# Optimal Control and Analysis of the SEIRS Model on the Problem of Online Game Addiction : A Case Study Among Class VIII Students of the State Junior High Schools in Makassar City

Syafruddin Side, Miftahul Jannah, Abdul Saman, Muhammad Isbar Pratama, Wahidah Sanusi

***Abstract*—This study aimed to develop a Suspected-Exposed-Infected-Recovered (SEIRS) mathematical model to solve addiction to online games. The model would use optimal control to analyze and predicts addiction cases among the State Junior High School students. The model was analyzed through determination of the equilibrium point, stability, and basic reproduction number ($R_0$). The optimal control problem was solved using the Pontryagin principle to minimize the number of addicted groups. The analysis without control gave an initial value of $R_0 = 1.94 > 1$, which indicated that online game addiction in students at a worrying level. The value R0 with 1% control was $R_0 = 1.892$, which showed that the problem is still worrying. Meanwhile, 50% and 90% controls gave $R_0 = 0.814$ and $R_0 = 0.556$, respectively, which indicated that the problems can be overcome because the values are less than one. This shows that the greater the control, the lower the transmission rate for online game addiction.**

***Index Terms*—SEIRS Model, Online Game Addiction, Optimal Control, Basic Reproductive Numbers,**

## I. Introduction

IN In the health sector, mathematical modeling is employed to determine whether a disease is epidemic, communicable, or non-communicable [1]. It can also be applied to model online games that have a negative impact on users when played excessively, leading to behaviors like laziness to engage in other activities and reduced social interaction within the community [2].

In 2017, the Indonesian Internet Service Providers Association (APJII) reported that the country had 143.26 million internet users (50% of the 262 million population) [3]. According to a marketing research institute in Amsterdam called Newzoo, the country had 43.7 million gamers in 2017, with 56% being male and 44% female. Indonesia ranked 17th globally for having the highest number of active game players and the most in Southeast Asia. These games are played on mobile phones, personal computers, and laptops [4]. However, excessive gaming can lead to aggressive behaviors, including physical violence such as fights and hitting friends when annoyed. Verbal aggression is also evident, manifesting as the use of offensive language and curses [5].

Studies related to the negative impact of online games have been carried out by [6], [7], [8], [9], [10] to determine the problem of addiction from a social perspective. Furthermore, there are also previous reports on mathematical modeling of SIR, SEIR, and SEIRS on the transmission of various diseases such as dengue hemorrhagic fever, tuberculosis, and COVID-19 [11], [12], [13], [14], [15], [16], [17], [18]. These were carried out to analyze and predict the number of populations infected with the diseases. Although mathematical models on online games addiction cases have been carried out by [19], [20], it's only focused on the analysis and simulation of uncontrolled models. The study by [6]-[20] has not examined the optimal control with the SEIRS mathematical model. Therefore, this study examines the optimal control and analysis of the SEIRS mathematical model as a solution to reduce the number of adolescents, specifically Junior High School students, who are addicted to online games.

## II. Method

A theoretical study was carried out on the SEIRS mathematical modeling using a qualitative approach in the case of online game addiction with optimal control. The SEIR model was built by adding control parameters such as parental supervision and analyzed using a generation matrix to obtain basic reproduction numbers and model stability [20]. The numerical simulation used primary data obtained from class VIII students of the State Junior High School in Makassar City and analyzed with Matlab software.

## III. Result and Discussion

### A. SEIRS Mathematical Model on the Problem of Online Game Addiction

The SEIRS model scheme in problems of online game addiction with optimal control is shown in Figure 1 below:

The variables in Figure 1 consist of a total population of N, which is divided into four compartments, namely Susceptible (S) class, which is a group of students who are prone to online game addiction, Exposed (E), those have started playing but have not become addicted, Infected (I), the addicted group, and Recovered (R), students who are free

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S.Side is a Professor in Mathematics Department, Universitas Negeri Makassar, Indonesia. (corresponding author to provide phone: +62 8134225-8671, email: syafruddin@unm.ac.id)

M. Jannah is a Postgraduare Student in Mathematics Department, Universitas Negeri Makassar, Indonesia. (email:miftahuljannah4027@gmail.com)

A.Saman is a Profesor in Guidance and Counseling Department, Universitas Negeri Makassar, Indonesia. (email:abdulsaman@unm.ac.id)

