A New Interval Arithmetic Approach to Solve the Trapezoidal Intuitionistic Fuzzy Linear Programming Problem

R. Sanjana and *G. Ramesh

Abstract—**Linear programming Problem (LPP) is intended to assist executives in making decisions. It is a mathematical technique to determine the best allocation of resources such as labour, employees, components, machinery, and other facilities in order to achieve a specific goal. The parameters in our real-life situations are hazy and imprecise. The Intuitionistic Fuzzy Set (IFS) is a tool for dealing with decision-making issues under uncertainty. Due to various types of complexity, determining the accurate Membership Function (MF) by an ordinary fuzzy set is not always possible. Interval numbers are thus used to describe the unpredictability. LPP is resolved in this study using novel interval arithmetic operations with Trapezoidal Intuitionistic Fuzzy Numbers (TrIFNs) as parameters. In order to obtain the optimal solution, LPP is solved using three methods: Simplex Method (SM), Robust Two Step Method (RTSM) and Alternative Solution Method (SOM-2). Additionally, the proposed methods are numerically shown, and results are compared and represented diagrammatically.**

Index Terms—**Linear Programming problem, TrIFNs, Interval Arithmetic, SM, RTSM, SOM-2.**

I. Introduction

LPP is used to determine the maximum or minimum value of any variable in a function. To handle complicated problems in wartime activities, George B. Dantzig [1] in 1947 developed Simplex method to solve LPP. The problem is typically presented as a linear function that are optimized with variety of constraints. LPP is widely employed in various industries, including resource management, wealth management, commercial funding, etc. Fuzzy Set (FS) is demonstrated as a beneficial technique for recognising situations in which the data appears to be unclear or imprecise. FS deals with these conditions by attributing a degree of membership to each object in the set. Furthermore, the FS is discovered in 1965 by Zadeh [2] is an extended form of the classical notation system. The Interval-valued fuzzy set is an extension of FS in which the membership values are intervals rather than crisp and this concept is first introduced by Zadeh in 1975. K. T. Atanassov offered a generalisation of the idea using FS known as Intuitionistic fuzzy sets (IFS) [3] at the beginning of 1983 in which the non-membership is a new degree that is added to the degree of membership, with the restriction that their sum be less than or equal to 1. The hesitation margin, a component of IFS, is a measure of hesitancy and is described as 1 less than the total of membership and non-membership, respectively. Additionally, it has been found that a single membership and non-membership degree do not adequately capture the state of volatility that arises in real-life problems, Atanassov and Gargov [4] created Interval-valued Intuitionistic Fuzzy sets (IVIFS), a generalisation of IFS in which membership and non-membership degrees are intervals rather than fixed real numbers, to improve the ability of an IFS to handle ambiguity and hesitancy. Therefore, the goal is to use IFS theory to solve LPP.

First, we can observe how the LPP is addressed through FS theory. Wan et al. [5] discovered a novel possibility for LPP with Trapezoidal Fuzzy Numbers (TrFNs) and presented the auxiliary multi-objective programming to solve the associated potential LPP with TrFNs for the imprecise objective coefficients and/or the imprecise technical parameters and/or materials. Saghi et al. [6] used hesitant cost factors to deal with the Hesitant Fuzzy Linear Programming Problem (HFLPP). Due to information loss, HFLPP cannot be turned into LPP or Fuzzy Linear Programming Problem (FLPP). SM can fix this problem as it retains data and is illustrated with two examples. Maximization problems are used in application to farm planning: Yano [7] introduced a decision-making approach for fuzzy multi-objective LPP with fuzzy goals and integrated two membership functions and ultimately, pareto optimal solution is obtained with maximizing profit. Akram et al. [8] demonstrated the Fully Fuzzy Linear Programming (FFLP) by the simplex approach with the new arithmetic operations of TrFNs and compared to the current techniques. Ghoushchi [9] resolved FFLP employing Triangular fuzzy numbers (TFNs) as decision variables and is based on modified TFNs and α-cut theory for obtaining the best optimal fuzzy solution. To manage uncertainty using interval numbers in LPP, G. Ramesh [10] defined an extension of the traditional LPP to an imprecise environment, by applying a new simple ranking and new extended interval arithmetic.

After that, we observe how an IFS can make LPP to address ambiguity. Arpita Kabiraj [11] suggested an approach to solve Intuitionistic fuzzy linear programming problem (IFLPP) using FLPP method and by ranking function, finally a comparative investigation are given. Nalla Veerraju [12] suggested a new ranking technique called the GM-R approach based on support and resulting membership of an Intuitionistic Fuzzy Numbers (IFNs).

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The proposed approach has been applied to IFLPP and is investigated for superior outcomes in providing an optimal solution. Considering a case study of fruit orchards in Baluchistan, Pakistan, Sajida Kousar [13] has been able to solve an IFLP model for fruit production using Triangular Intuitionistic Fuzzy Numbers (TIFNs), a tactical tool for controlling uncertainty. Sukhpreet Kaur Sidhu [14] addressed the shortcomings from the paper Parvathi et al. [15] in which by using a distinct ranking function and the linearity property, Intuitionistic fuzzy simplex algorithm for solving IFLPP is done with Symmetric Trapezoidal Intuitionistic Fuzzy Numbers (STrIFNs). Sidhu et al. [16] highlighted the disadvantages of current approaches, and a new method termed Mehar method is outlined to overcome the drawbacks with an example. As a result, IFS plays a significant role in solving LPP in many areas to deal with hesitancy. Following that, IVIFS is more effective in handling uncertainty than IFS, as IFS are very challenging to represent precise real numbers in several real-world decision-making consequences. E. Fathy [17] expressed the decision variables as Interval-valued IFNs and these numbers are split into nine distinct, crisp linear problems using the reduction methodology providing the most and least acceptable outcomes of the objective function and technique is shown numerically.

RTSM is designed by Y. R. Fan et al. [18] to deal with the Interval Linear Programming problem (ILPP). When compared to previous approaches, this approach produces a bigger solution space and prevents significant loss of decision-related knowledge. RTSM provides simpler solution process and does not require a large amount of computing. SOM-2 is designed by Lu et al. [19] to solve ILPP and this method is simple and useful in several cases. The numerical illustrations are obtained from Ritika Chopra [20] in which the notion of value and ambiguity indices for TIFNs is extended to TrIFNs by applying a novel ranking function, and it is apparent that the decision makers determine the outcome. Also, another example is observed from Nachammai et al. [21], in which centroid based distance is used for ranking IFNs and IFLPP is solved with the arithmetic operations of Generalized Trapezoidal Intuitionistic Fuzzy Numbers (GTrIFNs). The following are the major characteristics of our suggested methods:

- A unique approach to IFLPP is presented, in which all objective function and constraint coefficients are specified in terms of STrIFNs and GTrIFNs.
- The given STrIFNs and GTrIFNs are then turned into an interval number using the (α, β)-cut method.
- The suggested methods are demonstrated for solving LPP in an Intuitionistic fuzzy environment, and the findings are contrasted to the procedure of Chopra et al. and Nachammai et al. by using SM, RTSM and SOM-2.
- The specified intervals are then transformed using the midpoint and width in order to produce the most optimal outcomes.
- The optimal solution provided by Chopra's method is in crisp value whereas in Nachammai's method, it is STrIFNs. However, we compare the value in terms of intervals as it enables more adaptable value through the use of SM, RTSM and SOM-2, and a diagrammatic representation is presented.
- At last, the most effective solution for TrIFNs is displayed and contrasted with the current one.

Following the introduction in section 1, some fundamental ideas required for the creation of a system for resolving problems are presented including preliminaries, ranking function and novel arithmetic operations. There are six sections in the article.
(i) Section 2 describes the basic ideas relevant to our work in more detail.
(ii) Section 3 provides a complete explanation of the mathematical model of IFLPP.
(iii) The efficiency of SM, RTSM and SOM-2 to solve LPP is provided in Section 4.
(iv) The themes of this paper are studied numerically, and the obtained outcomes are analyzed in section 5.
(v) Section 6 includes the results and discussions of our current article.

II. Preliminaries

In this section, the fundamental definitions of FS, IVFS, IFS, IVIFS, ranking, and arithmetic operations are discussed in detail with references to the original study.

Definition 1: (Fuzzy set). [22] The Fuzzy set $\tilde{M}$ is constituted by a membership function that maps the constituents of a domain, space, or universe of discourse Z to the unit interval [0,1]. i.e., $\tilde{M}$: Z → [0,1], a generic element y ∈ Z and its grade of membership may be expressed as a set of ordered pairs to describe a fuzzy set $\tilde{M}$ in Z.

$$\tilde{M} = \{y, \mu_{\tilde{M}}(y) : y \in Z, \mu_{\tilde{M}}(y) \in [0, 1]\}$$

Definition 2: (Intuitionistic Fuzzy set). [3] A fuzzy set is said to be an IFS $\tilde{M}$ in Z, if it takes the following form:

$$\tilde{M} = \{y, \mu_{\tilde{M}}(y), \nu_{\tilde{M}}(y) : y \in Z\}$$

whereas this equation contains the functions, $\mu_{\tilde{M}}(y) : Z \to [0, 1]$ and $\nu_{\tilde{M}}(y) : Z \to [0, 1]$, determines the degree of membership and non-membership of the element y ∈ Z respectively.

The formula $\pi_{\tilde{M}}(y) = 1-\mu_{\tilde{M}}(y)-\nu_{\tilde{M}}(y)$ is characterized as the degree of non-determinacy (hesitation) of the element y in Z based on the IFS $\tilde{M}$.

Definition 3: (Intuitionistic Fuzzy Number). [11] The Intuitionistic fuzzy number ${}^{I}\tilde{M}$ should be an Intuitionistic fuzzy subset of the real line ℝ is conveyed below,

- ${}^{I}\tilde{M}$ is Normal: i.e., $y_0 \in \mathbb{R}$ in which the grades of membership and non-membership are $\mu_{{}^{I}\tilde{M}}(y_0) = 1$ and $\nu_{{}^{I}\tilde{M}}(y_0) = 0$ respectively.
- For the membership degree, ${}^{I}\tilde{M}$ is convex, i.e.,
$$\mu_{{}^{I}\tilde{M}}(\lambda y_1 + (1 - \lambda)y_2) \geq min(\mu_{{}^{I}\tilde{M}}(y_1), \mu_{{}^{I}\tilde{M}}(y_2)) \quad \forall y_1, y_2 \in \mathbb{R}, \lambda \in [0, 1]$$
- ${}^{I}\tilde{M}$ is concave for the non-membership degree, i.e.,
$$\nu_{{}^{I}\tilde{M}}(\lambda y_1 + (1 - \lambda)y_2) \leq max(\nu_{{}^{I}\tilde{M}}(y_1), \nu_{{}^{I}\tilde{M}}(y_2)) \quad \forall y_1, y_2 \in \mathbb{R}, \lambda \in [0, 1]$$

Definition 4: (i) (Symmetric TrIFNs). [20] A fuzzy number is known to be a Trapezoidal Fuzzy Number (TrFN) $\tilde{M}$

= (t_1, t_2, t_3, t_4) in $\mathbb{R}$ with the membership function $\mu_{\tilde{M}}$: Z $\rightarrow$ [0,1] is listed below.

$$\mu_{\tilde{M}}(\mathtt{x}) = \begin{cases} \frac{\mathtt{x}-t_1}{t_2-t_1}, & t_1 \le \mathtt{x} \le t_2 \\ 1 & t_2 \le \mathtt{x} \le t_3 \\ \frac{t_4-\mathtt{x}}{t_4-t_3} & t_3 \le \mathtt{x} \le t_4 \\ 0 & otherwise \end{cases}$$

The TrFN $\tilde{M}$ = (t_1', t_2, t_3, t_4') in $\mathbb{R}$ of the Non-Membership Function (NMF) $\nu_{\tilde{M}}$: Z $\rightarrow$ [0,1] is comprised of the following:

$$\nu_{\tilde{M}}(\mathtt{x}) = \begin{cases} \frac{t_2-\mathtt{x}}{t_2-t_1'}, & t_1' \le \mathtt{x} \le t_2 \\ 0 & t_2 \le \mathtt{x} \le t_3 \\ \frac{\mathtt{x}-t_3}{t_4'-t_3} & t_3 \le \mathtt{x} \le t_4' \\ 1 & otherwise \end{cases}$$

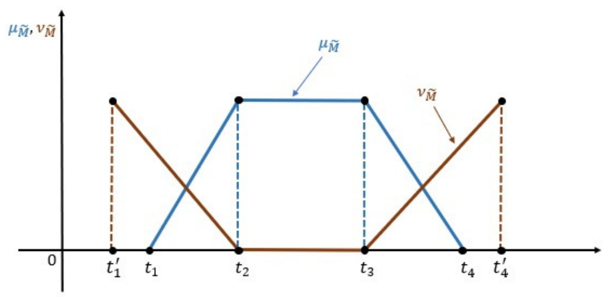

Fig. 1. Symmetric TrIFNs

where $t_1' \le t_1 \le t_2 \le t_3 \le t_4 \le t_4'$ and $\mu_{\tilde{M}}(\mathtt{x}) + \nu_{\tilde{M}}(\mathtt{x}) \le 1$. TrIFNs is an advanced form for the fusion of TrFN=(t_1, t_2, t_3, t_4) (membership) grade and TrFN=(t_1', t_2, t_3, t_4') (non-membership) grade, it is shown in **Fig. 1** and symbolised by,

$$^{T_I}\tilde{M} = \{(t_1, t_2, t_3, t_4); (t_1', t_2, t_3, t_4')\}$$

(ii) (GTrIFNs) [23] GTrIFNs is denoted by $^{T_I}\tilde{M} = (t_1, t_2, t_3, t_4; m_{\tilde{M}}), (t_1', t_2, t_3, t_4'; n_{\tilde{M}})$ with the membership and non-membership function $\mu_{\tilde{M}}, \nu_{\tilde{M}}$: Z $\rightarrow$ [0,1] is given as,

$$\mu_{\tilde{M}}(\mathtt{x}) = \begin{cases} m_{\tilde{M}}(\frac{\mathtt{x}-t_1}{t_2-t_1}), & t_1 \le \mathtt{x} \le t_2 \\ m_{\tilde{M}}, & t_2 \le \mathtt{x} \le t_3 \\ m_{\tilde{M}}(\frac{t_4-\mathtt{x}}{t_4-t_3}), & t_3 \le \mathtt{x} \le t_4 \\ 0 & otherwise \end{cases}$$

$$\nu_{\tilde{M}}(\mathtt{x}) = \begin{cases} n_{\tilde{M}}(\frac{t_2-\mathtt{x}}{t_2-t_1'}), & t_1' \le \mathtt{x} \le t_2 \\ n_{\tilde{M}}, & t_2 \le \mathtt{x} \le t_3 \\ n_{\tilde{M}}(\frac{\mathtt{x}-t_3}{t_4'-t_3}), & t_3 \le \mathtt{x} \le t_4' \\ 1 & otherwise \end{cases}$$

where $t_1' \le t_1 \le t_2 \le t_3 \le t_4 \le t_4'$ and $\mu_{\tilde{M}}(\mathtt{x}) + \nu_{\tilde{M}}(\mathtt{x}) \le 1$. The diagrammatic representation is shown in **Fig. 2**.

