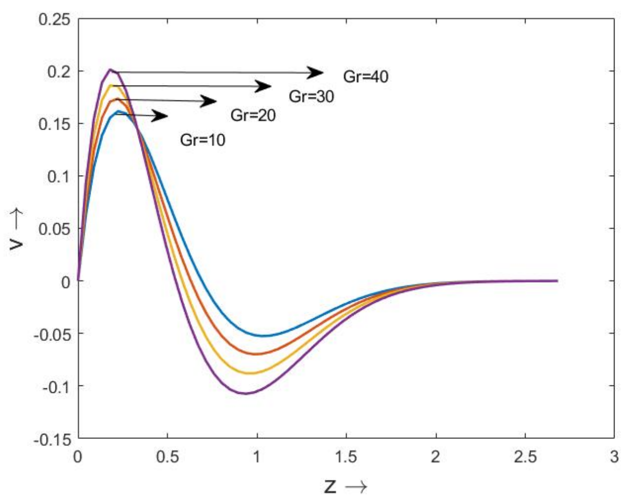


Figure 14. Secondary velocity in cosine oscillation for various $Gr$.

The pattern indicates that the velocity rises as the radiation parameter rises.

Figure 15 shows the primary velocity profiles for sine oscillation at various phase angles $(\omega t = \pi/6, \pi/3, \pi/2), R = 10, K = 0.5, Gr = 5, Gc = 5, Sc = 2.01, Pr = 7, \Omega = 0.5$ and $t = 0.2$.

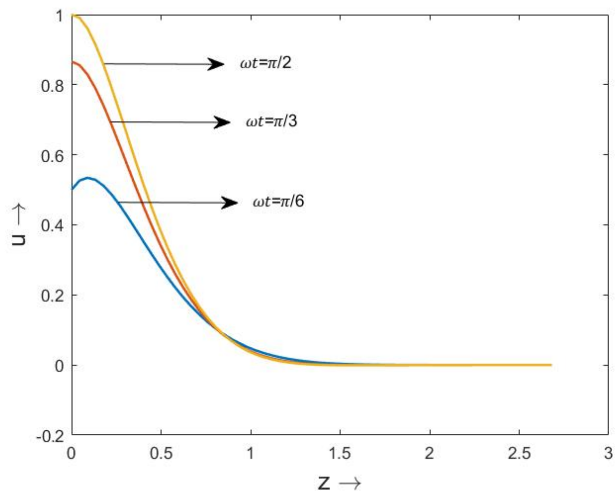


Figure 15. Primary velocity in sine oscillation for various $\omega t$.

It is evident that when phase angle$(\omega t)$ rises, velocity increases.

Figure 16 shows the secondary velocity profiles for sine oscillation at various phase angles $(\omega t = \pi/6, \pi/3, \pi/2), R = 10, K = 0.5, Gr = 5, Gc = 5, Sc = 2.01, Pr = 7, \Omega = 0.5$ and $t = 0.2$.

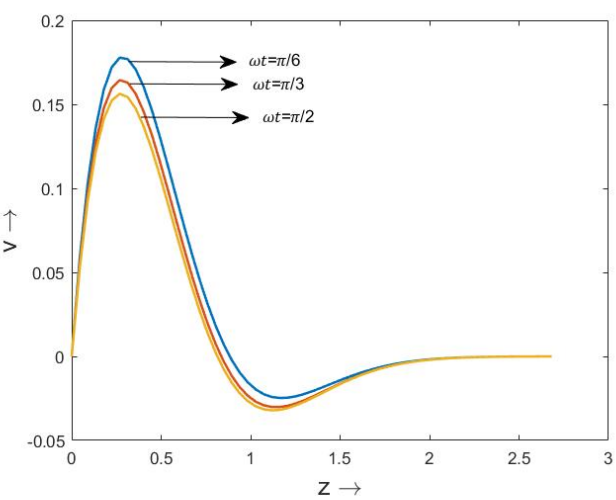


Figure 16. Secondary velocity in sine oscillation for various $\omega t$.

