Enhancing High-Frequency Order Placement Strategies with Fuzzy Logic and Fuzzy Inference

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Abstract — **The problem of optimal order execution has been a main concern for financial trading and brokerage firms for decades. The idea of executing a client's order to buy or sell a pre-specified number of shares at a price better than all other competitors seems intriguing. This paper introduces a system that utilises fuzzy logic in order to capture the current market condition generated by the accumulation of momentum. The proposed fuzzy logic momentum analysis system outperforms the traditional systems used in industry, which are often based on executing orders dependent on the weighted average of the current volume. The system proves that, on average, it increases profitability on orders on both the buy and sell sides.**

Index Terms — **Fuzzy Logic, High Frequency Trading, Momentum Analysis, Order Execution.**

I. INTRODUCTION

High frequency trading holds a rapidly growing interest both for researchers and financial investment entities. Finding better order execution rates is an intriguing problem. For brokers trading large orders, the effect of order size and the market's trend and volatility are crucial for order scheduling [10]. The cumulated order quantity of these institutional traders usually represents a big proportion of the daily trading volume, requiring sophisticated order splitting mechanisms to reduce market impact. This paper proposes a new framework for high frequency order execution using a novel way of momentum analysis which makes use of fuzzy logic reasoning mechanisms. The suggested order placement algorithm also considers the market's intraday volatility to minimize trading costs.

The modelling of financial systems continues to hold great interest not only for researchers but also for investors and policymakers. Many of the characteristics of these systems, however, cannot be adequately captured by traditional financial modelling approaches. Financial systems are complex, nonlinear, dynamically changing systems in which it is often difficult to identify interdependent variables and their values. However, this involves the implementation of a system that considers the whole price formation process from a different point of view. Financial brokers profit from executing clients' orders of buying and selling of certain

amounts of shares at the best possible price. Many mathematical and algorithmic systems have been developed for this task [7], yet most of them can not overcome a standard volume based system.

Time series models were first combined with fuzzy theory [20]-[21], resulting in fuzzy time-series, which is the fundamental framework of all of the investment systems. These authors detail five steps for such a system:
1. Definition and partition the universe of discourse.
2. Definintion the fuzzy sets.
3. Fuzzifying the observations.
4. Establishing the fuzzy relationships.
5. Forecasting and defuzzifying the results.

Researchers creating stock trading systems have implemented many variations of this model, of which the key adaptation primarily concerns the selection of appropriate observations, the definition of the fuzzy relationships, and the particular inference system used for forecasting.

Most systems use well-documented technical indicators from financial theory for their observations. For example, [9] used three technical indicators in their stock trading system: the rate of change, the stochastic momentum indicator and a support-resistance indicator that is based on the thirty-day price average. A convergence module then maps these indices as well as the closing price on to a set of inputs for the fuzzy system, thus providing a total of seven inputs. In some cases, such as the rate of change, an indicator maps to a single input. However, it is also possible to map one indicator to multiple inputs. Four levels of quantification for each input value are used: small, medium, big and large. Mamdani's form of fuzzy rules [18] can be used to combine these inputs and produce a single output variable with a value between 0 and 100. Low values indicate a strong sell, and high values a strong buy. The system is evaluated using three years of historical stock price data from four companies with variable performance during one period and employing two different strategies (risk-based and performance-based). In each strategy, the system begins with an initial investment of $10,000 and assumes a constant transaction cost of $10. Similarly, tax implications are not taken into consideration. The resulting system output is shown to compare favourably with stock price movement, outperforming the S&P 500 in the same period.

The application presented in this study differs from the above, as it introduces a fuzzy logic-based system for momentum analysis [17]. The system uses fuzzy reasoning to analyse the current market conditions according to which a certain equity's price is currently moving. This is then used as a trading application. First, the membership functions were decided by the expert-based method but then later optimized using ANFIS to further improve the trading performance.