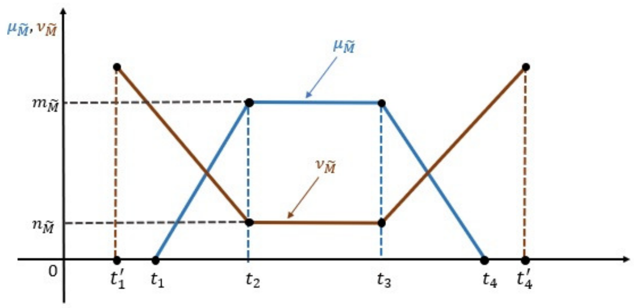

Fig. 2. Generalized TrIFNs

Definition 5: (i) (α, β-cut of TrIFNs [20]). The specification of the TrIFNs cut sets is as described in the following: The general representation of (α, β)-cut for IFN $^I\tilde{M}$ in $\mathbb{R}$ is stated as,

$$\tilde{M}^{\alpha}_{\beta} = \{\mathtt{x} : \mu_{\tilde{M}}(\mathtt{x}) \ge \alpha, \nu_{\tilde{M}}(\mathtt{x}) \le \beta, \quad 0 \le \alpha + \beta \le 1\}$$

On combining the above equation with **Def. 4 (i)**, we get an (α, β)-cut set of $^{T_I}\tilde{M} = (t_1, t_2, t_3, t_4); (t_1', t_2, t_3, t_4')$ is a crisp component of $\mathbb{R}$, the $\alpha - cut$ of lower and upper membership value is,

$$\begin{aligned} M_1(\alpha) &= \frac{\mathtt{x}-t_1}{t_2-t_1} \ge \alpha = \mathtt{x} - t_1 \ge \alpha(t_2 - t_1) \\ &= \mathtt{x} \ge t_1 + \alpha(t_2 - t_1) \\ M_2(\alpha) &= \frac{t_4-\mathtt{x}}{t_4-t_3} \ge \alpha = t_4 - \mathtt{x} \ge \alpha(t_4 - t_3) \\ &= \mathtt{x} \le t_4 - \alpha(t_4 - t_3) \\ \therefore [M_1(\alpha), M_2(\alpha)] &= [t_1 + \alpha(t_2 - t_1), t_4 - \alpha(t_4 - t_3)] \end{aligned}$$

Likewise, $\beta - cut$ of lower and upper non-membership value is,

$$\begin{aligned} M_1'(\beta) &= \frac{t_2-\mathtt{x}}{t_2-t_1'} \le \beta = t_2 - \mathtt{x} \le \beta(t_2 - t_1') \\ &= \mathtt{x} \ge t_2 - \beta(t_2 - t_1') \\ M_2'(\beta)&\frac{\mathtt{x}-t_3}{t_4'-t_3} \le \beta = \mathtt{x} - t_3 \le \beta(t_4' - t_3) \\ &= \mathtt{x} \le t_3 + \beta(t_4' - t_3) \\ \therefore [M_1'(\beta), M_2'(\beta)] &= [t_2 - \beta(t_2 - t_1'), t_3 + \beta(t_4' - t_3)] \end{aligned}$$

The general representation of TrIFNs with the (α, β)-cut is represented in the below equation.

$$\begin{aligned} ^{T_I}\tilde{M}^{\alpha}_{\beta} = \{[M_1(\alpha), M_2(\alpha)]; [M_1'(\beta), M_2'(\beta)]\}, \\ \alpha + \beta \le 1, \alpha, \beta \in [0, 1] \end{aligned}$$

$$\begin{aligned} M_1(\alpha) = t_1 + \alpha(t_2 - t_1) \quad & M_2(\alpha) = t_4 - \alpha(t_4 - t_3) \\ M_1'(\beta) = t_2 - \beta(t_2 - t_1') \quad & M_2'(\beta) = t_3 + \beta(t_4' - t_3) \end{aligned}$$

(ii) (α, β-cut of GTrIFNs [24]). α, β-cut of $^{T_I}\tilde{M} = (t_1, t_2, t_3, t_4; m_{\tilde{M}}), (t_1', t_2, t_3, t_4'; n_{\tilde{M}})$ is a crisp subset of real number $\mathbb{R}$ is defined as,

$$\tilde{M}^{\alpha}_{\beta} = \{\mathtt{x} : \mu_{\tilde{M}}(\mathtt{x}) \ge \alpha, \nu_{\tilde{M}}(\mathtt{x}) \le \beta\}$$

The equations in **Def. 4 (ii)** is combined with the above equation, the α-cut of lower and upper degree of membership is,

$$\begin{aligned}
M_1(\alpha) &= m_M\left(\frac{\mathtt{x}-t_1}{t_2-t_1}\right) \geq \alpha \\
&= m_M(\mathtt{x}-t_1) \geq \alpha(t_2-t_1) \\
&= \mathtt{x}-t_1 \geq \frac{\alpha}{m_M}(t_2-t_1) \\
&= \mathtt{x} \geq t_1 + \frac{\alpha}{m_M}(t_2-t_1) \\
M_2(\alpha) &= m_M\left(\frac{t_4-\mathtt{x}}{t_4-t_3}\right) \geq \alpha \\
&= m_M(t_4-\mathtt{x}) \geq \alpha(t_4-t_3) \\
&= t_4-\mathtt{x} \geq \frac{\alpha}{m_M}(t_4-t_3) \\
&= \mathtt{x} \leq t_4 - \frac{\alpha}{m_M}(t_4-t_3) \\
[M_1(\alpha), M_2(\alpha)] &= [t_1 + \frac{\alpha}{m_M}(t_2-t_1), t_4 - \frac{\alpha}{m_M}(t_4-t_3)]
\end{aligned}$$

The β-cut of lower and upper degree of non-membership is,

$$\begin{aligned}
M_1'(\beta) &= n_M\left(\frac{t_2-\mathtt{x}}{t_2-t_1'}\right) \leq \beta \\
&= n_M(t_2-\mathtt{x}) \leq \beta(t_2-t_1') \\
&= t_2-\mathtt{x} \leq \frac{\beta}{n_M}(t_2-t_1') \\
&= \mathtt{x} \geq t_2 - \frac{\beta}{n_M}(t_2-t_1') \\
M_2'(\beta) &= n_M\left(\frac{\mathtt{x}-t_3}{t_4'-t_3}\right) \leq \beta \\
&= n_M(\mathtt{x}-t_3) \leq \beta(t_4'-t_3) \\
&= \mathtt{x}-t_3 \leq \frac{\beta}{n_M}(t_4'-t_3) \\
&= \mathtt{x} \leq t_3 + \frac{\beta}{n_M}(t_4'-t_3) \\
[M_1'(\beta), M_2'(\beta)] &= [t_2 - \frac{\beta}{n_M}(t_2-t_1'), t_3 + \frac{\beta}{n_M}(t_4'-t_3)]
\end{aligned}$$

The equation below represents the general expression for TrIFNs having α, β-cut of ${}^{T_I}\tilde{M} = (t_1, t_2, t_3, t_4; m_{\tilde{M}}), (t_1', t_2, t_3, t_4'; n_{\tilde{M}})$ is given by,

$$\begin{gathered}
{}^{T_I}\tilde{M}^{\alpha}_{\beta} = \{[M_1(\alpha), M_2(\alpha)]; [M_1'(\beta), M_2'(\beta)]\}, \\
0 \leq \alpha+\beta \leq 1, 0 \leq \alpha \leq m_{\tilde{M}}, n_{\tilde{M}} \leq \beta \leq 1
\end{gathered}$$

$$M_1(\alpha) = t_1 + \frac{\alpha}{m_{\tilde{M}}}(t_2-t_1) \quad M_2(\alpha) = t_4 - \frac{\alpha}{m_{\tilde{M}}}(t_4-t_3)$$
$$M_1'(\beta) = t_2 - \frac{\beta}{n_{\tilde{M}}}(t_2-t_1') \quad M_2'(\beta) = t_3 + \frac{\beta}{n_{\tilde{M}}}(t_4'-t_3)$$

Definition 6: (Interval number). [25], [26] Consider an interval on the real line $\mathbb{R}$, the interval number $i = [i_1, i_2] = \{y \in \mathbb{R} : i_1 \leq y \leq i_2 \text{ and } i_1, i_2 \in \mathbb{R}\}$ If $\tilde{i} = i_1 = i_2 = i, then\ \tilde{i} = [i_1, i_2] = i$ is a real number or degenerate interval, then the midpoint and width of an interval $\tilde{i} = [i_1, i_2]$ can be written as $\tilde{i}_m = \left(\frac{i_1+i_2}{2}\right), \tilde{i}_w = \left(\frac{i_2-i_1}{2}\right)$ respectively. This interval number $\tilde{i}$ can be redefined with regard to its midpoint and width as $\tilde{i} = [i_1, i_2] = ((\tilde{i}_m), (\tilde{i}_w))$.

A. Ranking of Interval numbers

Sengupta et al. [27] demonstrated a straightforward approach for linking any two intervals on real numbers with considering decision-maker's opinion into analysis.
For a set of two intervals $\tilde{e} = [e_1, e_2]\ and\ \tilde{f} = [f_1, f_2]$, $\otimes$ denotes the extended order connection that exists among these intervals, then
(i) $If(\tilde{e}_m) < (\tilde{f}_m)$, then $\tilde{e} < \tilde{f}$ (or) $\tilde{e}$ is inferior to $\tilde{f}$.
(ii) $If(\tilde{e}_m) > (\tilde{f}_m)$, then $\tilde{e} > \tilde{f}$ (or) $\tilde{e}$ is superior to $\tilde{f}$.
(iii) $If(\tilde{e}_m) = (\tilde{f}_m)$, then $\tilde{e} = \tilde{f}$ (or) $\tilde{e}$ is equal to $\tilde{f}$.

Fig. 3. Interval Number

The Acceptibility Function (AF) can be used to formulate overlapping intervals, for example, if we consider two intervals [0.5,0.9] and [0.6,0.8], which is pictured in **Fig. 3** where the midpoints are equal but the intervals are not equal.
Let I be the set of all closed intervals on the real line $\mathbb{R}$. We define AF, $\mathbb{A}$: I x I $\rightarrow$ [0,∞) such that $\mathbb{A}(E \otimes$F) or $\mathbb{A}_{\otimes} = \frac{m(F)-m(E)}{w(F)+w(E)}$, where w(B)+w(A) $\neq$0, $\mathbb{A}(E \otimes$F) may be interpreted as the grade of acceptibility of the first interval to be inferior to the second interval (i.e., "E is inferior to F"). The grade of acceptability of $\mathbb{A}(E \otimes$F) may be classified and interpreted as,

$$\mathbb{A}(E \otimes F) = \begin{cases} = 0 & if\ m(E) = m(F), \\ > 0, < 1 & if\ m(E) < m(F) \\ \geq 1 & if\ m(E) < m(F) \end{cases}$$

- If $\mathbb{A}(E \otimes F) = 0$, then the interval numbers are equivalent or non-inferior to each other.
- If $0 < \mathbb{A}(E \otimes F) < 1$, then the premise $(E \otimes F)$ is accepted with different grades of satisfaction ranging from 0 to 1 (excluding 0 and 1).
- If $\mathbb{A}(E \otimes F) \geq 1$, the interpreter $(E \otimes F)$ is accepted.