M. I, Pratama is a lecturer in Mathematics Department, Universitas Negeri Makassar, Indonesia. (email: isbarpratama@unm.ac.id)

W. Sanusi. is senior lecturer in Mathematics Department, Universitas Negeri Makassar, Indonesia. (email:wahidah.sanusi@unm.ac.id)

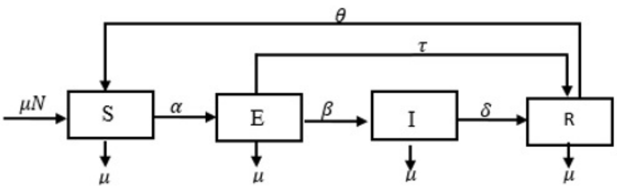


Fig. 1. SEIRS Model Schematic on the problem of online game addiction

TABLE I
DEFINITION OF PARAMETERS IN SEIRS MODEL OF ONLINE GAME ADDICTION

| Par | Definition |
|---|---|
| $\mu N$ | The rate of students vulnerable to online game addiction group |
| $\alpha$ | The rate of movement from vulnerable for online game addiction group (Suspected) to starting to play online games group |
| $\beta$ | The rate of movement from the starting to play online games group (exposed) to addicted to online games (infected) |
| $\tau$ | The rate of movement from starting to play online games group (exposed) to free from online game addiction group (recovered) |
| $\delta$ | The rate of movement from addicted to online games group (infected) to free from online game addiction group (recovered) |
| $\theta$ | The rate of movement of free from addiction (recovered) to vulnerable for online game addiction group (suspected) |
| $\mu$ | The rate of students leaving each compartment |

from online game addiction. The definition of the SEIRS model of online game addiction is shown in Table 1.

The formulation of the SEIRS model on the problem of online game addiction based on Figure 1 is a non-linear system of differential equations as expressed in Equations 1 below:

$$\begin{cases} \frac{dS}{dt} = \mu N - \theta R - \alpha SI - \mu S \\ \frac{dE}{dt} = \alpha SI - \beta E - \mu E - \tau E \\ \frac{dI}{dt} = \beta E + \mu I - \delta I \\ \frac{dR}{dt} = \delta I + \tau E - \mu R - \theta R \end{cases} \tag{1}$$

*1) Basic Reproduction Number:* The value of basic reproduction number ($R_0$) obtained using the Next Generation Matrix method from Equations (1) is:

$$F = \begin{pmatrix} \beta SI \\ 0 \end{pmatrix}$$

$$F' = \begin{pmatrix} 0 & \alpha\delta \\ 0 & 0 \end{pmatrix} \tag{2}$$

The values of $V^(-1)$ is determined using the equation below:

$$V = \begin{pmatrix} (\beta + \mu + \tau)E \\ \mu I + \delta I - \beta E \end{pmatrix}$$

$$V'{-1} = \begin{pmatrix} \frac{1}{\beta+\mu+\tau)} & 0 \\ \frac{\beta}{(\beta+\mu+\tau)(\delta+\mu)} & \frac{1}{\delta+\mu} \end{pmatrix} \tag{3}$$

therefore, the basic reproduction number is obtained as in Equation 4 below:

$$F'V'^{-1} = \begin{pmatrix} \frac{\alpha\delta\beta}{(\beta+\mu+\tau)(\delta+\mu)} & \frac{\alpha\delta}{\mu+\delta} \\ 0 & 0 \end{pmatrix}$$

$$R_0 = \frac{\alpha\beta}{(\beta+\mu+\tau)(\delta\mu)} \tag{4}$$

*2) Solution of SEIRS Model Optimal Control on Online Game Addiction:* The SEIRS model on the problem of online game addiction in Equations 1 with the addition of control is expressed in Equations 5 below:

$$\begin{cases} \frac{dS}{dt} = \mu N - \theta R - \alpha SI - \mu S \\ \frac{dE}{dt} = \alpha SI - \beta E - \mu E - \tau(1-u)E \\ \frac{dI}{dt} = \beta E + \mu I - \delta I \\ \frac{dR}{dt} = \delta I + \tau(1-u)E - \mu R - \theta R \end{cases} \tag{5}$$

