A Fractional Study for Solving the SIR Model and Chaotic System

Mohammed Berir, Rania Saadeh, Mohamed A. Abdoon, Ahmad Qazza and Dalal Almutairi

***Abstract*—In this study, a novel scheme for the Caputo-Fabrizio fractional derivative is employed to solve the problem of a fractional SIR model and a financial chaotic system. It is discovered that the proposed method yields equivalent solutions to some approximate results presented by the other techniques. Therefore, the strategy could be generalized to other systems to get more precise solutions. A novel fractional derivative scheme and algorithm presented here can be used to create and simulate fractional models for solving challenging problems in physics, biology, and engineering in the future.**

***Index Terms*— Numerical solutions, Numerical scheme, Simulation, Chaos.**

I. Introduction

In recent decades, fractional calculus has become increasingly popular for modeling diffusion, control, and viscoelasticity. Engineering and scientific disciplines utilize fractional differential equations [1], [2]. Fractional differential equations can be solved in a variety of ways [3], [4]. The models of hyper-chaotic and chaotic systems have been widely applied in different disciplines such as electrical circuits, biology, and physics [5], [6]. Extensive research has focused on understanding complex systems such as cancer tumor dynamics, Zika virus transmission, Chaplygin gas models, biological population migration, and drift-flux models. Various mathematical techniques have been employed, including Lie symmetry analysis and numerical methods, to investigate these systems and derive exact solutions, conservation laws, and analyze their behavior [7]-[11].

Chaos theory finds significant applicability in the field of electrical circuit modeling, as discussed in a number of papers [12]-[14]. Considering the difficulty of predicting many real-world events, it is justifiable to use chaotic models. Asymptotic stability is a new technique for assessing chaotic systems by describing how the model parameter affects the dynamics of chaotic models, and Lyapunov's inverse identifies the exact nature of chaos. Fractional calculation has a wide range of mathematical and scientific applications. Fractional computation has become increasingly common in science, mathematics, biology, and other fields for some of the latest research and applications [15]-[21]. This discovery is significant because fractional operators have multiple meanings [22]. The fractional derivative is particularly beneficial because it accounts for the effects of long-term memory.

This paper investigates numerical solutions of fractional order systems and fractional SIR models utilizing diverse approaches. We have employed mathematical approaches to thoroughly examine and develop sophisticated and optimized solutions for both the fractional SIR model and a chaotic system model. Hence, the proposed methodology holds the potential for conducting additional investigations on alternative models.

Recent research [23] has uncovered multiple compelling justifications for utilizing fractional derivatives in practical scenarios. The literature abounds with countless instances of chaotic systems, wherein it is widely acknowledged that chaotic systems exhibit tumultuous responses to the given initial conditions and even slight alterations in the parameters. Refer to the following fractional calculus papers [24] for study on the utilization of fractional derivatives in the modeling of chaotic systems. Various approaches, such as those derived from physics and engineering, have been utilized to address issues in management, economics, and biology, among other fields. [25]–[31]. The fractional SIR model has gained recent attention due to the proliferation of diseases like COVID-19 and others. observe [32]-[36].

The importance of this research lies in its ability to offer a numerical solution for a fractional SIR model and a chaotic fractional derivative system. This demonstration showcases the implementation of the Caputo-Fabrizio fractional derivative (CF fractional derivative e) scheme using MATLAB, a software platform for numerical computations. Therefore, the methodology presented is commendable for future investigations in different models. The presented study was done with the intention of making it relevant to future applications of fractional systems in the fields of physics and engineering.

The significance and use of this approach lie in its ability to provide numerical solutions in various models, that includes chaotic and disease models. Moreover, it can be extended to include additional models in the field of pathology, dynamical models, coding, and hyper-chaos. Moreover, it has demonstrated remarkable efficacy in precisely identifying images, whether by predicting diseases or showing attractor chaos.

The subsequent content presents the organization of this

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article: Section 2 provides definitions of fractional derivatives. In Section 3, we explain the steps of a novel scheme for The Caputo-Fabrizio fractional derivative. In Section 4, we implement the novel scheme to solve the problem of a fractional SIR model and a financial chaotic system. At the end of this article, we present the conclusion.

II. Preliminaries And Basic Definitions

In this section, we provide a concise summary of the fractional operators essential to our study.

