Path Optimization of Green Multimodal Transportation Considering Dynamic Random Transit Time

Yuzhao Zhang, Muchen Ye, Luyuan Deng, Jianling Yang, Yueqi Hu

***Abstract*— This paper focuses on optimizing the green multimodal transportation path under the condition of transit time subjected to random distribution with dynamic changes. The study utilizes the transition matrix of a homogeneous Markov chain and the theory of time-space network to simulate the transit time subjected to random distribution with dynamic changes. A time-space network path optimization model is established and solved using a commercial solver. The solutions are compared under different transit time scenarios and carbon tax rates. It is observed that the reliability of the same solution decreases in a transportation environment with higher randomness. Additionally, new solutions that did not appear in a single situation are generated under dynamic changes in transit time. As the carbon tax rate increases, the transportation strategy gradually shifts away from road transportation. Furthermore, the time-space network model may yield better solutions compared to the physical network model. The choice of different types of distributions to simulate transit times also influences the solution results.**

***Index Terms*— Multimodal transport, Route optimization, Time-space network, Time uncertainty, Carbon emission**

I. Introduction

Reference [1] indicates that subsequent to the commencement of the 13th Five-Year Plan, China's modern logistics system has undergone effective modifications in its transportation structure. A notable upsurge in the railway freight volume has been observed, characterized by an average annual growth rate surpassing 20% in the multimodal transport freight sector. To catalyze the progression and transformation of multimodal transportation, continued efforts in restructuring transportation are essential. This strategy should aim to capitalize on the unique benefits of various transportation modes and enhance the role of railway and waterway transport, aligning with policy goals focused on reducing structural expenses and advancing sustainable logistics. Nonetheless, the multimodal transport framework in China requires further refinement. Areas demanding attention include the improvement of mode conversion efficiency, operations segmentation, and the standardization of transport units, in conjunction with efforts to curtail the aggregate cost of the transport chain. Considering the array of uncertainties in the real-world transportation landscape, integrating these variables into decision-making models for multimodal transport plans is pivotal. Recent years have witnessed significant research interest in this domain from both national and international academics.

The simulation of transportation speed uncertainty is approached in [2] through the application of various probability distribution combinations. Correspondingly, studies in [3] and [4] have focused on the simulation of transportation time uncertainty. The aspect of transportation demand uncertainty has been explored in [5], [6], and [7], whereas [8] addresses the unpredictability of shipping arrivals. A more holistic approach is evident in [9], which examines an array of uncertain elements including material demand, demand priority, and traffic conditions within road networks, particularly in the context of emergency material multimodal transport deployment strategies. The study in [10] presents an analysis of the dual uncertainties inherent in transportation demand and carbon trading prices. Additionally, [11] considers both transportation time and unit freight rate uncertainties. The model developed in [12] for the reliable path optimization in emergency materials multimodal transport, integrates a comprehensive set of uncertainties encompassing demand, transportation environment, node epidemic infection risks, cost, scheduling, and capacity constraints. Further, [13] introduces concepts of fuzzy delivery times and stochastic cargo loss rates. In a similar vein, [14] addresses the uncertainties related to freight volumes, costs, timeframes, and carbon emissions. To manage these uncertainties, the application of stochastic programming and robust optimization theories is prevalent. Researchers frequently resort to commercial solvers for

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linear programming model resolutions or devise enhanced heuristic algorithms in scenarios where linear programming proves inapplicable. Reflecting the current focus on sustainability, numerous studies have expanded their objective functions to include carbon emission considerations, complementing the traditional focus on transportation costs and duration, and aligning with the broader agenda of promoting green, low-carbon multimodal transport alongside energy conservation and emission reduction policies.

The operational efficacy of multimodal transport systems is contingent upon the synergistic integration of diverse transportation modes at each transfer junction. This critical interplay is predominantly manifested through the transfer times in multimodal transport decision-making models. Acknowledging the inherent uncertainties in transit times, study [15] delves into a triad of uncertain elements: transportation time, transit time, and consumer demand. Building upon this, study [16] introduces an additional layer of complexity by incorporating the uncertainty of transit freight volume. Further extending this framework, [17] examines the uncertainties associated with highway transportation speed and transit time, making adjustments in response to various carbon policy scenarios. Furthermore, studies [18] and [19] integrate transportation and transit time uncertainties within the opportunity constraint model of multimodal transport. The methodologies predominantly employed in these studies encompass the use of probability distributions, scenario-based approaches, or robust interval techniques to effectively characterize the uncertainty associated with transit times.

It is critical to recognize that the uncertainty inherent in transit times is not static but rather varies dynamically over time. The parameters characterizing this uncertainty are, therefore, subject to temporal fluctuations. Consequently, the application of static probability distributions, scenario methods, or robust intervals may fall short in accurately representing the dynamic nature of transit time changes. In response to this challenge, we propose a multi-modal transportation time-space network path optimization model. This innovative model employs the transition matrix of a time-homogeneous Markov chain, coupled with stochastic simulation technology, to effectively model the time-space network while capturing the dynamic shifts in transit times. The resolution of this complex problem is facilitated through the use of a commercial solver, which not only yields a dependable transportation decision-making framework but also provides a valuable benchmark for the development of real-world transportation strategies.

II. Modal Building

A. Problem Description

The issue of path selection within a multimodal transport time-space network, particularly under the influence of dynamic transit time fluctuations, is conceptualized as follows: this network is constructed on the basis of established physical transport network parameters, complemented by a discrete-time framework. Within this network, each node symbolizes a unique state of a physical transport hub as defined in the discrete-time context. Concurrently, each time-space arc encapsulates the transition of physical paths or node transit processes across various transportation modes, again within the discrete-time framework. The transportation task is characterized by specific origin nodes, designated arrival node windows, and defined freight volumes. The execution of this task entails traversing through these space-time arcs to various nodes, necessitating adherence to the dynamic and uncertain intervals of transportation time. Furthermore, it requires the strategic coordination of multiple space-time arc traversals to optimize the overall transportation costs.

