A Critical Review of Statistical Modeling of Digital Images

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Abstract-Images may contain large number of patterns generated by various stochastic processes. Defining and modeling these patterns represents one of the fundamental importance of generic image processing tasks, such as perceptual grouping, segmentation, compression, restoration, and recognition. This paper summarizes the various techniques that are used in statistical modeling of images. Statistical analysis of images reveals two interesting properties: (i) invariance of image statistics to scaling of images, and (ii) non-Gaussian behavior of image statistics, i.e. high kurtosis, heavy tails, and sharp central cusps. In this paper we review some recent results in statistical modeling of natural images that attempt to explain these patterns. Two categories of results are considered: (i) studies of probability models of images or image decompositions, and (ii) discoveries of underlying image manifolds while restricting to natural images.

Index Terms—Digital Image Processing, Statistical Image Models and Markov Random Model

I. INTRODUCTION

Many successful methods in image processing and computer vision rely on statistical models of images, and it is thus of continuing interest to develop improved models, both in terms of their ability to precisely capture image structures, and in terms of their tractability when used in applications. Constructing such a model is difficult, primarily because of the intrinsic high dimensionality of the space of images. Two simplifying assumptions are usually made to reduce model complexity. The first is Markovianity in which the density of a pixel conditioned on a small neighborhood is assumed to be independent from the rest of the image. The second assumption is homogeneity in which the local density is assumed to be independent of its absolute position within the image. The set of models satisfying both of these assumptions constitute the class of homogeneous Markov random fields (hMRFs).

Probabilities are associated with the definitions of image patterns and are even derived from deterministic definitions. In statistical image processing, we view an image x as a realization of a random field with joint probability density function (pdf) g(x). Solutions to problems such as segmentation, compression, and restoration rely on g(x); the more accurately it can be specified, better the solution. Of course, we rarely have enough information to specify the joint pdf exactly. The main objective of this paper is to review the state of art of existing realistic image model that approximates g(x) which allows efficient image processing of algorithms. Image models become popular and indispensable when people realized the vision problems, typically the segmentation problems which are ill-posed. Extra information is needed to account for regularities in real-world scenes. All early models assumed simple smoothness (sometimes piece wisely) of surfaces or image regions, and they were developed from different perspectives. For example, physically-based models, regularization theory, and energy functional, etc.

This paper addresses the state of art of existing statistical models. Also, this paper reviews seven classes of models namely: (1) Markov random field models [1-23], (2) Hierarchical models [24-33], (3) Shape based models [34-46], (4) Finite mixture models[47-49], (5) AM-FM models [50-52], (6) Context models [53], [54], (7) Autoregressive models[81], [84], [85]. Organization of this paper is as follows. Section II discusses the Markov random filed modeling technique. Hierarchical models such as mutiresolution (Wavelet domain) models and multiscale models are analyzed in section III. In section IV, the shape based models such as deformable models, active contours (snakes) and its variants are reviewed. The other models such as autoregressive models, finite mixture models, context models and AM-FM models are discussed in section V. Conclusion is provided in section VI.

II. MARKOV RANDOM FIELD MODEL

A. State of art of MRF Modeling Technique

Based on the nature of potential function, MRF models may be classified into: causal MRF, non-causal MRF, Gaussian MRF, Ising model, pots models and hidden MRFs. The tree diagram for the MRF model is shown in Fig.1. Non-causal Markov models are widely used in early vision applications for the representation of images in high-dimensional inverse problem. For most non-causal representations, the graph associated to the Markov model is the rectangular lattice equipped with the nearest (or second nearest) neighborhood system. Non-causal MRFs do not impose unwanted directionality effects. However implementation of this model is not straightforward.

Commonly used Non-causal Markov random fields are not in fact capable of representing the moderate-to-large scale-clustering present in naturally occurring images and can be time consuming to simulate, requiring iterative algorithms which can take hundreds of thousands of sweeps of the image to converge. However the causal MRF such as Pickard random field, the mutually compatible MRFs and Markov chain image model can approximate the non-causal MRFs.

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Fig.1 Tree Diagram of the MRF based Models

Pickard random fields are known to represent only a limited class of spatial statistics and generally yield directional artifacts in the image plane. On the other hand, accurate causal approximations of non-causal MRFs can be obtained by Markov Chain image model.