Definition 2.1: Let $q \in [1,\infty)$ and ω be open subset of R, the Sobolev space $H^q(\omega)$ is defined by [1]:

$$H^q(\omega) = \{f \in L^2(\omega) : D^\beta f \in L^2(\omega), \text{for all} |\beta| \le q\}.$$

Definition 2.2: The Riemann-Liouville fractional integral operator of order $\alpha > 0$ for a function $y(t)$ is given by [1]:

$$I^\alpha y(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} y(\tau) d\tau, \quad t > 0. \tag{1}$$

Definition 2.3: The Caputo derivative of a function $y(t)$ of order α, where $0 \le n-1 < \alpha < n$ and with the lower limit zero, is defined as follows [34]:

$$\begin{aligned} D^\alpha y(t) &= \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} y^n(\tau) d\tau \\ &= I^{n-\alpha} y^n(t), \qquad t > 0. \end{aligned} \tag{2}$$

Definition 2.4: For $y \in H^1(0,t)$, $t > 0$, $T > 0$, $\alpha \in (0,1]$. The CF fractional operator [37] is defined as follows:

$$ {}_0^{CF}D_t^\alpha y(t) := \frac{B(\alpha)}{1-\alpha} \frac{d}{dt} \int_0^t y(\tau) exp\left(-\alpha \frac{t-\tau}{1-\alpha}\right) d\tau, \quad 0 < \alpha < 1. \tag{3}$$

The expression $B(\alpha)$ must satisfy the requirement $B(0) = B(1) = 1$.

Theorem 2.1: (Generalized Fundamental Theorem of the Fractional Calculus) [38]

Let $\alpha > 0$, $k = [\alpha] + 1$, and $(l^{k-\alpha} f)^{(k)}(x)$ be absolutely continuous on $[a,b]$. Then,

$$\begin{aligned} (l^\alpha D^\alpha f)(x) &= f(x) \\ &- \sum_{j=1}^k \frac{(j^{k-\alpha} f)^{(k-j)}(a)}{\Gamma(\alpha - j + 1)} (x-a)^{\alpha-j}. \end{aligned} \tag{4}$$

III. Numerical scheme for the fractional derivative

The objective of this section is to examine an innovative approach for the CF fractional derivative, which is expressed in the following form [39]:

$$ {}_0^{CF}D_t^\alpha y(t) = f(t, y(t)). \tag{5}$$

Applying Theorem 2.1 in Equation (5), we have the following

$$\begin{aligned} y(t) - y(0) &= \frac{(1-\alpha) f(t, y(t))}{M(\alpha)} \\ &+ \frac{\alpha \int_0^t f(\theta, y(\theta)) d\theta}{M(\alpha)}, \end{aligned} \tag{6}$$

where $M(\alpha) = \frac{2}{2-\alpha}$ is a normalization function such that $M(0) = M(1) = 1$. In this way

$$\begin{aligned} y(t_{n+1}) - y(0) &= \frac{(2-\alpha)(1-\alpha) f(t_n, y(t_n))}{2} \\ &+ \frac{\alpha(2-\alpha) \int_0^{t_{n+1}} f(t, y(t)) dt}{2}, \end{aligned} \tag{7}$$

and

$$\begin{aligned} y(t_n) - y(0) &= \frac{(2-\alpha)(1-\alpha)}{2} f(t_{n-1}, y(t_{n-1})) \\ &+ \frac{\alpha(2-\alpha)}{2} \int_0^{t_n} f(t, y(t)) dt. \end{aligned} \tag{8}$$

Replacing (8) in (7) we have

$$\begin{aligned} y(t_{n+1}) &= y(t_n) + \frac{(2-\alpha)(1-\alpha)}{2} \\ &\times \left(f(t_n, y(t_n)) - f(t_{n-1}, y(t_{n-1}))\right) \\ &+ \frac{\alpha(2-\alpha)}{2} \int_{t_e}^{t_{n+1}} f(t, y(t)) dt, \end{aligned} \tag{9}$$

where

$$\int_{t_n}^{t_{n+1}} f(t, y(t)) dt = \frac{3h}{2} f(t_n, y_n) - \frac{h}{2} f(t_{n-1}, y_{n-1}). \tag{10}$$

The numerical solution is derived using the following mathematical expression

$$\begin{aligned} y_{n+1} &= y_n + \left(\frac{(2-\alpha)(1-\alpha)}{2} + \frac{3h}{4} \alpha(2-\alpha)\right) f(t_n, y_n) \\ &- \left(\frac{(2-\alpha)(1-\alpha)}{2} + \frac{h}{4} \alpha(2-\alpha)\right) f(t_{n-1}, y_{n-1}). \end{aligned} \tag{11}$$

IV. Applications

This section analyzes the practicality of the innovative suggestion for CF-FD on the numerical resolution of a fractional SIR model and a Chaotic system.