Optimal control in form of parental supervision added to the SEIRS model of online game addiction is an effort to reduce the number of addicted individuals. Basic Reproductive Numbers $R_0 u$ with optimal control using the next-generation matrix method based on Equations 5 is stated in Equation 6 below:

$$R_0 u = \frac{\alpha\beta\delta}{(\beta + \mu + (\tau + u))(\mu + \delta)} \tag{6}$$

Based on the Pontryagin principle, the first step taken to obtain optimal control is to form a Hamilton function, according to the objective function obtained:

$$H(S, E, I, R, u, \sigma) = I(t) + \frac{c}{2}u^2 t + \sum_{i=1}^{4}(\sigma f) \tag{7}$$

Where $\sigma$ and $f$ represent the costate variables and the right-hand side of the system of equations, respectively. Equation 7 can be rewritten as:

$$H(S, E, I, R, u, \sigma) = I(t) + \frac{C}{2}u^2(t) + I(t) + \frac{C}{2}u(t) + \sigma_1(t)(\mu N + \theta R - \alpha SI - \mu S) + \sigma_2(t)[\alpha SI - (\beta + \mu)E - \tau(1-\epsilon)E] + \sigma_3(t)[\beta E - (\mu + \delta)I] + \sigma_4(t)[\delta I + \tau(1-u)E - (\mu + \theta)R]$$

According to Pontryagin's principle, the Hamilton function reaches an optimal solution when the state and costate equations, as well as stationary conditions, apply:

**State Equation**

$$\begin{cases} \frac{\partial H}{\partial \sigma_1} = \mu N - \theta R - \alpha SI - \mu S \\ \frac{\partial H}{\partial \sigma_2} = \alpha SI - \beta E - \mu E - \tau(1-u)E \\ \frac{\partial H}{\partial \sigma_3} = \beta E + \mu I - \delta I \\ \frac{\partial H}{\partial \sigma_4} = \delta I + \tau(1-u)E - \mu R - \theta R \end{cases} \tag{8}$$

**Co-State Equation**

$$\begin{cases} -\frac{\partial H}{\partial S} = -[-\sigma_1(t)\alpha I - \sigma_1(t)S + \sigma_2(t)\alpha I] \\ -\frac{\partial H}{\partial E} = -[-\sigma_2(t)(\beta + \mu) - \sigma_2(t)(\tau - \tau u) + \\ \quad \sigma_3(t)\beta + \sigma_4(t)(\tau - \tau u)] \\ -\frac{\partial H}{\partial I} = -[-\sigma_1(t)\alpha S + \sigma_2(t)\alpha S - \\ \quad \sigma_3(t)(\mu + \delta) + \sigma_4(t)\delta] \\ -\frac{\partial H}{\partial R} = -[\sigma_1(t)\theta + \sigma_4(t)(\mu + \theta)] \end{cases} \tag{9}$$

**Stationary Condition**

$$\frac{\partial H}{\partial \epsilon} = 0$$

$$C\epsilon(t) + \sigma_2(t)E(t) - \sigma_3(t)E(t) = 0$$

$$\epsilon(t) = \frac{\sigma_2(t)\tau E(t) + \sigma_4 \tau(t)E(t)}{C} \tag{10}$$

Where $0 \leq u \leq 1$, obtain:

$$U^* = \begin{cases} 0, & if \quad u'(t) \leq 0 \\ u'(t), & if \quad 0 \leq u'(t) \leq 1 \\ 1, & if \quad u'(t) \geq 1 \end{cases} \quad (11)$$

Therefore, the control form u(t)is obtained which is presented in Equation 12 below:
$u(t) = min(1, max(0, u'(t))$

$$u(t) = min(1, (max(0, \frac{\sigma_2(t)\tau E(t) + \sigma_4 \tau(t) E(t)}{C}))) \quad (12)$$

### B. SEIRS Model Simulation on the Problem of Online Game Addiction in Makassar City

The SEIRS model's numerical simulation was conducted using Matlab by substituting parameters and initial values from the primary data. This encompasses the count of students from State Junior High School in Makassar City who initiate playing online games, are susceptible to addiction, and engage in activities unrelated to online games. Meanwhile, the parameter values employed in the simulation were derived from questionnaire responses. The initial values for the model's variables and parameters are presented in Table 2 below.