Gaussian MRFs give rise to linear estimators, but the basic homogeneous Gaussian MRFs are well known to allow noise cancellation only at the expense of over smoothing the object. Generalized Gaussian MRFs preserve edges better while maintaining convex energies [4]. However none of these priors can give rise to maximum a posteriori (MAP) estimators truly accounting for the presence of both homogeneous parts and edges in the objects. Using pairwise interaction piecewise Gaussian MRFs (PG MRFs) with a non-interacting Boolean line process this problem is solved [15]. Compound Gauss-Markov random field (CGMRF) is used to model images by preserving the discontinuity [7].

A hidden Markov process (HMP) is a discrete-time finite state homogeneous Markov chain observed through a discrete time memory less invariant system [18]. The image is characterized by a finite set of transition densities indexed by the states of the Markov chain. Unlike hidden Markov fields, a hidden Markov chain uses one-dimensional set of pixels by scanning the two-dimensional sets. HMC-based segmentation methods can be competitive in some particular situations, and they are much faster than the HMRF based ones. The partially hidden Markov models (PHMM) combines the power of using the past as context and the power of hidden states in modeling. It differs from conventional hidden Markov models (HMMs) by conditioning the transition probabilities and emission/output probabilities on the previously observed symbols. Ising model attempts to minimize the boundary length between objects, which results in very high estimates for the MRF class transition costs and, thus, a strong favor for smooth boundaries. A non-stationary Ising model, with different parameters in uniform regions of pure region than at places where objects mix, might be a promising starting point. There are also methods that estimate Gibbs parameters with pre computed derivatives of log-partition functions. These algorithms were used primarily for learning MRF models with pair cliques, such as Ising models and Potts models. It has the advantage of taking the observations directly into account. Moreover, the study of the case of the homogeneous isotropic Potts model gives reasons dissuading from using the mean field approximation on the marginal field.

B. Markov Random Field Model

Gibbs distributions are used to explicitly write the distributions of MRFs. A Gibbs distribution is any distribution which can be expressed in the form

$$g(x) = 1/Z \exp\{-\sum_{c \in C} V_c(x)\}$$
 (1)

Where Z is a normalizing constant also known as partition function, $V_c(..)$ is any function of a local group of points c, and C is the set of all such local groups. The key to the definition of the Gibbs distribution is the specification of these local groups of points. A local set of points, c, is called a clique if $\forall s, r \in c, s$ and r are neighbors. The clique associated with first and second order neighborhood system η is shown in Fig.2



Fig.2 Cliques Associated with Neighborhood Systems: (a) First Order Neighborhood $\eta 1$; (b) Clique Types of $\eta 1$; (c) Second Order Neighborhood $\eta 2$; (d) Clique Types of $\eta 2$

If Gibbs distributions are restricted to use functions of cliques included by the neighborhood system ∂s , the random field X will have the property that

$$\forall s \in S \qquad p(x_s | x, r \neq s) = p(x_s | x, r \in \partial s) \tag{2}$$

This is the fundamental property of an MRF. In fact, the Hammersley-Clifford theorem states that if X has a strictly positive density function, then X is a MRF if and only if the distribution of X has the form of Gibbs distribution.

The potential function $V_c(x)$ may be is defined as

$$V_{c}(x) = -\sum_{\{s,r\}\in C} b_{sr} \rho(\lambda | x_{s} - x_{r}|)$$
(3)

Where $b_{sr} > 0$, λ is a scaling parameter, $\rho(.)$ is a monotone increasing function. The efficiency of the model highly depends of the selection of $\rho(.)$. Various $\rho(.)$ studied by different authors can be classified as convex, non- convex and others like generalized Gaussian, scalable potential functions. Some of the potential functions are listed below.

Non- Convex Log Prior Distributions

Non-Gaussian MRFs are interesting because they can potentially model both the edges and smooth regions of images. The simpler Gibbs distributions is of the form

$$\log g(x) = -\sum_{\{s,r\}\in C} b_{sr} \rho(\lambda |x_s - x_r|) + cons \tan t \quad (4)$$

Where λ is a scaling parameter, and ρ is monotone increasing but not convex function. A typical function used by Blake and Zissreman [4] is

$$\rho(\Delta) = \min\{|\Delta|, T\}^2 \tag{5}$$

Where T is a variable threshold parameter. This function is shown in Fig.3 for T= 0.5. Notice that the function is quadratic near zero, but the flat region beyond the value of T.