Fig.1 presents a dual representation of a multimodal transport system. On the left, a physical transportation network is depicted, comprising eight transfer hubs. The network supports three transportation modalities – highway, railway, and waterway – for connecting adjacent hubs, as indicated by directional arrows. This side of the figure exemplifies the route and transportation modality for a specific task, denoted as A-railway-B-waterway-E-highway-H. The right side of the figure showcases the corresponding space-time transportation network, with directed arrows illustrating the various space-time arcs utilized by goods within this network. The route and mode for this transportation task are more intricately detailed as: (A, T1) via Railway to (B, T3), followed by a waiting period at (B, T4), then via Waterway to (E, T6), another waiting period at (E, T7), and finally via Highway to (H, T8).

The problem is structured under the following assumptions:

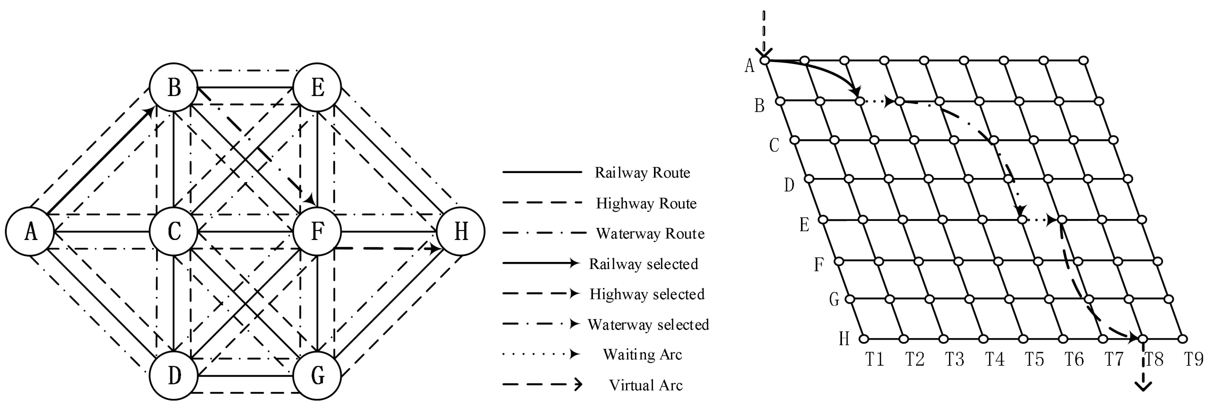

Fig. 1. Examples of physical network and time-space network

Indivisibility of Transportation Tasks: Each task encompasses the entirety of goods requiring transport, prohibiting subdivision into smaller segments.

Node-specific Modality Change: Alteration in the transportation method is permissible solely at nodes, with a maximum of one change allowed per node.

Conservation of Goods Quantity: The aggregate volume of goods remains constant throughout the transportation journey.

B. Modal Description

TABLE I
DESCRIPTION OF SETS, PARAMETERS AND DECISION VARIABLES(I)

Definition		Description
Sets	N	Time-space network nodes set
	A_R	Railway transportation arcs set
	A_H	Highway transportation arcs set
	A_W	Waterway transportation arcs set
	A_D	Waiting arcs set
	A_{vo}	Virtual departure arc set
	A_{vd}	Virtual arrival arcs set
	A	Time-space arcs set $A = A_R \cup A_H \cup A_W \cup A_D \cup A_{vo} \cup A_{vd}$
	A_n^+	Arcs set with node n as starting point
	A_n^-	Arcs set with node n as ending point
	H	Arc attribute set $r, h, w, v, s \in H$
	M	Arc transportation attribute set, $r, h, w \in M$
Parameters	T	Decision cycle
	O_V	Virtual start node
	D_V	Virtual end node
	r	Railway mode
	h	Highway mode
	w	Waterway mode
	v	Virtual mode
	s	Waiting mode
	o_a	Start node of arc a
	d_a	End node of arc a
	m_a	Attribute of arc a
	t_n^{m1m2}	The necessary waiting time for the transportation demand to switch from mode m1 to mode m2 at the node n
	c_{m2}^{m1}	Unit cost of transfer from mode m1 to mode m2
	c_a	Unit cost of arc a
	e_a	Unit carbon emission of arc a
	e_{m2}^{m1}	Unit carbon emission of transfer from mode m1 to mode m2
	θ	Carbon tax rate
	l_n	Entity location of node n
	t_n	Time of node n
	o	Start node
	$[d_e, d_l]$	Start node window
	q	Freight volume
Decision variables	x_a	0-1 variable to judge whether the arc is called
	y_n^{m1m2}	0-1 variable to judge whether the transportation task is converted arc attribute from m1 to m2 at the node n

$$\min C = C1 + C2 + C3 + C4 \quad (1)$$

$$C1 = \sum_{a\in A_R \cup A_H \cup A_W} x_a c_a q \quad (2)$$

$$C2 = \sum_{a\in A_D} \sum_{m1,m2\in H} \sum_{n\in N} q(x_a c_a + y_n^{m1m2} c_{m2}^{m1}) \quad (3)$$

$$C3 = \sum_{a\in A_g^{vd}} x_a e_a q \quad (4)$$

$$C4 = \sum_{a\in A} \sum_{m1,m2\in H} \sum_{n\in N} \theta q(x_a e_a + y_n^{m1m2} e_{m2}^{m1}) \quad (5)$$

Equation (1) delineates the model's objective function, designed to compute the aggregate cost entailed in the transportation process, encompassing various expense categories. Equation (2) is dedicated to the computation of transportation expenses, aggregating the costs associated with the utilization of transportation arcs throughout the journey. Equation (3) addresses the transit costs, accumulating expenses attributed to waiting arcs and the conversion of transportation modes at respective nodes. Equation (4) is formulated to calculate the penalty costs, which reflect the additional expenses incurred due to deviations from scheduled arrival times, either early or late. In the actual modeling process, this time-related penalty cost is quantified using a distinct virtual arrival arc cost, serving to distinguish it from standard waiting arc costs. Finally, Equation (5) is responsible for the assessment of carbon emission costs, which involves summing the expenses associated with carbon emissions produced during both transportation and transit phases.

