Comparative Study of 1-D Chaotic Generators for Digital Data Encryption

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Abstract—**The security of transmitted digital information through a channel, against passive or active attacks, becomes more and more important. The use of a 1-D chaotic signal to mask useful information and to make it unrecognizable by an eavesdropper is a field of research in full expansion. In order to obtain such high-level security; chaotic generators used to encrypt digital data, must have desirable dynamical statistical properties such as: noise-like autocorrelation, cross-correlation, uniformity, attractors, etc. In this paper; first, we design, improve and simulate using Matlab some 1-D chaotic generators: Logistic map, PWLCM (PieceWise Linear Chaotic Map) map, Perturbed PWLCM map, Frey map, perturbed Frey map, (n, t)-tailed shifts map and perturbed one. Second, we show the importance of perturbed chaotic orbit with a comparative study of the dynamical statistical properties obtained under simulation and Nist tests**.

Index Terms—**Chaotic sequences generators, encryption, dynamical degradation, perturbed chaotic orbit, security.**

I. INTRODUCTION

Recently, chaos has been widely studied in secure communications [1, 2], and the idea of using digital chaotic systems to construct cryptosystems has been extensively studied since 1990s, and attracts more and more attention in the last years [3, 4]. Chaotic output signal of one dimensional chaotic generator is used for both confusion and diffusion operations in a cryptosystem. In order to be used in all applications, chaotic sequences must seem absolutely random. Therefore, we need a digital chaotic generator with good cryptographic properties, such as: balance on {0, 1}; long cycle-length; high linear complexity; δ-like autocorrelation; cross-correlation near to zero; desired attractor (with free space).

In [3,5], the authors show that logistic map don't verify the randomness properties. In [2,6], we show some good chaotic maps for DS-CDMA communication systems. In [4], we prove the efficacy of perturbed PWLCM with our encryption algorithm. Also in [7], an analysis of another parametric PWL chaotic map and its utilization for secure transmission system based on the CSK principle is provided.

In this paper, we study and improve some existing techniques used to generate chaotic signals with desired statistical properties.

An experimental comparison of dynamical systems properties for the generators under test is made using Matlab. Such study will permit us choosing the best chaotic generator to be used in a cryptographic system. Digital chaotic systems working in a 2^N-dimensional finite space, introduce deterministic quantization (round-off, truncated or ceiled) errors in discrete iterations, and then pseudo orbits become different from theoretical ones even after a short number of iterations. Consequently, dynamical degradations of digital chaotic signal occur. Furthermore, all pseudo orbits are possibly periodic and their cycle lengths are shorter than 2^N. To avoid the dynamical degradation, two techniques can be used: cascading multiple chaotic systems and pseudo randomly perturbing the chaotic system. The second solution is better. We will explain and use it. Then, we will do a comparative analysis of these methods in order to conclude by choosing the best one. This paper is organized as follows. The second paragraph is presenting some chaotic maps. The third part describes the proposed perturbation technique. The fourth part presents some simulation results. Finally, some conclusions are drawn.

II. PRESENTATION OF SOME CHAOTIC 1D GENERATORS

A. Logistic map

The Logistic Map defines one of the simplest forms of a chaotic process. Basically, this map, like any one-

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dimensional map, is a rule for getting a number from a number [3, 5, 8].

$$x(n) = F[x(n-1)] = p \times x(n-1) \times (1 - x(n-1)) \quad (1)$$

The secret key always includes the initial condition $x(0)$ and the control parameter p within the interval (0; 4]; $x(n)$ is in [0; 1]. The Logistic map has been well-studied in the past, and it had been demonstrated that the control parameter p should be greater than the accumulation point 3.569945672 in order to maintain the highly chaotic state.

B. PWLCM map

Due to the poor balance property of Logistic map, some implementations use the following Zhou's map with better balance property [8].

A piecewise linear chaotic map (PWLCM) is a map composed of multiple linear segments (limited breaking points are allowed).

$$x(n) = F[x(n-1)] = \begin{cases} x(n-1) \times \dfrac{1}{p} & \text{if } 0 \le x(n-1) < p \\ [x(n-1) - p] \times \dfrac{1}{0.5 - p} & \text{if } p \le x(n-1) < 0.5 \\ F[1 - x(n-1)] & \text{if } 0.5 \le x(n-1) < 1 \end{cases} \quad (2)$$

where the positive control parameter p ϵ (0, 0.5) and x(i) ϵ (0, 1).