Fig. 3 Non - convex cost function

Convex Log Prior Distributions

Convex functions have been considered in [4]. They chose the Huber function by the following formula

$$\rho(\Delta) = \begin{cases} \Delta^2 & \text{if } |\Delta| \le T \\ T^2 + 2T |\Delta - T| & \text{if } |\Delta| > T \end{cases}$$
(6)

For values greater than T, the linear region of this function also allows sharp edges, yet convexity makes the MAP estimate efficient to compute. This function is shown in Fig.4



Fig. 4 convex cost function

Generalized Gaussian Markov Random Field

This distribution may be written in the form of log likelihood function represented by:

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$$\log g(x) = -\lambda^{p} \left(\sum_{s} a_{sr} \left| x_{sr} \right|^{p} + \sum_{\{s,r\} \in C} b_{sr} \left| x_{s} - x_{r} \right|^{p} \right) + cons \tan t \quad (7)$$

Where b is a symmetric positive definite matrix, $a_{sr} = \sum_{r \in S} B_{sr}$

and $b_{sr} = -B_{sr}$. $1 \le p \le 2$, and λ is the parameter which is inversely proportional to the scale of x. The choice of p is critical in determining the character of the model. Large values of p discourage abrupt discontinuities while smaller values of p allow them. This function determines the tendency of neighboring pixels to be attached and plays a role analogous to the influence function of robust statistics.

Scalable Potential Functions

Bouman and Sauer [22] showed that the following function characterize all scale invariant functions:-

$$\Delta = x_i - x_j$$
, this cost function is shown in Fig. 5.

$$\rho(x_i, x_j, p) = |x_i - x_j|^p, p > 0$$
 (8)

Pickard's theory enables us to construct isotropic GRFs, it does not answer the question of the uniqueness of such random fields. John K. Goutsias [16] answers these questions systematically by developing a unified theory for the mathematical description of GRFs. He derived the local transfer function as a function of local neighborhood.



Fig. 5 Scale invariant cost function

In [12], Sridhar Lakshmanan and Haluk Derin proposed a class of GRF called multilevel logistic (MLL) distribution. The MLL distribution defined as follows: A parameter is associated with each clique type, except for single pixel cliques say β_k with clique type k. The potential function for all cliques of that type is then defined as follows:-

$$\mathbf{V}_{ck}(x) = \begin{cases} \beta_k & \text{if all } \mathbf{x}_{ij} \text{ in } \mathbf{c}_k \text{ are equal} \\ -\beta_k & \text{otherwise} \end{cases}$$
(9)

Where c_k denotes any clique type k. For the single pixel cliques, the potential function is defined as

$$V_{ck}(x) = \alpha_m \quad \text{if } x_{ij} = g_m \quad \text{for } c = (i, j) \tag{10}$$

Where α_m is a parameter associated with region m. By assigning the same potential function to all cliques of a certain type, independent of their positions in the image, it is implicitly assumed that the random variable X is homogeneous. The values of β_k influence the sizes and shapes of the resulting regions, while those of α_m influence the relative likelihood of each region type. Other model parameters such as: the gray levels g_m , the number of region types M and noise parameters such as variance are assumed known.

C. Hidden Markov Models

Hidden Markov models are mixture models in which the populations from one observation to the next are selected according to an unobserved finite state-space Markov chain [18]. We assume that $(X_n)_{n \in N}$ is a Markov chain, with each $X_n \in \{w_1, \ldots, w_k\}$ and with stationary transition probabilities.

$$c_{ij} = P[X_n = w_i, X_{n+1} = w_j]$$
(11)

This does not depend on "n". Thus, the initial distribution is given by

$$\prod_{i} = P[X = w_j] = \sum_{1 \le j \le k} c_{ij}$$
(12)

and the transition matrix $A = [a_{ij}]$ has entries

$$a_{ij} = P[X_{n+1} = w_j \setminus X_n = w_i] = c_{ij} / \sum_{1 \le j \le k} c_{ij}$$
(13)

III. HIERARCHICAL IMAGE MODELS

Markov random fields discussed in the previous section is efficient and powerful framework for specifying nonlinear interactions between features of the same nature or of a different one. They help in combining and organizing spatial and temporal information by introducing strong generic knowledge about the features to be estimated. When they are associated with the MAP criterion, they lead to the minimization of a global energy function which may exhibit local minima. This minimization is generally performed using deterministic or stochastic relaxation algorithms. Stochastic algorithms may be drastically time consuming while deterministic schemes often get stuck in local minima of the energy function.

