Fast Bias Removal Equation-Error Adaptive Filtering Algorithms

Junghsi Lee, Yi-Wen Chiu, and Hsu Chang Huang

Abstract—**This paper presents a multi step-size monic normalization equation-error linear filter. We then extend the idea to nonlinear adaptive filter and derive a multi step-size monic normalization equation-error bilinear filter (MSS MNEEBF). The algorithms enjoy fast convergence behavior and can remove biased estimates associated with conventional equation-error adaptive filters. Simulation results validate the usefulness of our algorithms.**

Index Terms**—Bias-Removal Algorithms, Bilinear Adaptive Filters , Equation-Error Adaptive Filters, Nonlinear Adaptive Filters**

I. Introduction

Finite impulse response (FIR) least mean square (LMS) adaptive filter has been very popular due to its simplicity and good convergence characteristics. For applications such as acoustic echo cancellation which requires filters with thousands of coefficients to model the impulse response of echo path, time-domain FIR LMS may not be the proper solution. Infinite impulse response (IIR) adaptive filter has the potential to be a good alternative. The input-output relationship for a linear system is given as

$$y(n) = \frac{1}{a_0}\left(\sum_{k=0}^{K} b_k x(n-k) - \sum_{l=1}^{L} a_l y(n-l)\right) , \quad (1)$$

where without loss of generality it is assumed that $a_0 = 1$.

There are two approaches to adaptive IIR filtering: output-error and equation-error formulations. The output-error approach uses the past samples of the output of the adaptive filter to obtain the filter's current output. The equation-error approach uses the past samples of the desired response signal directly to compute the adaptive filter output. Because of the feedback structure, output-error IIR needs stability monitoring to assure it would not diverge. Also, its error surface is multimodal and therefore the output-error algorithm may converge to the local minimum.

The equation-error approach is very simple to design and implement. It does not need stability check. The error surface is unimodal. However, it results in biased system estimates in the presence of noisy observations. Consequently, such equation-error algorithms are useful only in applications in which the measurement noise is fairly small.

Early equation-error IIR algorithms tried to remove the bias through additional filtering or noise suppression schemes [1]. Some alternatives such as the unit-norm constraint approach [2], [3] and monic normalization method [4] have been proposed to produce unbiased parameter estimates in the presence of noise. The basic idea behind the monic normalization equation-error filter (MNEE) is to set the first denominator coefficient of a rational system to unity, which is implemented by having all its denominator coefficients normalized by the first coefficient after each iteration. However, the MNEE presented in [4] had to use a very small value for step-size parameter, therefore, it takes very long to converge even for filters with just two feed-forward and two feedback coefficients.

While linear filters have been very useful in a large variety of applications and are conceptually and implementationally very simple, there are applications in which they will not perform well at all. A very common system model that has been employed with relatively good success in nonlinear filtering applications is the Volterra system model [5], [6]. The main problem associated with such filters is the extremely large number of coefficients that is usually required to properly model the nonlinear system under consideration. Just as linear IIR filters can model many linear systems with great parsimony than FIR filters, there are a large number of nonlinear systems that can be approximated by nonlinear feedback models using a relatively small number of parameters. In such situations, one can expect that the corresponding adaptive filters can be implemented with good computational efficiency. Bilinear filters are among the simplest of recursive nonlinear systems [5]. The input-output relationship for a bilinear system is given as

$$y(n) = \frac{1}{a_0}\left\{\sum_{k=0}^{K} b_k x(n-k) - \sum_{l=1}^{L} a_l y(n-l) + \sum_{k=0}^{K}\sum_{l=1}^{L} c_{kl} x(n-k) y(n-l)\right\} . \quad (2)$$

Just as linear IIR filters, it is assumed that $a_0 = 1$.

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J. Lee is with the Department of Electrical Engineering, Yuan-Ze University, Chung-Li, Taoyuan, 32026, TAIWAN (Phone: +886-3-4638800 Ext. 2419; fax: +886-3-4639355; e-mail: eejlee@saturn.yzu.edu.tw.).

Y. W. Chiu .was with Department of Electrical Engineering, Yuan-Ze University, Chung-Li, Taoyuan, 32026, TAIWAN.

H. C. Huang is with Department of Electrical Engineering, Yuan-Ze University, Chung-Li, Taoyuan, 32026, TAIWAN, (Fax: +886-3-4639355; e-mail: s958501@mail.yzu.edu.tw.).

