A Mathematical Model for Evaluating the Risk of Airborne Infection Among Bus Passengers Using Ventilation Systems

Jenjira Sooknum, and Nopparat Pochai

Abstract—**Carbon dioxide from human breath contributes significantly to airborne diseases. Breathing can expose us to usually dangerous airborne infections, which rapidly spread. By using a bus, there is a chance of contracting an infection. This study takes into account a mathematical model of airborne infection caused by human breath. The purpose of this research is to evaluate the probability that passengers in a bus with ventilation systems may well get an airborne infection. The model can be divided into five submodels, such as an exhaled air concentration measurement model for a bus with a variable number of passengers, the volume fraction of exhaled air model, the concentration of airborne infectious particles model, the number of airborne infectious particles model, and the risk of airborne infection model. The model's solution might be used to determine the probability that susceptible people will get an airborne infection. An explicit forward-time centered-space finite difference method is used to approximate the solution. In order to reduce the risk of airborne infection and improve ventilation, the provided mathematical models were used to assess the risk of airborne infection among bus passengers using ventilation systems. Better air quality control that balances the number of passengers allowed to travel on a bus will be among the ventilation's main advantages.**

Index Terms—**Airborne, Bus, Mathematical model, Passengers, Ventilation system**

I. Introduction

THEY used a portable carbon dioxide monitor to measure carbon dioxide in the classroom under unstable conditions. Using the carbon dioxide risk equation, transmission criteria for TB were evaluated. It was concluded that, given the high smear-positive rate among high school adolescents in South Africa, a proposal to achieve a carbon dioxide level of 1000 ppm through natural ventilation was warranted. Additionally, it is a low-cost strategy to aid in TB outbreak management in high-prevalence regions [11]. They measured carbon dioxide levels at several sites in acute care hospitals to assess ventilation adequacy in [25], which could be a simple way for healthcare facilities to assess ventilation adequacy. They determined the fraction of air previously exhaled by people indoors. Then, a non-steady version of the Wells-Riley equation was obtained, which is particularly useful in poorly ventilated environments. They show that it is possible to have critical fractions of indoor air with inhalation. which is lower than the spread of common respiratory infections in the air, and the flu will not happen in [4].

They concentrated on developing and exhibiting adaptable mathematical models that can forecast the probability of infectious illnesses spreading through the air in both steady-state and non-steady settings. They employ the exhaled air accumulation rate idea to calculate the mean volume percentage of air directly breathed in a certain region by observing the air exhaled by an infected individual in a small space. They showed that there is a mathematical connection between the pace of airborne infectious particle formation and the likelihood of TB transmission. Rate of ventilation, the average volume proportion of exhaled air, the prevalence of TB, and the length of time spent close to an infected individual [13]. They proposed in [16] a mathematical model for analyzing the risk of airborne infectious disease in the outpatient setting. In which air quality control was simulated by adjusting the inlet and exhaust ventilation rates under the condition of the surrounding number of people with a personal classification factor, they used Runge-Kutta fourth order (RK4) to estimate the model solution. The presented numerical model can be used to describe the dynamic spread of airborne infectious diseases in outpatient rooms. Furthermore, they will be able to manage airborne infections in more intricate buildings. They studied a mathematical model for measuring carbon dioxide concentrations and assessing the risk of airborne infection in a ventilated outpatient room. The number of patients in each room fluctuates continuously. Additionally, they applied the fourth Runge-Kutta method. It is clear that the likelihood of infection changes with the presence of more people. Ventilation rate and each vaccine's effectiveness. These findings might be utilized to help reduce the risk of airborne infection to the target level if there is a government vaccination database system [24].

They report field measurements of naturally ventilated hospital wards in Hong Kong and present the possibility of using natural ventilation for infection control in hospital wards. It shows that natural ventilation can achieve high ventilation rates. Especially when opening windows and

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doors in the ward. When all openings are closed and the exhaust fan is turned on, the high ventilation rate of natural ventilation can reduce airborne cross-infection. Therefore, they recommend considering its use in suitable hospital wards for infection control. Their results also show the possibility of converting an existing ward using natural ventilation into a temporary isolation room by installing a mechanical exhaust fan [7]. Based on the severe acute respiratory syndrome (SARS) outbreak, they created a model for severe infectious disease epidemic occurrences, and they provided two approaches for estimating the model's parameters. The first technique is based on parameter estimates taken from the literature, whereas the second is based on parameters calculated for the traditional susceptible-infectious-removal (SIR) model. Similar outcomes are foreseen by our model and the SIR model. A considerably more serious epidemic is predicted by our model [9].