II. FUZZY LOGIC

Fuzzy logic is a field of artificial intelligence that extends conventional (Boolean) logic to handle concepts of partial truths, i.e. truth values that are neither absolutely true nor absolutely false. This means that fuzzy logic can be used for approximate reasoning rather than exact reasoning. The method used by fuzzy logic is similar to that of how the human brain performs its reasoning process, and it is built on the fact that most modes of human reasoning, especially common sense reasoning, are approximate in nature. The essential characteristics of fuzzy logic are as follows:

- Exact reasoning is a limiting case of approximate reasoning.
- Decisions or conclusions are a matter of degree.
- Any logical system can be fuzzified.
- Linguistic values are used to express knowledge as a collection of flexible variables.
- Process of propagation of these variables is known as defuzzification.

Fuzzy logic has been used in control systems that incorporate degrees of uncertainty due to its ability to mimic the human decision-making process. A common example from a financial point of view is making the decision of investing in a certain development. An investor wants to buy some shares for investment. He spots a new starting company that is offering a niche product (e.g. high tech devices). The investor concludes that this is a good investment opportunity. However, the action of choosing the right investment (number of shares) depends upon various factors, for example, the current market value of shares of current well performing companies as opposed to the new company, if there is a future potential in the type of technology the company is offering, and the expected reward from the investment. Hence, the investor takes the decision to invest a certain amount of money based on all the factors possible. The investor would not invest a large sum of money in a new untested product or company, nor would he like to miss the opportunity to invest in such a new and promising company. This reasoning all takes place subconsciously in the human brain. This is what fuzzy logic attempts to mimic — human common sense. Therefore, fuzzy logic deals with such uncertainty in decision making by considering all of the inputs and then defuzzifying them according to a predefined set of rules, and finally producing an output comprising a final decision of the amount if investment.

Zadeh [24] was the first to introduce fuzzy sets by proposing the use of a paradigm shift that first gained acceptance in the Far East. The ideas proposed by [24] were very successfully implemented and later consequently adopted worldwide. Fuzzy sets are an extension of classical set theory and are commonly employed in fuzzy logic. Usually, the membership of elements in relation to a set is assessed using binary terms, that is, an element either belongs to the set or not. On the other hand, fuzzy set theory makes the gradual assessment of the membership of elements in relation to a set possible. This is described with the aid of a membership function that is valued in the real unit interval [0,1]. A fuzzy set on a classical set X is defined as

$$\tilde{A} = \{(x, \mu_A(x)) | x \in X\}. \tag{1}$$

The membership function $\mu_A(x)$ quantifies the grade of membership of the elements x to the fundamental set X. Any element that is mapped to the value 0 means that it is not included in the given set. On the other hand, 1 describes a fully included member. Values that fall between 0 and 1 characterise the fuzzy members.

III. FUZZY INFERENCE

A fuzzy inference system is a rule-based fuzzy system that can be seen as an associative memory and is comprised of five components:

- A rule base which consists of the fuzzy if-then rules.
- A database which defines membership functions of the fuzzy sets used in the fuzzy rules.
- A decision-making unit which is the core unit and is also known as the inference engine.
- A fuzzification interface which transforms crisp inputs into degrees of matching linguistic values.
- A defuzzification interface which transforms fuzzy results into crisp output.

Many types of fuzzy inference systems have been proposed in literature [8]. However, in the implementation of an inference system, the most common is the Sugeno model [22], which makes use of if-then rules to produce an output for each rule. Rule outputs consist of the linear combination of the input variables as well as a constant term; the final output is the weighted average of each rule's output. The rule base in the Sugeno model has rules of the form

$$IF \ x \ is \ A_1 \ and \ y \ is \ B_1 \ , \ THEN \ f_1 = p_1 x + q_1 y + r_1 \tag{2}$$

$$IF \ x \ is \ A_2 \ and \ y \ is \ B_2 \ , \ THEN \ f_2 = p_2 x + q_2 y + r_2 \tag{3}$$