C. (n, t) – tailed shifts map

A well known family of PWAM (piece Wise Affine Markov) that generate chaotic sequences are the (n,t)-tailed shifts map [6, 9].

$$M(x) = \begin{cases} ((n-t)x)_{(\mathrm{mod}(n-t)/n)} + \dfrac{t}{n} & \text{if } 0 \le x < \dfrac{n-t}{n} \\ (t(x - \dfrac{n-t}{n}))_{(\mathrm{mod}\, t/n)} \end{cases} \quad (3)$$

It is known that these maps are exact and have a uniform invariant probability density function [9].

D. Frey map

An approach to generate chaos for secure communications has been demonstrated by Frey. The codec uses a non linear filter with finite precision (N bits) in conjunction with its inverse filter [1, 2]. The non linear function used is the left circulate function suited to hardware implementation.

In figure 1, we present the structure of Frey generator. The generator scheme consists in a non linear function $F_{NL}(x)$ with a delayed feedback loop. The general equation is defined by the following relation:

$$e_u(n) = F_{NL}\{k_u(n) + \sum_{i=1}^{m} [G_i \times e_u(n - D_i) + s(n)]\} \quad (4)$$

where

$$F_{NL}(x) = \begin{cases} x & \text{if } x < 2^{N-1} \\ x \mod(2^N) & \text{otherwise} \end{cases} \quad (5)$$

The index u in equation (4) denotes an unsigned number. Also, all additions are modulo 2^N. These operators are assumed to be generally nonlinear operations.

The coder uses the following particular case of the system driven by equation (4):

m=2, $G_1 = 1$, $G_2 = 2$, $D_1 = 1$, $D_2 = 2$, as shown in figure 2. All operations inside the loop work in the unsigned number representation modulo 2^N. The delivered chaotic signal $e_u(n)$ is composed by 2^N quantized levels including the interval between $[0, 2^N - 1]$, and having the duration of T_{ch} seconds for each chip:

$$e_u(n) = \mathrm{mod}\left\{k_u(n) + \mathrm{mod}\left\{e_u(n-1) + lcirc\left[e_u(n-2)\right]\right\}\right\} \quad (6)$$

where

$$lcirc\left[e_u(n-2)\right] = \mathrm{mod}\left\{2e_u(n-2) + s_u(n)\right\} \quad (7)$$

and

$$s_u[n] = \begin{cases} 0 & \text{if } e_u(n-2) < 2^{N-1} \\ 1 & \text{otherwise} \end{cases} \quad (8)$$

The carry bit function $s_u[n]$ plays the role of a noise source that is correlated in a nonlinear way to the response $e_u(n)$. The input signal $k_u(n)$ plays in our application the role of an additional key, which does not allow an unauthorized eavesdropper to recover the generated signal. However, $k_u(n)$ can be used as information signal to encrypt.

Using equations (6) and (7) we can rewrite equation (6) as following:

$$e_u(n) = \mathrm{mod}\left[k_u(n) + e_u(n-1) + 2e_u(n-2) + s_u(n)\right] \quad (9)$$

where the system states are given by:

$$x_1(n) = e_u(n-1) \text{ and } x_2(n) = e_u(n-2)$$

In order to reduce the signal's mean power, and to make its amplitude independent of the number of levels (in fact, on N), the generated signal $e_u(n)$ is, first converted into a signed signal $e_s(n)$ (The index s means «signed») in the 2's complement in the 2^N representation set, $[C2, 2^N]$, andthen normalized by the maximum absolute value of the quantized levels. Hence, the effective generated signal is:

$$e_s^{'}(n) = \sum_n q_n \times p_{T_{ch}}(t - nT_{ch}) \qquad (10)$$

with

$$\frac{-2^{N-1}}{2^{N-1}} \le q_n < \frac{2^{N-1}-1}{2^{N-1}} \Leftrightarrow -1 \le q_n < 1 \text{ and}$$

$$p_{T_{ch}} = \begin{cases} 1 & \text{if} \quad 0 \le t < T_{ch} \\ 0 & \text{otherwise} \end{cases}$$