It is evident that when the phase angle$(\omega t)$ rises, the velocity drops.

Figure 17 depicts the impacts of velocity for sine oscillation with different radiation parameter values $(R = 1, 1.5, 2, 2.5), \omega t = \pi/4, K = 0.2, Gr = 10, Gc = 10, Pr = 7, Sc = 2.01, \Omega = 0.5$ and $t = 0.8$.

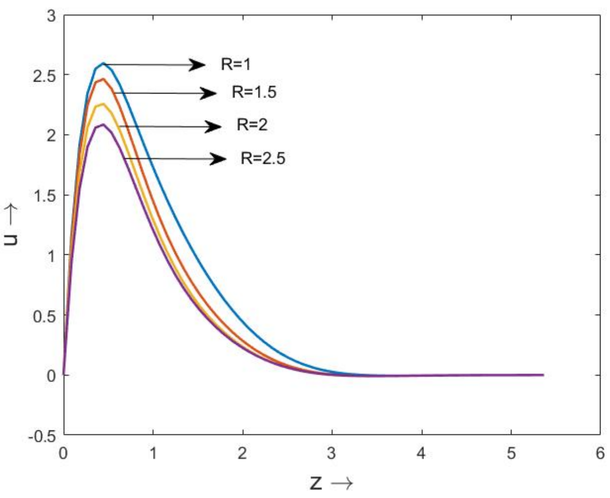


Figure 17. Primary velocity in sine oscillation for various $R$.

The pattern indicates that when the radiation parameter increases, the velocity drops.

Figure 18 depicts the effects of secondary velocity for different radiation parameter values, such as $(R = 1, 1.5, 2, 2.5), \omega t = \pi/4, K = 0.2, Gr = 10, Gc = 10, Pr = 7, Sc = 2.01, \Omega = 0.5$ and $t = 0.8$ for sine oscillation.

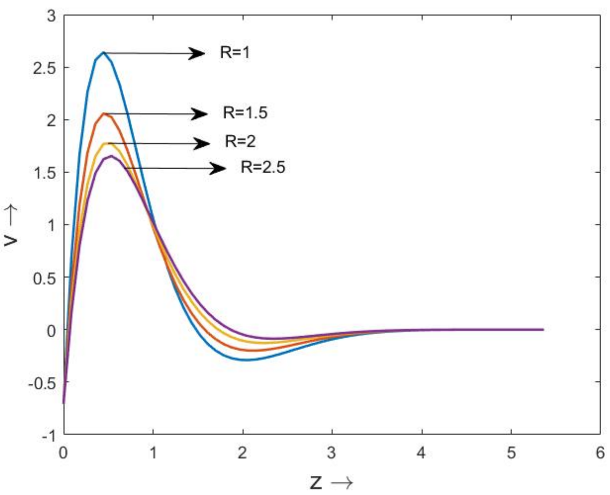


Figure 18. Secondary velocity in sine oscillation for various $R$.

The pattern indicates that the velocity drops as the radiation parameter rises.

The primary velocity profiles for various values are shown in Figure 19 for the rotational parameters. The values of the rotational parameters are $\Omega = 1, 1.5, 2, 2.5$ with $R = 10, K = 0.5, Gr = 5, Gc = 5, Sc = 2.01, t = 0.2, \omega t = \pi/6$ and $Pr = 7$ for sine oscillation.