TABLE II
INITIAL VALUE OF VARIABLES AND PARAMETERS OF SEIR MODEL FOR ONLINE GAME ADDICTION PROBLEMS

| Variable & Parameters | Initial Value |
|---|---|
| $S_0$ | 0.27 |
| $E_0$ | 0.57 |
| $I_0$ | 0.12 |
| $R_0$ | 0.04 |
| $N$ | 364 |
| $\mu$ | 0.03 |
| $\alpha$ | 0.70 |
| $\beta$ | 0.3 |
| $\delta$ | 0.27 |
| $\tau$ | 0.03 |
| $\theta$ | 0.16 |

When the parameter values are substituted into Equation (4), the basic reproduction number value is $R_0 = 1.944 > 1$. This showed that online game addiction in students is a concern. The results of the SEIRS model simulation on the problem of online game addiction for grade VIII students are shown in Figures 3 and 4 below.

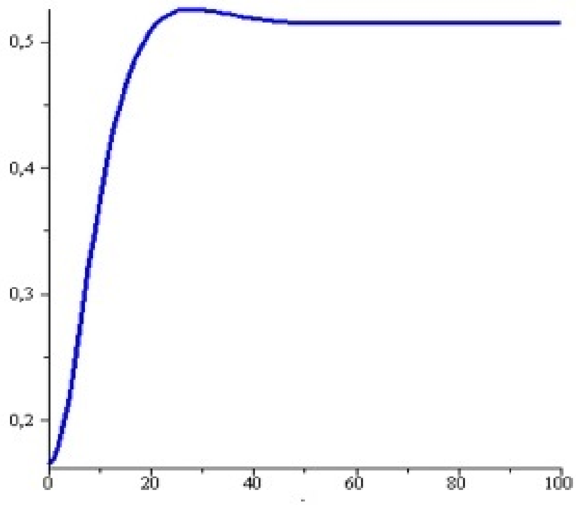


Fig. 2. Predict the number of vulnerable and start playing online games of junior high school students

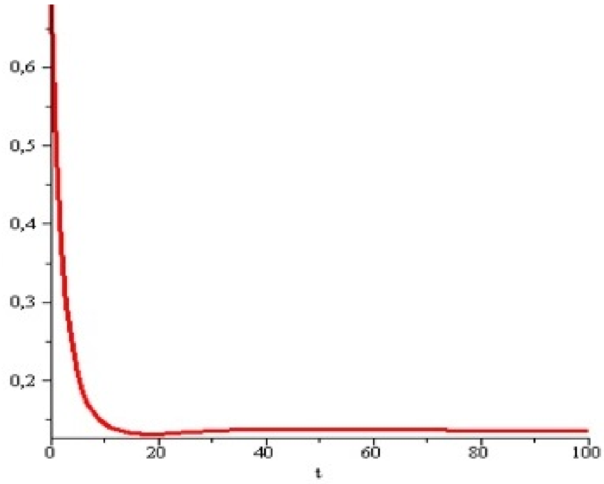


Fig. 3. Predict the number of vulnerable and start playing online games of junior high school students

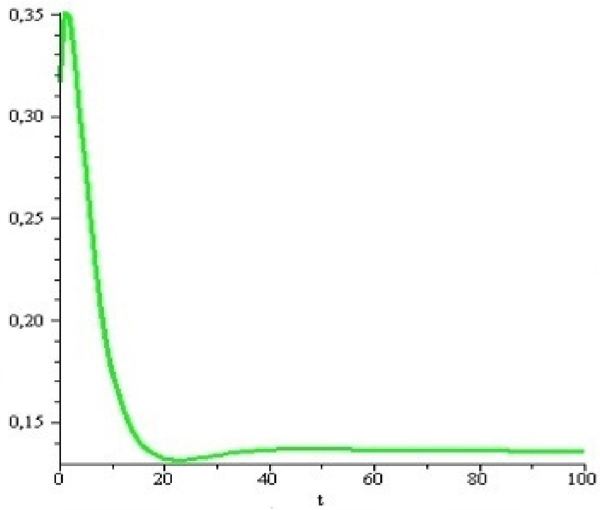


Fig. 4. Predict the number of junior high school students who are addicted to and detached from online games