There is little research on the airborne spread of respiratory illnesses on subway platforms. They focused on computational fluid dynamics (CFD) models to examine airflow patterns and infection risks on island platforms under conventional ventilation modes. This research could provide practical guidance for preventing respiratory illnesses on subway platforms [22]. They studied to quantify the public health risks associated with inhaling airborne contaminants in public vehicles during transportation using a mathematical model [6]. The Wells-Riley model was then used to assess the influence of environmental variables. The duration of contact and the number of passengers will vary. If an infected person refuses to use a surgical mask, all infected people will wear a mask. The risk of transmission is reduced. They concluded that improving ventilation appears to be an effective and feasible way to prevent influenza infection in public trains.

How can we assess the risk of airborne infection at that time and place if there is an epidemic of infectious diseases in the air or at a site where people are affected? We employed a mathematical approach in this study to evaluate the risk of airborne disease among bus passengers with ventilation systems.

A. Simulation 1: The Probability of Airborne Infection Risk for Susceptible Individuals on the Bus with Reduced Ventilation Rates

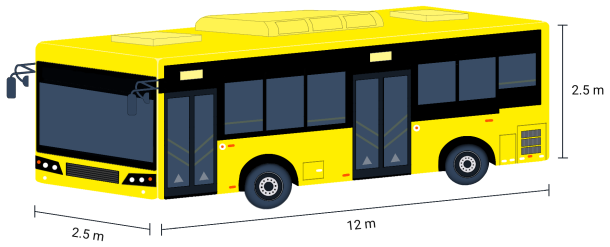

Fig.1. Model of a bus.

The seating plan in Figure 2 depicts the 48-seater air-conditioned bus, and the front and back of the vehicle each have two exhaust fans.

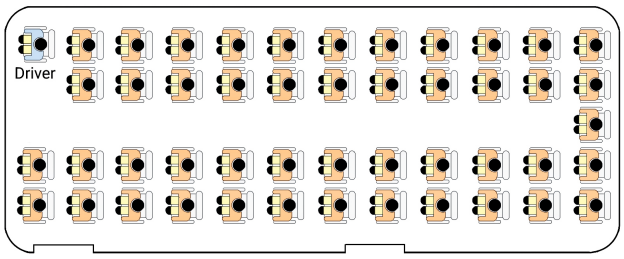

Fig.2. Seating plan.

II. Governing Equation

In general, the rate of ventilation and exhalation per person affects the elevated concentration of indoor carbon dioxide [2, 3, and 10]. The exhaled air of an infected person includes airborne infectious particles [2], [3], [5], [11], and [12]. Carbon dioxide levels can be utilized as a surrogate for exhaled air. Typically, the atmosphere contains 400 parts per million of carbon dioxide [3], [4], and [11].

A. A One-Dimensional Exhaled Air Concentration Measurement Model: A Bus with a Constant Number of Passengers

We assume that a bus's interior volume is V. The velocity of ventilation Q and the number of passengers on the bus can influence the concentration of exhaled air that contains airborne infectious particles, which may tend to increase in the presence of infectors.

Modeling several environmental problems using the advection-diffusion equation may be done for [14], [15], [18], [19], [20], [21], [26], and [27]. We simply assume that considerable amounts of carbon dioxide, a marker of exhaled air, are produced by bus passengers. The exhaled air rate generated by passengers plus the carbon dioxide diffusion rate combined with air flow velocity and minus the ventilation rate that removes exhaled air is the primary equation of the accumulation rate of exhaled air concentration in a bus with carbon dioxide:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} + u\frac{\partial C}{\partial x} + npC_a - QC\ , \qquad (1)$$

for all $(x,t) \in \Omega$, where $\Omega = [0,L]\times[0,T]$, C is the air concentration within the exhaled bus (ppm), D is the carbon dioxide diffusion coefficient (m^2/s), u is airflow velocity (m/s), p is the breathing rate (L/s) for each passenger in the bus, C_a is the carbon dioxide fraction of exhaled air and Q is the ventilation rate, t is the amount of time, T is the stationary simulation time, and L is the length of a considered bus. Initial condition $C(x,0) = C_0$ where C_0 is the latent carbon dioxide concentration. The boundary conditions are given by $\frac{\partial C}{\partial x} = C_F$ where $x = 0$ and C_F is a given constant and $\frac{\partial C}{\partial x} = C_B$ where $x = L$ and C_B is a given constant.