Problem 4.1: We start with the fractional SIR model. In the early 20th century, Kermack and McKendrick published [40]. SIR, a quantitative model, was introduced for the first time. The SEIR model was created by adding Exposed (E) as a fourth compartment to the SIR model [41] in order to describe how an outbreak disease

$$\begin{aligned} \frac{dS}{dt} &= -aSI, \\ \frac{dI}{dt} &= aSI - r\,\mathrm{I}, \\ \frac{dR}{dt} &= rI. \end{aligned} \tag{12}$$

In Table I below, we provide numerical approximate solutions of system (12) the fractional SIR model considering the values: $\alpha = 1$, $a = 2\mathrm{e}-3/7$, $r = 0.15$, $S_0 = 10e^3$, $I_0 = 1$, $R_0 = 1$ and $t = 1$. Applying the RK4 approach at $t = 1$ and a numerical strategy for the FC operator. Moreover, when the step size h is short enough, the proposed numerical outcomes in this study are noticed to be in good agreement with that solutions derived by the RK4 method. We gave precise, near-identical solutions that closely resembled RK4.

TABLE I
SOLUTIONS OF EQUATION (12), WHERE $\alpha = 1$.

h	***S***	***I***	***R***
1/320	9996.95318699669	3.886830948692717	0.159982054607931
1/640	9996.96173824264	3.878728779002160	0.159532978368678
1/1280	9996.96603533265	3.874657354516271	0.159307312828754
1/2560	9996.96818923873	3.872616562754912	0.159194198503111
1/5120	9996.96926753133	3.871594897687368	0.159137570989833
1/10240	9996.9698070125	3.871083747944641	0.159109239650452
1/20480	9996.97007683664	3.870828093782068	0.159095069585691
1/40960	9996.97021176968	3.870700246879310	0.159087983454617
$RK4$	9996.97034666668	3.870572434101	0.159080899213

In Table II, The numerical solutions were provided using fractional order, with a values of $\alpha = 0.95$, $a = 2\mathrm{e}-3/7$, $r = 0.15$, $S_0 = 10e^3$, $I_0 = 1$, $R_0 = 1$ and $t = 40$, and this demonstrates that the technique works well and is adaptable to other fractional systems.

TABLE II
SOLUTIONS OF EQUATION (12), WHERE $\alpha = 0.95$.

h	***S***	***I***	***R***
1/320	0.0004686065557601543	822.26585208617	9178.73367930738
1/640	0.0004683504784449073	822.366986579088	9178.63254507064
1/1280	0.0004682303486661944	822.422783614156	9178.57674815549
1/2560	0.0004681733217616004	822.45318900555	9178.5463428211
1/5120	0.000468146115540744	822.469636926625	9178.52989492752
1/10240	0.000468133122363914	822.478491758963	9178.52104010755
1/20480	0.000468126924699345	822.483242155794	9178.51628971674
1/40960	0.000468123976367115	822.485783604041	9178.51374827362

In Figure 1, we demonstrate the numerical solution of the fractional order model described in equation (12) using the fractional CF approach. This method is derived via evaluating the dynamics of the susceptible, infected, and recovered classes over a period of 40 days, with a fractional order value of $\alpha = 0.95$.

Problem 4.2: The financial chaotic system discussed in [42] is made up of product, capital, labor force, and debt. The model takes into account the interest rate $x(t)$, which is influenced by two factors: the surplus of savings over investment and the structural adjustment based on products pricing. The investment demand $y(t)$, which is directly related to the investment rate, and the price exponent $z(t)$, which is determined by the market's conflict between supply and demand in the commercial sector [40]

$$\begin{aligned} \frac{dx}{dt} &= z(t) + (y(t) - a)x(t), \\ \frac{dy}{dt} &= 1 - by(t) - x^2(t), \qquad (13) \\ \frac{dz}{dt} &= -x(t) - cz(t), \end{aligned}$$

the cost of the investment, and $c \geq 0$ represents the demand elasticity of the commercial market.