In addition, it is known that hierarchical methods can improve significantly the convergence rate of iterative schemes. They are useful when the energy function to be minimized presents many local minima. It has indeed been conjectured that multiresolution analysis may, to certain extent, smooth the energy landscape. Deterministic relaxation schemes can then be used at coarse scales to get a good initial guess, which may be refined over increasing resolution. Thus combination of Markovian models and hierarchical methods such as Gaussian pyramids, wavelets decomposition gives consistent and tractable statistical models. Hence, in this section we will discuss the hierarchical models such as multiresolution and multiscale models. Tree diagram of the hierarchical image models is shown in Fig. 6.



Fig.6 Tree diagram of the hierarchical models

A. State of art of Hierarchical Image Models

Concept of hierarchical processing received an increasing attention from both the computer vision and signal processing communities [24-33]. Many multiresolution models have been developed to represent statistical dependence among image pixels, one such multiresolution model is based on HMM is proposed for wavelet coefficients in [25]. In that paper, wavelet coefficients across resolutions are modeled as HMM and resolution are represented by the time-like role in the Markov chain. If we view wavelet coefficients as special cases of features, the model in [26] considers features observed at multiple resolutions.

The wavelet transform nearly decorrelates many images and can be viewed as an approximate Karhunen-Loève transform (KLT). This basic property is exploited by early wavelet coders and wavelet denoising algorithms. Nevertheless, significant dependencies still exist between wavelet coefficients. Each statistical wavelet model in the literature focuses on a certain type of dependencies, which attempts to capture using a relatively simple and tractable model. These models are classified into following three categories [23]: 1) Interscale Models in which the magnitudes of wavelet coefficients in typical images are strongly correlated across scales. Consider a quadtree representation of wavelet coefficients, if a parent node has small magnitude, its children are very likely to be small too. This property is exploited in Shapiro's embedded zerotree wavelet (EZW) coder. Combining the self-similarity across scales with a clever scheme for set partitioning in hierarchical trees (SPIHT). Said and Pearlman developed an even better coder. The hidden Markov tree model (HMT) also captures the dependencies between a parent and its children. A hidden state is associated with each wavelet coefficient; conditioned on their hidden states, the coefficients are Gaussian, independent and identically distributed (iid). 2) Intrascale Models: Strong dependencies in the form of spatial clusters exist between wavelet coefficients inside each subband. Compression algorithms such as the morphological coder exploit the spatial clustering of these wavelet coefficients. The EQ coder models wavelet coefficients as independent generalized Gaussian distributed (GGD) with zero mean and slowly varying variance. Local statistics are estimated from the data. This model has recently found applications to denoising. 3) Composite Dependency Models: Both types of dependencies above may be combined [32].

Multiscale processing is an old but powerful idea. It is usually applicable whenever one wishes to implement an image processing algorithm that is iterative in nature and requires many successive updates. The basic principle is to construct an image pyramid and to start applying the procedure at the coarsest level on a very small version of the image. Upon convergence, the solution is propagated to the next finer level where it is used as starting condition. One then proceeds with this coarse-to-fine iteration strategy until one reaches the finest level of the pyramid which corresponds to the image itself. There are numerous examples of the application of this principle in the literature. Mallat et al [30] employ a quadtree like pyramid for unsupervised texture segmentation and Lovell et al [31] uses a multiscale relaxation algorithm applied to image classification.

B. Multiresolution and Multiscale Models

Histogram and log-histogram of the wavelet coefficients in one subband is shown in Fig.7. The dotted line is a generalized Gaussian approximation. The solid line is a two-component Gaussian mixture model fitted to the data. Although the generalized Gaussian density is a better fit, by using only two states in the Gaussian mixture model, one can achieve a close fit to the histogram. The Gaussian mixture model is not exact, but it allows simple and efficient algorithms, especially for capturing dependencies between wavelet coefficients.