$$\sum_{m1,m2\in H} y_{d_a}^{m1m2} = x_a, \forall a \in \{a | d_a \in N\} \quad (6)$$

$$\sum_{m2\in H} y_{d_a}^{m_a m2} = x_a, \forall a \in \{a | d_a \in N\} \quad (7)$$

$$\sum_{m1\in H} y_{o_a}^{m1 m_a} = x_a, \forall a \in \{a | o_a \in N\} \quad (8)$$

Equations (6) through (8) delineate the constraints related to arc attribute transformations within the transportation task. Specifically, Equation (6) imposes a restriction ensuring that any transformation of an arc's attribute is permissible only once per node. Equation (7) stipulates that the attribute transformation of any arc at a given node must maintain consistency with the attributes of the preceding arc. Conversely, Equation (8) mandates that the attribute transformation of arc segments at each node should be in harmony with the attributes of the subsequent arc invoked. In essence, Equations (6) to (8) collectively guarantee the coherence and continuity of arc attribute transformations throughout the transportation task, ensuring alignment with the attributes of the arcs invoked both prior to and subsequent to each node.

$$\begin{aligned} & x_{a1} + \sum_{a2\in A_n^+} x_{a2} \le 1 \\ & \forall a1 \in A_R \cup A_H \cup A_W, \forall g \in G \\ & \forall a2 \in \{a | (m_a \ne m_{a1}) \wedge m_a \in M\} \\ & \forall n \in \{n | (l_n = l_{d_{a1}}) \wedge t_n \in [t_{d_{a1}}, t_{d_{a1}} + t_{d_{a1}}^{m_{a1}m_{a2}})\} \end{aligned} \quad (9)$$

$$\begin{aligned} & x_{a1} + \sum_{a2 \in A_n^+} x_{a2} \le 1 \\ & \forall a1 \in A_R \bigcup A_H \bigcup A_W, \forall g \in G \\ & \forall a2 \in \left\{ a \middle| m_a = m_{a1} \right\} \\ & \forall n \in \left\{ n \middle| \left(l_n = l_{d_{a1}} \right) \wedge t_n \in (t_{d_{a1}}, T] \right\} \end{aligned} \tag{10}$$

Equations (9) and (10) are formulated to articulate the constraints associated with transit times within the transportation task. Specifically, Equation (9) ensures that the actual waiting time during transit at each node – particularly when there is a change in the transportation mode – adheres to a minimum required time interval. This stipulation is designed to ensure sufficient time for the mode change process. On the other hand, Equation (10) mandates that in cases where the transportation mode remains constant at a node, the transportation task must proceed immediately, thereby precluding any unnecessary delays. This ensures operational efficiency and timeliness in the transit process.

$$\sum_{a \in A_n^+} x_a \le 1, \forall n \in N \tag{11}$$

$$\sum_{a \in A_n^-} x_a \le 1, \forall n \in N \tag{12}$$

Equations (11) and (12) delineate the constraints related to the indivisibility of transportation tasks within the network. Specifically, Equation (11) asserts that a singular arc is utilized when the transportation task initiates departure from any given node. In contrast, Equation (12) stipulates that only a single arc may be employed by the transportation task for arriving at any node. Collectively, these equations (11 and 12) play a pivotal role in maintaining the space-time consistency of the transportation tasks, ensuring that the process adheres to a coherent and sequential flow without any divergence or bifurcation at the nodes.

$$\sum_{a \in A_n^+} x_a - \sum_{a \in A_n^-} x_a = \begin{cases} 1, n = O_V \\ 0, n \in N \\ -1, n = D_V \end{cases}, \forall n \in N \tag{13}$$

Equation (13) establishes the constraints governing both the initiation and termination points of transportation, as well as the continuity of the transportation process. This ensures that each transportation task is clearly demarcated with specific start and end points, and that it follows an uninterrupted trajectory through both time and space. The effective initiation of transportation tasks is crucial, particularly in the context of minimizing the model's objective function.

$$\begin{aligned} & x_a = 0 \\ & \forall a \in \left\{ a \middle| a \in A_D \wedge \left(l_{o_a} = l_{d_e} \right) \right\} \end{aligned} \tag{14}$$

Equation (14) is formulated to serve as a delivery constraint for the transportation of goods. It specifically mandates that the transportation task should not engage any waiting arcs in cases where it reaches the destination earlier than planned. This provision plays a significant role in streamlining the computation of the penalty cost. By precluding the activation of waiting arcs for early arrivals, the equation effectively reduces the complexity involved in calculating any additional costs that may be incurred due to deviations from the scheduled timeline.

$$x_a \in \{0,1\}, \forall a \in A \tag{15}$$

$$y_n^{m1m2} \in \{0,1\}, \forall n \in N, \forall m1, m2 \in H \tag{16}$$

Equations (15) and (16) establish the constraints for the decision variables within the model, stipulating that their values are confined within the range of 0 or 1.

In traditional physical transportation networks, route optimization models predominantly focus on the selection of the transportation route and mode. This approach necessitates that transportation demand swiftly progresses through intermediate nodes, adhering to transit or scheduling constraints. Conversely, within space-time transportation networks, the decision variable in the routing optimization model pertains to the selection of disparate arcs. Notably, the waiting time for transportation demand at any given node is variable and can be tailored by choosing the appropriate waiting arc, a feature that is ensured by Equation (9). While this characteristic might appear redundant in scenarios where node parameters are static, it gains significance in contexts where these parameters are subject to dynamic changes or uncertainty. Under such conditions, the model's objective function benefits from an expanded solution space, thereby enhancing the potential for optimal solutions. The model in question is structured as a 0-1 linear programming model and is amenable to resolution through a commercial solver.