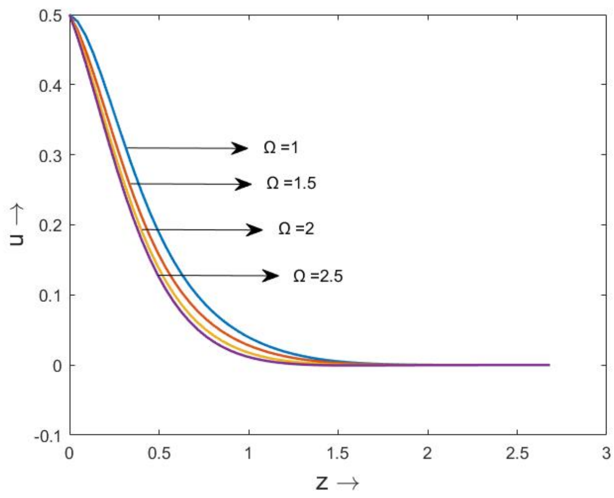


Figure 19. Primary velocity in sine oscillation for various $\Omega$.

The primary velocity increases as the rotation parameter $\Omega$ decreases.

Figure 20 depicts the secondary velocity patterns for different values of the rotational parameter. The values of the rotational parameters are $\Omega = 1, 1.5, 2, 2.5$ with $R = 10, K = 0.5, Gr = 5, Gc = 5, Sc = 2.01, t = 0.2, \omega t = \pi/6$ and $Pr = 7$ for sine oscillation.

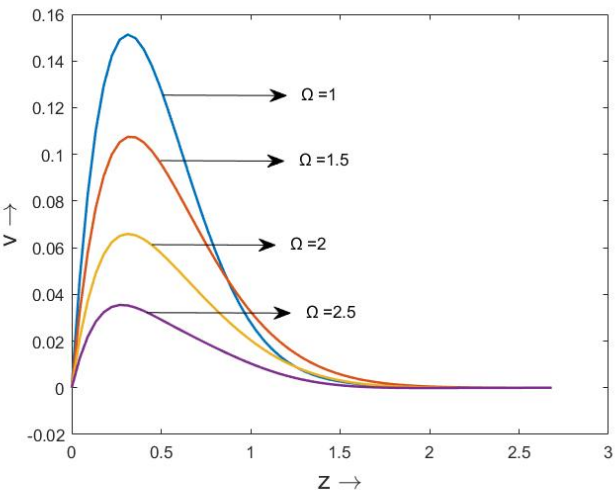


Figure 20. Secondary velocity in sine oscillation for various $\Omega$.

As a result of the rotation parameter decreases, the secondary velocity increases.

Figure 21 displays the Chemical reaction parameter's primary velocity profiles for various values. The values of the rotational parameters are $(K = 1, 1.5, 2, 2.5)$ with $R = 5, \Omega = 0.5, Gr = 10, Gc = 10, Sc = 2.01, t = 0.8, \omega t = \pi/4$ and $Pr = 7$ for sine oscillation.

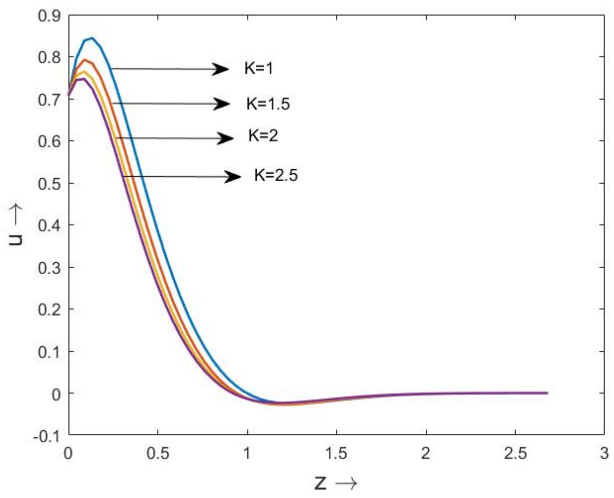


Figure 21. Primary velocity in sine oscillation for various $K$.

As a result of the chemical reaction parameter's influence, the primary velocity decreases.

Figure 22 depicts the effect of secondary velocity for several radiation parameter values, such as $(K = 1, 1.5, 2, 2.5), \omega t = \pi/4, R = 5, Gr = 10, Gc = 10, Pr = 7, Sc = 2.01, \Omega = 0.5$ and $t = 0.8$ for sine oscillation.