B. New Interval Arithmetic Operations

Ming Ma [28] projected a new interval fuzzy arithmetic which is built on both "location index and fuzziness index functions". For any two intervals, $\tilde{E} = [e_1, e_2],\ \tilde{F} = [f_1, f_2]$, the arithmetic operations on $\tilde{E}$ and $\tilde{F}$ are well established and justified as,

$$(i)\tilde{E}+\tilde{F}=\langle\tilde{e}_m,\tilde{e}_w\rangle+\langle\tilde{f}_m,\tilde{f}_w\rangle = \langle\tilde{e}_m+\tilde{f}_m, max(\tilde{e}_w,\tilde{f}_w)\rangle$$
$$(ii)\tilde{E}-\tilde{F}=\langle\tilde{e}_m,\tilde{e}_w\rangle-\langle\tilde{f}_m,\tilde{f}_w\rangle = \langle\tilde{e}_m-\tilde{f}_m, max(\tilde{e}_w,\tilde{f}_w)\rangle$$
$$(iii)\tilde{E}\times\tilde{F}=\langle\tilde{e}_m,\tilde{e}_w\rangle\times\langle\tilde{f}_m,\tilde{f}_w\rangle = \langle\tilde{e}_m\times\tilde{f}_m, max(\tilde{e}_w,\tilde{f}_w)\rangle$$
$$(iv)\tilde{E}\div\tilde{F}=\langle\tilde{e}_m,\tilde{e}_w\rangle\div\langle\tilde{f}_m,\tilde{f}_w\rangle = \langle\tilde{e}_m\div\tilde{f}_m, max(\tilde{e}_w,\tilde{f}_w)\rangle$$

III. Intuitionistic Fuzzy LPP

The major aim of linear optimization is to minimize or maximize a linear objective function that is relevant to linear constraints, which can be equalities or inequalities. The primary goal of LPP is to find the most effective solution, which includes estimating loss or gain. It is a method for analysing various inequalities in a situation and assessing the cheapest option that must be obtained in those circumstances. LPP in intuitionistic fuzzy environment has four key elements: objective function, decision variables, constraints, and parameters.

- LPP is comprised of a statement of fact that the objective is recognised as the objective function. The objective might be profit maximization or loss minimization.
- These variables determine the unknown value that must be calculated. The decision maker has the ability to control the objective through the use of decision variables.
- Constraints are limitations or restrictions that arise from various sources that helps a decision maker's ability to choose the values of the decision variables. There are three classes, and they are as follows.
  - *System Constraints*: It involves several decision variables.
  - *Individual Constraints*: There is only one variable involved.
  - *Non-negative Constraints*: No variable is allowed to assume a negative value.
- The objective function and constraints are composed of symbols that symbolize the decision variables as well as numerical values known as parameters.

LPP states, "A mathematical method to define the problem using a linear objective function and linear inequality constraints and to distribute finite resources to competing activities in an optimal way."
Among some of the descriptions used mostly in LPP are as follows:

Definition 7: In the maximization problem, a simplex solution is **optimal** if the ${}^I\tilde{c}_j - {}^I\tilde{x}_j$ row is completely composed of zeros and negative numbers that is, there aren't any positive numbers in the rows.

Definition 8: A **basic solution** of a LPP in general form is a solution $({}^I\tilde{x}_1, {}^I\tilde{x}_2, ...{}^I\tilde{x}_n; s_1, s_2, ..., s_k)$ of the constraint equations in which at least m variables are nonzero; these variables are known as basic variables. A **basic feasible solution** is one that has all variables that are non-negative.

Definition 9: In a basic feasible solution, basic and non-basic variables have a non-zero and zero coefficient respectively. These variables classifies an optimization problem's decision variables.

Definition 10: To change inequality constraints into equality constraints, different variables known as **slack variables** are added to the given constraints of a LPP.
The model of the LPP is represented in this section, along with some fundamental ideas for the solution.

A. Mathematical Formulation of Intuitionistic Fuzzy LPP

1) Simplex method: The general way of formulating an optimization model with an Intuitionistic fuzzy objective function and constraints are given below:

$$\text{maximize or minimize } {}^I\tilde{z} = \sum_{j=1}^{n} {}^I\tilde{c}_j {}^I\tilde{x}_j$$

$$\text{subject to, } \sum_{j=1}^{n} {}^I\tilde{a}_{ij} {}^I\tilde{x}_j \preceq, \approx, \succeq {}^I\tilde{b}_i,\ where\ {}^I\tilde{x}_j \geq 0$$

where i = 1,2,....,m ; j = 1,2,....,n and ${}^I\tilde{c}_j, {}^I\tilde{a}_{ij}, {}^I\tilde{b}_i$ are TrIFNs. Here,

- ${}^I\tilde{x}_j$ represents the j^{th} decision variable,
- ${}^I\tilde{c}_j$ represents the coefficient of the objective function for the j^{th} variable,
- ${}^I\tilde{a}_{ij}$ represents the constraint i's coefficient on ${}^I\tilde{x}_j$,
- ${}^I\tilde{b}_i$ represents the individual constraints i's right-side coefficient,
- ${}^I\tilde{x}_j \geq 0$ represents the non-negativity constraints.

The Simplex method entails starting with a feasible solution and iteratively improving it until an optimal solution can be found. This method guides the search for an optimal solution by using a collection of theorems. The following are the theorems connected to LPP and the simplex method.

Theorem 1. Given a basic feasible solution to the LPP, ${}^I\tilde{x}_{\tilde{B}} = \tilde{B}^{-1}b = ({}^I\tilde{x}_{\tilde{B}_1}, {}^I\tilde{x}_{\tilde{B}_2}, ..., {}^I\tilde{x}_{\tilde{B}_m})$ and ${}^I\tilde{Z} = {}^I\tilde{Z}^*$ such that ${}^I\tilde{c}_j - {}^I\tilde{z}_j \leq 0$ for each column ${}^I\tilde{a_j}$ in $\tilde{A}$ but not in $\tilde{B}$. Then ${}^I\tilde{Z}$ is the objective function's maximum value, and ${}^I\tilde{x}_{\tilde{B}}$ is the optimal basic feasible solution.

proof: Consider, ${}^I\tilde{x}_{\tilde{B}} = \tilde{B}^{-1}b$ is the feasible solution of LPP and ${}^I\tilde{Z}$= ${}^I\tilde{c}_{\tilde{B}} {}^I\tilde{x}_{\tilde{B}}$ is the corresponding objective function.
And, ${}^I\tilde{x}_j \geq 0$ (j=1,2,...,n) is any feasible solution of LPP. The equation $A^I\tilde{x}$=b can be written as follows using column vectors of $\tilde{A}$:

$${}^I\tilde{a_1} {}^I\tilde{x_1} + {}^I\tilde{a_2} {}^I\tilde{x_2} + ... + {}^I\tilde{a_n} {}^I\tilde{x_n} = b \quad (1)$$

The objective function's value for this solution is determined by,

$${}^I\tilde{Z}^* = {}^I\tilde{c} {}^I\tilde{x} = {}^I\tilde{c_1} {}^I\tilde{x_1} + {}^I\tilde{c_2} {}^I\tilde{x_2} + ... + {}^I\tilde{c_n} {}^I\tilde{x_n}$$

Any column vector ${}^I\tilde{a_j}$ of $\tilde{A}$ can be represented by a linear combination of column vectors β_i of $\tilde{B}$, which is as follows:

$${}^I\tilde{a_j} = \sum_{i=1}^{m} {}^I y_{ij}\beta_i$$

Substituting ${}^I\tilde{a_j}$ value in eqn.(1) we get,

$$ {}^Ix_1\sum_{i=1}^{m}{}^I\tilde{y}_{i_1}\beta_i + {}^Ix_2\sum_{i=1}^{m}{}^I\tilde{y}_{i_2}\beta_i + ... + {}^Ix_n\sum_{i=1}^{m}{}^I\tilde{y}_{i_n}\beta_i = b $$

Substituting the limits,

$$ \left\{\sum_{i=1}^{m}{}^Ix_j{}^I\tilde{y}_{i_j}\right\}\beta_1 + \left\{\sum_{i=1}^{m}{}^Ix_j{}^I\tilde{y}_{2_j}\right\}\beta_2 + ...+ \left\{\sum_{i=1}^{m}{}^Ix_j{}^I\tilde{y}_{m_j}\right\}\beta_m = b $$

Let for every column ${}^I\tilde{a_j}$ which is in $\tilde{A}$ but not in $\tilde{B}$, ${}^I\tilde{c}_j -{}^I \tilde{z}_j \leq 0$. Now we prove, ${}^I\tilde{Z}$ ¿ ${}^I\tilde{Z}^*$ for any other feasible solution. For all column vectors of $\tilde{A}$ which is in $\tilde{B}$, ${}^I\tilde{a_j} \in \tilde{B}$, we get,

$$ {}^I\tilde{y_j} = \tilde{B}^{-1I}\tilde{a_j} = \tilde{B}{-1}\beta_i = {}^I\tilde{u_i} \text{ (unit vector)} $$

provided ${}^I\tilde{a_j}$ in column i for $\tilde{B}$. then,

$$ {}^I\tilde{c_j} - {}^I\tilde{z_j} = {}^I\tilde{z_j} - {}^I\tilde{c}_{\tilde{B}}{}^I\tilde{y_j} = {}^I\tilde{c_j} - {}^I\tilde{c}_{\tilde{B}}{}^I\tilde{u_i} = {}^I\tilde{c_j} - {}^I\tilde{c_j} = 0 $$

Then ${}^I\tilde{c_j} - {}^I\tilde{z_j} = 0$ for all columns of $\tilde{A}$ in $\tilde{B}$. Applying the assumption that ${}^I\tilde{c_j} - {}^I\tilde{z_j} \leq 0$ for all columns in $\tilde{A}$. Then from eqn. (1), we have,

$$ \sum_{i=1}^{m}({}^I\tilde{c_j} - {}^I\tilde{z_j}){}^I\tilde{x_j} \leq 0 $$

$$ \sum_{j=1}^{n}{}^I\tilde{c_j}{}^I\tilde{x_j} \leq \sum_{j=1}^{n}{}^I\tilde{z_j}{}^I\tilde{x_j} = \sum_{j=1}^{n}{}^I\tilde{x_j}\left\{\sum_{i=1}^{m}{}^I\tilde{c}_{\tilde{B_i}}{}^I\tilde{y}_{ij}\right\} $$

$$ = \left\{\sum_{j=1}^{n}{}^I\tilde{x_j}{}^I\tilde{y_{1j}}\right\}{}^I\tilde{c}_{\tilde{B_1}} + \left\{\sum_{j=1}^{n}{}^I\tilde{x_j}{}^I\tilde{y_{2j}}\right\}{}^I\tilde{c}_{\tilde{B_2}} + ...+ \left\{\sum_{j=1}^{n}{}^I\tilde{x_j}{}^I\tilde{y_{mj}}\right\}{}^I\tilde{c}_{\tilde{B_m}} $$

$$ = {}^I\tilde{x}_{\tilde{B_1}}{}^I\tilde{c}_{\tilde{B_1}} + {}^I\tilde{x}_{\tilde{B_2}}{}^I\tilde{c}_{B_2} + ... + {}^I\tilde{x}_{\tilde{B_m}}{}^I\tilde{c}_{\tilde{B_m}} = {}^I\tilde{Z} $$

$$ \therefore {}^I\tilde{Z}^* \leq {}^I\tilde{Z} $$

This concludes the theorem's proof.

Theorem 2. A sufficient condition for a Basic feasible solution ${}^I\tilde{x}_{\tilde{B}}$ to a LPP

$$ \begin{aligned} maximize\ {}^I\tilde{z} &= {}^I\tilde{c}^I\tilde{x} \\ subject\ to\ \tilde{A}^I\tilde{x} &= b \\ {}^I\tilde{x} &\geq 0 \end{aligned} $$

to be optimal is that ${}^I\tilde{z_j} -{}^I\tilde{c_j} \geq 0$ for all the column vector ${}^I\tilde{a_j}$ of $\tilde{A}$.

Theorem 3. A Linear programming problem,

$$ \begin{aligned} maximize\ {}^I\tilde{z} &= {}^I\tilde{c}^I\tilde{x} \\ subject\ to\ \tilde{A}^I\tilde{x} &= b \\ {}^I\tilde{x} &\geq 0 \end{aligned} $$

will have no finite optimal solution if there exists atleast one column vextor ${}^I\tilde{a_j}$ corresponding to a non-basic variable ${}^I\tilde{x_j}$ such that ${}^I\tilde{z_j} -{}^I\tilde{c_j} < 0$ and ${}^I\tilde{y}_{ij}$ for all i.

Theorem 4. If there is an optimal basic feasible solution to the LPP,

$$ \begin{aligned} maximize\ {}^I\tilde{z} &= {}^I\tilde{c}^I\tilde{x} \\ subject\ to\ \tilde{A}^I\tilde{x} &= b \\ {}^I\tilde{x} &\geq 0 \end{aligned} $$

and at the optimal stage of the simplex algorithm, ${}^I\tilde{z_j} -{}^I\tilde{c_j} = 0$ for some non-basic vector ${}^I\tilde{a_j}$ with ${}^I\tilde{y}_{ij} > 0$ for atleast one i, then there exists more than one optimal solution.