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In this paper, we first introduce a multi step-size (MSS) monic normalization equation-error linear filter (MSS-MNEE). The proposed method has the comparable bias removal capability as that of the MNEE while converges very fast. In addition, we extend the idea to nonlinear filters with feedback and propose a multi step-size monic normalization equation-error bilinear filter (MSS-MNEEBF).

II. MNEE WITH MULTI- STEP SIZE

The problem of multi step-size monic normalization equation-error linear filtering may be formulated as follows. Given a desired response signal $d(n)$ and an input signal $x(n)$, we want to estimate $d(n)$ adaptively as

$$\hat{d}(n)=\sum_{k=0}^{K}\hat{b}_k x(n-k)-\sum_{l=1}^{L}\hat{a}_l d(n-l)\ . \tag{3}$$

The objective of the adaptive filter is to update the coefficients at each time so that a convex function of the estimation error

$$\alpha(n)=d(n)-\hat{d}(n) \tag{4}$$

is minimized or at least reduced. For simplicity of presentation, vector representation will be employed. Define the K-element feed-forward input vector $X(n)$ and L+1-element feedback input vector $D(n)$, respectively, as

$$X(n)=[x(n),x(n-1),\cdots,x(n-K)]^T\ , \tag{5a}$$

and

$$D(n)=[d(n),d(n-1),\cdots,d(n-L)]^T\ , \tag{5b}$$

where the superscript T denotes transportation. Also the corresponding feed-forward and feedback coefficient vectors are defined, respectively, as

$$\hat{B}(n)=[\hat{b}_0(n),\hat{b}_1(n),\cdots,\hat{b}_K(n)]^T\ , \tag{6a}$$

and

$$\hat{A}(n)=[\hat{a}_0(n),\hat{a}_1(n),\cdots,\hat{a}_L(n)]^T\ . \tag{6b}$$

The problem consider in this section is that of finding a gradient descent solution for the coefficients of the adaptive filter which attempts to reduce the squared error cost function given by

$$J(n)=\left(d(n)-\hat{d}(n)\right)^2=\left(e_g(n)\right)^2\ , \tag{7}$$

where

$$e_g(n)=\hat{A}^T(n)D(n)-\hat{B}^T(n)X(n) \tag{8}$$

is the generalized equation error. Note that $e_g(n)$ reduces to the conventional equation error $e_e(n)$ when $\hat{a}_0(n)$=1.

Because $X(n)$ and $D(n)$ do not depend on $\hat{B}(n)$ and $\hat{A}(n)$, it is straightforward to write update equations

$$\hat{B}(n+1)=\hat{B}(n)+\mu_b e_g(n)X(n)\ , \tag{9a}$$

$$\hat{A}(n+1)=\hat{A}(n)-\mu_a e_g(n)D(n)\ , \tag{9b}$$

where μ_a and μ_b are convergence parameters that control the rate at which the adaptive filter converges. The monic normalization proposed in [4] is to normalize $\hat{A}(n+1)$ as

$$\hat{A}(n+1)=\frac{1}{\hat{a}_0(n+1)}\hat{A}(n+1)\ , \tag{9c}$$

so that we can maintain $\hat{a}_0(n)$=1 at all times. The filter output is then produced as

$$\hat{d}(n)=\sum_{k=0}^{K}\hat{b}_k(n+1)x(n-k)-\sum_{l=1}^{L}\hat{a}_l(n+1)\hat{d}(n-l)\ . \tag{10}$$

And the *a-posterior* error signal is

$$e(n)=d(n)-\hat{d}(n)\ . \tag{11}$$

The MNEE algorithm proposed in [4] is capable of reducing the biased coefficients estimates. However, the analysis provided by Kim is based on the usage of very small step-size. As a result, this fairly small step-size utilized in the simulations made the filters converge extremely slow even for the very simple case that K=1 and L=2 . Obviously, such slow convergence rate is simply not acceptable in most applications. In order to solve the slow convergence problem while maintaining the bias removal capability, we propose a multi step-size (MSS) scheme in the paper.

Without loss of generality, we just present a two step-size scheme in the paper. The idea is to utilize two values of step-size in response to mean squared error indication function defined as

$$\varepsilon(n,N)=\frac{1}{N}\sum_{i=n-N+1}^{n}e^2(i)\ , \tag{12}$$

where N is a memory factor. The MSS scheme selects bigger step size when $\varepsilon(n,N)$ is greater than some pre-chosen threshold, and uses smaller step size otherwise. The analysis of stationary points made in [4] can be applied to MSS-MNEE since our method also utilizes a small step size later in the process.

