Minimum ev-Dominating Energy of Semigraph

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***Abstract*—This paper established the idea of minimum ev-dominating matrix of semigraph and calculated its energy. The minimum ev-dominating energy $E_{meD}(G)$ of a semigraph G is the sum of the absolute values of the eigenvalues of the minimum ev-dominating matrix. Here some results are also derived in connection with the energy of minimum ev-dominating matrix. Some lower bounds are also established.**

***Index Terms*—Semigraph, adjacency matrix, minimum ev-dominating matrix, minimum ev-dominating energy.**

I. Introduction

ENERGY of graph conceptualized by I. Gutman [1] and defined as- let G be a graph of order n and A be its adjacency matrix, then energy of graph $E(G)$ is the sum of the absolute values of the eigenvalues of the adjacency matrix of the graph (G). The energy of graph is extensively studied in [2], [3]. Followed to Gutman [1], a number of researchers studied on different types of energies of graph. In this regard, colour energy [4], [5], [6], minimum covering energy [7], distance energy [8], minimum Clique-clique dominating Laplacian energy [9], minimum covering maximum reverse degree energy [10], etc., are reported.

Gupta [11] studied the dominations in graph and stated a number of application applying the concept. Harshita *et. al.,* [12] studied the different domination number of graph and obtained relation between the complements of a graph. Various domination energies in graph were studied substancially in [13], [14].

A. *Minimum Dominating Energy of graph*

Rajesh Kanna *et al.*, [15] introduced the concept of minimum dominating energy of graph and defined as-

Let G be a simple graph of order n with vertex set V and edge set E. Let D be a minimum dominating set of G. The minimum dominating matrix of G is the $n \times n$ matrix, defined by $A_D(G) := (a_{ij})$, Where,

$$a_{ij} = \begin{cases} 1 & \text{if} \quad v_i, v_j \in E \\ 1 & \text{if} \quad i = j \text{ and } \quad v_i \in D \\ 0 & \text{otherwise} \end{cases}$$

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The minimum dominating energy $E_D(G)$ is the sum of the absolute values of the eigenvalues of the adjacency matrix of the dominating set D of graph G and is defined as

$$E_D(G) := \sum_{i=1}^{n} |\lambda_i|$$

Where, $\lambda_i = \lambda_1, \lambda_2, \ldots .., \lambda_n$ are the minimum dominating eigenvalues of the minimum dominating matrix $A_D(G)$ of the graph G.

B. *Semigraph*

Semigraph, a new approach to graph theory introduced by Sampathkumar [16] is a generalization of graph. A semigraph $G(V, X)$ is an ordered pair of two sets V and X, where V is a non-empty set of the elements, which are called vertices of G and X is the set of n-tuples, which are called edges of distinct vertices of G satisfying the following conditions:

SG-1 Any two edges have at least one vertex in common.

SG-2 Two edges $(u_1, u_2, ..., u_n)$ and $(v_1, v_2, ..., v_m)$ are considered to be equal if and only if-

i) $m = n$, and

ii) either, $u_i = v_i$ for $1 \leq i \leq n$, or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

Accordingly, the edges $(u_1, u_2, ..., u_n)$ and $(u_n, u_{n-1}, ..., u_1)$ are same. For an edge $e = (u_1, u_2, ..., u_n)$, u_1 and u_n are the end vertices and $u_2, u_3, ..., u_{n-1}$ are the $middle$ vertices [16], [17], [18].

*1) **Representation of semigraph**:* [19] The geometrical representation of semigraph on plane is as follows-

i) Dots for end vertices and circles for middle vertices on an edge.

ii) Edge of a semigraph is the curve passing through all the distinct vertices (dots/circles) contained on the edge.

iii) When middle vertex of an edge is the end vertex of an other edge then small tangent is drawn to the circle representing the middle vertex to specify it as the end vertex of the other edge.