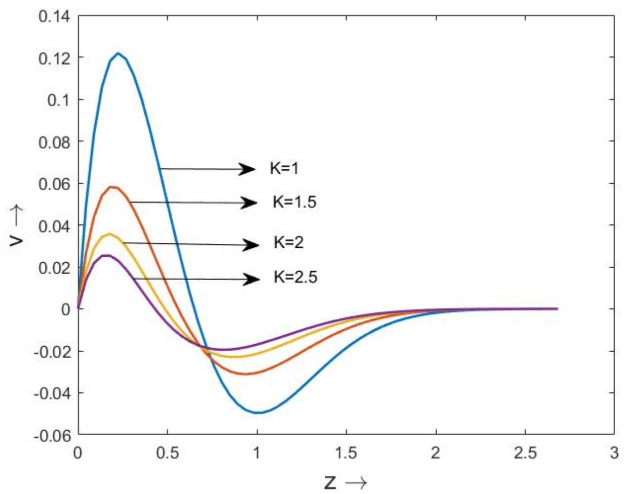


Figure 22. Secondary velocity in sine oscillation for various $K$.

As a result of the chemical reaction parameter's influence, the secondary velocity decreases.

Figure 23 depicts the effect of secondary velocity for several thermal Grasshof number $(Gr = 10, 20, 30, 40), \omega t = \pi/2, K = 0.5, R = 5, Gc = 5, Pr = 7, Sc = 2.01, \Omega = 0.5$ and $t = 0.2$ for sine oscillation.

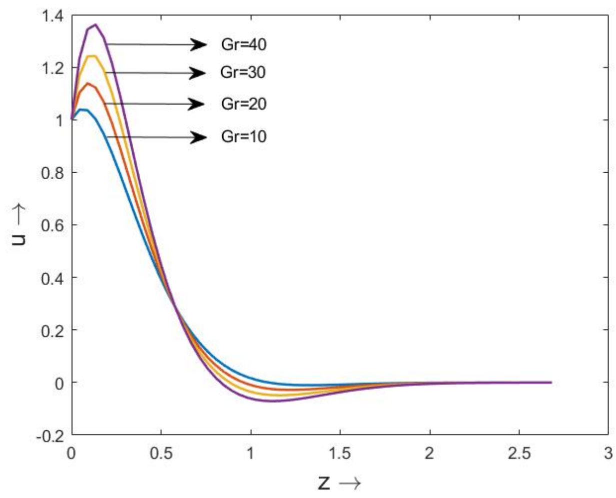


Figure 23. Primary velocity in sine oscillation for various $Gr$.

It is evident that when the thermal Grasshof number rises, the velocity rises.

Figure 24 displays the thermal Grasshof number's primary velocity profiles for various values. The values of the thermal Grasshof number are $(Gr = 10, 20, 30, 40), \omega t = \pi/2, K = 0.5, R = 5, Gc = 5, Pr = 7, Sc = 2.01, \Omega = 0.5$ and $t = 0.2$ for sine oscillation.

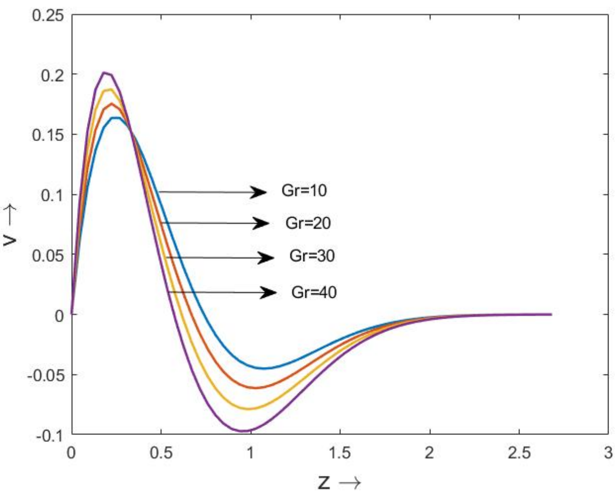


Figure 24. Secondary velocity in sine oscillation for various $Gr$.