B. A One-Dimensional Exhaled Air Concentration Measurement Model: A Bus with a Variable Number of Passengers

In a simple scenario, if the number of passengers is unstable, then the number of passengers depends on the time assumed by $n(x,t)$. In this study, we preferred to use (1) as follows:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} + u\frac{\partial C}{\partial x} + n(x,t)pC_a - QC \ , \qquad (2)$$

for all $(x,t) \in \Omega$.

C. The Exhaled Air's Volume Fraction

We obtain the exhaled air sample's concentration, $C(x,t)$, in the designated area. By the volume fraction of exhaled air, $f(x,t)$, given by the concentration of sampled exhaled air divided by the fraction of carbon dioxide included in breathed air, we get

$$f(x,t) = \frac{C(x,t)}{C_a} \ , \qquad (3)$$

for all $(x,t) \in \Omega$.

D. The Airborne Infectious Particle Concentration

If infectious particles formed by infectors reach the host's target infection site at a particular level, the probability of infection for susceptible individuals is extremely high [8]. Some infectious particles, on the other hand, can get stuck in the upper respiratory tract and spread to other parts of the body. Let β represent all the infectious particle generation rates the infector releases into the air (particles/s), and let μ represent the infection particle death rate produced by the infector that does not reach the alveoli (particles/s).

The volume fraction of air that infectors rebreathed multiplied by the concentration of airborne infectious particles they generated in the area that reached the target infection site in the respiratory tract is the concentration of airborne infectious particles, N, that induce infection:

$$N(x,t) = \frac{If(x,t)(\beta-\mu)}{n(x,t)p} \ , \text{ and } \beta - \mu \geq 1 \ , \qquad (4)$$

where I is the possible infectors rate in the bus and for all $(x,t) \in \Omega$.

E. The Number of Infectious Particles in the Air

Because not all infected particles can reach and deposit in the alveoli, let θ represent the fraction of airborne infectious particles that settle in the host's targeted infection site by respiratory deposition. As a result, the number of infectious particles in the air, $\lambda(x,t)$, inhaled by a susceptible individual that causes infection is equal to the product of the volume of air inhaled by the susceptible, the respiratory deposition fraction of airborne infectious particles, θ, and the concentration of airborne infectious particles released by infectors,

$$\lambda(x,t) = pt\theta N(x,t) \ , \ 0 < \theta < 1 \ , \qquad (5)$$

where t is the duration of stay in the area of infection and for all $(x,t) \in \Omega$.

F. The Risk of Airborne Infection

The probability of airborne infection risk for susceptible individuals:

$$P(x,t) = 1 - e^{-\lambda(x,t)} \ , \qquad (6)$$

where $(x,t) \in \Omega$.

III. Numerical Techniques

A continuous approximation to the solution $C(x,t)$ will not be obtained; instead, approximations to C will be generated at various values, called mesh points, in the interval $[0,T]$.

Once the approximate solution at other points in the interval can be found by interpolation. We first make the stipulation that the mesh points are equally distributed throughout the interval $[0,T]$. This condition is ensured by choosing a positive integer M and selecting the mesh point $t_n = nl$, for each $n = 0,1,2,...,M$ where $l = (T-0)/M$ is called the time step. This condition is ensured by choosing a positive integer N and $x_m = mh$, for each $m = 0,1,2,...,N$. The common distance between points $h = (L-0)/N$ is called the step size.

A. Initial Condition Setting

$$C(x,0) = f(x) \ , \qquad (7)$$

for all $(x,t) \in \Omega$ and $f(x)$ is a given function of the remaining exhaled air concentration in an empty bus.

B. Boundary Conditions Setting

Assuming that there is no absorbance mechanism on the front and the back of the considered bus. The left boundary condition (LBC):

$$\frac{\partial C}{\partial x} = C_F \ , \qquad (8)$$

for all $t > 0$ and $x = 0$.

The right boundary condition (RBC):

$$\frac{\partial C}{\partial x} = C_B \ , \tag{9}$$

for all $t > 0$ and $x = L$.

C. A Forward-Time Centered-Space Finite Difference Method for a One-Dimensional Exhaled Air Concentration Measurement Model: A Bus with a Variable Number of Passengers

Retaining in $\frac{\partial C}{\partial t}$, $\frac{\partial C}{\partial x}$, and $\frac{\partial^2 C}{\partial x^2}$ in (2).