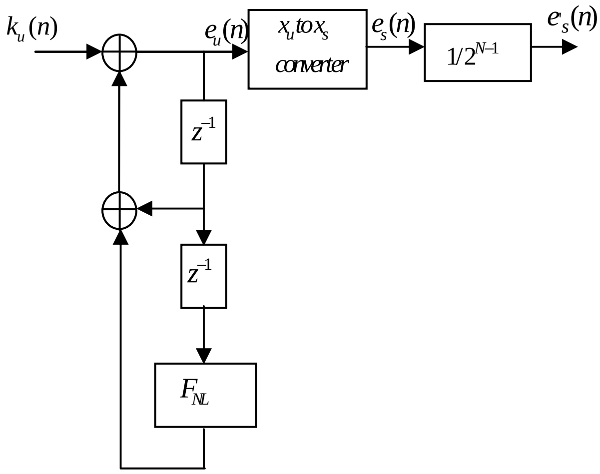


Fig. 1: Frey structure generator under test

III. PROPOSED PERTURBATION TECHNIQUE

A. On the Periodicity of chaotic orbit

Since digital chaotic iterations are constrained in a discrete space with 2^N elements, it is obvious that every chaotic orbit will eventually be periodic [10] and will finally go to a cycle with limited length not greater than 2^N (Fig. 2). Generally, each digital chaotic orbit includes two connected parts: $x_1, x_2, ..., x_{l,}$ and $x_l, x_{l+1}, ..., x_{l+n,}$, which are respectively called "transient branch" and "cycle". Accordingly, *l* and *n+1* are respectively called "transient length" and "cycle period", and *l+n* is called "orbit length". Conceptually, there is only a small number of limit cycles for all pseudo-orbits, which means the digital phase space will contrast to an attractor whose size is smaller than 2^N. Apparently, such a collapsed phase space will destroy the ergodicity of the continuous systems.

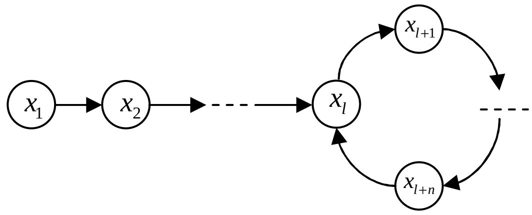


Fig. 2: A pseudo orbit of a digital chaotic system.

Then, some questions might arise: how to estimate the maximal and (mean) transient lengths, cycle periods and the number of limit cycles? Are the lengths large enough to ensure the dynamical properties of continuous chaotic systems? Unfortunately, as B.V. Chirkikov and F. Vivaldi demonstrated in [11], rigorous studies of such estimations (especially the average lengths) are notoriously difficult and the difficulties are actually from the lack of an ergodic theory for discrete chaotic systems. Since the theoretical analysis is not possible, statistical experiments are widely used to explore this issue.

B. Perturbed chaotic orbit

To improve the dynamical degradation, a perturbation based algorithm is used. Indeed, in this case as demonstrated in appendix, the cycle length is expanded and so a good statistical properties are obtained [12, 13]. Considering a one dimensional chaotic generator defined by:

$$x(n) = F[x(n-1)] \in [0,1] \qquad x(n) \in [0,1] \qquad n = 1,2,... \qquad (11)$$

Here, for computing precision *N*, each sample *x* can be represented as:

$$x(n) = 0.x_1(n)x_2(n)...x_i(n)...x_N(n) \qquad \begin{matrix} x_i(n) \in \{0,1\} \\ i = 1,2,...,N \end{matrix} \qquad (12)$$

The fundamental basis of the perturbing method consists in that the chaotic system run away from the cycle after iterations, i.e. the chaotic system having entered a cycle can be driven to leave the cycle immediately by a perturbance. The choose of the perturbance is done according to the following principles: it should have controllable long cycle length and uniform distribution; it should not degrade the good statistical properties of chaos dynamics, so the magnitude of the perturbing signal must be much smaller than that of the chaotic signal. The signal-to-noise ratio is defined as:

$$\text{SNR} = 10 \times \log\left(\frac{\text{maximum magnitude of chaotic signal}}{\text{maximum magnitude of perturbing signal}}\right) \qquad (13)$$