III. MSS-MNEEBF

In this section, we derive the LMS adaptive bilinear filter using multi step-size monic normalization procedure. Follow our work in section 2, now for a desired response signal $d(n)$ and an input signal $x(n)$, we want to estimate $d(n)$ adaptively as

$$\hat{d}(n)=\sum_{k=0}^{K}\hat{b}_k x(n-k)-\sum_{l=1}^{L}\hat{a}_l d(n-l)+\sum_{k=0}^{K}\sum_{l=1}^{L}\hat{c}_{k,l}x(n-k)d(n-l)\ . \tag{13}$$

Define input vectors $X(n)$, $D(n)$ as (5a) and (5b) and define *Q(n)* as

$$Q(n) = [x(n)d(n-1), x(n)d(n-2), \cdots, x(n)d(n-L), \\ \cdots, x(n-K)d(n-1), \cdots, x(n-K)d(n-L)]^T \quad (14)$$

Define the corresponding coefficient vectors $\hat{B}(n)$ and $\hat{A}(n)$ as in (6a) and (6b), and define $\hat{C}(n)$ as

$$\hat{C}(n) = [\hat{c}_{0,1}(n), \hat{c}_{0,2}(n), \cdots, \hat{c}_{K,L}(n)]^T \quad (15)$$

Define the generalized equation error

$$e_g(n) = \hat{A}^T(n)D(n) - \hat{B}^T(n)X(n) - \hat{C}^T(n)Q(n) \quad (16)$$

The update equations for $\hat{B}(n)$ and $\hat{A}(n)$ are as that in (9a) and (9b), respectively, and

$$\hat{C}(n+1) = \hat{C}(n) + \mu_c e_g(n)Q(n). \quad (17)$$

The monic normalization is to normalize $\hat{A}(n+1)$ as

$$\hat{A}(n+1) = \frac{1}{\hat{a}_0(n+1)}\hat{A}(n+1). \quad (18)$$

The filter output is then produced as

$$\hat{d}(n) = \sum_{k=0}^{K}\hat{b}_k(n+1)x(n-k) - \sum_{l=1}^{L}\hat{a}_l(n+1)\hat{d}(n-l) \\ + \sum_{k=0}^{K}\sum_{l=1}^{L}\hat{c}_{k,l}(n+1)x(n-k)\hat{d}(n-l). \quad (19)$$

The a-posterior error signal is

$$e(n) = d(n) - \hat{d}(n). \quad (20)$$

Utilizing the idea of MSS scheme proposed in section 2 to the equation-error bilinear filter, we have a multi step-size monic normalization equation-error bilinear filter which is capable of reducing the biased estimates and has fast convergence performance.

IV. SIMULATION RESULTS

In this section, we compare the performance of our filters to that of the conventional equation-error filters with monic normalization scheme. The experiment results are ensemble average of 10 independent runs. For the purpose of smoothing the curves, the demonstrated learning curves are averaged over 1000 points.

A. Example 1: Linear System

The problem considered is that of estimating the parameters of a linear system governed by the equation

$$y(n) = 0.847x(n) - 0.423x(n-1) + y(n-1) - 0.5y(n-2). \quad (21)$$

The excitation signal $x(n)$ is white Gaussian with zero mean and unit variance. This setup makes $y(n)$ with a power close to unit. A white Gaussian noise which is independent to input signal and has variance 0.1 is added to the system. This setup was first used in [4]. We compared the proposed MSS-MNEE to the MNEE introduced in [4]. MNEE uses fixed step-size of 0.001. The MSS-MNEE selects the step-size from two values, 0.001 and 0.01, depends on the indication function $\varepsilon(n, N)$ defined in (12). The threshold employed in the experiment is 0.13. We have used the squared norm of the coefficient error vectors to evaluate the performance of the algorithms. The norm is defined as

$$\| V_L(n) \| = 10\log\frac{\sum_{k=0}^{1}(\hat{b}_k(n)-b_k)^2 + \sum_{l=1}^{2}(\hat{a}_l(n)-a_l)^2}{\sum_{k=0}^{1}b_k^{\,2} + \sum_{l=1}^{2}a_l^{\,2}}. \quad (22)$$

Figure 1 shows the evolution of $\| V_L(n) \|$. It is noted that our MSS-MNEE exhibits faster convergence behavior than the MNEE. Both algorithms have very close steady state performance which matches our analysis.