2) RTSM and SOM-2: The following is a representation of an ILPP model:

$$ Maximize^I\tilde{\mathtt{Z}}^{\pm} = \sum_{j=1}^{n}{}^I\tilde{\mathtt{C}}^{\pm I}\tilde{\mathtt{X}}^{\pm} $$

$$ subject\ to\ ,\sum_{j=1}^{n}{}^I\tilde{\mathtt{A}}^{\pm I}\tilde{\mathtt{X}}^{\pm} \leq {}^I\tilde{\mathtt{B}}^{\pm},\ where\ {}^I\tilde{\mathtt{X}}^{\pm} \geq 0 $$

where, $\mathbb{R}^{\pm}$ represents a collection of interval numbers,

$$ \begin{aligned} {}^I\tilde{\mathtt{A}}^{\pm} &= ({}^I\tilde{a}_{ij})_{m\times n} \in (\mathbb{R}^{\pm}) \\ {}^I\tilde{\mathtt{C}}^{\pm} &= (\mathtt{c}_1^{\pm}, \mathtt{c}_2^{\pm}, ..., \mathtt{c}_n^{\pm}) \in (\mathbb{R}^{\pm})^{q\times n} \\ {}^I\tilde{\mathtt{B}}^{\pm} &= (\mathtt{b}_1^{\pm}, \mathtt{b}_2^{\pm}, ..., \mathtt{b}_m^{\pm}) \in (\mathbb{R}^{\pm})^{m\times q} \\ {}^I\tilde{\mathtt{X}}^{\pm} &= (\mathtt{x}_1^{\pm}, \mathtt{x}_2^{\pm}, ..., \mathtt{x}_n^{\pm}) \in (\mathbb{R}^{\pm})^{n\times q} \end{aligned} $$

IV. Computational method

In order to find the optimal solution, the two methods are employed to solve the given LPP.

A. Simplex method

One of the most widely used approaches to resolving LPP is the simplex method. The procedures that are involved in computing an optimal solution is given below,

Step 1: Construct a mathematical model of the problem's objective function and constraints by formulating a Intuitionistic fuzzy linear programming model.

Step 2: Add the slack variable to every inequality formulation to turn the provided inequalities into equations and generate a zero coefficient in the objective function.

Eg: The given problem becomes,

$$ Max^I\tilde{z} = {}^I\tilde{c}_1^I\tilde{x}_1 + {}^I\tilde{c}_2^I\tilde{x}_2 + ... + {}^I\tilde{c}_n^I\tilde{x}_n + 0s_1 + 0s_2 + ... + 0s_k \tag{2} $$

Subject to,

$$ \begin{gathered} {}^I\tilde{a}_{11}^I\tilde{x}_1 +{}^I\tilde{a}_{12}^I\tilde{x}_2 + ... +{}^I\tilde{a}_{1n}^I\tilde{x}_n + s_1 ={}^I\tilde{b}_1 \\ {}^I\tilde{a}_{21}^I\tilde{x}_1 + {}^I\tilde{a}_{22}^I\tilde{x}_2 + ... + {}^I\tilde{a}_{2n}^I\tilde{x}_n + s_2 = {}^I\tilde{b}_2 \\ \cdots\cdots\cdots \\ {}^I\tilde{a}_{m1}^I\tilde{x}_1 + {}^I\tilde{a}_{m2}^I\tilde{x}_2 + ... + {}^I\tilde{a}_{mn}^I\tilde{x}_n + s_k = {}^I\tilde{b}_k \end{gathered} $$

Here, ${}^I\tilde{x}_1, {}^I\tilde{x}_2, ...{}^I\tilde{x}_n$ and $s_1, s_2, ..., s_k$ are slack variables to change inequality constraints to equality constraints as well as non-negative.

Step 3: Build the initial feasible solution. Setting the decision variables to zero, results in an initial basic feasible

solution. Then, the objective function in (2) turns into, ${}^I\tilde{x}_1 = {}^I\tilde{x}_2 = ... = {}^I\tilde{x}_n = 0$, Then we have, $s_1 = {}^I\tilde{b}_1$, $s_2 = {}^I\tilde{b}_2$,...., $s_k = {}^I\tilde{b}_k$
The **Table I** appears to be,

TABLE I
GENERAL REPRESENTATION OF ITERATIONS IN PROPOSED METHOD

			${}^I\tilde{c}_j \rightarrow$	
Basic variables	${}^I\tilde{C}_B$	${}^I\tilde{X}_B$	${}^I\tilde{x}_1, {}^I\tilde{x}_2 .... {}^I\tilde{x}_k$	${}^I\tilde{s}_1, {}^I\tilde{s}_2 .... {}^I\tilde{s}_k$
${}^I\tilde{s}_1$	0	${}^I\tilde{b}_1$	${}^I\tilde{a}_{11}, {}^I\tilde{a}_{12}, ..., {}^I\tilde{a}_{1n}$	$1\ 0\ 0 \dots 0$
${}^I\tilde{s}_2$	0	${}^I\tilde{b}_2$	${}^I\tilde{a}_{21}, {}^I\tilde{a}_{22}, ..., {}^I\tilde{a}_{2n}$	$0\ 1\ 0 \dots 0$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$
${}^I\tilde{s}_k$	0	${}^I\tilde{b}_k$	${}^I\tilde{a}_{m1}, {}^I\tilde{a}_{m2}, ..., {}^I\tilde{a}_{mn}$	$0\ 0\ 0 \dots 1$
${}^I\tilde{z}_j$	...	...	...	...
${}^I\tilde{c}_j - {}^I\tilde{z}_j$	...	...	...	...

Step 4: Analyze the ${}^I\tilde{c}_j - {}^I\tilde{z}_j$ values. These three scenarios are possible:

- The basic feasible solution is optimal if all ${}^I\tilde{c}_j - {}^I\tilde{z}_j \leq 0$
- If the coefficients matrix contains at least one column for which ${}^I\tilde{c}_j - {}^I\tilde{z}_j > 0$ and all other elements are negative (i.e., $a_{ij} < 0$), then the presented problem has an unbounded solution.
- If there is at least one ${}^I\tilde{c}_j - {}^I\tilde{z}_j > 0$ and each of these columns contains at least one positive element (i.e., $a_{ij} > 0$) for some row, this suggests that the objective function ${}^I\tilde{z}$ is feasible and it is continued in next step.

The procedure that needs to be followed to solve the problem is explained in the flowchart of **Fig. 4**.

Step 5: If there exists multiple positive solutions in the bottom of the table, select the largest positive among them, then the corresponding column is considered as the entering column which is denoted as (C_E).
Step 6: Calculate the ratio $\frac{{}^I\tilde{C}_B}{(C_E)}$ in which ${}^I\tilde{C}_B$ is the objective function's cost value and is written in the separate column, then choose the value that is the least of the values and is represented as the leaving row. The key element of the table is the component that is presented common in both the entering column and the exiting row.
Step 7: After tracking down the key element, divide its row by the maximum element so that the outcome which is equal to 1, and can make all of the other elements in its column equal to 0 by deducting the proper multiples of this new row from the other rows.
Step 8: Once completing these steps, if either one of the numbers in the ${}^I\tilde{c}_j - {}^I\tilde{z}_j$ row are still positive, repeat steps from (5-7) once again until the optimal solution is discovered. Otherwise, the given problem came up with the finest optimal solution.

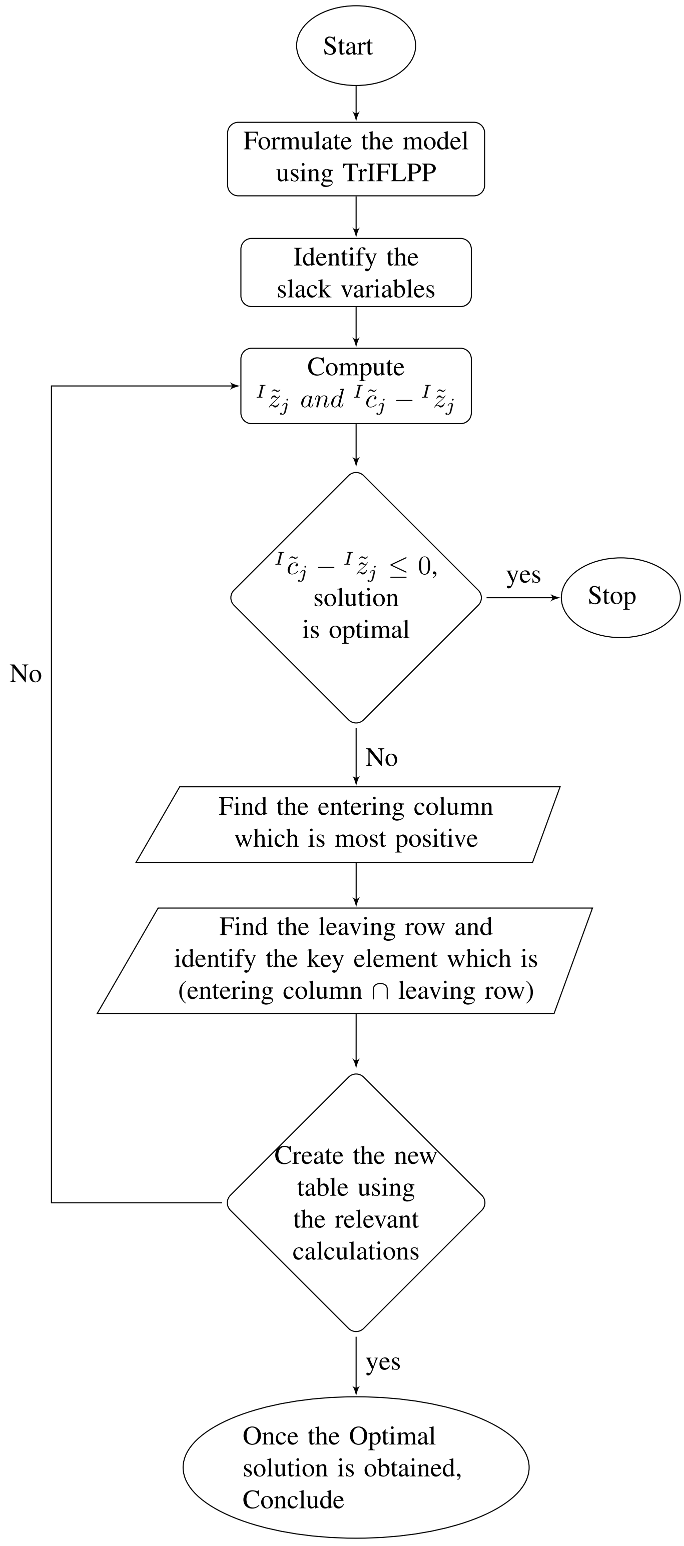

Fig. 4. Flowchart of Simplex method

B. Robust two step method

For the purpose of solving interval LPP, a robust two-step method (RTSM) is developed and separated into two submodels that correspond to z. This is done in order to prevent a total violation of the constraints as the decision variables in the generated decision space changed. It's probable that the RTSM approach won't require much calculation. The RTSM method's solution space is completely feasible.
In this technique, when the objective function is to be maximised, the primary sub-model is formulated to correspond to ${}^I\tilde{z}^-$, and the second sub-model is formulated to correspond to ${}^I\tilde{z}^+$, utilising the results of the first sub-model. The RTSM method's sub-models can be defined below: (assume ${}^I\tilde{z}^{\pm} > 0$, ${}^I\tilde{b}_i^{\pm} > 0$)

1) Sub-model 1:

$$Max\ {}^I\tilde{z}^- = \sum_{j=1}^{p} {}^I\tilde{c}_j^{-\,I}\tilde{x}_j^- + \sum_{j=p+1}^{n} {}^I\tilde{c}_j^{-\,I}\tilde{x}_j^+$$

Subject to

$$\sum_{j=1}^{p} |{}^I\tilde{a}_{ij}|^+\ sign(\tilde{a}_{ij}^{\pm})^I\tilde{x}_j^- + \sum_{j=p+1}^{n} |{}^I\tilde{a}_{ij}|^-\ sign(\tilde{a}_{ij}^{\pm})^I\tilde{x}_j^+ \le^I \tilde{b}_i^-, i = 1, 2, ..., m,$$

${}^I\tilde{x}_j^- \ge 0$, j = 1, 2, ...,p and ${}^I\tilde{x}_j^+ \ge 0$, j = p+1, p+2,....,n

The values of $|{}^I\tilde{a}_{ij}|^{\pm}$ and sign$(\tilde{a}_{ij}^{\pm})$ are,

$$|{}^I\tilde{a}_{ij}|^- = \begin{cases} {}^I\tilde{a}_{ij}^-, & {}^I\tilde{a}_{ij}^{\pm} \ge 0 \\ -{}^I\tilde{a}_{ij}^+, & {}^I\tilde{a}_{ij}^{\pm} < 0 \end{cases}$$

$$|{}^I\tilde{a}_{ij}|^+ = \begin{cases} {}^I\tilde{a}_{ij}^+, & {}^I\tilde{a}_{ij}^{\pm} \ge 0 \\ -{}^I\tilde{a}_{ij}^-, & {}^I\tilde{a}_{ij}^{\pm} < 0 \end{cases}$$

$$sign(\tilde{a}_{ij}^{\pm}) = \begin{cases} 1, & {}^I\tilde{a}_{ij}^{\pm} \ge 0 \\ -1, & {}^I\tilde{a}_{ij}^{\pm} < 0 \end{cases}$$

The optimal solution that sub-model 1 may yield is ${}^I\tilde{x}_{jop}^-$, and ${}^I\tilde{z}_{op}^-$ is the optimal objective function for this model. Using the solution from the first sub-model, the second sub-model is presented and is defined as