*2) **Cardinality of semigraph**:* [20] Cardinality is the number of vertices present in an edge e_i of a semigraph G, and is represented by k.

*3) **Adjacency in semigraph**:* [21], [22] Any two vertices u_i and u_j in a semigraph is said to be

i) adjacent if they are belongs to the same edge.

ii) consecutively adjacent if they are adjacent and consecutive in order as well.

iii) e-adjacent if they are the end vertices of the edge.

*4) **Adjacency matrix of a semigraph**:* The adjacency matrix $A(G)$ of a semigraph G of order n is a square matrix whose elements are determined by [20]. Let a semigraph $G = (V, E)$ with $V = \{v_1, v_2, ..., v_k\}$ be the vertex set and $E = \{e_1, e_2, ..., e_n\}$ be the edge set, then the adjacency matrix is $A(G) = (a_{ij})_{n\times n}$. Where-

$$a_{ij} = \begin{cases} 1 & \text{if } v_i,\ v_j \text{ belong to the edge} \\ 0 & \text{otherwise} \end{cases}$$

*5) **Energy of Semigraph**:* Gaidhani *et al.*, [17] introduced the concept of energy of semigraph in two ways viz. matrix energy (E_m) as the sum of the singular values of the adjacency matrix and polynomial energy (E_{re}) as the characteristics energy of polynomial of the adjacency matrix of the semigraph. Accordingly, Nath *et al.*, [23] introduced the distance energy of semigraph, Murugesan and Narmatha [22] introduces e and n energy of semigraph and Nandi *et al.*, [24] introduces minimum covering energy of semigraph. Nandi *et al.*, [25] also studied on the color matrix and energy of semigraph.

*6) **Matrix energy of a semigraph**:* [17] If $\sigma_1,\ \sigma_2, ...,\ \sigma_n$ are the singular values of adjacency matrix $A(G)$ of a semigraph G, then the matrix energy of semigraph is denoted by $E_m(G)$, is defined as the sum of the singular values, *i.e.*,

$$E_m(G) = \sum_{i=1}^{n} \sigma_i$$

*7) **Polynomial energy of semigraph**:* [17] Let G be a semigraph of n vertices with eigenvalues $\lambda_1, \lambda_2, ..., \lambda_n$. The polynomial energy of semigraph denoted by $E_{re}(G)$, is defined as

$$E_{re}(G) = \sum_{i=1}^{n} |Re\,(\lambda_i)|$$

*8) **Domination in semigraph**:* Kamath and Bhat [21] introduced the domination in semigraph and defined as-

Let G be a semigraph and V_e be the set of all end vertices of G. Then,

i) A dominating set $D \subseteq V$ is called adjacency dominating set (ad-set) if for every $v \in V{-}D$ there exist a $u \in D$ such that u is adjacent to v in G.
ii) A dominating set $D \subseteq V_e$ is an end vertex adjacency dominating set (ead-set) if
   (a) D is an ad-set and
   (b) Every vertex $v \in V{-}D$ is e-adjacent to some of the vertex $u \in D$ in G.

Followed to Kamath and Bhat [21], Gomathi *et al.*, [26] introduced the *(m,e)*-domination in semigraph. Biradar *et al.*, [27] stated the application of domination in semigraph in the study of traffic routing and traffic density at junctions of road networks. Praba *et al.*, [28], [29] examine semigraph based edge domination in wireless sensore networks.

In the present study a new matrix of semigraph called minimum ev-dominating matrix is introduced. Further, the singular values of the minimum ev-dominating matrices are determined and minimum ev-dominating energies are calculated.