C. Space-time Variability Simulation

Transit time at a transfer hub is a composite of the operation time and the waiting period during transit, with its duration predominantly determined by the latter, which, in turn, is influenced by the hub's operating load. Traditional simulation approaches often assign a static value to transit time or employ a fixed distribution model, thereby overlooking the dynamic variability of node operating loads. In our study, this variability is incorporated by modeling transit time distributions across five distinct operating load scenarios: fastest, fast, average, slow, and slowest. This is achieved by categorizing node operation load levels into five gradations (0-4). Fig.2 showcases an example where transit time is modeled as following a uniform distribution. Here, the interval [t3, t5] denotes the average transit time distribution, whereas [t0, t8] encompasses the full range of transit time variability. However, this approach does not account for the dynamic fluctuations in transit time attributable to changing node operating loads, thus curtailing the model's practical utility. To mitigate this, we have delineated [t0, t2] to represent the range for the fastest transit time, [t1, t4] for fast, [t5, t7] for slow, and [t6, t8] for the slowest transit durations. The entire spectrum is segmented into five intervals corresponding to the aforementioned categories. We then simulate the time-variant distribution of transit times by randomly amalgamating these intervals. The assumption here is that the time-varying probability of node operating load levels adheres to a time-homogeneous Markov chain. The evolution of node operating loads over time is modeled using stochastic simulation techniques, utilizing the transition matrix P (outlined in equation 17).

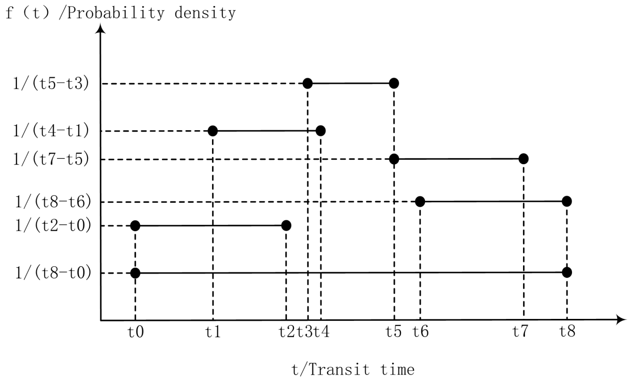

Fig. 2. Diagram of transit time distribution under different situations

$$P=\begin{bmatrix} P_{00},P_{01},P_{02},P_{03},P_{04} \\ P_{10},P_{11},P_{12},P_{13},P_{14} \\ P_{20},P_{21},P_{22},P_{23},P_{24} \\ P_{30},P_{31},P_{32},P_{33},P_{34} \\ P_{40},P_{41},P_{42},P_{43},P_{44} \end{bmatrix} \tag{17}$$

In implementing the aforementioned method, it is crucial to consider the gradual transition of space-time transformations, ensuring that the amplitude of changes is not excessively volatile. As illustrated in Fig. 3, on the same space-time axis, for each unit of time delay in the arrival of a transportation task at a node, the parameters on which the necessary waiting time depends undergo a change. These parameters shift progressively and evenly from one scenario's parameters to another's. The determination of these parameters follows the methodologies outlined in Equations (18) to (23).

$$t_a \sim N(\mu_a, \sigma_a^2) \tag{18}$$

$$t_b \sim N(\mu_b, \sigma_b^2) \tag{19}$$

$$t_1 \sim N(\mu, \sigma^2), \begin{cases} \mu = \mu_a + (\mu_b - \mu_a)/t_{gra} \\ \sigma = \sigma_a + (\sigma_b - \sigma_a)/t_{gra} \end{cases} \tag{20}$$

$$t_{t_{gra}-1} \sim N(\mu, \sigma^2)$$

$$\begin{cases} \mu = \mu_a + (t_{gra}-1)(\mu_b - \mu_a)/t_{gra} \\ \sigma = \sigma_a + (t_{gra}-1)(\sigma_b - \sigma_a)/t_{gra} \end{cases} \tag{23}$$

III. Example Analysis

A. Parameter Description

In our study, we developed a multimodal transportation physical network comprising 15 nodes, as depicted in Fig. 4. The network initiates with generation nodes, denoted as (A, 0), tasked with meeting specific transportation requirements. Delivery node windows are defined as [(O, 45), (O, 55)], and the total cargo weight is set at 100 tons. To facilitate model resolution, we established the minimum time interval in the time-space network at 0.1 hours and assumed a consistent speed for various transportation vehicles, thereby fixing the consumption time for each transportation arc. Our model incorporated parameter values from reference [9], supplemented by updated statistical data from the "China Statistical Yearbook 2022," "China Logistics Yearbook (2022)," and "2020 Statistical Bulletin on the Development of the Transportation Industry." We established the time-based unit transportation costs for highways, railways, and waterways at 40, 6, and 1.5 yuan/(t·h), respectively. Corresponding unit carbon emissions were set at 3.836, 0.505, and 0.52 kg/(t·h). The cost incurred for early or delayed delivery of goods, considered as warehousing/penalty costs, was determined to be 15/30 yuan/(t·h). The cost for each transit waiting arc was set at 5 yuan/t, and the carbon tax rate was fixed at 0.1 yuan/kg. Tab.2 illustrates the transportation parameters for nodes under various scenarios, while Tab.3 displays the values of the stochastic matrix for node operation load levels. We programmed the model using Python 3.7 and solved it using Cplex12.10.