Letting that

$$C \approx C_m^n \ , \tag{10}$$

$$C_m^n = C(x_m, t_n) \ , \tag{11}$$

$$C_{m+1}^n = C(x_m + h, t_n) \ , \tag{12}$$

$$C_m^{n+1} = C(x_m, t_n + l) \ , \tag{13}$$

$$C_{m-1}^n = C(x_m - h, t_n) \ . \tag{14}$$

Then

$$\frac{C_m^{n+1} - C_m^n}{l} = D\left(\frac{C_{m-1}^n - 2C_m^n + C_{m+1}^n}{h^2}\right) + u\left(\frac{C_{m+1}^n - C_{m-1}^n}{2h}\right) + n(x,t)pC_a - QC_m^n \ ,$$

$$C_m^{n+1} = \frac{Dl}{h^2}\left(C_{m-1}^n - 2C_m^n + C_{m+1}^n\right) + \frac{ul}{2h}\left(C_{m+1}^n - C_{m-1}^n\right) + n(x,t)pC_a l - QC_m^n l + C_m^n \ ,$$

$$C_m^{n+1} = \alpha C_{m-1}^n - 2\alpha C_m^n + \alpha C_{m+1}^n + n(x,t)pC_a l - QC_m^n l + \frac{r}{2}C_{m+1}^n - \frac{r}{2}C_{m-1}^n + C_m^n \ ,$$

from (15), we get the FTCS finite difference equation becomes

$$C_m^{n+1} = \left(\alpha - \frac{r}{2}\right)C_{m-1}^n - \left(2\alpha + Ql - 1\right)C_m^n + \left(\alpha + \frac{r}{2}\right)C_{m+1}^n + n(x,t)pC_a l \ , \tag{15}$$

where $\alpha = \frac{Dl}{h^2}$ is the diffusion number, $r = \frac{ul}{h}$ is the convection number, and $p = 0.12$ (L/s) is the breathing rate.

The stability condition of (15) is [1] $0 < \alpha \le \frac{1}{2}$.

IV. Numerical Experiments and Results

Assuming that the volume of a bus is $V = 75$ (m^3), each passenger's breathing rate is assumed to be $p = 0.12$ (L/s), the diffusion coefficient of carbon dioxide is $D = 0.732$ (m^2/s), the airflow velocity is $u = 0.025$ (m/s), the carbon dioxide fraction included in the breathed air is $C_a = 0.04$, and the rate of change of the carbon dioxide at the frontend and the backend of the bus is neglected, which are $C_F = C_B = 0$.

A. Simulation 1: The Probability of Airborne Infection Risk for Susceptible Individuals on the Bus with Reduced Ventilation Rates

Table I displays the number of passengers in each row. The bus employs an air vent rate of 75, 37.5, 18.75, and 0. $C_0 = 0.1$ is the initial value of the carbon dioxide concentration in the air on a bus (ppm). We achieve the approximate solutions presented in Figures 4 to 8 obtained by employing the forward time centered space (FTCS) method (10) to (15) and (3) indicated in Figures 7 to 8.

TABLE I
The Number of Passengers on the Bus in Each Row

Time (min)	The Number of Passengers in Each Row												
	0	1	2	3	4	5	6	7	8	9	10	11	12
0-10	0	1	1	1	1	1	1	1	1	1	1	1	0
10-20	0	3	3	3	3	3	3	3	3	3	3	3	3
20-30	0	2	4	4	4	4	4	4	4	4	2	2	2
30-40	0	1	1	1	1	1	1	1	1	1	1	1	1
40-50	0	2	2	2	2	2	2	2	2	2	2	2	2
50-60	0	2	2	2	2	2	2	2	2	2	0	0	0

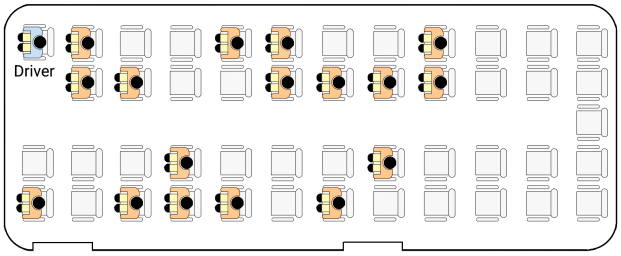

Fig.3. Seat map depicting the number of passengers seated on the bus for 50 to 60 minutes.

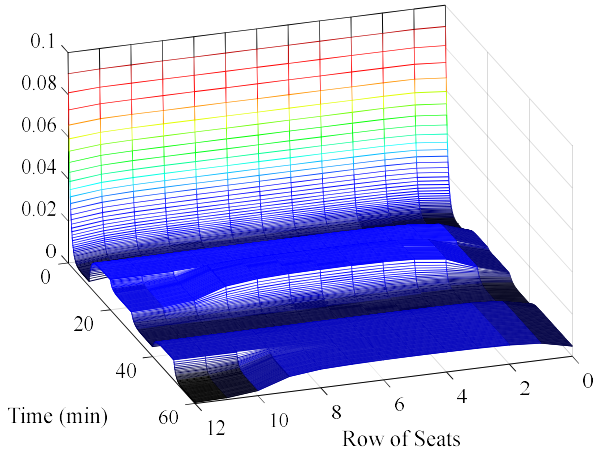

Fig.4. The approximated carbon dioxide concentration in the air on a bus with a ventilation system $l = 0.1$ $T = 60$.