II. Main Results

A. *Minimum ev-dominating energy of semigraph*

Let $G\,(V, X)$ be a semigraph with $|V| = n$ and $|X| = m$ and D be an ev-dominating set. Then the minimum ev-dominating matrix of G, and defined as $A_{meD}(G) = (a_{ij})$ of order n, where,

i) For every edge e_i of X of cardinality say k, let $e_i = (i_1, i_2, i_3, \ldots, i_k)$ such that $e_i = i_1, i_2, i_3, \ldots, i_k$ are distinct vertices in V, for all $i_r \in e_i$, $r = 1, 2, \ldots, k$
   - $a_{i_1 i_r} = r - 1$
   - $a_{i_k i_r} = k - r$
ii) $a_{ij} = 1$ if i = j and $V_j \in D$, where D is ev-dominating set.
iii) All the remaining entries of $A_{meD}(G)$ are zero.

Example: Let us consider the following semigraph **Figure 1**: $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $E = \{e_1 = (v_1, v_2, v_3, v_4), e_2 = (v_4, v_6), e_3 = (v_2, v_5, v_6)\}$. Here we get two minimum ev-domination set, $D_1 = \{v_2, v_4\}$ and $D_2 = \{v_4, v_6\}$.

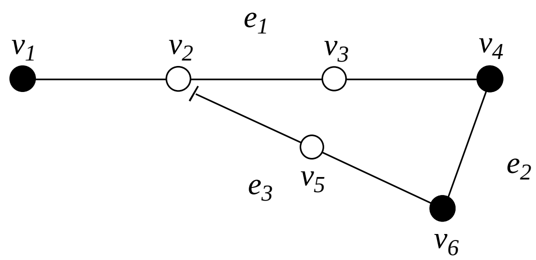

Figure 1. Semigraph

Corresponding to D_1, the $A_{meD_1}(G)$ is given by

$$A_{meD_1}(G) = \begin{pmatrix} 0 & 1 & 2 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Characteristic equation of the matrix $A_{meD_1}(G)A'_{meD_1}(G)$ is $x^6 - 42x^5 + 495x^4 - 2074x^3 + 2556\lambda^2 = 0$. The eigenvalues are: $x_1 = x_2 = 0, x_4 = 5.280, x_5 = 8.832$ and $x_6 = 25.760$. The singular values are: $\lambda_3 = \sqrt{x_3} \approx 1.458, \lambda_4 = \sqrt{x_4} \approx 2.297, \lambda_5 = \sqrt{x_5} \approx 2.971$ and $\lambda_6 = \sqrt{x_6} \approx 5.075$.

And for D_2, the $A_{meD_2}(G)$ is given by

$$A_{meD_2}(G) = \begin{pmatrix} 0 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Characteristic equation of the matrix $A_{meD_2}(G)A'_{meD_2}(G)$ is similar above.

Thus for D_1,

$$A_{meD_1}(G)A'_{meD_1}(G) = \begin{pmatrix} 14 & 1 & 0 & 7 & 0 & 5 \\ 1 & 6 & 0 & 4 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 4 & 0 & 16 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 5 & 0 & 6 \end{pmatrix}$$

The eigen values are:

$\lambda_1 = 0$ with multiplicity of 2.
$\lambda_2 = 0$ with multiplicity of 2.
$\lambda_3 \cong 2.127$
$\lambda_4 \cong 5.280$
$\lambda_5 \cong 8.831$
$\lambda_6 \cong 25.760$

Thus, minimum ev-dominating energy for the minimum ev-dominating set D_1 is-

$$\therefore E_{meD_1}(G) = 11.801$$
$$\sum \lambda_i = 41.992 \cong 42$$

And for D_2

$$A_{meD_2}(G)A'_{meD_2}(G) = \begin{pmatrix} 14 & 0 & 0 & 7 & 0 & 5 \\ 0 & 5 & 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 2 & 0 & 16 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 6 & 0 & 7 \end{pmatrix}$$

The eigen values are:

$\lambda_1 = 0$ with multiplicity of 2.
$\lambda_2 = 0$ with multiplicity of 2.
$\lambda_3 = 1.886$
$\lambda_4 = 5.845$
$\lambda_5 = 8.557$
$\lambda_6 = 25.710$

Thus, minimum ev-dominating energy for the minimum ev-dominating set D_2 is-

$$\therefore E_{meD_2}(G) = 11.785$$
$$\sum \lambda_i = 41.998 \cong 42$$

The minimum ev-dominating energy of D_1=11.801 and of D_2=11.785. Thus, from the above example it can be state that minimum ev-dominating set effects the energy of an element of a semigraph.