2) Sub-model 2:

$$Max\ {}^I\tilde{z}^+ = \sum_{j=1}^{p} {}^I\tilde{c}_j^{+\,I}\tilde{x}_j^+ + \sum_{j=p+1}^{n} {}^I\tilde{c}_j^{+\,I}\tilde{x}_j^-$$

Subject to

$$\sum_{j=1}^{p} |{}^I\tilde{a}_{ij}|^-\ sign(\tilde{a}_{ij}^{\pm})^I\tilde{x}_j^+ + \sum_{j=p+1}^{n} |{}^I\tilde{a}_{ij}|^+\ sign(\tilde{a}_{ij}^{\pm})^I\tilde{x}_j^- \le^I \tilde{b}_i^+, i = 1, 2, ..., m$$

$$\sum_{j=1}^{q_{i1}} |{}^I\tilde{a}_{ij}|^{-\,I}\tilde{x}_j^+ + \sum_{j=q_{i1}+1}^{p} |{}^I\tilde{a}_{ij}|^{-\,I}\tilde{x}_{jop}^- + \sum_{j=p+1}^{q_{i2}} |{}^I\tilde{a}_{ij}|^{-\,I}\tilde{x}_j^- + \sum_{j=q_{i2}+1}^{n} |{}^I\tilde{a}_{ij}|^{-\,I}\tilde{x}_{jop}^+ \le^I \tilde{b}_i^+$$

${}^I\tilde{x}_j^+ \ge {}^I\tilde{x}_{jop}^-$, j = 1, 2, ..., p
${}^I\tilde{x}_j^- \le {}^I\tilde{x}_{jop}^+$, j = p+1, p+2, ..., n
${}^I\tilde{x}_j^+ \ge 0$, j = 1, 2, ...,p and ${}^I\tilde{x}_j^- \ge 0$, j = p+1, p+2,....,n

where $\tilde{c}_j^{\pm} \ge 0$ for j = 1, 2, ..., p and $\tilde{c}_j^{\pm} \le 0$ for j = p+1, p+2, ..., n; ${}^I\tilde{a}_{ij} \ge 0$ for j = 1, 2, ... q_{i1}; j = $q_{i1}+1, q_{i1}+2$, ..., n and ${}^I\tilde{a}_{ij} \le 0$ for j = $q_{i1}+1, q_{i1}+2, ..., q_{i2}$, where $q_{i1} \le$ p, and $q_{i2} \ge$ p.

The optimal solution that sub-model 2 can generate is ${}^I\tilde{x}_{jop}^+$, and the best objective function for this model is ${}^I\tilde{z}_{op}^+$. As a result, the combined conclusions of Sub-models 1 and 2 are interpreted as, ${}^I\tilde{x}_{jop}^{\pm} = [{}^I\tilde{x}_{jop}^-, {}^I\tilde{x}_{jop}^+]$ and ${}^I\tilde{z}_{jop}^{\pm} = [{}^I\tilde{z}_{op}^-, {}^I\tilde{z}_{op}^+]$.

C. Alternative Solution Method

An alternative solution method (SOM-2) for solving interval LPP model is introduced by Lu et al. [19] for minimising the objective function (i.e., min $z^{\pm} = \sum_{j=1}^{p} {}^I\tilde{c}_j^{\pm\,I}\tilde{x}_j^{\pm}$) with the help of its sub-models. This method can produce alternative solution that are flexible to the practical circumstances of decision makers. This paper solves the maximization problem and considers the SOM-2 sub-models as,

1) Sub-model 1:

$$Max\ {}^I\tilde{z}^- = \sum_{j\in A_1} {}^I\tilde{c}_j^{-\,I}\tilde{x}_j^- + \sum_{j=p+1}^{n} {}^I\tilde{c}_j^{-\,I}\tilde{x}_j^+$$

Subject to

$$\sum_{j\in A_1} |{}^I\tilde{a}_{ij}|^+\ sign(\tilde{a}_{ij}^{\pm})^I\tilde{x}_j^- + \sum_{j=p+1}^{n} |{}^I\tilde{a}_{ij}|^-\ sign(\tilde{a}_{ij}^{\pm})^I\tilde{x}_j^+ \le^I \tilde{b}_i^+, i = 1, 2, ..., m,$$

${}^I\tilde{x}_j^- \ge 0$, $j \in A_1$ and ${}^I\tilde{x}_j^+ \ge 0$, $j \in A_2$,
$A_1 = \{j : \tilde{c}_j^{\pm} \ge 0\}, A_2 = \{j : \tilde{c}_j^{\pm} \le 0\}$, A_1,A_2 are index sets.
The optimal solution of sub-model 1 is ${}^I\tilde{x}_{jop}^-$ for $j \in A_1$, ${}^I\tilde{x}_{jop}^+$ for $j \in A_2$ and ${}^I\tilde{z}_{op}^-$ is the optimal objective function for sub-model 1. Using the solution from the first sub-model, the sub-model 2 is defined as

2) Sub-model 2:

$$Max\ {}^I\tilde{z}^+ = \sum_{j\in A_1}^{p} \tilde{c}_j^{+\,I}\tilde{x}_j^+ + \sum_{j=p+1}^{n} {}^I\tilde{c}_j^{+\,I}\tilde{x}_j^-$$

Subject to

$$\sum_{j\in A_2} |{}^I\tilde{a}_{ij}|^-\ sign(\tilde{a}_{ij}^{\pm})^I\tilde{x}_j^+ + \sum_{j=p+1}^{n} |{}^I\tilde{a}_{ij}|^+\ sign(\tilde{a}_{ij}^{\pm})^I\tilde{x}_j^- \le^I \tilde{b}_i^-, i = 1, 2, ..., m,$$

${}^I\tilde{x}_j^+ \ge {}^I\tilde{x}_{jop}^-$, for $j \in A_1$, $0 \le {}^I\tilde{x}_j^- \le {}^I\tilde{x}_{jop}^+$, for $j \in A_2$.

As a result, the combined solutions of Sub-models 1 and 2 are interpreted as, ${}^I\tilde{x}_{jop}^{\pm} = [{}^I\tilde{x}_{jop}^-, {}^I\tilde{x}_{jop}^+]$ and ${}^I\tilde{z}_{jop}^{\pm} = [{}^I\tilde{z}_{op}^-, {}^I\tilde{z}_{op}^+]$.

V. Numerical Illustration

A. Example 1:

This section uses the numerical demonstration from [20] to validate the trapezoidal IFLPP's suggested numerical method. The challenge in the actual world is determining the production's maximum profit. Consider the subsequent maximization LPP. A businessman who wishes to increase profits with two separate products, A and B, each of which has a net profit per item close to Rs. 5 and Rs. 3, respectively. He can only spend a certain amount of money on labour and supplies. Product A took 4 hours of labour to produce each unit, while Product B just needs 3 hours. While each unit of product A only needs one unit of raw materials, each unit of product B needs three units. The maximum number of raw material units and labour hours permitted are 12 and 6, respectively.

$$\max\ {}^{I}\widetilde{5}\mathtt{x}_1 + {}^{I}\widetilde{3}\mathtt{x}_2$$
$$\text{Subject to } {}^{I}\widetilde{4}\mathtt{x}_1 + {}^{I}\widetilde{3}\mathtt{x}_2 \leq {}^{I}\widetilde{12}$$
$${}^{I}\widetilde{1}\mathtt{x}_1 + {}^{I}\widetilde{3}\mathtt{x}_2 \leq {}^{I}\widetilde{6}$$
$$\mathtt{x}_1, \mathtt{x}_2 \geq 0$$

The objective function and constraints are expressed in STrIFNs in this particular instance.

$$\mathtt{c}_1 = {}^{I}\widetilde{5} = (4, 5, 5.5, 6); (4, 5, 5.5, 6.1);$$
$$\mathtt{c}_2 = {}^{I}\widetilde{3} = (2.5, 3, 3.1, 3.2); (2, 3, 3.1, 3.5)$$
$$\mathtt{a}_{11} = {}^{I}\widetilde{4} = (3.5, 3.8, 4, 4.1); (3, 3.8, 4, 5);$$
$$\mathtt{a}_{12} = {}^{I}\widetilde{3} = (2.5, 3, 3.1, 3.5); (2.4, 3, 3.1, 3.6)$$
$$\mathtt{a}_{21} = {}^{I}\widetilde{1} = (0.8, 1, 1.2, 2); (0.5, 1, 1.2, 2.1);$$
$$\mathtt{a}_{22} = {}^{I}\widetilde{3} = (2.8, 2.9, 3, 3.2); (2.5, 2.9, 3, 3.2)$$
$$\mathtt{b}_1 = {}^{I}\widetilde{12} = (11, 11.8, 12.2, 13); (11, 11.8, 12.2, 14);$$
$$\mathtt{b}_2 = {}^{I}\widetilde{6} = (5.5, 6, 6.2, 7.5); (5, 6, 6.2, 8.1)$$

The flowchart in **Fig. 5** explains the process that must be maintained throughout the problem.

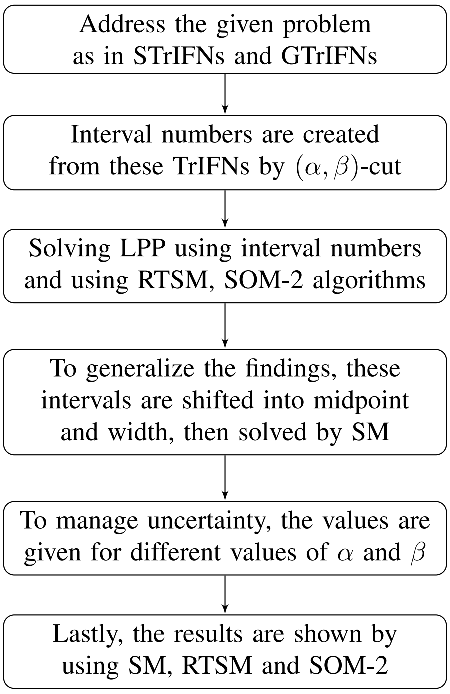

Fig. 5. Process flow diagram

1) Simplex method: The solution of the given problem is: With the use of the (α, β)-cut, these STrIFNs are turned into interval numbers and the values are tabulated in **Table II**.

In order to explicitly handle the uncertainty, we obtain the Interval valued Intuitionistic fuzzy numbers (IVIFNs) by changing the TrIFNs to IVIFNs by assigning α and β values amongst 0 and 1.

Let us characterize all interval parameters as $\tilde{E} = [e_1, e_2]$ *and* $\tilde{F} = [f_1, f_2]$ in aspects of midpoint and width, respectively, as

$$\tilde{E} = \langle \tilde{e}_m \rangle, \langle \tilde{e}_w \rangle \ and\ \tilde{F} = \langle \tilde{f}_m \rangle, \langle \tilde{f}_w \rangle$$

TABLE II
INTERVAL VALUES USING (α, β)-CUT

Linear values	Result using (α, β)-cut
$\mathtt{c}_1$	[4+α, 6-0.5α], [5-β, 5.5+0.6β]
$\mathtt{c}_2$	[2.5+0.5α, 3.2-0.1α], [3-β, 3.1+0.4β]
$\mathtt{a}_{11}$	[3.5+0.3α, 4.1-0.1α], [3.8-0.8β, 4+β]
$\mathtt{a}_{12}$	[2.5+0.5α, 3.5-0.4α], [3-0.6β, 3.1+0.5β]
$\mathtt{a}_{21}$	[0.8+0.2α, 2-0.8α], [1-0.5β, 1.2+0.9β]
$\mathtt{a}_{22}$	[2.8+0.1α, 3.2-0.2α], [2.9-0.4β, 3+0.2β]
$\mathtt{b}_1$	[11+ 0.8α, 13-0.8α], [11.8-0.8β, 12.2+1.8β]
$\mathtt{b}_2$	[5.5+0.5α, 7.5-1.3α], [6-β, 6.2+1.9β]

The IVIFNs is then converted into numbers which is represented in terms of midpoint and width with the help of new interval arithmetic operations.

When α, β value is 0.5 we acquire for the **membership value** as,

$$\max\ \langle 5.125, 0.625 \rangle \mathtt{x}_1 + \langle 2.95, 0.2 \rangle \mathtt{x}_2$$
$$\text{Subject to } \langle 3.85, 0.2 \rangle \mathtt{x}_1 + \langle 3.025, 0.275 \rangle \mathtt{x}_2 \leq \langle 12, 0.6 \rangle$$
$$\langle 1.25, 0.35 \rangle \mathtt{x}_1 + \langle 2.975, 0.125 \rangle \mathtt{x}_2 \leq \langle 6.3, 0.55 \rangle$$
$$\mathtt{x}_1, \mathtt{x}_2 \geq 0$$

Accordingly, the **non-membership value** for α, β = 0.5 is,

$$\max\ \langle 5.15, 0.65 \rangle \mathtt{x}_1 + \langle 2.9, 0.4 \rangle \mathtt{x}_2$$
$$\text{Subject to } \langle 3.95, 0.55 \rangle \mathtt{x}_1 + \langle 3.025, 0.325 \rangle \mathtt{x}_2 \leq \langle 12.25, 0.85 \rangle$$
$$\langle 1.2, 0.45 \rangle \mathtt{x}_1 + \langle 2.9, 0.2 \rangle \mathtt{x}_2 \leq \langle 6.325, 0.825 \rangle$$
$$\mathtt{x}_1, \mathtt{x}_2 \geq 0$$

By solving through the simplex method, the solution for the above LPP problem in terms of midpoint and width is,

For membership function, the solution is, $\mathtt{x}_1 = \langle 3.1169, 0.6 \rangle, \mathtt{x}_2 = \langle 0, 0 \rangle$ and $\tilde{Z} = \langle 15.974, 0.625 \rangle$

It is then converted into interval and the value we got is, $\mathtt{x}_1$ = [2.5,3.7], $\mathtt{x}_2$ = [0,0] and $\tilde{Z}$ = [15.3,16.65] which satisfies **Theorem 3.2** as ${}^{I}\tilde{z}_j -{}^{I} \tilde{c}_j \geq 0$ and the Intuitionistic basic feasible solution is obtained.