where x and y are predefined membership functions, A_i and B_i are membership values, and p_i , q_i and r_i are the consequent parameters. When we calculate the equation of first-order Sugeno model [22], the degree of membership variable of x_1 in membership function of A_i are multiplied by the degree of membership variable of x_2 and in membership function B_i, and the product is weight W_i. Finally, the weighted average of f_1 and f_2 is deemed the final output Z, which is calculated as

$$Z = \frac{W_1 \cdot f_1 + W_2 \cdot f_2}{W_1 + W_2}. \tag{4}$$

In the case of designing a fuzzy system for financial modelling, one should opt to use the model introduced by [18], which is based on linguistic variables and linguistic output.

A. Defuzzification of Output

In many instances, it is desired to produce a single crisp output from a fuzzy inference system (FIS); this is called defuzzification. There are various types of defuzzification [16]:

Maximum Defuzzify: This method finds the mean of the maximum values of a fuzzy set as the defuzzification value. It is important to note that this does not always work well due to the presence of x ranges where the y value is constant at the maximum value and other places where the maximum value is only reached for a single x value. In this case, the single value is given too much weight in the defuzzified value.

Moment Defuzzify: This represents the fuzzy set by an inference that produces a floating point. The first moment of area of a fuzzy is calculated on the y axis. The set is further subdivided into different shapes by breaking down each point in the set. This produces rectangular, triangular and trapezoidal membership functions. The centre of gravity (moment) and area of each subdivision is then calculated using respective formulas, depending on the shape.

Center of Area: This differs from the above two methods in the sense that half of the area under the fuzzy set is located on each side of the centre of the defuzzified function. In other words, a mid-point in the distribution exists where the areas on both the left and right side are equal to one another.

Once the process of defuzzification is complete, a fuzzy decision follows. A fuzzy decision is a decision-making model that is suitable for fuzzy environments and involves the characterisation of the object function and constraints such as their membership functions, the intersection of fuzzy constraints and fuzzy objection function. However, the choice of membership functions can be a difficult task.

B. Deciding the Membership Functions

There are various approaches that can be taken to determine the shape of the membership functions. This depends on many factors such as the problem domain, the number of variables and the interdependability amongst those variables. These can be based on direct methods of inquiry made from observations in the problem domain and corrected using various methods, through which one attempts to eliminate the casual and systematic deformations affecting the membership functions. There are three ways of determining membership functions [5]:

Automatic Methods: These are used when no expert is available or in the case when there is a large amount of data that can be automatically processed. So far, the most common approaches are based on neural networks and/or on genetic algorithms in order to obtain and modify the membership functions. This happens in three steps:

1. Choice of various membership functions, and testing them while comparing the results of all approaches.
2. Adjustment of these primary functions using optimisation.
3. Once membership functions have been chosen and the parameters optimised, the system will run on the selected set of optimised membership functions.

Statistical Methods: The data expressed in the form of frequency histograms or other probability curves are used as a reference to create the membership functions. It must be noted that membership functions are not probability distributions. There are various conversion methods, and each one has its advantages and disadvantages. Some of these statistical methods include the "Yes-No" method and the set estimate proposed by [23].

Expert-Based (Psychological) Methods: These methods are based on human knowledge acquisition and are performed using the direct natural extraction of membership functions by interviewing experts in the problem domain. This leads to drawing the membership curve appropriate to the given problem, according to the expert's opinion. As there is an infinite array of possible membership functions for a given problem, the choice is often restricted to a certain predefined set of membership functions.

Fuzzy Logic provides a reasoning-like mechanism that can be used for decision making. However when combined with a neural network architecture, the resulting system is called a neuro-fuzzy system. Such systems are used for optimisation since they combine the reasoning mechanism that fuzzy logic offers together with the pattern recognition capabilities of neural networks.