Table III below, provides numerical results of the fractional derivative FC to solve the financial chaotic system in Equation (13), where $\alpha = 1$, $a = 1$, $b = 0.1$, $c = 1$, $x(0) = 3$, $y(0) = 1$, $z(0) = 1$ and $t = 1$. Applying the RK4 approach at $t = 1$ and a numerical strategy for the FC operator. Furthermore, when the magnitude of the step size h is sufficiently small, our computational solutions are observed to be in exceptional concurrence with those obtained by the RK4 methodology. We provided accurate and nearly comparable results that closely approximated the RK4 method.

TABLE III
SOLUTIONS OF EQUATION (13) WHERE $\alpha = 1$, $a = 1$, $b = 0.1$, $c = 1$, $x(0) = 3$, $y(0) = 1$, $z(0) = 1$ AND $t = 1$

h	***S***	***I***	***R***
1/320	1.392480462893589	-1.456089389634900	-0.304025735714919
1/640	1.395421662728035	-1.455404284769252	-0.303162624174348
1/1280	1.396886454030830	-1.455074812429870	-0.302732773906679
1/2560	1.397617390759015	-1.454913350981111	-0.302518275688362
1/5120	1.397982493535476	-1.454833439527427	-0.302411133373569
1/10240	1.398164953419043	-1.454793688692286	-0.302357588922827
1/20480	1.398256160471136	-1.454773864506897	-0.302330823375111
1/40960	1.398301758273062	-1.454763965223381	-0.302317442270793
$RK4$	1.398347391565288	-1.454753677519081	-0.302304022169948

In Table IV, the solutions were presented in fractional order $\alpha = 0.95$, $a = 1$, $b = 0.1, c = 1$, $x(0) = 3$, $y(0) = 1$, $z(0) = 1$ and $t = 10$, and this demonstrates that the technique works well and is adaptable to other fractional systems.

TABLE IV
SOLUTIONS OF EQUATION (13), WHERE $\alpha = 0.95$, $a = 1$, $b = 0.1$, $c = 1$, $x(0) = 3$, $y(0) = 1$, $z(0) = 1$ AND $t = 10$.

h	***S***	***I***	***R***
1/320	-0.114945270518440	2.461003090354902	0.078920896300135
1/640	-0.114886963398490	2.460443007000058	0.078898975169522
1/1280	-0.114855046856798	2.460158283451454	0.078886471458949
1/2560	-0.114838399904129	2.460014754790493	0.078879834662401
1/5120	-0.114829904519515	2.459942699184568	0.078876420136375
1/10240	-0.114825613883149	2.459906598618383	0.078874688854980
1/20480	-0.114823457833114	2.459888530151504	0.078873817211395
1/40960	-0.114822377125635	2.459879491372964	0.078873379889088

Figure 2 and Figure 3, display the numerical results for the financial chaotic system to Equation (13), where $(a, b, c) = (3, -1, -1)$, and when $\alpha = 1$ and $\alpha = 0.95$. We display the Equation (13) attractors that were calculated with this numbers. For specific values of the parameters, the numerical scheme of the FC operator can display a similar type of attractors in chaos as its fractional order $\alpha = 0.95$. and integer orders $\alpha = 1$. This method's advantage is that it displays chaos in an understandable and efficient way, much like the integer order, and its pictures are accurate