B. *Theorems related to minimum ev-dominating energy of semigraph:*

Theorem 1. *Let G be a semigraph and $A_{meD}(G)$ be its minimum ev-dominating matrix. If $\lambda_1, \lambda_2, ..., \lambda_n$ are the eigen values of $A_{meD}(G)A'_{meD}(G)$, then,*

$$\sum_{i=1}^{n} \lambda_i = 2\sum_{e\in X}(1^2 + 2^2 + \cdots + k_e^2) + |D|$$

Where an edge $e \in X$ of G has the cardinality $k_e + 1$ and $k_e \geq 1$.

Proof: In the matrix $A_{meD}(G)$, a sequence $1, 2, \ldots.., k_e$ will appear in the rows corresponding to the end vertex of every edge $e \in E$, with cardinality $k_e + 1$ and $k_e \geq 1$. Also, in the diagonal of $A_{meD}(G)$ there exist $|D|$ no. of $1's$, where D is the minimum ev-dominating set in G. So, every edge contributes $2\sum_e (1^2 + 2^2 + \cdots + k_e^2)$ and the diagonal elements contributes $|D| \times 1^2$ in the trace of $A_{meD}(G)A'_{meD}(G)$.

Therefore,

$$trace\ (A_{meD}A'_{meD}) = 2\sum_{e\in X}\left(1^2 + 2^2 + \cdots + k_e^2\right) + |D| \times 1^2$$

Hence,

$$\sum_{i=1}^{n} \lambda_i = 2\sum_{e\in X}(1^2 + 2^2 + \cdots + k_e^2)\ + |D|$$

■

Theorem 2. *The minimum ev-dominating energy $E_{meD}(G)$ of a semigraph G is a square root of an even or odd integer accordingly as $|D|$ is even or odd.*

Proof: If $\sigma_1, \sigma_2, ..., \sigma_n$ be the singular values of minimum ev-dominating matrix $A_{meD}(G)$ of the semigraph G, then

$$(\sigma_1 + \sigma_2 + \ldots + \sigma_2)^2 = \sum_{i=1}^{n} {\sigma_i}^2 + \sum_{i<j} \sigma_i\sigma_j$$

Thus,

$$[E_{meD}(G)]^2 = \sum_{i=1}^{n} \lambda_i + 2\sum_{i<j} \sigma_i\sigma_j$$
$$= 2\sum_{e\in X}\left(1^2 + 2^2 + \ldots + {k_e}^2\right) + |D| + 2\sum_{i<j}\sigma_i\sigma_j$$
$$= 2\left[\sum_{e\in X}\left(1^2 + 2^2 + \ldots + {k_e}^2\right) + \sum_{i<j}\sigma_i\sigma_j\right] + |D|$$
$$\therefore E_{meD}(G) =$$
$$\sqrt{2\left[\sum_{e\in X}\left(1^2 + 2^2 + \ldots + {k_e}^2\right) + \sum_{i<j}\sigma_i\sigma_j\right] + |D|}$$

Thus, the minimum ev-dominating energy $E_{meD}(G)$ of a semigraph G is a square root of an even or odd integer accordingly as $|D|$ is even or odd. ■

Theorem 3. *The minimum ev-dominating energy $E_{meD}(G)$ of a semigraph G is*

$$[E_{meD}(G)]^2 = |D|(mod2)$$

Proof: Following Theorem 2, the minimum ev-dominating energy $E_{meD}(G)$ of a semigraph G is a square root of an even or odd integer accordingly as $|D|$ is even or odd. i.e.