IV. Adaptive Neuro Fuzzy Inference System

The ANFIS [15] is an adaptive network of nodes and directional links with associated learning rules. The approach learns the rules and membership functions from the data [22]. It is called adaptive because some or all of the nodes have parameters that affect the output of the node. These networks identify and learn relationships between inputs and outputs, and have high learning capability and membership function definition properties. Although adaptive networks cover a number of different approaches, for our purposes, we will conduct a detailed investigation of the method proposed by [16] with the architecture shown in Figure 1.
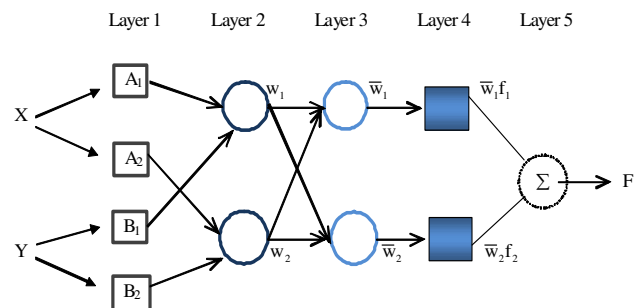

Figure 1: ANFIS architecture for a two rule Sugeno system

As mentioned above, ANFIS is used later in this chapter to optimise the membership functions of the Fuzzy Logic Momentum Analysis System to produce a further improved system that provides higher performance. ANFIS can also be used to design forecasting systems [2].

The circular nodes have a fixed input-output relation, whereas the square nodes have parameters to be learnt. Typical fuzzy rules are defined as a conditional statement in the form:

$$\text{If } X \text{ is } A_1, \quad \text{then } Y \text{ is } B_1 \ . \qquad (5)$$
$$\text{If } X \text{ is } A_2, \quad \text{then } Y \text{ is } B_2 \ . \qquad (6)$$

X and Y are linguistic variables; A_i and B_i are linguistic values determined by fuzzy sets on the particular universes of discourse X and Y respectively. However, in ANFIS we use the 1st-order Takagi-Sugeno system [22], which is described in Equation (2) and (3). We briefly discuss the 5 layers in the

following:

1. The output of each node in Layer 1 is:

$$O_{1,i} = \mu_{A_i}(x) \qquad for \ i = 1,2$$
$$O_{1,i} = \mu_{B_{i-2}}(x) \qquad for \ i = 3,4 \qquad (7)$$

Hence, $O_{1,i}(x)$ is essentially the membership grade for x and y. Although the membership functions could be very flexible, experimental results lead to the conclusion that for the task of financial data training, the bell-shaped membership function is most appropriate [1]. We calculate

$$\mu_A(x) = \frac{1}{1 + \left|\frac{x - c_i}{a_i}\right|^{2b_i}} \ , \qquad (8)$$

where a_i, b_i, c_i are parameters to be learnt. These are the premise parameters.

2. In Layer 2, every node is fixed. This is where the t-norm is used to 'AND' the membership grades, for example, the product:

$$O_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(y), \quad i = 1,2 \ . \qquad (9)$$

3. Layer 3 contains fixed nodes that calculate the ratio of the firing strengths of the rules:

$$O_{3,i} = \overline{w_i} = \frac{w_i}{w_1 + w_2} \ . \qquad (10)$$

4. The nodes in Layer 4 are adaptive and perform the consequent of the rules:

$$O_{4,i} = \overline{w_i} f_i = \overline{w_i}(p_i x + q_i y + r_i) \ . \qquad (11)$$

The parameters (p_i, q_i, r_i) in this layer are to be determined and are referred to as the consequent parameters.

5. In Layer 5, a single node computes the overall output:

$$O_{5,i} = \sum_i \overline{w_i} \, f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \ . \qquad (12)$$

This is how the input vector is typically fed through the network layer by layer. We then consider how the ANFIS learns the premise and consequent parameters for the membership functions and the rules.