Our time-space network path selection model generates solutions involving specific time-space arcs, accounting for precise arrival and departure times, a notable deviation from the conventional physical network model that includes only nodes and transportation methods. When dealing with uncertain transit times, a single time-space arc solution does not suffice for multiple random simulations. To address this,

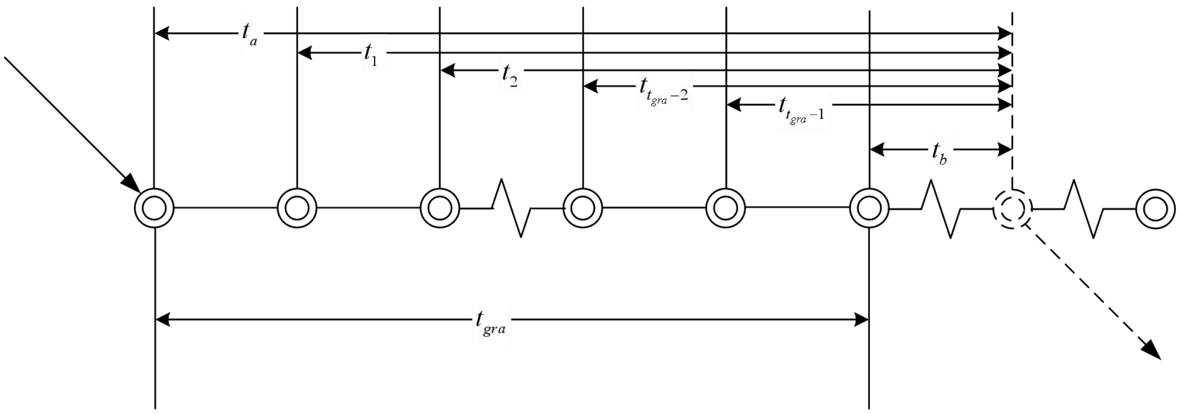

Fig. 3. gradual change of spatio-temporal transformation

TABLE II
TRANSIT TIME PARAMETERS

Conversion mode	Determined transit time (h)/Uncertain transit time distribution (h)					Unit cost (yuan/t)	Carbon emissions factors (kg/t)
	Fastest	Fast	Average	Slow	Slowest		
rail-road	2/[2,1^2]	3/[3,1^2]	4/[4,1^2]	5/[5,1^2]	6/[6,1^2]	8	0.0324
rail - water	5/[5,1^2]	6/[6,1^2]	4/[7,1^2]	8/[8,1^2]	9/[9,1^2]	12	0.0424
road - rail	4/[4,1^2]	5/[5,1^2]	6/[6,1^2]	7/[7,1^2]	8/[8,1^2]	8	0.0324
road -water	3/[3,1^2]	4/[4,1^2]	5/[5,1^2]	6/[6,1^2]	7/[7,1^2]	10	0.0424
water - rail	6/[6,1^2]	7/[7,1^2]	8/[8,1^2]	9/[9,1^2]	10/[10,1^2]	12	0.0424
water - road	1/[1,1^2]	2[2,1^2]	3/[3,1^2]	4/[4,1^2]	5/[5,1^2]	10	0.0424

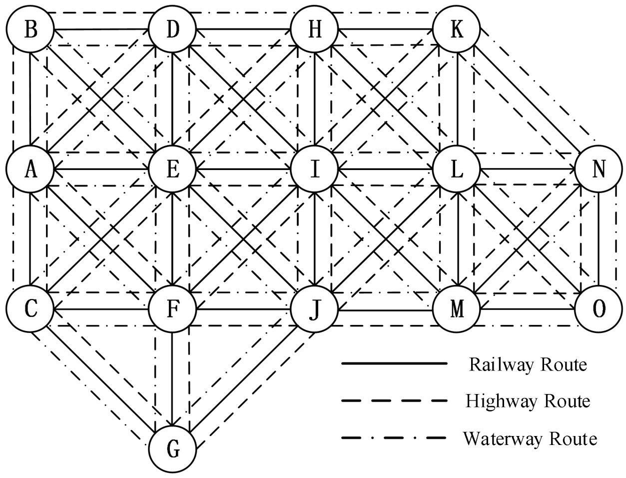

Fig. 4. Multimodal transport physical network

TABLE III
TRANSITION MATRIX PARAMETERS

P_{ij}	j=0	j=1	j=2	j=3	j=4
i=0	0.25	0.4	0.2	0.1	0.05
i=1	0.1	0.2	0.4	0.2	0.1
i=2	0.1	0.2	0.4	0.2	0.1
i=3	0.1	0.2	0.4	0.2	0.1
i=4	0.05	0.1	0.2	0.4	0.25

we consider multiple time-space arc solutions, sharing the same transit node and mode, as a singular solution, tallying their occurrences across various simulations. We conducted the simulation 500 times to compute the expected cost for each solution.

B. Result Analysis

The results of our study are delineated in Tab. 4 and Fig. 5. Regarding the transportation modes within the model, numerical representations are assigned as follows: 1 for highway, 2 for railway, and 3 for waterway. In examining the five different transit time scenarios, we identified that the solution which consistently exhibited a predominant frequency under the condition of uncertain transit time distribution also coincided with the solution for scenarios where transit time was constant. This congruence suggests that such a solution stands as the most dependable in these specific circumstances. It is important to note, however, that solutions manifesting a lower frequency are not devoid of relevance. Despite their potentially reduced reliability, they retain their utility as viable references in certain transportation contexts. In the scenario characterized by the fastest transit time, the model operates under minimal constraints, allowing for a solution that integrates all three transportation modes – highway, railway, and waterway. This integration capitalizes on the efficiencies of multimodal transportation, thereby minimizing the overall costs. As the scenario shifts to 'fast' transit time, the model encounters increased constraints on transportation decision-making. Consequently, the most reliable solution adapts by reducing the reliance on time-space arcs, leading to fewer transitions between transportation modes. This change effectively lowers transit costs, but it also results in a rise in the total cost due to more stringent constraints. In the 'average' transit time scenario, the constraints become even more pronounced, prompting the incorporation of time penalty costs into the model. Notably, the most reliable solution remains consistent in both 'slow' and 'slowest' transit time scenarios. Intriguingly, under conditions of uncertain transit time distribution, this solution's frequency escalates in tandem with the increasing rigidity of transportation decision constraints.

In scenarios characterized by five distinct single transit times, it is observed that the overall cost escalates with the prolongation of transit time. Concurrently, the rising transportation constraints curtail the ability of the multimodal transportation system to leverage the unique advantages of each mode. As transit times lengthen, the strategy shifts away from favoring waterway transport, which, despite its extended duration, offers cost-effectiveness, towards a preference for road-rail intermodal transport. This transition results in a uniform increase across all four cost metrics. Specifically, transit costs rise in response to the increased waiting times, while transportation and carbon emission costs escalate due to a higher reliance on road and railway modes, known for their relatively higher cost and carbon footprint. Simultaneously, the time penalty cost also starts to grow, initially from zero. This gradual increase is indicative of the system's diminishing capability to ensure timely delivery of goods within the predefined time windows, especially as transit times become more prolonged.