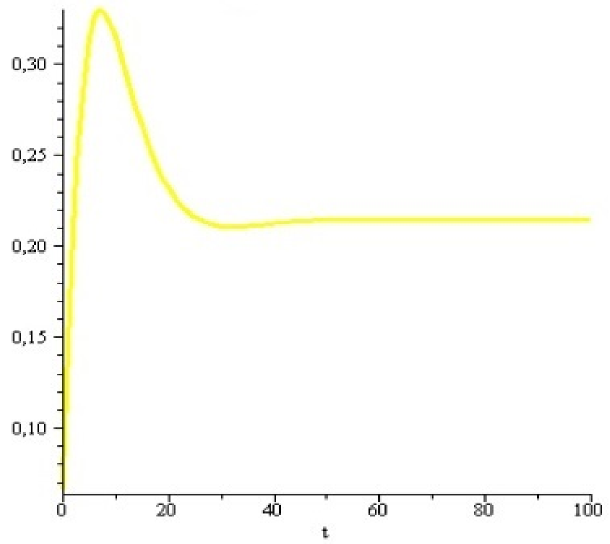


Fig. 5. Predict the number of junior high school students who are recovered from online games addicted

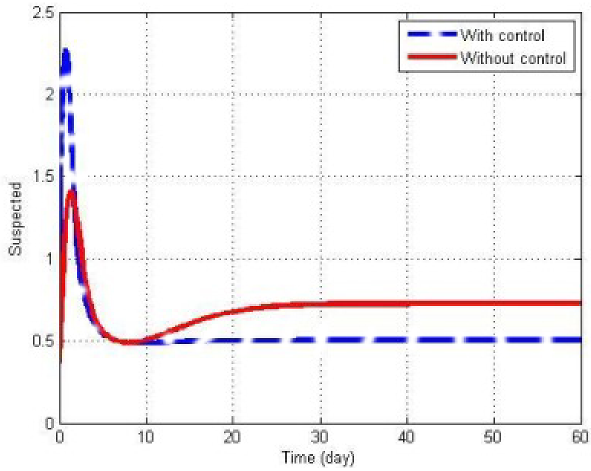


Fig. 6. Predict the number of vulnerable students and start playing online games with and without control

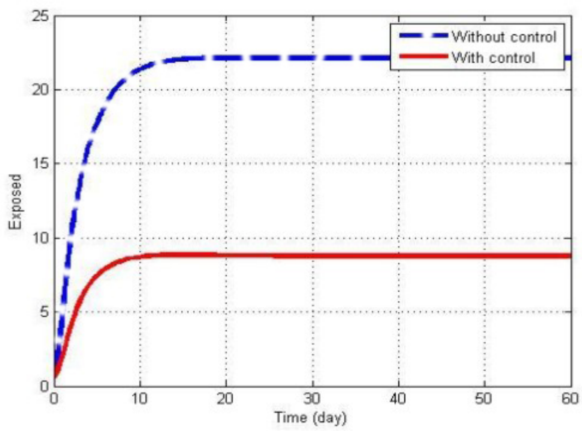


Fig. 7. Predict the number of vulnerable students and start playing online games with and without control

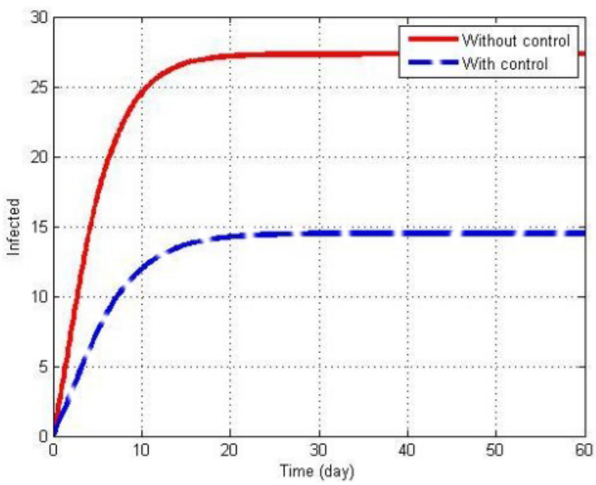


Fig. 8. Predict the number of students who are addicted to and regardless of playing online games with and without control

Figures 2 and 3 show that the suspected class or students who are prone to online game addiction continued to increase from the 1th month, but remained stable in the 50th month. In the exposed class, students who started playing online games continued to decline and stabilize in the 70th month. The infected class experienced a significant increase in the first month and stabilized in the 50th month. Meanwhile, the recovered class increased at the beginning, but decreased in the 15th month and stabilized in the 50th month.