V. FUZZY LOGIC MOMENTUM ANALYSIS SYSTEM

Creating a fuzzy inference system to detect momentum is a complex task. The identification of various market conditions has been a topic subject to various theories and suggestions [14]. This study proposes a fuzzy inference system that categorises the market conditions into seven categories based on price movement, using the current volume to determine the participation rates (PR) of the trading system each time [17]. The participation rate is the amount of volume that will be traded at each instance.

The first step in designing the Fuzzy Logic Momentum Analysis System (FULMAS) involves defining the "market conditions" that the fuzzy system has to identify. The following seven market conditions are used to cover all possible movements of the price series:

- Rallying
- Strong up
- Slightly up
- Average
- Slightly down .

- Strong down
- Crashing

These conditions are considered linguistic values for the fuzzy logic system, and they will be used to determine the current state of the price formation and its momentum. As momentum builds, the system considers the previous x amount of ticks and performs an inference procedure by adding all of the movements of the current price to the previous price in order to determine whether the general trend has been up or down after x points. The momentum is detected as displayed in the pseudo-code in Listing 1.

```
BEGIN
P = Price;
K = 0;
Start data feed (i);
REPEAT
  if P(i) > P(i-1)
    then k(i) = k + 1;
  else if P(i) < P(i-1)
    then k(i) = k - 1;
  else
    then k(i) = 0;
END
```

Listing 1: Calculating momentum

P_i denotes the current price, P_{i-1} is the previous price and k_i is a fluctuating counter that goes up or down according to the movement of the price. Whenever price goes up, it adds 1, and when the price goes down, it subtracts 1. Hence, this can be used to identify market conditions for x amount of ticks (where tick is a price observations), where if the market is moving strongly upwards, it will be detected by having more +1 than -1 or 0. This can be explained in the following equation:

$$Momentum(x) = \sum_{i=1}^{x} k_i \ . \qquad (13)$$

For example, if we want to detect the momentum of the last 100 ticks, we add all the up, down fluctuations and then feed the resulting number to the fuzzy system, which would lie somewhere in the membership functions. The choice of triangular membership functions was made after using the expert based method, where it was suggested that triangular membership functions should be used due to their mathematical simplicity. Triangular shapes require three parameters and are made of a closed interval and a kernel comprised of a singleton. This simplifies the choice of placing the membership functions. The expert merely has to choose the central value and the curve slope on either side.

The same procedure is applied for calculating the linguistic variable "volatility", where the linguistic values are as follows:

- Very high
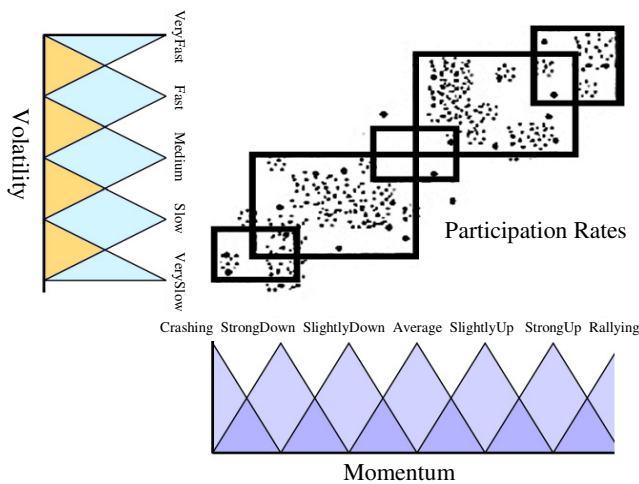- High
- Medium
- Low
- Very low .

Figure 2: Extracting fuzzy rules

The fuzzy logic system considers both market momentum and volatility. It generates the rules and then takes a decision based on the amount of market participation. This is illustrated in Figure 2.

VI. EMPIRICAL ANALYSIS

The main objective of the Fuzzy Logic Momentum Analysis System (FULMAS) implemented in this chapter is to outperform the industry standard volume system that has been used by brokerage firms to execute large orders of buying or selling a certain stock. Although many systems have used quantum modelling and analysis to determine the various participation rates (PR), they usually fail to outperform the standard volume system in the long term [4]. In particular, FULMAS will be applied to determine the PR in the market according to the current momentum. For example, for a buy order, it is preferable to increase the PR when the price is low and to decrease the participation when the price is high.