$$E_{meD}\,(G) = \sqrt{2k + |D|}$$

where k is a positive integer.

$$[E_{meD}(G)]^2 = 2k + |D|$$

Thus,

$$[E_{meD}(G)]^2 = |D|\,(mod 2)$$

■

C. Some bounds on minimum ev-dominating energy of semigraph

Theorem 4. *For a semigraph G of n vertices and m edges,*

$$\sqrt{2\sum_{e\in X}\left(1^2+2^2+\cdots+{k_2}^2\right)+|D|} \leq E_{meD}(G) \leq \sqrt{n\left[2\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)+|D|\right]}$$

Proof: Let $\sigma_i,\ i=1,2,\ldots,n$ be the singular values of A_{meD}, and $\lambda_i,\ i=1,2,\ldots,n$ be the eigenvalues of $A_{meD}(G)A'_{meD}(G)$. By Cauchy-Schwarz's inequality on two vectors $(\sigma_1,\ \sigma_2,\ldots,\sigma_n)$ and $(1,1,\ldots,1)$, we have

$$(\sigma_1+\sigma_2+\ldots+\sigma_n)^2 \leq n\sum_{i=1}^{n}\sigma_i^2 = n\sum_{i=1}^{n}\lambda_i$$

Thus,

$$[E_{meD}(G)]^2 \leq n\left[2\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)+|D|\right]$$

Again we have

$$[E_{meD}(G)]^2 = \left(\sum_{i=1}^{n}\sigma_i\right)^2 \geq \sum_{i=1}^{n}\sigma_i^2 = \sum_{i=1}^{n}\lambda_i$$

Thus,

$$[E_{meD}(G)]^2 \geq 2\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)+|D|$$

Hence,

$$\sqrt{2\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)+|D|} \leq E_{meD}(G) \leq \sqrt{n\left[2\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)+|D|\right]}$$

■

Theorem 5. *If G be a semigraph having n vertices and m edges, then*

$$[E_{meD}(G)]^2 \geq 2\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)+|D|+ n(n+1)\Delta^{\frac{1}{n}}$$

Where, $\triangle$=*det*($A_{meD}A'_{meD}$)

Proof: Let $\sigma_i,\ i=1,2,\ldots,n$ be the singular values of $A_{meD}(G)$, then we have

$$[E_{meD}(G)]^2 = \left(\sum_{i=1}^{n}\sigma_i\right)^2 = \sum_{i=1}^{n}\sigma_i^2 + 2\sum_{i<j}\sigma_i\sigma_j = \sum_{i=1}^{n}\lambda_i + \sum_{i\neq j}\sigma_i\sigma_j$$

As $\sigma_i,\ i=1,2,\ldots,n$ are non-negative numbers, $\sigma_i\sigma_j$ are also non-negative numbers. Therefore, applying $AM > GM$ on $n(n-1)$ nos. of non-negative numbers $\sigma_i\sigma_j$.
We have

$$\frac{1}{n(n-1)}\sum_{i\neq j}\sigma_i\sigma_j \geq \left(\prod_{i\neq j}\sigma_i\sigma_j\right)^{\frac{1}{n(n-1)}} = \left(\prod_{i=1}^{n}{\sigma_i}^{2(n-1)}\right)^{\frac{1}{n(n-1)}}$$

i.e.

$$\sum_{i\neq j}\sigma_i\sigma_j \geq n\,(n-1)\left(\prod_{i=1}^{n}\lambda_i^{n-1}\right)^{\frac{1}{n(n-1)}} = n(n-1)\left(\prod_{i=1}^{n}\lambda_i\right)^{\frac{1}{n}}$$