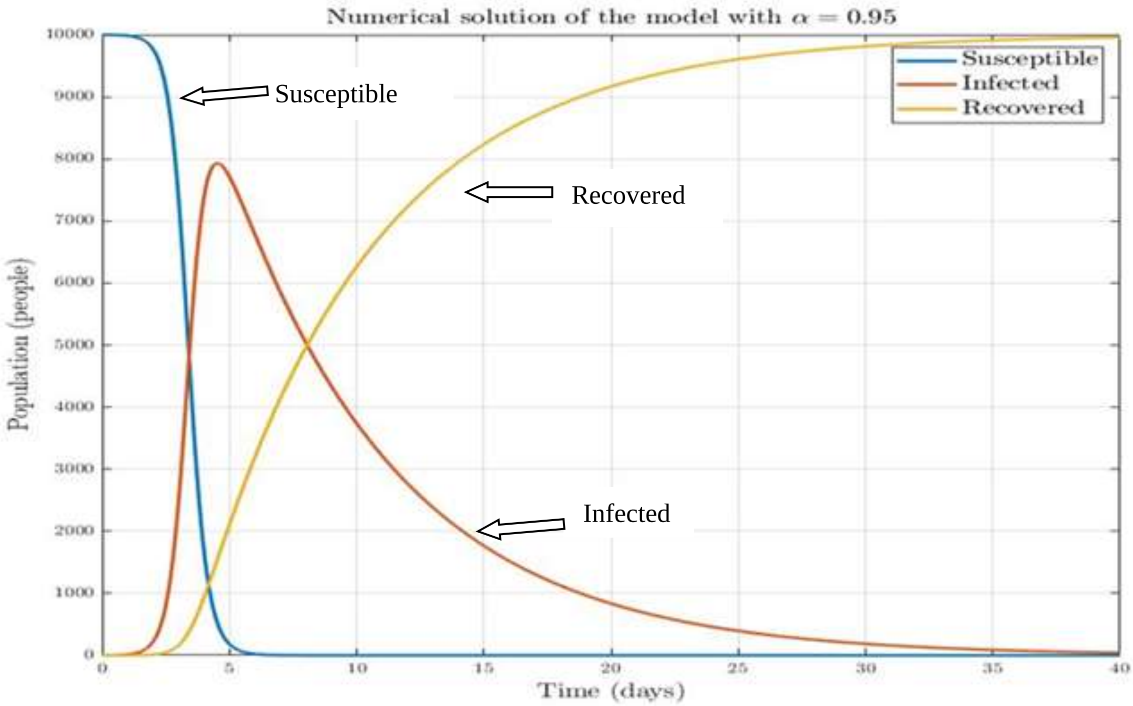

Fig. 1. The numerical solution of SIR model.