The pattern indicates that the velocity rises as the thermal Grasshof number rises.

## IV. Conclusions

The effects of thermal radiation and chemical reaction in a rotating fluid on unsteady flow in the presence of changing temperature and mass diffusion are studied by simulating the motion of a vertically infinite oscillating plate. To solve dimensionless governing equations, the Laplace transform method is used. We investigated $\omega t, R, K, Gr, Gc, Sc$ and $t$ to see how they vary with temperature, velocity for cosine

and sine, and concentration. High thermal radiation causes the temperature to decrease. Additionally, it has been observed that concentration increases as the chemical reaction parameter reduces.
For cosine oscillation,

1) The primary velocity rises with a dropping phase angle $\omega t$.
2) The secondary velocity drops with increasing phase angle $\omega t$.
3) The primary velocity drops with rising radiation parameter R.
4) The secondary velocity drops with rising radiation parameter R.
5) The primary velocity increases with increasing rotation parameter $\Omega$.
6) The secondary velocity increases with the rising rotation parameter $\Omega$.
7) The primary velocity drops with rising Chemical reaction parameter K.
8) The secondary velocity drops with rising Chemical reaction parameter K.
9) The primary velocity increases with increasing thermal Grasshof number Gr.
10) The secondary velocity increases with increasing thermal Grasshof number Gr.

For sine oscillation,

1) The primary velocity rises with increasing phase angle $\omega t$.
2) The secondary velocity decreases with decreasing phase angle $\omega t$.
3) The primary velocity decreases with rising radiation parameter R.
4) The secondary velocity decreases with rising radiation parameter R.
5) The primary velocity rises with decreasing rotation parameter $\Omega$.
6) The secondary velocity increases with decreasing rotation parameter $\Omega$.
7) The primary velocity drops with rising Chemical reaction parameter K.‘
8) The secondary velocity drops with rising Chemical reaction parameter K.
9) The primary velocity increases with increasing thermal Grasshof number Gr.
10) The secondary velocity increases with increasing thermal Grasshof number Gr.

## V. Mathematical Representations

$a^*$ - absorption coefficient
$A$ - constant
$C'_w$ - concentration of the plate
$C'_\infty$ - concentration in the fluid far away from the plate
$C'$ - species concentration in the fluid
$C$ - dimensionless concentration
$c_p$ - specific heat at constant pressure
$D$ - mass diffusion coefficient
$g$ - acceleration due to gravity
$Gr$ - thermal Grashof number
$Gc$ - mass Grashof number
$Pr$ - Prandtl number
$q_r$ - radiative heat flux in the y- direction
$R$ - radiation parameter
$Sc$ - Schmidt number
$T'_\infty$- temperature of the fluid far away from the plate
$T'_w$ - temperature of the plate
$T'$ - temperature of the fluid near the plate
$t'$ - time
$t$ - dimensionless time
$u'$ - velocity of the fluid in the x'- direction
$u_0$ - velocity of the plate
$u$ - dimensionless velocity
$v'$ - velocity of the fluid in y'- direction
$v$ - dimensionless velocity
$y'$ - coordinate axis normal to x'- axis
$z'$ - coordinate axis normal to the plate
$z$ - dimensionless coordinate axis normal to the plate
$\beta$ - volumetric coefficient of thermal expansion
$\beta^*$ - volumetric coefficient of concentration expansion
$\mu$ - coefficient of viscosity
$\vartheta$- kinematic viscosity
$\Omega'$ - rotation parameter
$\Omega$ - dimensionless rotation parameter
$\rho$ - density
$\theta$ - dimensionless temperature
$erfc$ - complementary error function

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