Experiments in this study have been carried out on high-frequency tick data obtained from ICAP plc of both Vodafone Group plc (VOD) and Nokia Corporation (NOK). A very important characteristic of this type of data is its irregular spacing in time, which means that the price observations (ticks) are taken in real-time (as they arrive). Both price and traded volume are available as data. The application is designed for an interdealer broker[1], which means that they have the ability to create orders with any amount of volume.

For both stocks, two months of high-frequency tick data between 2 January 2009 and 27 February 2009 has been obtained (see Figure 3 and 4), simulations are terminated whenever 1 million shares have been bought or sold. Since we are comparing two approaches, each simulation starts at the point where the last simulation of either system has been terminated. Whenever the fuzzy logic system receives the

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[1] An interdealer broker is a member of a major stock exchange who is permitted to deal with market makers, rather than the public, and can sometimes act as a market maker.

first batch of data and performs all buy and sell actions on it, the same procedure is repeated using the standard volume-based system, allowing a comparison of the performance of both systems.

Figure 5 shows how the data is split after each simulation in order to avoid any possible similarities or autocorrelation in the price. As illustrated, each simulation starts at the point where the last simulation ended from either system. This is done to avoid overlapping on the tested data. Once the observation is obtained, the system starts again where the last simulation got terminated, each time noting the performance of both systems. It must be mentioned that two months of high-frequency tick data is a significantly large amount of data, considering every iteration, the system analyses the momentum of the past 100 ticks.
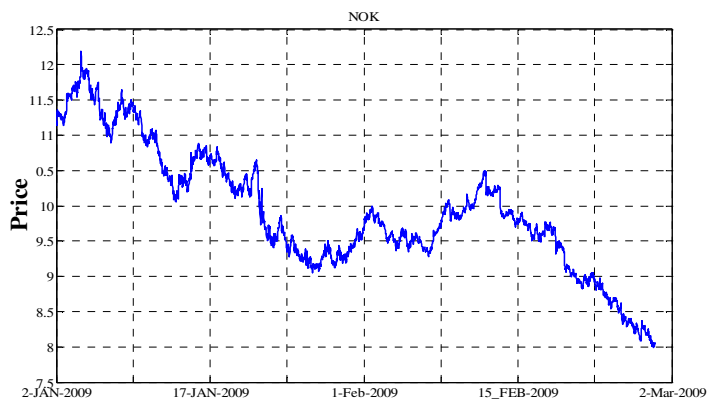

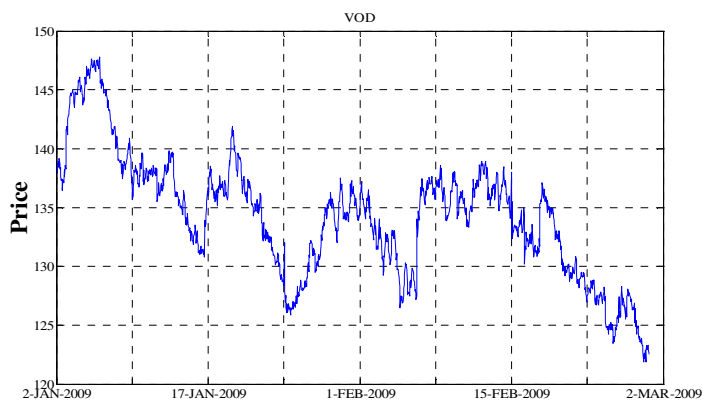*Figure 3: Price for NOK from 2 January 2009 and 27 February 2009*