In the scenario where random combinations are applied to the five distinct single transit time scenarios, each of the four previously identified solutions manifests with a specific frequency. Remarkably, this stochastic approach also results in the emergence of a novel solution, not observed in the earlier deterministic scenarios. This newly identified

TABLE IV
INFLUENCE OF DIFFERENT TRANSIT TIME SCENARIOS ON TRANSPORTATION DECISIONS(I)

Transit time scenario	Constant Value			Probability Distribution		
	Transportation Route	Transportation Mode	Frequency (%)	Transportation Route	Transportation Mode	Frequency (%)
Fastest	A-B-D-I-M-O	1-2-3-3-1	100	A-B-D-I-M-O	1-2-3-3-1	76.3
				A-E-I-M-O	2-2-3-1	20.4
				A-E-J-M-O	1-1-3-1	3.3
Fast	A-E-I-M-O	2-2-3-1	100	A-B-D-I-M-O	1-2-3-3-1	25.8
				A-E-I-M-O	2-2-3-1	60.4
				A-E-J-M-O	1-1-3-1	13.8
Average	A-E-J-M-O	1-1-3-1	100	A-E-I-M-O	2-2-3-1	17.6
				A-E-J-M-O	1-1-3-1	65.4
				A-E-J-L-O	1-2-2-1	17
Slow	A-E-J-L-O	1-2-2-1	100	A-E-I-M-O	2-2-3-1	8.9
				A-E-J-M-O	1-1-3-1	16.9
				A-E-J-L-O	1-2-2-1	74.2
Slowest	A-E-J-L-O	1-2-2-1	100	A-E-I-M-O	2-2-3-1	4.2
				A-E-J-M-O	1-1-3-1	11.5
				A-E-J-L-O	1-2-2-1	84.3
Random Transformation	A-B-D-I-M-O	1-2-3-3-1	10.8	A-B-D-I-M-O	1-2-3-3-1	16.8
	A-B-D-I-L-O	1-2-3-1-1	5.5	A-B-D-I-L-O	1-2-3-1-1	9.6
	A-E-I-M-O	2-2-3-1	20.7	A-E-I-M-O	2-2-3-1	19.6
	A-E-J-M-O	1-1-3-1	33.7	A-E-J-M-O	1-1-3-1	28.6
	A-E-J-L-O	1-2-2-1	29.3	A-E-J-L-O	1-2-2-1	25.4

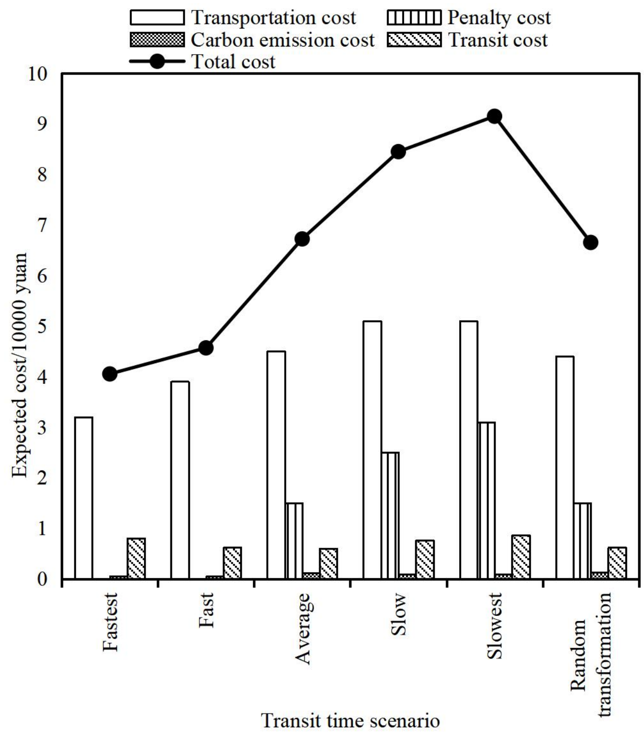

Fig. 5. Impact of different transit time scenarios on various costs

solution, characterized by its lowest frequency, exclusively emerges in the context of random transformation. A notable shift occurs when the transit time is modeled from a fixed value to a probabilistic distribution: the distribution of frequencies across different solutions undergoes a significant alteration. The previously less frequent solutions gain in frequency, leading to a more balanced distribution among all solutions. This pattern indicates a reduction in the reliability of the solutions that were initially more frequent, especially in a context marked by increased uncertainty in transit times. This observation underscores the necessity of considering a broader spectrum of solutions while devising practical transportation plans, particularly in environments characterized by higher unpredictability.

C. *Impact of different carbon tax rates*

Reference [18] highlights that the existing low-carbon policies in China implement a carbon tax rate of 10 yuan/t, a figure that falls below the global average. This comparatively low rate is presently insufficient to significantly influence the structural transformation of the transportation sector's supply side. However, with the anticipated maturation of China's carbon pricing mechanism, a rise in carbon tax rates is likely in the offing. In light of these expected developments, it becomes imperative to investigate how alterations in the carbon tax rate could impact decision-making processes in transportation. Our study addresses this by resolving and analyzing a transportation problem, taking into account a carbon tax increment of 0.4 yuan/kg and the variable distribution of transit times. Within our analysis, transportation modes are numerically designated, with 1, 2, and 3 denoting highway, railway, and waterway modes, respectively.