The form for the marginal distribution of a wavelet coefficient w_i comes directly from the efficiency of the wavelet transform in representing real-world images: a few wavelet coefficients are large, but most are small. Gaussian mixture modeling runs as follows:-



Fig. 7 Histogram and log-histogram of the wavelet coefficients

Associate with each wavelet coefficient w_i an unobserved hidden state variable $S_i \in \{S, L\}$. The value of S_i dictates which of the two components in the mixture model generates

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 W_i . State S corresponds to a zero-mean, low-variance Gaussian. If we let

$$g(x,\mu,\sigma^2) = 1/\sqrt{2\pi}\sigma \exp\{-(x-\mu)^2/2\sigma^2\}$$
 (14)

Denote the Gaussian pdf, then we can write

$$f\left(w_{i}/S_{i}=S\right) = g\left(w_{i},0,\sigma_{S;i}^{2}\right)$$
(15)

State L, in turn, corresponds to a zero-mean, high-variance Gaussian

$$f\left(w_{i}/S_{i}=L\right) = g\left(w_{i},0,\sigma_{L;i}^{2}\right)$$
(16)

With $\sigma_L^2 > \sigma_s^2$. The marginal pdf f(w_i) is obtained by a convex combination of the conditional densities

$$f(w_i) = p_i^{S} g(w_i; 0, \sigma_{S;i}^{2}) + p_i^{L} g(w_i, 0, \sigma_{L;i}^{2})$$
(17)

The P_i^S and P_i^L can be interpreted as the probability that w_i is small or large (in the statistical sense), respectively.



Fig. 8. Illustrating the parent-child relationship between regions at a coarse-scale and a fine-scale segmentation of an image.

Wavelet coefficients have often been modeled as realizations from a zero-mean GGD. In fact, the GGD models the marginal densities of the wavelet coefficients more accurately than the Gaussian mixture, as shown in Fig. 7, especially in the tails of the distribution. However, the Gaussian mixture model discussed above can approximate the generalized Gaussian density arbitrarily well by adding more hidden states. Of course, as the number of states in the model increases, the model becomes more computationally complex and less robust. As can be seen in Fig. 7, Justin K. Romberg et al [24] are matching the marginal histogram very closely using only two states. One can think of this two-state mixture model as an approximation to the generalized Gaussian model. The parent child relationship is shown in Fig. 8 and a result of segmentation by multiscale method for three different scales (1 to 3) is shown in Fig. 9.



Fig. 9. (a) Actual image and (b)–(d) three different scales (1 to 3) of segmentation given by the multiscale segmentation algorithm

IV. SHAPE BASED MODELS

In image analysis, graphical models are most often referred to as Markov random fields. Much of the statistical literature in image segmentation has used Markov random field models, not for shape variables, but for the "true" image underlying the observed gray value image. Although these image models are well suited for the description of textures, they can represent only little a priori information about the shape of the displayed object and are most often limited to describe some smoothness in shape. This necessitates the need for shape based models. In this section shape based models such as deformable models, algebraic curves and active contours (or snakes) are discussed. The concept of deformable templates was introduced as a means of modeling spatial patterns/shapes [34]. They define the template as a prototype for a desired/ideal pattern and they associate a deformation mechanism with the template graph. Markov chain to describe the deformation mechanism of nodes in a grid is used in [35]. Max Mignotte et al [36] presented an original statistical classification method using a deformable template model to separate natural objects from man-made objects in an image provided by high resolution sonar. L.H. Staib and J.S. Duncan [37] for example, have used a parametric template model to locate the road boundary in radar images where the two straight, parallel edges of a road are parameterized. A similar approach for shape matching is proposed [38] which combines, in the same manner, both the available knowledge of the shape properties (as prior model) and an observation model (as likelihood model). Cootes et al. [39] uses deformable shape descriptors built from a training set of images. The deformations are modeled using linear combinations of the eigenvectors of variations from the mean shape, thus defining the shape class and allowing deformation reflecting the variations in the training set.

Sclaroff [40] use linear deformations equivalent to the modes of vibration of the original shape. However, these modes are based on a generic elastic model that is not likely to be representative of the real variations which occur in a class of shapes. The Bayesian fitness measure they proposed is dependent on a shape model, a gray-level foreground model, and a gray-level background model. Yongmei Wang, Lawrence H. Staib [41] proposed a unified framework for boundary finding, where a Bayesian formulation, based on prior knowledge and the edge information of the input image (likelihood), is employed.