Thus,

$$\sum_{i\neq j}\sigma_i\sigma_j \geq n(n-1)\Delta^{\frac{1}{n}}$$

Where,

$$\triangle = \left(\prod_{i=1}^{n} = det(A_{meD}A^{/}_{meD})\right)$$

Therefore, we get

$$[E_{meD}(G)]^2 \geq \sum_{i=1}^{n}\lambda_i + n(n-1)\triangle^{\frac{1}{n}}$$

■

By **Theorem 1** we can get

$$[E_{meD}(G)]^2 \geq 2\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)+|D|+ n(n-1)\triangle^{\frac{1}{n}}$$

Lemma 6. *[22] If $A=[a_{ij}]$ is any non-constant matrix and its norm defined as*

$$||A||_2 = \sqrt{\sum_{ij}a_{ij}^2}$$

Suppose $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n$ are singular values of A, then

$$E(A) \geq \sigma_1 + \frac{||A||_2^2 - \sigma_1^2}{\sigma_2}$$

Thus, we evaluate a lower bound for $E_{meD}(G)$.

Theorem 7. *For a semigraph G of n vertices, if σ_1 and σ_2 are respectively largest and second largest singular values of its minimum ev-dominating matrix $A_{meD}(G)$. Then we have*

$$[E_{meD}(G)] \geq \sigma_1 + \frac{2\sum_{e\in X}(1^2+2^2+...+k^2)+|D|-\sigma_1^2}{\sigma_2}$$

Proof: By **Lemma 6** for the minimum ev-dominating matrix $A_{meD}(G)$ of G, we have

$$E_{meD}(G) \geq \sigma_1 + \frac{||A_{meD}||_2^2 - \sigma_1^2}{\sigma_2}$$

Clearly, from definition of norms of a matrix we ahve

$$||A_{meD}(G)||_2^2 = trace(A_{meD}(G)A'_{med}(G)) = 2\sum_{e\in X}(1^2+2^2+...k_e^2)+|D|-\sigma_1^2$$

$$E_{meD}(G) \geq \sigma_1 + \frac{2\sum_{e\in X}(1^2+2^2+...k_e^2)+|D|-\sigma_1^2}{\sigma_2}$$

■

Which gives another lower bound of $E_{meD}(G)$.

D. Relation between minimum ev-dominating energy and energy of semigraphs:

Theorem 8. *Let $G(V,X)$ be a semigraph of vertices n and edges m, then*

$$E_{meD}(G) \geq \frac{E(G)}{\sqrt{n}}$$

Where, $E(G)$ is the energy of the semigraph G.

Proof: If $G(V,X)$ be a semigraph of vertices n and edges m, then by Theorem 2 of [23] we have

$$\sqrt{2\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)} \leq E(G) \leq \sqrt{2n\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)}$$

$$2\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right) \leq [E(G)]^2 \leq 2n\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)$$

Thus,

$$[E(G)]^2 \leq 2n\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)$$

Therefore,

$$\frac{[E(G)]^2}{n} \leq 2\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)$$

If $E_{meD}(G)$ be the minimum ev-dominating energy of a semigraph $G(V,X)$, by **Theorem 5** we get

$$[E_{meD}(G)]^2 \geq 2\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)+|D| + n(n+1)\Delta^{\frac{1}{n}}$$

i.e.

$$[E_{meD}(G)]^2 \geq 2\sum_{e\in X}\left(1^2+2^2+\ldots+{k_e}^2\right)$$

Thus

$$[E_{meD}(G)]^2 \geq \frac{[E(G)]^2}{n}$$

Hence

$$E_{meD}(G) \geq \frac{E(G)}{\sqrt{n}}$$

■

Theorem 9. *For a semigraph $G(V,X)$ of vertices n and edges m, if σ_1 and σ_2 are respectively largest and second largset singular values of its minimum ev-dominating matrix $A_{med}(G)$. Then we have*

$$nE_{med}(G) \geq \frac{[E(G)]^2 - n\sigma_1^2}{\sigma_2}$$

Where, E(G) is the energy of semigraph.