A suitable candidate for the perturbing signal generator is the maximal length LFSR because its generated sequences have the following advantages: 1) definite cycle length $(2^k - 1)$ (k is the polynomial degree); 2) uniform distribution; 3) delta like autocorrelation function; 4) easy implementation; 5) controllable maximum signal magnitude given by $2^{-N} \times (2^k - 1)$ when used in N-precision system.

The perturbing bit for every n clock time can be generated as following:

$$Q_{k-1}^{+}(n) = Q_k(n) = g_0 Q_0(n) \oplus g_1 Q_1(n) \oplus ... \oplus g_{k-1} Q_{k-1}(n) \quad \text{with } n = 0,1,2,... \tag{14}$$

Where $\oplus$ represents 'exclusive or', $g_0\, g_1 ... g_{k-1}$ are the tap coefficients of the primitive polynomial generator, and $Q_0\, Q_1 ... Q_{k-1}$ are the initial register values of which at least one is non zero.

The perturbance starts at $n = 0$, and the next ones occur periodically every Δ iterations (Δ is a positive integer), with $n = l \times \Delta$, $l = 1,2,\ldots$, The perturbed sequence is given by the equation (15):

$$x_i(n) = \begin{cases} F[x_i(n-1)] & 1 \le i \le N-k \\ F[x_i(n-1)] \oplus Q_{N-i}(n) & N-k+1 \le i \le N \end{cases} \tag{15}$$

where $F[x_i(n)]$ represents the ith bit of $F[x(n)]$.

The perturbance is applied on the last k bits of $F[x(n)]$.

When $n \ne l \times \Delta$, no perturbance occurs, and then $x(n) = F[x(n-1)]$.

The system cycle length is given by the following relation (see appendix):

$$T = \sigma \times \Delta \times \left(2^k - 1\right) \tag{16}$$

where σ is a positive integer. The lower bound of the system cycle length is:

$$T_{\min} = \Delta \times \left(2^k - 1\right) \tag{17}$$

IV. EXPERIMENTAL RESULTS

In order to verify the proposed method and compare cryptographic properties of different generators, some experiments were performed. The finite computing precision is $N = 16$ bits. Both initial conditions and control parameters are generated randomly. A large number of sampled values are simulated (100000 samples).

For all generators under tests, we found that the time domain variation of output signals, the spectrums (DFT), the autocorrelation functions and cross-correlation functions are clearly noise-like.

Figure 3 shows the first cycle length of perturbed PWLCM versus Δ with two different keys. The primitive polynomial is given by: $x^8 + x^6 + x^5 + x^4 + 1$ ($k = 8$). In this figure, the continuous line presents the minimal cycle length that is proportional to delta as verified in section 3. The experimental results of cycle length are always bigger or equal to the expected one. Figure 4 shows the cycle length of perturbed PWLCM (*) and the (n,t)-tailed shifts map (+) versus the perturbation degree k. Clearly, we can see that the experimental results verify the theoretical one. We found a very large cycle length with a lower bound given by equation (17) and represented by the continuous curve. The perturbation of Frey map gives a large cycle length ($>10^5$) with different values of k and Δ. In the other hand, to verify the balance property, a set of 500 sequences for each chaotic generator are computed, each sequence containing 100000 chaotic values. It is already verified that the balance property of PWLCM is better than the case of Logistic map [3], being slightly different from uniform one. Figures 5(a) and 5(b) show the percentages of zeros and ones of PWLM and perturbed one, and verify that the sequences based on the PWLCM map have a visibly larger percentage of ones.

Also for perturbed Frey map and (n,t)-tailed shifts map, the percentage of zeros and ones is closely the same. Then, perturbed maps have a much better uniformity relatively to the original maps. In figure 6(a), the cross-correlation of two sequences with identical initial conditions but slightly different control parameters, is given for the perturbed Frey map. And it is the same result for the other maps. Perturbed Frey generator gives close results as perturbed PWLCM, its attractor shown in figure 6.c contains more free space than that of the perturbed PWLCM and (n, t)- tailed shifts map and shown in figure 5.b and 5.d, and then redundancy code can be used.