Other shape based models are active contour or snakes, algebraic curves and polynomials, piecewise and local image models. A snake or active contour is a virtual object (living on the image plane) which can deform elastically (thus possessing internal energy), and which is immersed in a potential field (thus having external/potential energy), which is a function of certain features of the image. An algebraic curve is defined as zero set of polynomials in two variables. Other related representations such as the quadric surface (e.g., cones, ellipsoids, hyperboloids, etc.) admit both a parametric and an implicit form. Piecewise image model and local image models are based on well-defined local image characteristics. Piecewise image model (PIMs), which model images as every where obeying a certain property (such as constancy or linearity) in a piecewise manner, and local image model (LIM's), which characterize images as obeying a certain property (such as monotonically or convexity) over every sub-image of specified geometry.

From a Bayesian perspective, snakes are interpretable as maximum a posteriori (MAP) contour estimators, where the internal and external energies are associated with the a priori probability function (or prior) and the likelihood function (observation model), respectively[44- 46]. The same is true for deformable templates where the prior biases the estimate toward the template shape. Let v be the contour to be estimated on the observed image I, a Bayesian approach requires the following steps:

1) Specification of a prior p(v) capturing a priori information/ constraints on v;

2) Derivation of a likelihood function p(I/v) modeling the observed image conditioned on the true contour;

3) Specification of a loss function L(v, v') measuring how much loss is incurred by an estimate v' when the true contour is in fact v.

Once these elements are in place, an optimal Bayes rule is the function of the data, called an estimator, and denoted $\dot{V}(I)$ that minimizes the a posteriori expected loss

$$v(I) = \arg\min_{v} \int L(v, v') p(v/I) dv$$
(18)

Where p(v/I) is the a posteriori probability density function obtained via Bayes law.

V. OTHER STATISTICAL IMAGE MODELS

In this section, statistical image models such as finite mixture model, AM-FM model, context model and autoregressive models are discussed.

A. Finite Mixture Models

In finite mixture modeling each pixel of observed image is a sample from a mixture of distributions. Assume that the data $X=(x_1,\ldots,x_T)$ are drawn independently and generated by a mixture density model[47-49]. The likelihood of the data is given by the joint density

$$p(x/\theta) = \prod_{i=1}^{r} p(x_i/\theta)$$
(19)

Where the mixture density is

$$p(x_i/\theta) = \sum_{k=1}^{K} p(x_i/C_k, \theta_k) p(C_k)$$
(20)

Where $\theta = (\theta_1..., \theta_K)$ are the unknown parameters for each $p(x/C_k, \theta_k)$, called the component densities. C_k denotes the class k and it is assumed that the number of classes, K, is known in advance.

B. AM-FM Models

The solutions of the reaction diffusion partial differential equations are in the form of amplitude modulated and frequency modulated (AM-FM) function which is computable model. AM-FM functions which admit non-stationary amplitude and frequency modulations. A 2-D AM-FM $\mu(x, y)$ function takes the form

$$\mu(x, y) = a(x, y) \exp[j\psi(x, y)]$$
(21)

Where a(x, y) and $\psi(x, y)$ are arbitrary real-valued functions [50, 51]. Without loss of generality, we assume that a(x, y) > 0. The AM and FM components of interest that are contained in $\mu(x, y)$ in (21) are the instantaneous amplitude a(x, y) and the instantaneous frequency vector $\nabla \psi(x, y) = [u(x, y), v(x, y)]^T$. The functions u(x, y)and v(x, y) are the horizontal and vertical instantaneous frequencies of $\mu(x, y)$.

Given $\mu(x, y)$, the AM and FM functions may be calculated using the straightforward demodulation formulae

$$\nabla \psi(x, y) = \operatorname{Re} \left[\nabla \mu(x, y) / j \mu(x, y) \right]$$
(22)

and

$$a(x, y) = |\mu(x, y)| \tag{23}$$

These formulae yield exact solutions at all points where $\mu(x, y) \neq 0$. The frequency equation (22) may be interpreted as a specialized instance of a Poletti equation; its use is motivated by the fact that the exponential function in (21) is invariant under differentiation. Oriented, highly repetitive images such as fingerprints are well suited for AM-FM modeling because they are dominated by nonstationary, locally narrowband processes and contain locally quasiperiodic patterns.