*1) SEIRS Model Simulation on Online Game Addiction Problems with Optimal Control:* The SEIRS model simulation of online game addiction problem with optimal control aims to determine the maximum effort of supervision or control from parents on addicted students. The numerical simulation with optimal control was solved using the Runge Kutta back and forth scheme of order 4. The state equations were solved using the forward scheme, while the costate equations were estimated using the backward scheme. The state equation is defined as $S_0 = y(1), E_0 = y(2), I_0 = y(3)$, and $R_0 = y(4)$, while the costate equation is defined as $\sigma(1) = y(1), \sigma(2) = y(2), \sigma(3) = y(3)$, and $\sigma(4) = y(4)$. The model simulation used the initial value of S=99,E=206,I=45,R=14, and N=364, with the given weight value of C=0.75, while the parameter values were used to refer to Table 2. This study contains only one control variable, therefore, the simulation was carried out in one scenario when there is parental supervision on children playing online games.

Comparison of simulation results on students who are addicted to online games without and with parental supervision are shown in Figures 8 below:

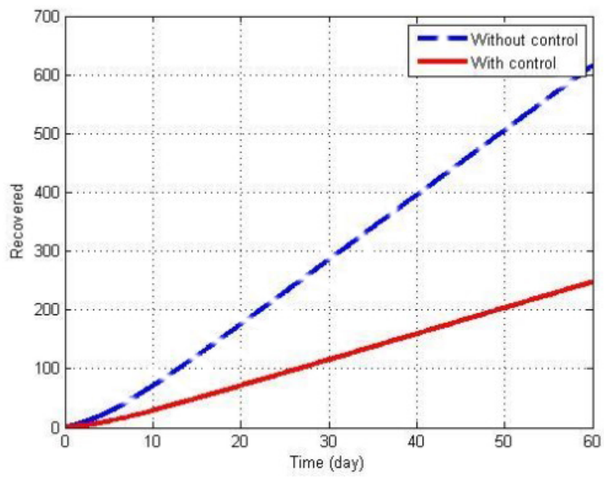


Fig. 9. Predict the number of students who are addicted to and regardless of playing online games with and without control

Figures 6-9 depict changes in the curves of the suspected, exposed, infected, and recovered classes before and after implementation of control measures. The results of the optimal control simulation indicate a noteworthy increase in the number of addicted students in the 1st month prior to control, stabilizing at 7.5 infected individuals by the 28th month. However, post-control implementation, the count of addicted students reduced to 4, reaching stability by the 10th month. The decline in the count of students who play online games and had developed online addiction was more rapid with the introduction of control measures, particularly in the form of parental supervision. This underscores the efficacy of this form of control in minimizing the number of addicted students optimally.

## IV. DISCUSSION

The results showed that the value of the basic reproduction number in the absence of control is $R_0 = 1.944 > 1$. This indicated that online game addiction among students is of great concern The simulation results after giving 1% control obtained a value of $R_0 = 1.892$, showing it is still in an alarming state. Meanwhile, 50% and 90% obtained the value of $R_0 = 0.814 < 1$ and $R_0 = 0.556 < 1$, respectively. This shows that giving 50% and 90% control can change the situation of students to be less worried about online game addiction or have greater control to reduce the number of addicted students.

A previous report [19] regarding cases of online game addiction among students at State Junior High School 3 Makassar suggests that optimal management can be achieved through guidance and counseling. This aligns with the current study, as the presence of control measures like parental supervision significantly contributes to reducing the count of students addicted to online games [6]. The study also highlights the importance of addressing students' dependency on these games, which corresponds with the current research where efforts to manage addiction through parental control during online gaming resulted in reduced addiction cases. An examination of SEIRS mathematical modeling of online game addiction in college students revealed an increase in addicted students over an 8-year period [20]. This trend mirrors the outcomes of the analysis and model simulation in the present study, where the count of addicted students in the state junior high school initially continued to rise, but subsequently decreased following the implementation of optimal control measures in the form of parental supervision.

## V. CONCLUSION

The findings demonstrate the feasibility of characterizing online game addiction using the SEIRS model. Through analysis and simulation of the model, it is evident that the incidence of cases among students diminishes when incorporating parameters, such as parental supervision, as an optimal control measure. Consequently, the SEIRS mathematical model, when employed with optimal control strategies, presents a viable solution to address the issue of online game addiction among State Junior High School students in Makassar City.

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