B. Example 2: Linear System

Consider another linear system governed by

$$y(n) = 4.1x(n) - 2.06x(n-1) + 0.8y(n-1) - 0.12y(n-2) \quad (23)$$

Signal $x(n)$ is white Gaussian with zero mean and variance 0.055. The additive noise is white Gaussian with variance 0.1. We have used the same setup for the step sizes as in Example 1. The threshold employed is 0.3. Figure 2 shows the evolution of $\| V_L(n) \|$. Again, our MSS-MNEE outperforms the MNEE with faster convergence behavior.

C. Example 3: Bilinear System

Consider a bilinear system governed by

$$y(n) = x(n) + x(n-1) + x(n-2) + 0.5y(n-1) - 0.5y(n-2) \\ + 0.3x(n)y(n-1) + 0.1x(n)y(n-2) \\ - 0.2x(n-1)y(n-1) - 0.2x(n-1)y(n-2) \\ + 0.1x(n-2)y(n-1) + 0.3x(n-2)y(n-2) \quad (24)$$

Signal $x(n)$ is white Gaussian with zero mean and variance 0.17. The additive noise is white Gaussian with variance 0.1. We have compared the MSS-MNEEBF to the MNEEBF with the same setup for the step sizes as in Example 1. The threshold employed is 0.3. Figure 3 shows the evolution of corresponding squared norm of the coefficient error vectors. The proposed MSS-MNEEBF clearly outperforms the MNEEBF with much faster convergence behavior.

V. CONCLUSIONS

This paper presented a multi step-size monic normalization equation-error linear filter. The algorithm enjoys fast convergence behavior and has the capability of removing biased estimates due to noisy observations. We also extended the idea to equation-error bilinear filter. Simulation results validated the usefulness of our algorithms.

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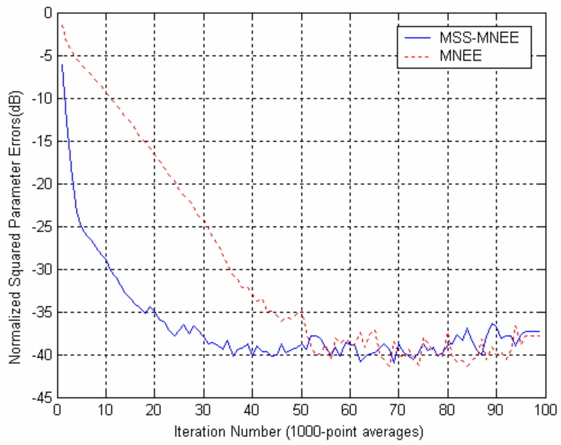

Figure 1. Plots of the normalized squared parameter errors of Example 1

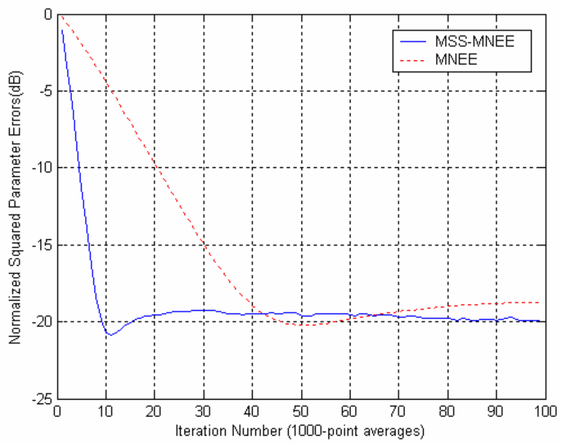

Figure 2. Plots of the normalized squared parameter errors of Example 2

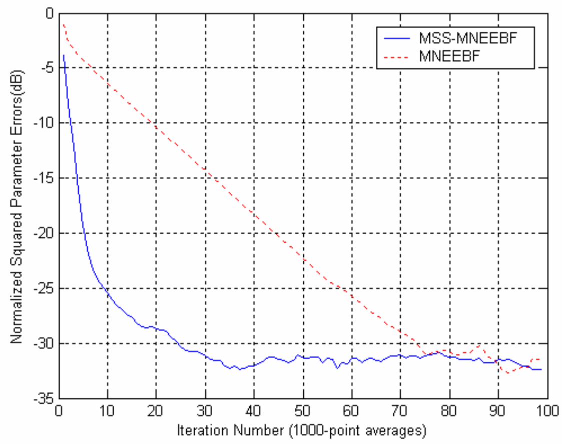

Figure 3. Plots of the normalized squared parameter errors of Example 3