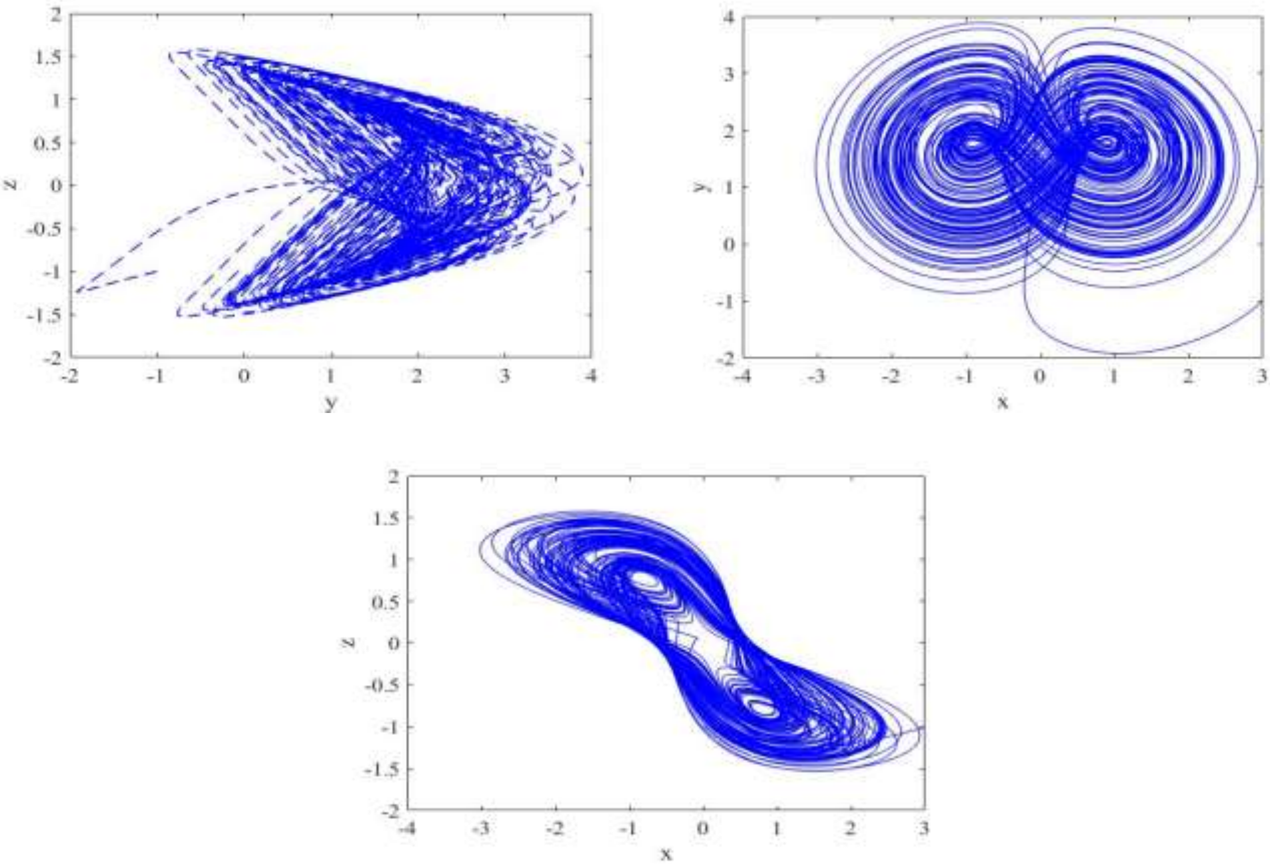

Fig. 2. The financial chaotic system (13), where $(a,b,c)=(3,-1,-1)$, when $\alpha=1$.

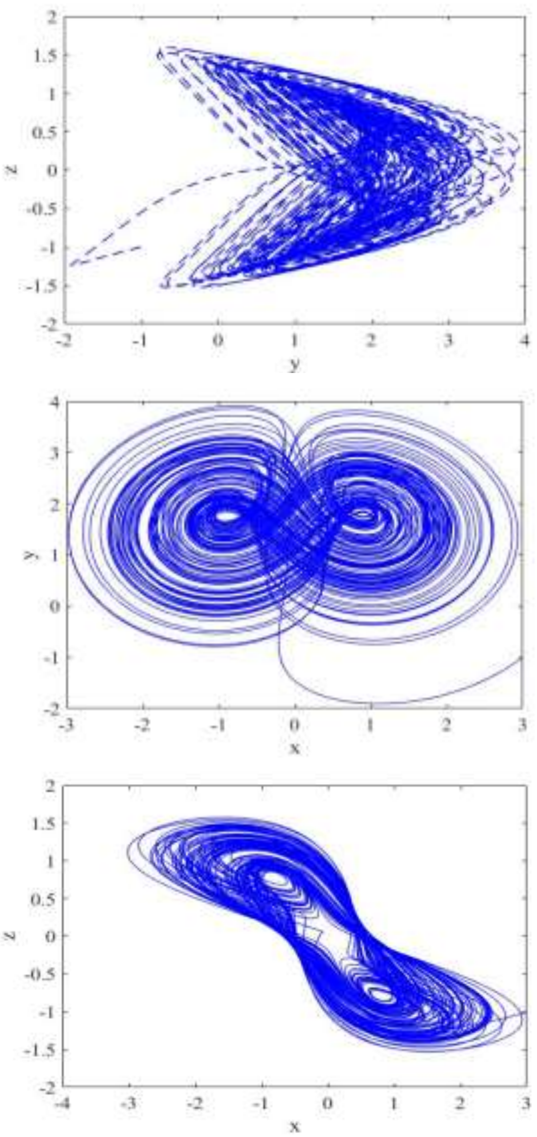

Fig. 3. The financial chaotic system (13), where $(a,b,c)=(3,-1,-1)$, when $\alpha=0.95$.

V. Conclusion

This study solved a fractional SIR model and a chaotic system using a novel method. A numerical strategy for the Caputo-Fabrizio fractional derivative is provided using MATLAB. Decreasing the step size h in numerical simulations reveals that the numerical technique for the CF fractional derivative yields numerical outcomes that closely resemble the precise solutions or RK4 solutions when dealing with integer orders. The numerical results obtained clearly indicate that our approach executes fractional operations with numerical stability. The phase portrait of the model exhibits chaotic behavior when specified values are supplied to the attached parameters. Therefore, the approaches presented are highly relevant for future research on different models. The purpose of this research was to create a valuable resource for future applications of fractional systems. The complexity of physics and engineering problems is growing, so we recommend a broader application of this technique. This method is widely applicable to other chaotic and hyperchaotic systems. Moreover, in the future, our method can be applied to modeling diseases and predicting disease spread. Additionally, it possesses the capability to solve Chaos systems, and we anticipate the method to be successful in hyperchaotic systems as well as its potential for application in engineering and coding. The purpose of this study is to solve some novel fractional models such as those described in [44]–[45], and to compare them with other numerical methods [46]–[50].

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