From Figure 4, we can observe that the concentration of carbon dioxide in the air on the bus decreases as there are fewer passengers in each row when passengers get off the bus at a stop.

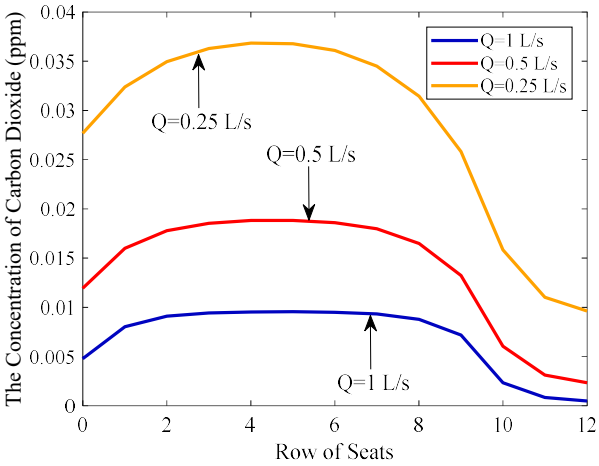

Fig.5. The approximated carbon dioxide concentration in the air on a bus at 60 minutes with a ventilation system $l = 0.1 \quad T = 60$.

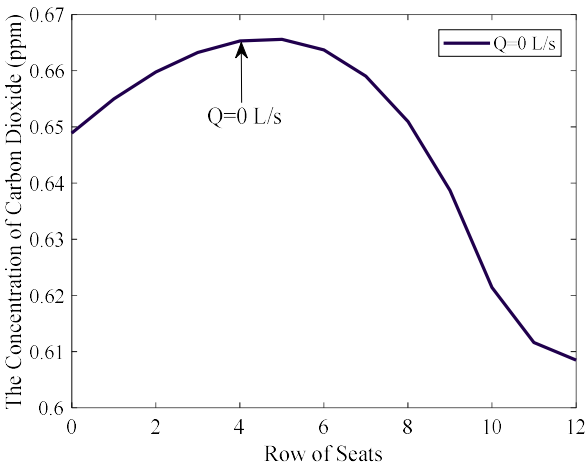

Fig.6. The approximated carbon dioxide concentration in the air on a bus at 60 minutes with a ventilation system $l = 0.1 \quad T = 60$.

We can observe that the concentration of carbon dioxide in the air on the bus increases as the ventilation rate reduces.

TABLE II
THE CONCENTRATION OF CARBON DIOXIDE IN THE AIR ON A BUS AT 60 MINUTES

Row of Seats	The Concentration of Carbon Dioxide			
	$Q = 1$	$Q = 0.5$	$Q = 0.25$	$Q = 0$
0	0.0048	0.0119	0.0277	0.6489
1	0.0080	0.0160	0.0324	0.6550
2	0.0091	0.0178	0.0350	0.6598
3	0.0094	0.0185	0.0363	0.6633
4	0.0095	0.0188	0.0368	0.6653
5	0.0096	0.0188	0.0368	0.6656
6	0.0095	0.0186	0.0361	0.6637
7	0.0093	0.0180	0.0345	0.6591
8	0.0088	0.0165	0.0314	0.6510
9	0.0072	0.0132	0.0258	0.6387
10	0.0023	0.0060	0.0158	0.6214
11	0.0008	0.0031	0.0110	0.6116
12	0.0005	0.0023	0.0096	0.6085

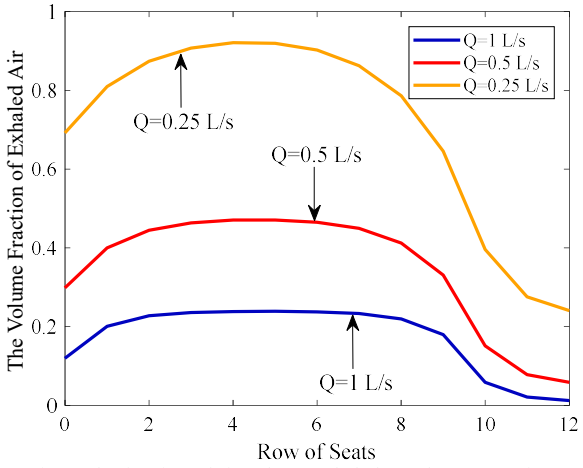

Fig.7. The fraction of air volume exhaled on a bus at 60 minutes.