Tab. 5 reveals that at a carbon tax rate ranging from 0 to 0.4, the transportation solutions align with those presented in Tab. 2, indicating that within this tax range, the carbon tax exerts no significant influence on transportation decisions. However, a notable shift is observed when the carbon tax ranges from 0.8 to 1.6. Here, the transportation mode changes, although the same nodes are traversed. Specifically, some solutions transition from road-water to rail-water intermodal transportation, while others move from road-rail to exclusively railway transportation. This shift away from road transportation and the marked increase in the frequency of these altered solutions suggest that the carbon tax within this bracket begins to significantly influence transportation decisions. When the carbon tax increases to between 1.6 and 2, there is a further evolution in the transportation mode choices, still keeping the node passage consistent. At this juncture, all solutions have abandoned road transportation, predominantly opting for rail-water intermodal transportation. This gradual shift away from highway transportation, attributed to its higher carbon emission costs, underscores the critical importance of incorporating carbon tax considerations into transportation planning.

TABLE V
IMPACT OF DIFFERENT CARBON TAX RATES ON TRANSPORTATION DECISIONS

Carbon tax	Transportation Route	Transportation Mode	Frequency (%)
0	A-B-D-I-M-O	1-2-3-3-1	17.6
	A-B-D-I-L-O	1-2-3-1-1	12.6
	A-E-I-M-O	2-2-3-1	17.1
	A-E-J-M-O	1-1-3-1	30.4
	A-E-J-L-O	1-2-2-1	22.3
0.4	A-B-D-I-M-O	1-2-3-3-1	16.1
	A-B-D-I-L-O	1-2-3-1-1	8
	A-E-I-M-O	2-2-3-1	22.1
	A-E-J-M-O	1-1-3-1	26.6
	A-E-J-L-O	1-2-2-1	27.2
0.8	A-B-D-I-M-O	1-2-3-3-1	13.6
	A-B-D-I-L-O	1-2-3-1-1	5.3
	A-E-I-M-O	2-2-3-1	17
	A-E-J-M-O	2-3-2-2	32.6
	A-E-J-L-O	2-2-2-2	31.5
1.2	A-B-D-I-M-O	1-2-3-3-1	11.4
	A-B-D-I-L-O	1-2-3-1-1	4.4
	A-E-I-M-O	2-2-3-1	16.5
	A-E-J-M-O	2-3-2-2	34.3
	A-E-J-L-O	2-2-2-2	33.4
1.6	A-B-D-I-M-O	2-2-3-3-2	16.5
	A-B-D-I-L-O	2-2-3-2-2	8.6
	A-E-I-M-O	2-2-3-2	21.6
	A-E-J-M-O	2-3-2-2	26.9
	A-E-J-L-O	2-2-2-2	26.4
2	A-B-D-I-M-O	2-2-3-3-2	17.3
	A-B-D-I-L-O	2-2-3-2-2	7
	A-E-I-M-O	2-2-3-2	22.2
	A-E-J-M-O	2-3-2-2	26.5
	A-E-J-L-O	2-2-2-2	27

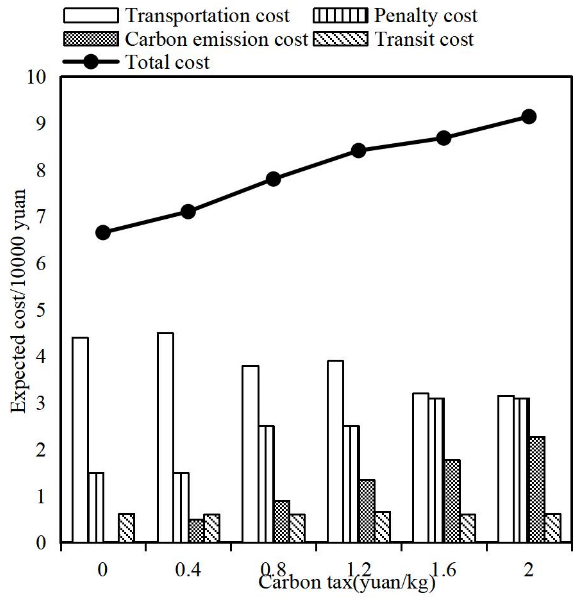

Fig. 6. Impact of different carbon tax rates on various costs

As depicted in Fig. 6, the overall cost escalates with the increasing carbon tax. In the 0 to 0.4 carbon tax bracket, the total cost increment is solely due to the carbon emission costs, with other costs remaining constant. At a carbon tax of 0.8, however, the change in transportation solutions offsets the rise in carbon tax through reduced reliance on road transportation, albeit leading to an elevated time penalty cost. A further modification in the transportation strategy is evident at a carbon tax of 1.6, where road transportation is completely abandoned to counter the surging carbon emission costs, resulting in an increased time penalty cost."

D. Model comparison

Developing a physical network path model, derived from the space-time network path model outlined in section 1.3, enables a comparative analysis of the solutions generated by both models. This physical network path model represents a streamlined version of the initially established space-time network path model, with the primary distinction being the omission of the time dimension.

A critical difference between this model and the space-time network model, as established earlier, lies in its approach to transportation demand. This physical network path model mandates the commencement of transportation at the earliest possible time for the selected mode, inherently leading to a more constrained and thus smaller solution space.

As depicted in Tab. 6, the results of the physical network model are presented. In comparison to the space-time network model shown in Tab. 4, one of the solutions in the physical network model undergoes a change. The path nodes remain unchanged, but there is a shift in the mode of transportation for a certain segment of the journey, transitioning from railway to road transportation. Despite this alteration, the solution continues to be reliable in slower and slowest scenarios. However, its frequency of occurrence decreases across all scenarios, with a greater decline as time constraints intensify and a lesser decline as the randomness of time scenarios increases. The total cost

TABLE VI
INFLUENCE OF DIFFERENT TRANSIT TIME SCENARIOS ON TRANSPORTATION DECISIONS(II)