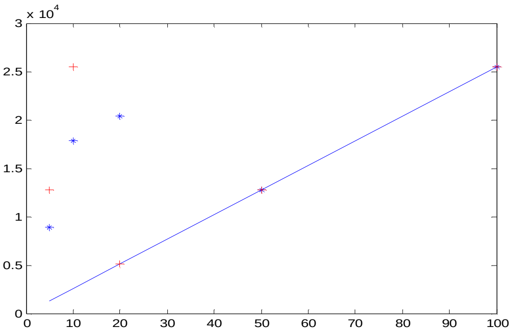


Fig. 3: Perturbed PWLCM cycle length versus Δ with two different keys; (-): Theoretical results. (*): Experimental results with an initial condition equal to 0.4 and control parameter equal to 0.3. (+): Experimental results with an initial condition equal to 0.6 and control parameter equal to 0.4.

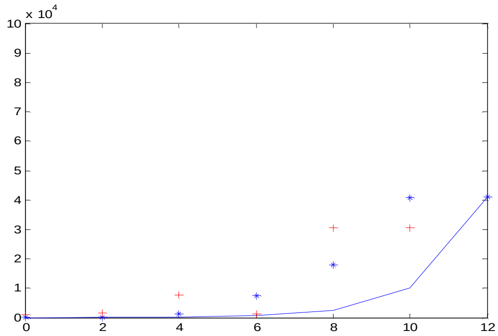
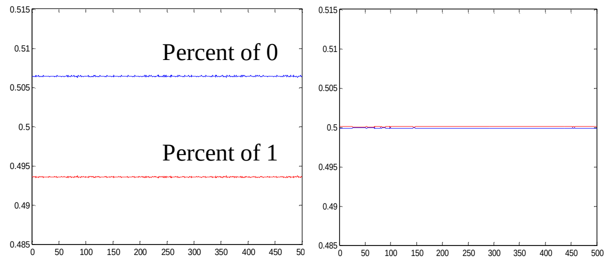
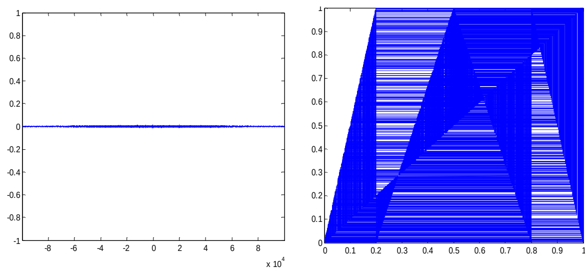


Fig. 4: Cycle length versus the perturbation degree k with Δ =10 for Perturbed PWLCM (*) and (n,t)-tailed shifts map (+).

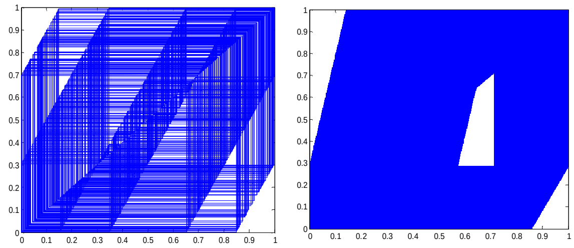


(a) (b)

Fig. 5: Balance property (a)PWLCM map, (b) Perturbed map .



(a) (b)



(c) (d)

Fig. 6: (a) Cross-correlation of perturbed PWLCM map, (b) and (c) Attractors of Perturbed PWLCM map, Frey map and (n,t)-tailed shifts map respectively.

Nist Statistical Test

Among the numerous standard tests for pseudo-randomness, a convincing way to show the randomness of the produced sequences is to confront them to the NIST Statistical Test (National Institute of Standards and Technology). The NIST statistical test [14] is a statistical package consisting of 16 tests that were developed to test the randomness of arbitrary long binary sequences produced by either hardware or software based cryptographic random or pseudorandom number generators. These tests focus on a variety different types of non-randomness that could exist in a sequence.