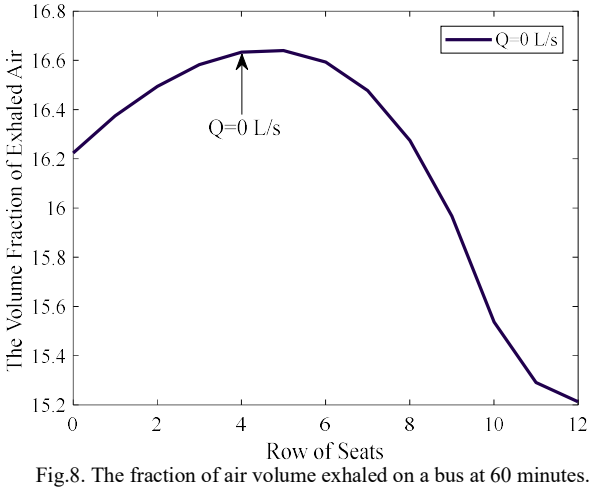

Fig.8. The fraction of air volume exhaled on a bus at 60 minutes.

We can see that at 60 minutes, the fraction of exhaled air volume increases owing to the reduction in ventilation rate.

TABLE III
PHYSICAL PARAMETERS

I	θ	β	μ
1.1151	0.25	100	87

Table III shows the physical characteristics. We achieve the approximate solutions illustrated in Figures 9 to 15 by using (4) shown in Figures 9 to 10, (5) shown in Figures 11 to 12, and (6) shown in Figures 13 to 15.

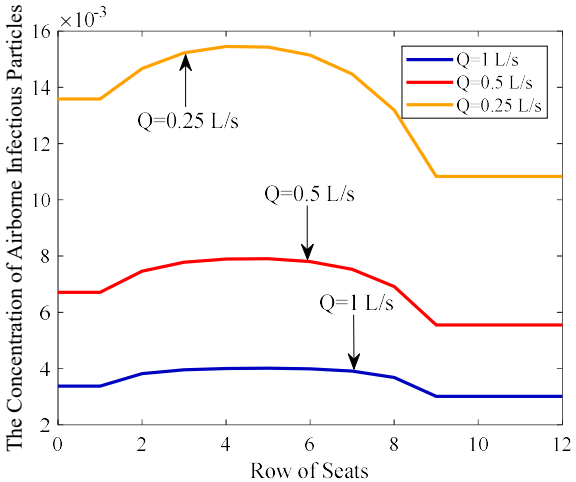

Fig.9. The concentration of airborne infectious particles on a bus at 60 minutes.

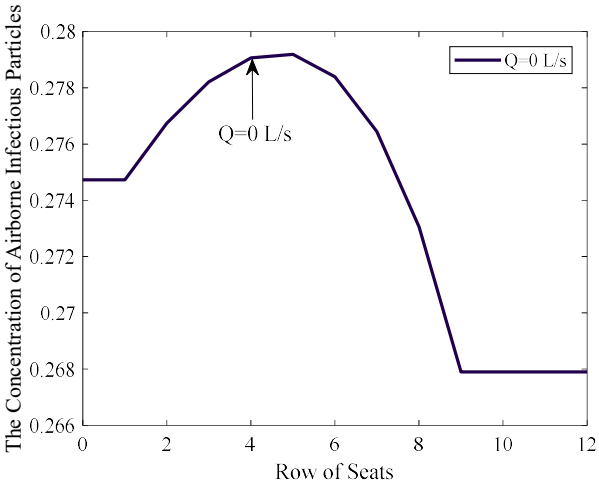

Fig.10. The concentration of airborne infectious particles on a bus at 60 minutes.

We can see that at 60 minutes, the concentration of infectious airborne particles increases due to the decreasing ventilation rate.

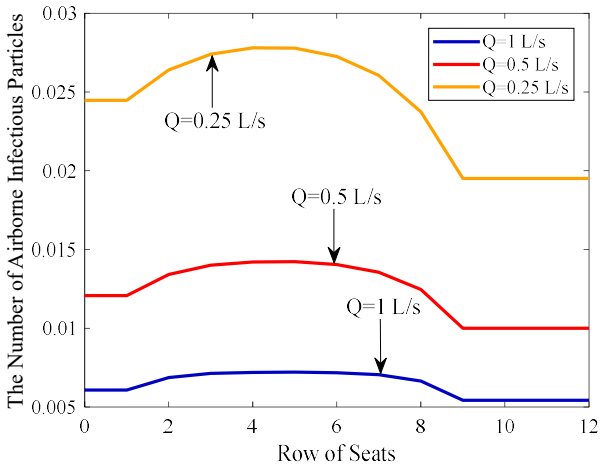

Fig.11. The number of airborne infectious particles on a bus at 60 minutes.