Transit time scenario	Constant Value			Probability Distribution		
	Transportation Route	Transportation Mode	Frequency (%)	Transportation Route	Transportation Mode	Frequency (%)
Fastest	A-B-D-I-M-O	1-2-3-3-1	100	A-B-D-I-M-O	1-2-3-3-1	76.3
				A-E-I-M-O	2-2-3-1	20.4
				A-E-J-M-O	1-1-3-1	3.3
Fast	A-E-I-M-O	2-2-3-1	100	A-B-D-I-M-O	1-2-3-3-1	25.8
				A-E-I-M-O	2-2-3-1	60.4
				A-E-J-M-O	1-1-3-1	13.8
Average	A-E-J-M-O	1-1-3-1	100	A-E-I-M-O	2-2-3-1	19.8
				A-E-J-M-O	1-1-3-1	68.6
				A-E-J-L-O	1-2-1-1	11.6
Slow	A-E-J-L-O	1-2-1-1	100	A-E-I-M-O	2-2-3-1	14.1
				A-E-J-M-O	1-1-3-1	25.1
				A-E-J-L-O	1-2-1-1	60.8
Slowest	A-E-J-L-O	1-2-1-1	100	A-E-I-M-O	2-2-3-1	8.6
				A-E-J-M-O	1-1-3-1	26.7
				A-E-J-L-O	1-2-1-1	64.7
Random Transformation	A-B-D-I-M-O	1-2-3-3-1	11.4	A-B-D-I-M-O	1-2-3-3-1	17.1
	A-B-D-I-L-O	1-2-3-1-1	6.4	A-B-D-I-L-O	1-2-3-1-1	10.3
	A-E-I-M-O	2-2-3-1	21.8	A-E-I-M-O	2-2-3-1	20.5
	A-E-J-M-O	1-1-3-1	35.2	A-E-J-M-O	1-1-3-1	29.7
	A-E-J-L-O	1-2-1-1	25.2	A-E-J-L-O	1-2-1-1	22.4

of this solution increases compared to its prior state, indicating that the space-time network model indeed has the potential to yield better solutions than the physical network model.

E. Comparison of the distribution obeyed

Prevailing studies in the field often overlook the specific distribution type when examining scenarios with uncertain time characterized by randomness. A common trend among researchers is the presumption that such uncertain time adheres to a normal distribution for simulation purposes. Nonetheless, to gain a more holistic understanding of these uncertain time scenarios, it is crucial to consider the assumption that the uncertain duration conforms to a uniform distribution and to thoroughly analyze the outcomes this assumption yields.

Tab. 7 showcases the solutions derived under the premise that uncertain time follows a uniform distribution. This scenario exhibits a markedly higher degree of randomness compared to the constant time and normal distribution scenarios presented in Tab. 4. Although the transportation route or mode remained unaltered in these solutions, a notable shift was observed in the frequency distribution of various solutions across different scenarios. Specifically, there was a more balanced spread in solution frequencies, and notably, the occurrence of reliable solutions diminished as the randomness in the time scenarios intensified. This observation underscores the critical need to consider a range of different scenarios and alternative solutions in the formulation of practical transportation plans.

TABLE VII
INFLUENCE OF DIFFERENT TRANSIT TIME SCENARIOS ON TRANSPORTATION DECISION(III)

Transit time scenario	Transportation Route	Transport ation Mode	Frequency (%)
Fastest	A-B-D-I-M-O	1-2-3-3-1	66.3
	A-E-I-M-O	2-2-3-1	26.3
	A-E-J-M-O	1-1-3-1	7.4
Fast	A-B-D-I-M-O	1-2-3-3-1	30.5
	A-E-I-M-O	2-2-3-1	51.4
	A-E-J-M-O	1-1-3-1	18.1
Average	A-E-I-M-O	2-2-3-1	23.6
	A-E-J-M-O	1-1-3-1	53
	A-E-J-L-O	1-2-1-1	23.4
Slow	A-E-I-M-O	2-2-3-1	13.9
	A-E-J-M-O	1-1-3-1	22.8
	A-E-J-L-O	1-2-1-1	63.3
Slowest	A-E-I-M-O	2-2-3-1	9
	A-E-J-M-O	1-1-3-1	16.7
	A-E-J-L-O	1-2-1-1	74.3
Random Transformation	A-B-D-I-M-O	1-2-3-3-1	20..3
	A-B-D-I-L-O	1-2-3-1-1	12.6
	A-E-I-M-O	2-2-3-1	20.5
	A-E-J-M-O	1-1-3-1	25.6
	A-E-J-L-O	1-2-1-1	21

IV. CONCLUSION

1) Building upon the foundation of traditional intermodal transportation costs, we have developed a multimodal transport time-space network path optimization model. This model minimizes the total cost by accounting for the dynamic randomness of transit times and incorporating carbon emission costs. This approach enhances the model's realism and facilitates a comprehensive evaluation of the impacts of transit time scenarios and carbon tax rates on path selection.

2) As transit times extend, the constraints on transportation decisions intensify, making it increasingly difficult to fully exploit the advantages of various multimodal transportation modes. The rail-road intermodal transportation mode gradually assumes a dominant role in decision-making as transit time constraints become more stringent. This indicates that rail-road intermodal transportation is a more cost-effective mode under tighter time constraints.

3) Different transit time scenarios significantly impact the generation of solutions: increased randomness in transit time scenarios reduces the reliability of previously dominant solutions. Uncertain time scenarios, as opposed to certain time scenarios, and dynamic change scenarios, as opposed to single scenarios, may lead to the emergence of new solutions. Therefore, considering various transportation scenarios is crucial for making informed transportation decisions.

4) The increase in carbon tax value begins to influence transportation decisions after reaching a certain level: the proportion of road transportation decreases, while the proportion of rail-water intermodal transportation increases. This indicates that rail-water intermodal transportation becomes a more economical mode under high carbon tax conditions.

5) Compared to the physical network model, the time-space network model, with its flexible departure interval constraints, has the potential to yield better solutions. This underscores the practical significance of simulating the temporal variability of transit times.

6) The choice of random distribution type used to simulate uncertain transit times also affects the solutions. Analyzing real transportation situations to identify the most appropriate distribution type is beneficial for making the most rational transportation decisions.

7) This model primarily focuses on the dynamic randomness of transit times. Future research needs to incorporate additional uncertain or time-varying factors, and the methods for describing uncertainty and time variability also require improvement to establish a more realistic transportation planning model.

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