For non-membership function, the solution is, $\mathtt{x}_1 = \langle 3.1013, 0.85 \rangle, \mathtt{x}_2 = \langle 0, 0 \rangle$ and $\tilde{Z} = \langle 15.9715, 0.85 \rangle$.

It is then converted into interval and the value we got is, $\mathtt{x}_1$ =[2.2,3.95], $\mathtt{x}_2$ = [0,0] and $\tilde{Z}$ = [15.15,16.85].

2) RTSM: If α and β are both 0.5, the interval numbers for **MF** are expressed as,

$$\max\ {}^{I}\tilde{z}^{\pm} = [4.5, 5.75]\mathtt{x}_1^{\pm} + [2.75, 3.15]\mathtt{x}_2^{\pm}$$
$$\text{Subject to } [3.65, 4.05]\mathtt{x}_1^{\pm} + [2.75, 3.3]\mathtt{x}_2^{\pm} \leq [11.4, 12.6]$$
$$[0.9, 1.6]\mathtt{x}_1^{\pm} + [2.85, 3.1]\mathtt{x}_2^{\pm} \leq [5.75, 6.85]$$
$$\mathtt{x}_1^{\pm}, \mathtt{x}_2^{\pm} \geq 0$$

The following is an expression for the RTSM method's sub-models:

Sub-model 1:

$$\max\ {}^{I}\tilde{z}^{-} = 4.5\mathtt{x_1}^{-} + 3.15\mathtt{x_2}^{+}$$

$$\text{Subject to } 4.05\mathtt{x_1}^{-} + 2.75\mathtt{x_2}^{+} \leq 11.4$$

$$1.6\mathtt{x_1}^{-} + 2.85\mathtt{x_2}^{+} \leq 5.75$$

$$\mathtt{x_1}^{+}, \mathtt{x_2}^{-} \geq 0$$

The solution of the initial Submodel through RTSM are: $\mathtt{x_1}^{-}_{op} = 2.335, \mathtt{x_2}^{+}_{op} = 0$ and ${}^{I}\tilde{z}^{-}_{op} = 12$.

Sub-model 2:

$$\max\ {}^{I}\tilde{z}^{+} = 5.75\mathtt{x_1}^{+} + 2.75\mathtt{x_2}^{-}$$

$$\text{Subject to } 3.65\mathtt{x_1}^{+} + 3.3\mathtt{x_2}^{-} \leq 12.6$$

$$0.9\mathtt{x_1}^{+} + 3.1\mathtt{x_2}^{-} \leq 6.85$$

$$\mathtt{x_1}^{+} \geq 2.335,\ \mathtt{x_2}^{-} \leq 0$$

$$0.9\mathtt{x_1}^{+} \leq 4.836$$

$$\mathtt{x_1}^{+}, \mathtt{x_2}^{-} \geq 0$$

The solution of the final Submodel through RTSM are: $\mathtt{x_1}^{+}_{opt} = 3.452, \mathtt{x_2}^{-}_{opt} = 0$ and ${}^{I}\tilde{z}^{+}_{opt} = 20$.

The optimal Membership solutions using RTSM for α, β = 0.5 in terms of interval are:
$\mathtt{x_1}^{\pm} = [2.335, 3.452],\ \mathtt{x_2}^{\pm} = 0$ and ${}^{I}\tilde{z}^{\pm} = [12, 20]$.

If α and β are both 0.5, the interval numbers for **non-Membership** are expressed as,

$$\max\ {}^{I}\tilde{z}^{\pm} = [4.5, 5.8]\mathtt{x_1}^{\pm} + [2.5, 3.3]\mathtt{x_2}^{\pm}$$

$$\text{Subject to } [3.4, 4.5]\mathtt{x_1}^{\pm} + [2.7, 3.35]\mathtt{x_2}^{\pm} \leq [11.4, 13.1]$$

$$[0.75, 1.65]\mathtt{x_1}^{\pm} + [2.7, 3.1]\mathtt{x_2}^{\pm} \leq [5.5, 7.15]$$

$$\mathtt{x_1}^{\pm}, \mathtt{x_2}^{\pm} \geq 0$$

Subsequently, the procedure for membership is likewise applied for the non-membership values.

As a result, the optimal non-Membership solutions for α and β are both 0.5 using RTSM are:
$\mathtt{x_1}^{\pm} = [2.07, 3.85],\ \mathtt{x_2}^{\pm} = 0$ and ${}^{I}\tilde{z}^{\pm} = [12, 23]$.

3) SOM-2: The value considered for α and β is 0.5. The problem in terms of interval numbers has already been represented in the previous method (RTSM). The sub-models for SOM-2 is discussed below,

Sub-model 1:

$$\max\ {}^{I}\tilde{z}^{-} = 4.5\mathtt{x_1}^{-} + 3.15\mathtt{x_2}^{+}$$

$$\text{Subject to } 4.05\mathtt{x_1}^{-} + 2.75\mathtt{x_2}^{+} \leq 12.6$$

$$1.6\mathtt{x_1}^{-} + 2.85\mathtt{x_2}^{+} \leq 6.85$$

$$\mathtt{x_1}^{+}, \mathtt{x_2}^{-} \geq 0$$

The solution of the initial Submodel through RTSM are: $\mathtt{x_1}^{-}_{op} = 2.3903, \mathtt{x_2}^{+}_{op} = 1.0616$ and ${}^{I}\tilde{z}^{-}_{op} = 14.2$.

Sub-model 2:

$$\max\ {}^{I}\tilde{z}^{+} = 5.75\mathtt{x_1}^{+} + 2.75\mathtt{x_2}^{-}$$

$$\text{Subject to } 3.65\mathtt{x_1}^{+} + 3.3\mathtt{x_2}^{-} \leq 11.4$$

$$0.9\mathtt{x_1}^{+} + 3.1\mathtt{x_2}^{-} \leq 5.75$$

$$0.9\mathtt{x_1}^{+} \leq 2.3903$$

$$\mathtt{x_1}^{+}, \mathtt{x_2}^{-} \geq 1.0616$$

The solution of the final Submodel through RTSM are: $\mathtt{x_1}^{+}_{opt} = 3.1233, \mathtt{x_2}^{-}_{opt} = 0$ and ${}^{I}\tilde{z}^{+}_{opt} = 17.9$.
The optimal MF solutions using SOM-2 for α, β = 0.5 in terms of interval are: $\mathtt{x_1}^{\pm} = [2.39, 3.123],\ \mathtt{x_2}^{\pm} = [0, 1.06]$ and ${}^{I}\tilde{z}^{\pm} = [14.2, 17.9]$.

Subsequently, the procedure for membership is likewise applied for the non-membership values.
As a result, the optimal non-Membership solutions for α and β are both 0.5 using SOM-2 are:
$\mathtt{x_1}^{\pm} = [2.08, 3.35],\ \mathtt{x_2}^{\pm} = [0, 1.37]$ and ${}^{I}\tilde{z}^{\pm} = [13.9, 19.4]$.

B. Example 2:

Another numerical depiction is presented to solve IFLPP which is extracted from Nachammai et al. [21]. The illustrations helps us to understand the significance of our findings. A toy manufacturing company wishes to maximize profits by determining the number of units required to make two types of toys, Toy A and Toy B. The number of units of Toy A to make each month and the number of units of Toy B to be manufactured each month are represented by the decision variables $\mathtt{x_1}$ and $\mathtt{x_2}$, respectively. Maximizing the monthly profit is the primary goal. Toy A makes Rs. 4 per unit, also Toy B makes Rs. 4 per unit. The total labour hours used in creating Toy A (1.8 hours) and Toy B (3 hours) shouldn't be more than 11 hours per day provided. The total amount of raw material utilised to make Toy A (5.5 kg) and Toy B (4.6 kg) shouldn't go above 12 kg every day. The quantity produced cannot be negative. The toy production in the form of IFLPP is given as,

$$\max\ \widetilde{{}^{I}4}\mathtt{x_1} + \widetilde{{}^{I}4}\mathtt{x_2}$$

$$\text{Subject to } \widetilde{{}^{I}1.8}\mathtt{x_1} + \widetilde{{}^{I}3}\mathtt{x_2} \leq \widetilde{{}^{I}11}$$

$$\widetilde{{}^{I}5.5}\mathtt{x_1} + \widetilde{{}^{I}4.6}\mathtt{x_2} \leq \widetilde{{}^{I}12}$$

$$\mathtt{x_1}, \mathtt{x_2} \geq 0$$

The objective function and constraints are expressed in GTrIFNs in this illustration.

$$\mathtt{c_1} = \widetilde{{}^{I}4} = (3, 4, 5, 5.5; 0.3); (2, 4, 5, 6; 0.6);$$

$$\mathtt{c_2} = \widetilde{{}^{I}4} = (3.5, 4, 4.5, 6; 0.3); (3, 4, 4.5, 7; 0.6)$$

$$\mathtt{a_{11}} = \widetilde{{}^{I}1.8} = (1, 1.8, 2, 3; 0.4); (0.5, 1.8, 2, 4; 0.6);$$

$$\mathtt{a_{12}} = \widetilde{{}^{I}3} = (2, 3, 5, 5.8; 0.4); (1, 3, 5, 6; 0.5)$$

$$\mathtt{a_{21}} = \widetilde{{}^{I}5.5} = (4, 4.5, 5, 8; 0.7); (3, 4.5, 5, 9; 0.2);$$

$$\mathtt{a_{22}} = \widetilde{{}^{I}4.6} = (3.8, 4, 6, 9; 0.7); (3, 4, 6, 10; 0.2)$$

$$\mathtt{b_1} = \widetilde{{}^{I}11} = (10, 11, 13, 15; 0.3); (9, 11, 13, 17; 0.6);$$

$$\mathtt{b_2} = \widetilde{{}^{I}12} = (11, 12, 15, 17; 0.6); (8, 12, 15, 19; 0.3)$$

1) Simplex method: The solution of the given problem by using SM is:
These GTrIFNs are converted into interval numbers using the (α,β)-cut, and the values are tabulated in **Table III**.

TABLE III
(α,β)-CUT OF GTRIFN INTO INTERVAL VALUES

Linear values	Result using (α,β)-cut
c_1	$\left[3+\frac{\alpha}{0.3}, 5.5-\frac{0.5\alpha}{0.3}\right], \left[4-\frac{2\beta}{0.6}, 5+\frac{\beta}{0.6}\right]$
c_2	$\left[3.5+\frac{0.5\alpha}{0.3}, 6-\frac{1.5\alpha}{0.3}\right], \left[4-\frac{\beta}{0.6}, 4.5+\frac{2.5\beta}{0.6}\right]$
a_{11}	$\left[1+\frac{0.8\alpha}{0.4}, 3-\frac{\alpha}{0.4}\right], \left[1.8-\frac{1.3\beta}{0.6}, 2+\frac{2\beta}{0.6}\right]$
a_{12}	$\left[2+\frac{\alpha}{0.4}, 5.8-\frac{0.8\alpha}{0.4}\right], \left[3-\frac{\beta}{0.5}, 5+\frac{\beta}{0.5}\right]$
a_{21}	$\left[4+\frac{0.5\alpha}{0.7}, 8-\frac{3\alpha}{0.7}\right], \left[4.5-\frac{1.5\beta}{0.2}, 5+\frac{4\beta}{0.2}\right]$
a_{22}	$\left[3.8+\frac{0.2\alpha}{0.7}, 9-\frac{3\alpha}{0.7}\right], \left[4-\frac{\beta}{0.2}, 6+\frac{4\beta}{0.2}\right]$
b_1	$\left[10+\frac{\alpha}{0.3}, 15-\frac{2\alpha}{0.3}\right], \left[11-\frac{2\beta}{0.6}, 13+\frac{4\beta}{0.6}\right]$
b_2	$\left[11+\frac{\alpha}{0.6}, 17-\frac{2\alpha}{0.6}\right], \left[12-\frac{4\beta}{0.3}, 15+\frac{4\beta}{0.3}\right]$

To explicitly handle uncertainty, we generate IVIFNs by converting GTrIFNs to IVIFNs by assigning α and β values between 0 and 1.

In terms of midpoint and width, let us characterize all interval parameters as $\tilde{E}=[e_1,e_2]$ and $\tilde{F}=[f_1,f_2]$.

$$\tilde{E}=\langle\tilde{e}_m\rangle,\langle\tilde{e}_w\rangle \ and \ \tilde{F}=\langle\tilde{f}_m\rangle,\langle\tilde{f}_w\rangle$$

When dealing with uncertainty, the midpoint and width representation of interval numbers is beneficial in optimization problems. Furthermore, interval arithmetic with a midpoint improves in capturing imprecision and uncertainty in calculation. After the conversion to midpoint and width, the given problem is displayed in **Table IV** for different values of α,β.