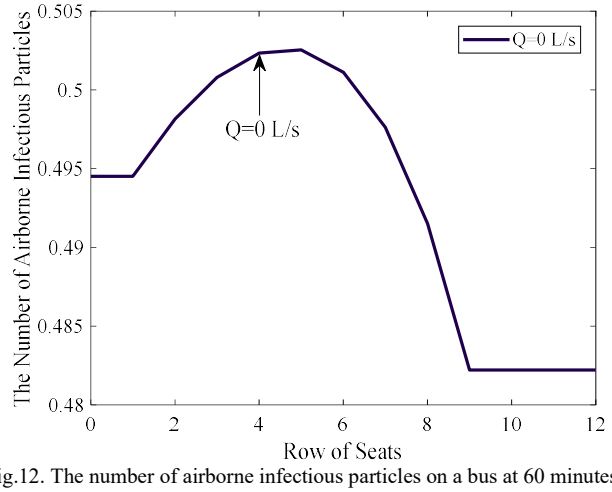

Fig.12. The number of airborne infectious particles on a bus at 60 minutes.

We can see that in 60 minutes, the number of airborne infectious particles increases as the respiratory rate decreases.

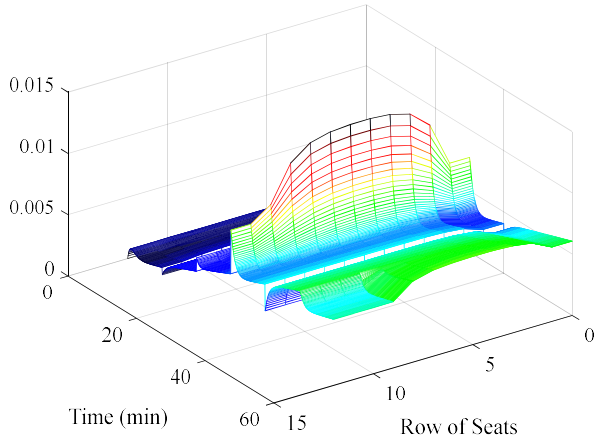

Fig.13. The probability of airborne infection risk for susceptible individuals.

From Figure 13, we can observe that passengers seated at 0 to 10 minutes and 30 to 40 minutes are at high risk of airborne infection.

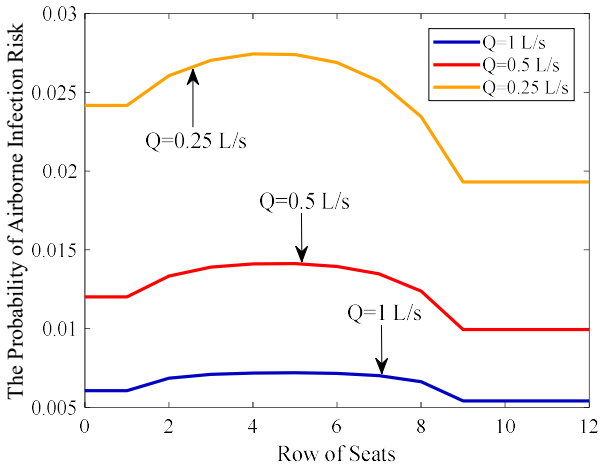

Fig.14. The probability of airborne infection risk for susceptible individuals on a bus at 60 minutes.

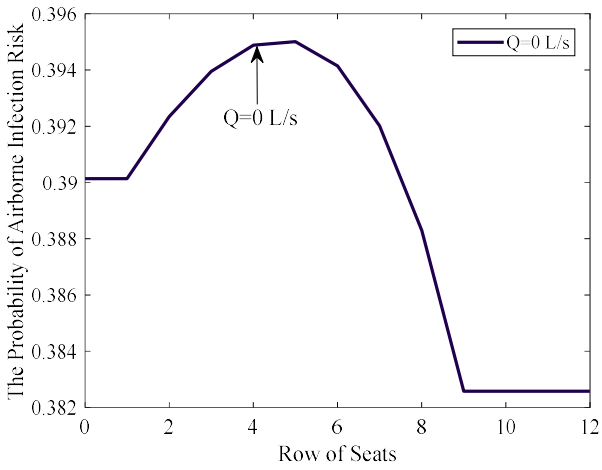

Fig.15. The probability of airborne infection risk for susceptible individuals on a bus at 60 minutes.