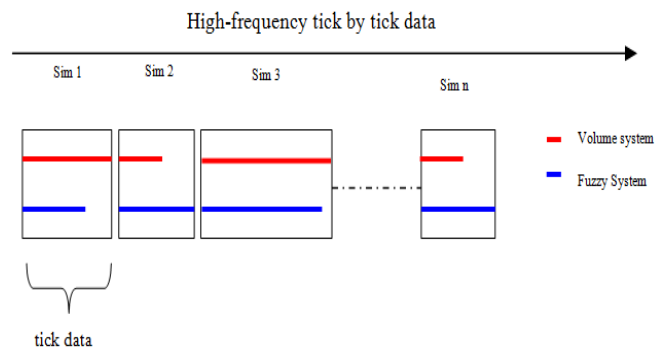*Figure 4: Price for VOD from 2 January 2009 and 27 February 2009*

Figure 5: Tick data-splitting mechanism, the new simulation starts where the last simulation has ended

A. Standard Volume System (SVS)

A standard brokerage and trading mechanism for executing large orders is a simple volume-based system that parses the volume being traded whenever a certain number of shares (a threshold) have been traded; the system will buy or sell (depending on the order) a certain percentage. In other words, if there is an order to trade one million shares of a certain stock, the threshold could be, for example, 10,000 shares. Whenever 10,000 shares have been traded and if the PR is set to 25%, the system will buy or sell 25% of the average volume. If the accumulated sum of the volume exceeds the predefined threshold, then the amount of shares traded is equal to the participation rate multiplied by the current volume:

$$Total\ SVS\ Cost\ =\ \sum_{i=1}^{n} Price_i * (\ amount\ of\ shares_i\ )$$

where n is the number of operations required to reach the target order (for example, 1 million shares), with a fixed PR, for example, 25% whenever the threshold is exceeded. The above system has proven to be efficient and is being adopted by various financial brokerage and market maker institutions [13].

B. FULMAS for Trading

The idea here is to use the momentum analysis system to identify in what market condition we are currently residing in. This will enable us to vary the PR (%). This provides an advantage, since the system can trade aggressively when the condition is at an extreme. It would also minimise its trading when the condition is at another extreme. In other words, if we are selling one million shares, the system will make a trade whenever the threshold of volume has been exceeded. However, if the current market condition indicates that the price is very high or rallying, then we know that this is a suitable time to sell a lot of shares, for example, 40% of the current volume. The same concept applies when the momentum indicates that the price is strong down, which means that the system should sell a lower volume at this low price, for example, 15%. The reverse mechanism applies for buying shares. When the market is crashing, this is a good indicator that we should buy a large volume (40%), and when the price is at an average point, it would behave like the SVS system, i.e. buying 25% of the volume. This is shown in Table I. The same procedure is applied to volatility and then combined with volume to produce the fuzzy rules.

TABLE I. PARTICIPATION RATES FOR BUY SIDE AND THE SELL SIDE OF FULMAS

Market Condition	Buying PR	Selling PR
Rallying	10%	40%
Strong up	15%	35%
Slightly up	20%	30%
Average	25%	25%
Slightly down	30%	20%
Strong down	35%	15%
Crashing	40%	10%

C. Benchmark Performance Measures

The aim is to prove that FULMAS outperforms this type of system in the long run; this is assessed using the order execution costs for buy and sell orders. When implementing SVS and FULMAS, the benchmark at which both systems will be compared against each other will be the outperformance of FULMAS on the SVS in basis points[2]. The basis point is a unit of measure often used to describe the percentage at which a change in the value or rate of a financial instrument has occurred. To calculate the improvement (imp) for the buy and sell sides in basis points, the following formulas are used:

$$impBuy\ =\ \left(1 - \frac{FULMAS\ price}{SVS\ price}\right) * 10^4\ bps \qquad (14)$$

$$impSell\ =\ \left(\frac{FULMAS\ price}{SVS\ price} - 1\right) * 10^4\ bps \qquad (15)$$

where $FULMAS\ price$ is the total cost of buying x amount of shares using FULMAS, and $SVS\ price$ is the total cost of buying the same number of shares using the traditional SVS.