Proof: If $G(V,X)$ be a semigraph of vertices n and edges m, then Theorem 2 of [23] we have

$$\sqrt{2\sum_{e\in X}(1^2+2^2+...+k_e^2)} \leq E(G) \leq \sqrt{2n\sum_{e\in X}(1^2+2^2+...+k_e^2)}$$

Thus,

$$[E(G)]^2 \leq 2n\sum_{e\in X}(1^2+2^2+...+k_e^2)$$

By **Theorem 8** we have,

$$E_{meD}(G) \geq \sigma_1 + \frac{2\sum_{e\in X}(1^2+2^2+,,,+k_e^2)+|D|-\sigma_1^2}{\sigma_2}$$

Thus,

$$\sigma_2 E_{meD}(G) - \sigma_1\sigma_2 + \sigma_1^2 \geq 2\sum_{e\in X}(1^1+2^2+...+k_e^2)+|D|$$

i.e.,

$$\sigma_2 E_{meD}(G) - \sigma_1\sigma_2 + \sigma_1^2 \geq 2\sum_{e\in X}(1^2+2^2+...+k_e^2)$$

i.e.,

$$n(\sigma_2 E_{meD}(G) - \sigma_1\sigma_2 + \sigma_1^2) \geq 2n\sum_{e\in X}(1^2+2^2+...=k_e^2)$$

i.e.,

$$n(\sigma_2 E_{meD}(G) - \sigma_1\sigma_2 + \sigma_1^2) \geq [E(G)]^2$$

i.e.,

$$nE_{meD}(G) \geq \frac{[E(G)]^2}{\sigma_2} - n\frac{\sigma_1^2}{\sigma_2} + n\sigma_1$$

i.e.,

$$nE_{meD}(G) \geq \frac{[E(G)]^2}{\sigma_2} - n\frac{\sigma_1^2}{\sigma_2}$$

Hence,

$$nE_{meD}(G) \geq \frac{[E(G)]^2 - n\sigma_1^2}{\sigma_2}$$

■

III. Observation

It is observed that minimum ev-dominating set governs the energy of a semigraph. If a semigraph G contains more than one ev-dominating set as such in the example taken in the present study (**Figure** 1: D_1 and D_2), then the ev-dominating set possessing minimum common dominating vertices is found to possess minimum energy.

In the minimum ev-dominating set $D_1 = \{v_4, v_2\}$ in the semigraph (Fig. 1), v_4 dominating the vertices $\{v_3,\ v_2,\ v_1,\ v_6\}$ and v_2 dominating the vertices $\{v_1,\ v_3,\ v_4,\ v_5,\ v_6\}$. It is found that common vertices dominated by v_4 and v_2 in the ev-dominating set $D_1 = \{v_4, v_2\}$ are $\{v_1,\ v_3,\ v_6\}$ with $E_{meD_1} = 11.801$.

Whereas, in the minimum ev-dominating set $D_2 = \{v_4, v_6\}$ in the semigraph (Fig. 1), v_4 dominating the vertices $\{v_3,\ v_2,\ \ v_1,\ v_6\}$ and v_6 dominating the vertices $\{v_4,\ v_5,\ v_2\}$. It is found that common vertices dominated by v_4 and v_6 in the ev-dominating set $D_2 = \{v_4, v_6\}$ is v_2 with $E_{meD_2} = 11.785$.

Dominating set D_1 with three common vertices possesses higher minimum ev-dominating energy compared to D_2 with one common vertax possessing lower minimum ev-dominating energy.

IV. Conclusion

The present study concludes that minimum ev-dominating set reign over the energy of the elements of a semigraph. If a semigraph have more than one ev-dominating sets, then the ev-dominating set possessing minimum common dominating vertices shows minimum energy. Thus, the present study of minimum ev-dominating matrix and energy on semigraph model is very fascinating and exhibits emerging area of research which can address a wide range of real world problem in near future.

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