TABLE IV
CONVERSION OF INTERVALS INTO MIDPOINT, WIDTH FOR VARIOUS α AND β VALUES

α,β	c_1	c_2	a_{11}	a_{12}	a_{21}	a_{22}	b_1	b_2
0	$\langle 4,1.3\rangle$	$\langle 5,1.3\rangle$	$\langle 2,1\rangle$	$\langle 4,1.9\rangle$	$\langle 6,2\rangle$	$\langle 6,2.6\rangle$	$\langle 12,2.5\rangle$	$\langle 14,3\rangle$
	$\langle 4,0.5\rangle$	$\langle 4,0.3\rangle$	$\langle 2,0.1\rangle$	$\langle 4,1\rangle$	$\langle 5,0.3\rangle$	$\langle 5,1\rangle$	$\langle 12,1\rangle$	$\langle 13,1.5\rangle$
0.2	$\langle 4.5,0.8\rangle$	$\langle 4,0.6\rangle$	$\langle 2,0.5\rangle$	$\langle 4,1.5\rangle$	$\langle 5.6,1.5\rangle$	$\langle 6,2.1\rangle$	$\langle 12,1.5\rangle$	$\langle 13.8,2.5\rangle$
	$\langle 4.3,1\rangle$	$\langle 4.5,0.8\rangle$	$\langle 2,0.6\rangle$	$\langle 3.8,1.6\rangle$	$\langle 6.5,2.5\rangle$	$\langle 6.5,3.5\rangle$	$\langle 12,1.8\rangle$	$\langle 13.5,4.2\rangle$
0.4	$\langle 4.6,0.2\rangle$	$\langle 4,0.1\rangle$	$\langle 1.9,0.1\rangle$	$\langle 4,1\rangle$	$\langle 5.3,1\rangle$	$\langle 5.6,1.7\rangle$	$\langle 12,0.5\rangle$	$\langle 13.7,2\rangle$
	$\langle 4.2,1.5\rangle$	$\langle 5,1.4\rangle$	$\langle 2,1.2\rangle$	$\langle 3.6,2.2\rangle$	$\langle 8.3,4.8\rangle$	$\langle 8,6\rangle$	$\langle 12.9,3.2\rangle$	$\langle 13.5,7\rangle$
0.6	$\langle 4.8,0.3\rangle$	$\langle 4,0.8\rangle$	$\langle 1.9,0.4\rangle$	$\langle 4,0.5\rangle$	$\langle 4.9,0.5\rangle$	$\langle 5.2,1\rangle$	$\langle 11.5,0.5\rangle$	$\langle 13.5,1\rangle$
	$\langle 4,2\rangle$	$\langle 5,2\rangle$	$\langle 2.3,2\rangle$	$\langle 3.4,2.8\rangle$	$\langle 10,7\rangle$	$\langle 9.5,8.5\rangle$	$\langle 13,4\rangle$	$\langle 13.5,9.5\rangle$
0.8	$\langle 5,0.7\rangle$	$\langle 3.4,1.4\rangle$	$\langle 1.8,1\rangle$	$\langle 4,0.1\rangle$	$\langle 4.6,0\rangle$	$\langle 4.8,1\rangle$	$\langle 11.2,1.5\rangle$	$\langle 13.3,1\rangle$
	$\langle 3.8,2\rangle$	$\langle 5.3,2\rangle$	$\langle 2.4,2.3\rangle$	$\langle 3,3.4\rangle$	$\langle 11.8,9.2\rangle$	$\langle 11,11\rangle$	$\langle 13.2,5\rangle$	$\langle 13.5,12\rangle$
1	$\langle 5,1.2\rangle$	$\langle 3,2.1\rangle$	$\langle 1.8,1.3\rangle$	$\langle 4,0.4\rangle$	$\langle 4.2,0.5\rangle$	$\langle 4.4,0.3\rangle$	$\langle 11,2.5\rangle$	$\langle 13.2,0.5\rangle$
	$\langle 3.7,3\rangle$	$\langle 5,3.2\rangle$	$\langle 2.5,2.9\rangle$	$\langle 3,4\rangle$	$\langle 13.5,11\rangle$	$\langle 12,13\rangle$	$\langle 13.9,6\rangle$	$\langle 13.5,15\rangle$

By following the steps mentioned in illustration 1, the optimal solution of IFLPP is acquired by using SM, RTSM and SOM-2 are discussed in the next section. As ${}^I\tilde{z}_j - {}^I\tilde{c}_j \geq 0$ satisfies **Theorem 3.2**, the Intuitionistic basic feasible solution is obtained.

2) RTSM: The solution of RTSM for example 2 is obtained by considering the interval values acquired using α,β-cut (as it is mentioned in Table III). It also follows the procedures of illustration 1.

3) SOM-2: The solution of SOM-2 for example 2 is also obtained by considering the interval values acquired using α,β-cut (shown in Table III). It also follows the procedures of illustration 1.

VI. RESULTS AND DISCUSSIONS

In example 1, STrIFNs is converted into interval values whereas in example 2, GTrIFNs is converted into intervals rather than crisp conversion. As intervals offer more adaptable manner to convey uncertainty and imprecision, they are frequently preferred. It can be challenging to identify an accurate number for a parameter in our actual world. Because of their adaptability and exactness, intervals are frequently selected over crisp values for portraying uncertainty and imprecision in a variety of applications. Results from SM, RTSM and SOM-2 are presented from 2 illustrations and examined with the previous findings. Lastly, the diagram displays the comparison.

A. Results using Simplex method of example 1

The optimal solution ${}^I\tilde{z}$ obtained using SM is in intervals, which provide more information than crisp numbers while simultaneously increase the complexity of data processing and computations. **Table V** contains the values that have been calculated for various α and β interval parameters.

TABLE V
SOLUTION OF ${}^I\tilde{z}$ FOR VARIOUS VALUES OF α & β OF EXAMPLE 1 USING SM

α	β	Existing (Z^*)	Proposed membership (${}^I\tilde{z}_m$)	Proposed non-membership (${}^I\tilde{z}_{nm}$)
0	0		[14.8,16.9]	[16.15,16.35]
0.1	0.1		[14.9,16.85]	[15.95,16.45]
0.2	0.2		[15,16.8]	[15.75,16.55]
0.3	0.3		[15.1,16.75]	[15.55,16.65]
0.4	0.4		[15.2,16.7]	[15.35,16.75]
0.5	0.5	6.8509	[15.3,16.65]	[15.15,16.85]
0.6	0.6		[15.4,16.6]	[14.95,16.95]
0.7	0.7		[15.5,16.55]	[14.75,17.05]
0.8	0.8		[15.6,16.5]	[14.55,17.15]
0.9	0.9		[15.7,16.45]	[14.35,17.25]
1	1		[15.8,16.4]	[14.15,17.25]

Fig. 6 and 7 compare the derived lower and upper membership and non-membership values of α and β from 0 to 1 to the existing crisp value. By examining the graph, it is evident that the larger α and β value, the greater the profit, i.e., when α and β are from 0 to 1, considering the lower value, the maximum profit attained is Rs. 15.8, whereas the customer's unsatisfactory rate is 14.15. The optimal outcome which is acquired by Chopra [20] is Rs. 6.8509. As a result, employing the simplex method and interval arithmetic yields the best result.

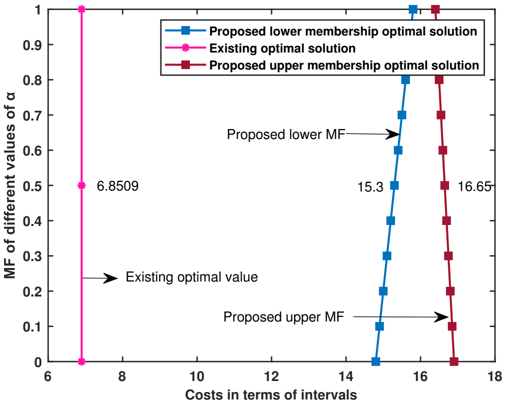

Fig. 6. Comparing MF of α and β from 0 to 1 with existing using SM

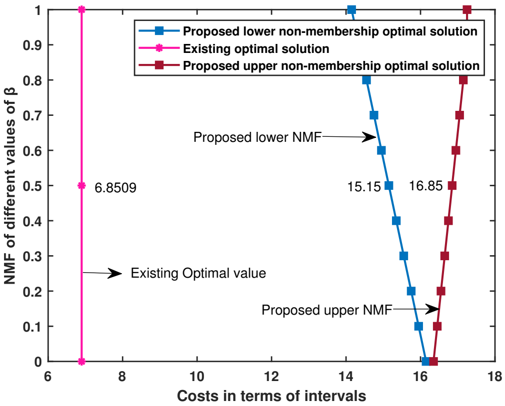

Fig. 7. Comparing NMF of α and β from 0 to 1 with existing using SM

B. Results using Simplex method of example 2

GTrIFNs can be used to represent uncertain parameters with the degrees of membership and non-membership. The SM approach deals with GTrIFNs, allowing the modelling of real-world problems when data is unclear or inaccurate. With the help of interval arithmetic and considering the degrees of MF and NMF associated with GTrIFNs, enabling uncertainty to be handled throughout the optimization process. Intervals are frequently considered to be a more adaptable and practical manner of representing uncertainty or imprecision and the optimal solution ${}^{I}\tilde{z}$ have been calculated for various α and β in terms of intervals by using (α, β) cut. **Table VI** represents the optimal solution of SM in terms of intervals rather than converting into crisp.

TABLE VI
SOLUTION OF ${}^{I}\tilde{z}$ FOR VARIOUS VALUES OF α & β USING SM OF EXAMPLE 2

α	β	Proposed membership (${}^{I}\tilde{z}_m$)	Proposed non-membership (${}^{I}\tilde{z}_{nm}$)
0	0	[7,13]	[11.3,14.3]
0.1	0.1	[7.6,13.1]	[7.9,13.5]
0.2	0.2	[8.5,13.5]	[5.1,13.5]
0.3	0.3	[9.1,13.6]	[3.2,14.2]
0.4	0.4	[9.8,13.8]	[1.2,14.8]
0.5	0.5	[10.6,14.1]	[-0.6,15.8]
0.6	0.6	[11.6,14.6]	[-2.4,16.6]
0.7	0.7	[12.3,14.8]	[-4.1,17.5]
0.8	0.8	[13.3,15.3]	[-5.8,18.6]
0.9	0.9	[14.2,16.2]	[-7.3,19.7]
1	1	[14.6,17.1]	[-8.9,20.7]

For various values of α, β, if α value increases, the profit also increases for toy manufacturing, i.e., the maximum profit is Rs. 14.6 to 17.1 per toy.

C. Results using RTSM of example 1

The RTSM produces an optimal solution ${}^{I}\tilde{z}$ in intervals. **Table VII** shows the computed values for various α and β parameters in intervals.

TABLE VII
SOLUTION OF ${}^{I}\tilde{z}$ FOR VARIOUS VALUES OF α, β USING RTSM OF EXAMPLE 1

α	β	Existing (Z^*)	Proposed MF (${}^{I}\tilde{z}_m$)	Proposed NMF (${}^{I}\tilde{z}_{nm}$)
0	0		[10.8,22.5]	[14.5,18]
0.1	0.1		[11.2,22]	[14,19]
0.2	0.2		[11.6,21.5]	[13.5,20]
0.3	0.3		[12,21]	[13,21]
0.4	0.4		[12.4,20.5]	[12.5,22]
0.5	0.5	6.8509	[12.8,20]	[12,23]
0.6	0.6		[13.2,19.5]	[11.5,24]
0.7	0.7		[13.6,19]	[11,25]
0.8	0.8		[14,18.5]	[10.5,26]
0.9	0.9		[14.5,18]	[10,27]
1	1		[14.8,17.5]	[9.5,28]

Fig. 8 and 9 displays the lower and upper membership and non-membership optimal values for α and β from 0 to 1 and is compared with the existing value. The graph demonstrates that the benefit increases with increasing α, β values; for example, when α and β are between 0 and 1, the maximum profit is Rs. 14.8 and the rejection rate is 9.5. Chopra achieves the outcome, which is Rs. 6.8509. As a result, when compared to the existing value, the RTSM and interval arithmetic produce the best results.