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C. Context Models

A context model assumes that the distribution of the current symbol depends only on the context in which it occurs [53],[54]. That is, given its context, the current symbol is conditionally independent of past data symbols. Associated with a given context model is a finite set of contexts or conditioning events C, a context determining rule or function that assigns a context C to each data sequence x_1, \ldots, x_i , and a finite set of pdfs, one for each context. Each context is characterized by a finite subset of past variables and a subset of their possible outcomes. Associated with each context C is the conditional pdf p(./C), and the average encoding rate is approximately given by

$$H(X/C) = -\sum_{x} p(x/C) \log_2 p(x/C)$$
(24)

When X appears in context C, and the overall average rate is approximately

$$H(X/C) = \sum_{c \in C} p(C)H(X/C)$$
(25)

Where P(C) is the probability of context C occurring. It is useful to note that this approach will achieve rate H(X/C)even if the current symbol is not conditionally independent of the past, given the contexts. Usually the number of contexts, i.e., the cardinality of the set C, is much less than the number of possible past sequences. If the pdfs, P(./C), are a priori unknown, they can usually be estimated by maintaining counts of symbol occurrences within each context or by estimating the parameters of an assumed pdf.

D. Autoregressive Model

Here the image is modeled as autoregressive process. The model parameters are estimated by solving Yule-walker equations or by using Kalman filters. Assume that the original image, S (m; n); is modeled by a 2-D autoregressive (AR) process [81], [84], [85] with a non-symmetric half plane (NSHP) region of support

$$S(m,n) = \sum c_{ij} S(m-i, n-j) + w(m,n)$$
(26)

Where c_{ij} are the model coefficients, and w(m; n) is a zero-mean white Gaussian random field with finite variance which drives the process.

VI. CONCLUSION

Among the various image models Markov random field model is widely used technique. A variety of distinct models exists within the class of MRFs, depending on the choice of potential function that assigns cost differences between neighboring pixels. Markov random field models are efficient and powerful framework for specifying nonlinear interactions between features of the same nature or of a different one. They help to combine and organize spatial and temporal information by introducing strong generic knowledge about the features to be estimated.

When they are associated with the MAP criterion, they lead to the minimization of a global energy function which may exhibit local minima. This minimization is generally performed using deterministic or stochastic relaxation algorithms. Stochastic algorithms may be drastically time consuming while deterministic schemes often get stuck in local minima of the energy function. In addition, it is known that hierarchical methods can improve significantly the convergence rate of iterative schemes. They are useful when the energy function to be minimized presents many local minima. It has indeed been conjectured that multiresolution analysis may, to certain extent, smooth the energy landscape. Deterministic relaxation schemes can then be used at coarse scales to get a good initial guess, which may be refined over increasing resolution. Thus, combination of Markovian models and hierarchical methods such as Gaussian pyramids, wavelets decomposition gives consistent and tractable statistical models.

Much of the statistical literature in image segmentation has used Markov random field models, not for shape variables, but for the "true" image underlying the observed gray value image. Although these image models are well suited for the description of textures, they can represent only little a priori information about the shape of the displayed object and are most often limited to describe some smoothness in shape. Shape based models such as deformable templates, algebraic curves and active contours are used to mitigate this problem.

Shape based models typically consider only global or local deformations. While global templates involve large structural interactions and contain less parameters to be optimized, these global parameters cannot exercise local control along the contour and their physical meaning are sometimes obscure. In contrast, local models such as snakes contain more parameter and exert local control, but they are ill-suited incorporation of global contour model. Hence a model is needed for representing any arbitrary shape, accounts for global changes due to rigid motions, and retains ability for local control. Which may be achieved by means of the contour model is based on a stable and regenerative shape matrix which is invariant and unique under rigid motions. Combined with the local characteristics of the Markov random field to model local deformations, this yields prior distribution that exerts influence over a global model while allowing for deformations.

Markov random filed models are more suitable for texture kind of images, where as hierarchical models provide faster computational methods, and shape based models are much useful when there is structural information along with the statistical data is available for modeling. If the probability density function cannot be represented by single distribution alone then finite mixture model is useful. If modeling is based on the context especially for compression kind of applications then context modeling is appropriate. AM-FM models are employed for fingerprint kind of images.

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