To verify our results, we employ the above tests suite to test the randomness of 100 binary sequences of length 38912. Note that the 100 binary sequences were generated with randomly selected secret keys. For each test, the default significance level α=0.01 was used, at the same time a set of P-values, which is corresponding to the set of sequences, is produced. Each sequence is called *success* if the corresponding P-value satisfies the condition P-value $\geq \alpha$, and is called *failure* otherwise.

Table1.

Performed tests and number of sequences passing each test in a sample of 100 sequences.

	Number of Passed Sequences					
Map name Name of Test	(n,t)-tailed shifts map	Perturbed (n,t)-tailed shifts map	Frey	Perturbed Frey	PWLCM	Perturbed PWLCM
Frequency	99	100	96	100	97	100
Block Frequency	100	100	96	99	96	99
Cumulative Sums	98	100	92	100	97	98
Runs	99	100	97	98	100	99
Rank	100	100	100	97	99	99
Discrete Fourier Transform	100	95	0	99	99	99
Serial1	97	100	98	100	98	100
Serial2	98	99	99	100	100	100
Approximate Entropy	97	99	94	97	97	100

In [5], the authors showed that logistic map does not satisfy the requirements as a good random source.
In Table 1, we show the results obtained with the application of NIST test on PWLCM, Frey and (n,t)-tailed shifts maps and the comparison with the results of perturbed ones.
As we can see, the perturbed maps are much better than original maps; that the sequences passed all the most of NIST tests. Then, the obtained results demonstrate the strength of the perturbation technique proposed. Since the perturbed maps pass all chosen NIST tests.

V. CONCLUSION

In a cryptographic system, the use of a good chaotic generator, with desirable dynamical statistical properties, is very important. We proved this idea in our previous paper [4]. In this paper, we have implemented six one dimensional chaotic generators (PWLCM, perturbed PWLCM, Frey, perturbed Frey, (n,t)-tailed shifts map and perturbed one). And we tested all generators with the application of NIST statistical tests. A comparison among these maps using standard criteria proves the efficiency of the perturbed technique. Indeed, both theoretical and experimental analyses demonstrated that perturbed maps have desired cryptography properties. As prospect of this work, we propose the use of 3 and 4 delays in Frey generator to obtain better dynamical statistical properties. And finally test another chaotic maps perturbed in the same way to find the best cryptographic properties.

APPENDIX

Theoretical analysis of expanded cycle length

Assume that the system has entered a period T state after n_0 iterations, i.e. $x_i(n+T) = x_i(n)$ $\left(\text{for n} > \text{n}_0\ ;\ 1 \le i \le N\right)$ and $n_1 = l_1 \times \Delta > n_0$ (l_1 is a positive integer), then $x_i(n_1+T) = x_i(n_1)$ for $1 \le i \le N$. If $T \ne l \times \Delta$ (l is a positive integer), the above equation implies $F[x_i(n_1-1+T)] = F[x_i(n_1-1)] \oplus Q_{N-i}(l_1)$ $(\mathit{for}\ N-k+1 \le i \le N)$. Since period T is defined as $F[x_i(n_1-1+T)] = F[x_i(n_1-1)]$ for $(1 \le i \le N)$, thus, $Q_{N-i}(l_1) = 0$ $(\mathit{for}\ N-k+1 \le i \le N)$. Because the initial sequences $Q_0, Q_1, ..., Q_{k-1}$ are not all zeros, the previous case will not occur. This implies that we only have $T = l \times \Delta$, which means $F[x_i(n_1-1+T)] \oplus Q_{N-i}(l+l_1) = F[x_i(n_1-1)] \oplus Q_{N-i}(l_1)$ $(\text{for } N-k+1 \le i \le N)$. As a result, we find $Q_{N-i}(l+l_1) = Q_{N-i}(l_1)$ (for $N-k+1 \le i \le N$). This implies: $l = \sigma\left(2^k - 1\right)$ where σ is a positive integer. Therefore the system cycle length is given by: $T = \sigma \times \Delta \times (2^k - 1)$ and $T_{\min} = \Delta \times (2^L - 1)$ is the lower bound of the system cycle length.

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