We can see that in 60 minutes, at a ventilation rate of 1 (L/s), the passenger is at the lowest risk of airborne infection.

TABLE IV
THE PROBABILITY OF AIRBORNE INFECTION RISK FOR SUSCEPTIBLE INDIVIDUALS ON A BUS AT 60 MINUTES

Row of Seats	The Probability of Airborne Infection Risk for Susceptible Individuals			
	$Q=1$	$Q=0.5$	$Q=0.25$	$Q=0$
0	0.0061	0.0120	0.0242	0.3901
1	0.0061	0.0120	0.0242	0.3901
2	0.0068	0.0133	0.0261	0.3923
3	0.0071	0.0139	0.0270	0.3939
4	0.0072	0.0141	0.0274	0.3949
5	0.0072	0.0141	0.0274	0.3950
6	0.0071	0.0139	0.0269	0.3941
7	0.0070	0.0135	0.0257	0.3920
8	0.0066	0.0124	0.0235	0.3883
9	0.0054	0.0099	0.0193	0.3826
10	0.0054	0.0099	0.0193	0.3826
11	0.0054	0.0099	0.0193	0.3826
12	0.0054	0.0099	0.0193	0.3826

TABLE V
THE PERCENTAGE RISK OF AIRBORNE INFECTION AMONG BUS PASSENGERS AT 60 MINUTES

Row of Seats	The Percentage Risk of Airborne Infection Among Bus Passengers			
	$Q=1$	$Q=0.5$	$Q=0.25$	$Q=0$
0	0.61	1.20	2.42	39.01
1	0.61	1.20	2.42	39.01
2	0.68	1.33	2.61	39.23
3	0.71	1.39	2.70	39.39
4	0.72	1.41	2.74	39.49
5	0.72	1.41	2.74	39.50
6	0.71	1.39	2.69	39.41
7	0.70	1.35	2.57	39.20
8	0.66	1.24	2.35	38.83
9	0.54	0.99	1.93	38.26
10	0.54	0.99	1.93	38.26
11	0.54	0.99	1.93	38.26
12	0.54	0.99	1.93	38.26

V. DISCUSSION

As indicated in the demonstrated simulation, we approximated the airborne carbon dioxide concentrations on the bus with different ventilation rates and according to the number of passengers in each row in Table I. Figure 4 shows the estimated carbon dioxide concentrations at which the bus ran for 60 minutes. Figures 5 to 6 show the concentration of carbon dioxide after 60 minutes. It can be seen that the concentration of carbon dioxide increases with the decreasing ventilation rate. Table II shows the approximate concentration of carbon dioxide in the air on a bus with a ventilation system.

The physical parameters in Table III are estimated using (4) to (6), as shown in Figures 7 to 15. The number of passengers on the bus in each row is shown in Table I. When estimated using the forward time-centered space (FTCS) method, the surface graph is obtained as shown in Figure 13. It can be seen that the concentration of carbon dioxide at the starting point decreases and increases gradually from about row 2 to row 10. Over a period of about 10 to 30 minutes, it gradually decreases again as the number of passengers has already disembarked the bus and fewer passengers sit in each row.

Figures 7 to 15 show the various ventilation rates on the bus at 60 minutes. The volume fraction of exhaled air, the concentration of airborne infectious particles, the quantity of airborne infectious particles, and the probability of airborne infection risk for susceptible individuals all increase as the ventilation rate falls. Table IV shows the probability of receiving an infection from the air for those who are susceptible individuals on a bus. Table V shows the percentage risk of airborne infection among bus passengers at 60 minutes. Passengers had the lowest average risk when the ventilation rate was 1 (L/s).

VI. CONCLUSION

It is visible that when a ventilation system's ventilation rate drops, the volume of carbon dioxide in the air on a bus rises. When the ventilation rate reduces, the volume fraction of exhaled air, the concentration of airborne infectious particles, the quantity of airborne infectious particles, and the probability of airborne infection risk for susceptible people all rise. We can determine the balance between the number of passengers permitted to take the bus due to the proposed air quality model. A numerical mathematical model is used to evaluate the risk of airborne infection on a ventilated bus, even if there are steps that may be taken to reduce the risk of airborne infection. The provided mathematical models were used to estimate the risk of airborne infection among bus passengers utilizing ventilation systems in order to control the possibility of airborne infection and enhance ventilation. Among the main advantages of ventilation will be improved air quality management, which will balance the number of passengers permitted to travel on a bus.

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