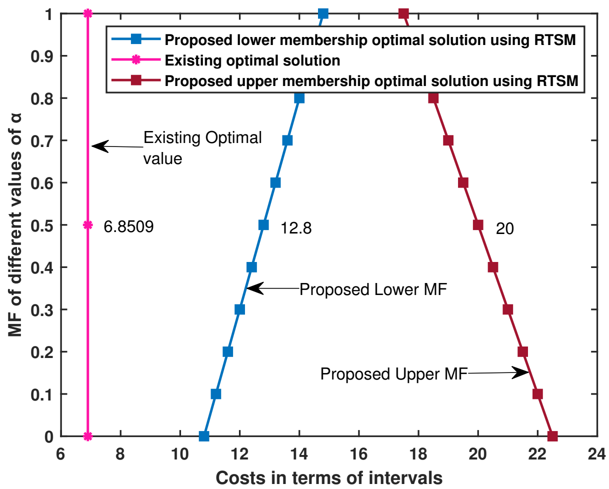

Fig. 8. Comparing MF of α and β from 0 to 1 with existing using RTSM

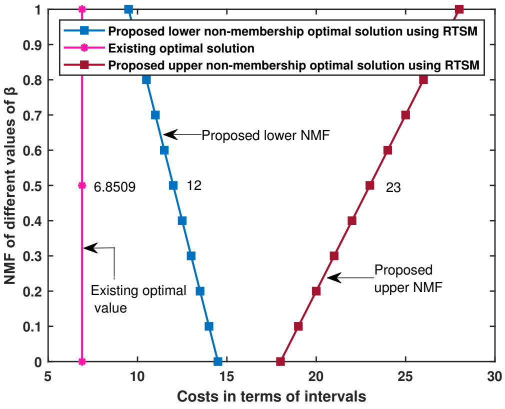

Fig. 9. Comparing NMF of α and β from 0 to 1 with existing using RTSM

D. Results using RTSM of example 2

By using RTSM, the optimal solution ${}^{I}\tilde{z}$ is presented in terms of intervals. **Table VIII** represents the interval values for various parameters of α, β.

TABLE VIII
SOLUTION OF ${}^{I}\tilde{z}$ FOR VARIOUS VALUES OF α & β OF EXAMPLE 2 BY USING RTSM

α	β	Proposed MF (${}^{I}\tilde{z}_{m}$)	Proposed NMF (${}^{I}\tilde{z}_{nm}$)
0	0	[17.4,22.5]	[13.5,16.6]
0.1	0.1	[16,22]	[15,19.8]
0.2	0.2	[15,20]	[16.4,23.5]
0.3	0.3	[13,19]	[18.6,28]
0.4	0.4	[12.6,17.5]	[21,33]
0.5	0.5	[13,16]	[23,39]
0.6	0.6	[13.5,17]	[28,46]
0.7	0.7	[13.5,17]	[40,59]
0.8	0.8	[13,17.7]	[46,64]
0.9	0.9	[12,18]	[55,78]
1	1	[11.5,23]	[66,94]

The solutions are shown for various values of α, β. The maximum profit of toy production is Rs. 11.5 to 23 per toy.

E. Results using SOM-2 of example 1

The SOM-2 produces an optimal solution ${}^{I}\tilde{z}$ in intervals. **Table IX** shows the computed values for various α and β parameters in intervals.

TABLE IX
SOLUTION OF ${}^{I}\tilde{z}$ FOR VARIOUS VALUES OF α, β USING SOM-2 OF EXAMPLE 1

α	β	Existing (Z^*)	Proposed MF (${}^{I}\tilde{z}_{m}$)	Proposed NMF (${}^{I}\tilde{z}_{nm}$)
0	0		[13.2,18.8]	[15.2,17]
0.1	0.1		[13.4,18.6]	[14.8,17.5]
0.2	0.2		[13.6,18.4]	[14.4,17.9]
0.3	0.3		[13.8,18.3]	[14.1,18.4]
0.4	0.4		[14,18.1]	[14,18.9]
0.5	0.5	6.8509	[14.2,17.9]	[13.9,19.4]
0.6	0.6		[14.4,17.7]	[13.8,19.9]
0.7	0.7		[14.6,17.5]	[13.7,20.5]
0.8	0.8		[14.8,17.3]	[13.6,21.1]
0.9	0.9		[15,17.1]	[13.6,21.7]
1	1		[15.2,17]	[13.5,22.3]

Fig. 10 and 11 displays the lower and upper membership and non-membership optimal values for α and β from 0 to 1. The graph demonstrates that the benefit increases with increasing α, β values; for example, when α and β are between 0 and 1, the maximum profit is Rs. 14.8 and the rejection rate is 9.5. Chopra achieves the outcome, which is Rs. 6.8509. As a result, when compared to the existing value, the RTSM and interval arithmetic produce the best results.

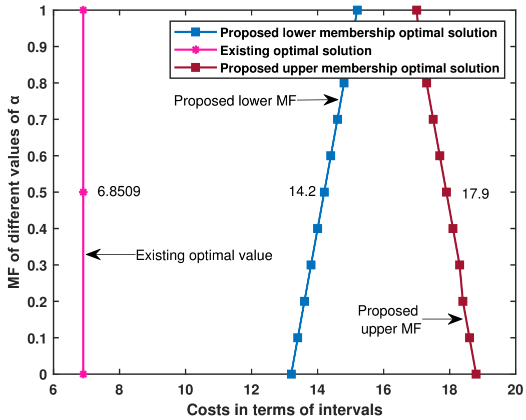

Fig. 10. Comparing MF of α and β from 0 to 1 with existing using SOM-2

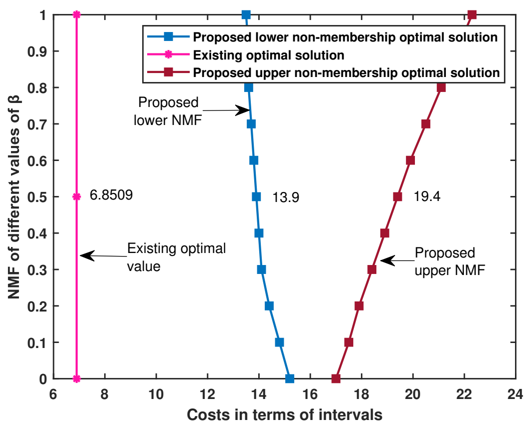

Fig. 11. Comparing NMF of α and β from 0 to 1 with existing using SOM-2

F. Results using SOM-2 of example 2

By using SOM-2, the optimal solution ${}^{I}\tilde{z}$ is presented in terms of intervals. **Table X** represents the interval values for various parameters of α, β.

TABLE X
SOLUTION OF ${}^{I}\tilde{z}$ FOR VARIOUS VALUES OF α & β OF EXAMPLE 2 BY USING RTSM

α	β	Proposed MF (${}^{I}\tilde{z}_{m}$)	Proposed NMF (${}^{I}\tilde{z}_{nm}$)
0	0	[15,27]	[13.3,20.3]
0.1	0.1	[14.6,24]	[13,22.8]
0.2	0.2	[14,21]	[12.4,28.5]
0.3	0.3	[13.7,18.5]	[11.7,36.3]
0.4	0.4	[13,16]	[10.9,45]
0.5	0.5	[12,16]	[9.5,52.8]
0.6	0.6	[13.6,16.1]	[8,66]
0.7	0.7	[14.3,16.2]	[6,83]
0.8	0.8	[15.2,16.3]	[3,100]
0.9	0.9	[16.4,18.3]	[0,131]
1	1	[17.4,22]	[0,174]

The solutions are shown for various values of α, β. The maximum profit of toy production is Rs. 17.4 to 22 per toy.

Table XI provides the optimal solutions taken from [20] in terms of TrIFNs for the value of α and β = 0.5, by substituting the values of the obtained x_1 and x_2 in the given objective function. This is because in the existing method, Chopra et al. demonstrated the value for α = 0.5 and compared the results to the existing optimal solution.

TABLE XI
OPTIMAL SOLUTION IN TERMS OF TRIFNS FOR EXAMPLE 1

Computational methods	Costs in terms of TrIFNs
Existing method for α = 0.5	(13.036, 16.475, 18.1225, 19.77); (13.036, 16.475, 18.1225, 20.0995)
Proposed method using SM for α, β = 0.5	(14.8, 18.5, 20.35, 22.2); (15.6, 19.5, 21.45, 23.79)
Proposed method using RTSM for α, β = 0.5	(13.6, 17, 18.7, 20.4); (15.2, 19, 20.9, 23.2)
Proposed method using SOM-2 for α, β = 0.5	(14.5, 18, 19.6, 21.2); (17, 21.2, 23, 25)

The optimal solution acquired in the original problem taken from [20] for α = 0.5 is obtained as: (13.036, 16.475, 18.1225, 19.77);(13.036, 16.475, 18.1225, 20.0995). Since the result of the existing technique is acquired for α = 0.5, we compare the value for α = 0.5 among all of the possible values. **Fig. 12** displays an analysis graph of the MF with already existing and proposed techniques.

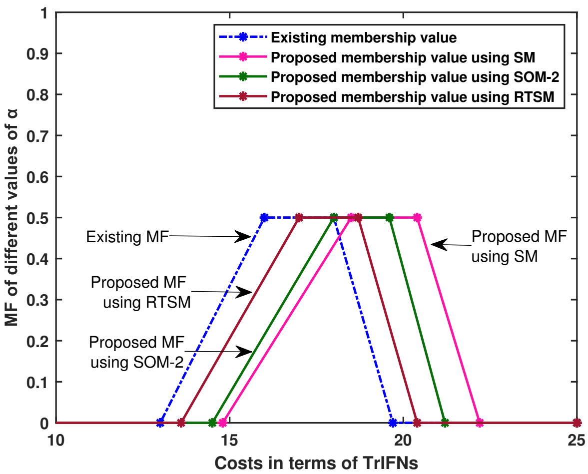

Fig. 12. Comparing Optimal membership solution of proposed approaches in terms of TrIFNs with exising value for example 1

- The maximum profit of MF obtained by using SM, RTSM and SOM-2 in terms of interval are Rs. 15.8, Rs. 14.8 and Rs. 15.2 respectively.
- In terms of TrIFNs, the highest profits for MF using SM, RTSM and SOM-2 are (14.8, 18.5, 20.35, 22.2), (13.6, 17, 18.7, 20.4) and (14.5, 18, 19.6, 21.2), respectively.
- The profit generated by the proposed approaches is significantly higher than that of the current approach.

Table XII shows the optimal result of the proposed method which is taken from [21] in terms of TrIFNs for the value α = 0.3, by substituting the values of the obtained x_1 and x_2 values in the given objective function. This is because, in the existing method, the degree of the membership value in the objective function is 0.3.

TABLE XII
OPTIMAL SOLUTION IN TERMS OF TRIFNS FOR EXAMPLE 2

Computational methods	Costs in terms of TrIFNs
Existing method for MF of 0.3 For NMF of 0.6	(9.9, 24.4, 32.5, 82.5); (4.5, 24.4, 32.5, 204)
Proposed method using SM for α = 0.3 For β = 0.6	(14.4 19 24 26); (38.5, 44, 49.5, 66)
Proposed method using RTSM for α = 0.3 For β = 0.6	(22 27 33 39); (38, 48, 58, 68)
Proposed method using SOM-2 for α = 0.3 For β = 0.6	(33, 39, 45, 57); (31.5, 36, 40.5, 54)

The optimal solution obtained in the original problem retrieved from [21] is as follows: (9.9, 24.4, 32.5, 82.5);(4.5, 24.4, 32.5, 204). Because the existing technique yields the membership degree of 0.3, we compare the value for α = 0.3 to all other potential values. **Fig. 13** depicts a comparison graph of the MF using existing and proposed methodologies.

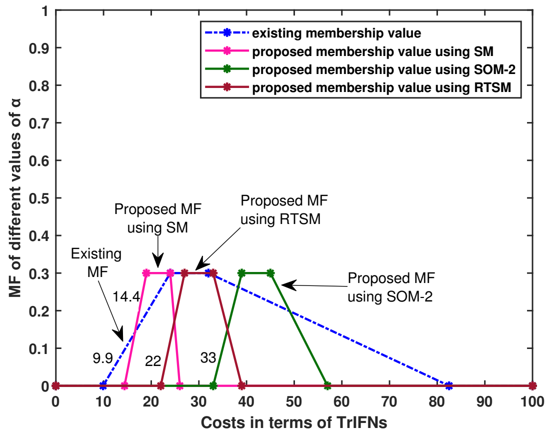

Fig. 13. Comparing Optimal membership solution of proposed approaches in terms of TrIFNs with exising value for example 2

- The graph clearly shows that the lower bound of the proposed methods' results is most significant when compared to the existing result.
- The existing result has a lower value of 9.9, however employing SM, RTSM and SOM-2, the values are 14.4, 22 and 33, respectively.
- So, we can conclude that our suggested approaches are practical means of dealing with uncertainty in real-life circumstances, such as administration and all kinds of production problems.

G. Advantages and Limitation of suggested Approach

- It is important to appropriately design ambiguous variables in decision-making situations because they vary depending on the problem. This paper makes the process easier to do.

- The intuitionistic fuzzy SM, RTSM and SOM-2 are used to handle uncertain parameters and are both easy to understand and apply.
- Like the special case, TrIFLPP is not turned into a crisp problem throughout this research, the optimal solution to the given LPP is achieved as Intervals and TrIFNs.
- Some of the outcomes developed by the RTSM approach may be non-optimal.
- The SOM-2 method's solution space is not absolutely optimal for some cases.

VII. Conclusion

In this paper, the parameters of the LPP is regarded as STrIFNs and GTrIFNs to deal with the unpredictable situations in which the decision makers experience when attempting to anticipate costs in LPP. The three methods SM, RTSM and SOM-2 are used to get the optimal solution, the results of these methods are contrasted with other approaches and it is shown by using two illustrations. The efficient optimal solution using intervals are shown in tables 5-10 and in figures 6-11 and it is achieved by using the arithmetic operations of interval values. For the specified value of α, β, the results using TrIFNs also provided in tables 11 and 12, also figures in 12 and 13. LPP is developed in this work and it may be applied to any actual problem when the parameters are ambiguous and uncertain. As a conclusion, TrIFLPP is evaluated using SM, RTSM and SOM-2, and the optimum values that maximizes the provided objective function is achieved.

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