D. Initial Results - FULMAS vs SVS

Initially simulations have been carried out on the described data, using triangular membership functions. The average price of the entire set for each simulation is also displayed; this price is calculated by taking the average price of the equity throughout the time of each simulation (see Figures 8-11 below). Finally the improvement of FULMAS against SVS is displayed. This improvement rate can be either positive, when FULMAS has outperformed SVS, or negative, when FULMAS is outperformed by SVS.

Table II first shows the average outperformance rate of buying one million shares of NOK using FULMAS, which turns out to be a positive of 2.98 basis points. This means that, on average, using FULMAS saves approximately 3 basis points whenever we buy one million shares of NOK. For VOD, there is an average outperformance of approximately 12.5 basis points. Another measurement mechanism was to observe the median of the results, which for both NOK and VOD were positive, indicating that, on average, FULMAS outperforms SVS for all of the buying simulations. Other descriptive statistics such as the standard deviation, skewness and kurtosis are also included in Table II. Next, we suggest the use of ANFIS for optimising the membership functions used by FULMAS.

TABLE II. ANALYSIS OF *INITIAL* RESULTS OF BUYING AND SELLING 1 MILLION SHARES OF NOK AND VOD WITH THE DESCRIPTIVE STATISTICS FOR THE IMPROVEMENT INDICATORS

	Mean	Median	Std Dev	Skewness	Kurtosis
Buying NOK	2.98	4.63	12.39	-0.055	2.56
Buying VOD	12.48	1.58	36.25	1.74	4.86
Selling NOK	1.68	2.92	8.79	-1.43	6.25
Selling VOD	2.73	2.46	27.71	0.70	8.84

[2] One basis point is equal to 1/100th of a percent or 0.01 percent. It is also equivalent to 0.0001 in decimal form.

E.   Optimising the Membership Functions

The complimentary characteristics of neural networks and fuzzy inference systems have been recognised and the methodologies have been combined to create neuro-fuzzy techniques.   Indeed, earlier work by [23] described an artificial neural network with processing elements that could handle fuzzy logic and probabilistic information, although the preliminary results were less than satisfactory.

In this study, ANFIS is used to optimise the membership functions implemented in FULMAS. This is performed by feeding the ANFIS system both the training data, the desired output, and tuning the ANFIS in order to reach the target result by modifying the membership functions. At each instance, ANFIS is fed the results currently obtained from the fuzzy system in the previous section, together with a target price or data. This target price will be an optimal price that is far better than the current one (a cheaper price if on buy mode or a higher price if in sell mode). The system runs and modifies the membership functions in each epoch in order to get as close to the optimal price as possible. This has been tested on two types of membership functions: the bell-shaped membership function and the triangular membership function [1]. Figure 6 and 7 show these membership functions before and after ANFIS was initialised to reach targeted "optimal" results.

Comparing the results of both optimised membership functions, an improvement in the original system was discovered. The optimised triangular membership functions have also outperformed the optimised bell-shaped membership functions; this confirms the experts' opinion presented previously concerning the choice of the triangular membership functions. The next section presents the new results using the membership functions optimised with ANFIS.

F.   Optimised Results

Here we present the final results of using both the optimised FULMAS and SVS to buy one million shares of VOD and NOK. Similar to the initial results, for each symbol, approximately 30 simulations have been carried on the tick dataset using a rolling windows approach described above. The average price of the whole set is also displayed, and finally, the improvement of FULMAS against SVS is displayed. Again, this improvement rate can be either positive, when FULMAS has outperformed SVS, or negative, when FULMAS was outperformed by SVS (see Figures 12-15 below).

Table III provides a summary showing the average outperformance rate of buying or selling one million shares of NOK using the optimised FULMAS. Here, we see a much higher improvement than the previous system, which confirms that the use of ANFIS to optimise the membership functions has increased the performance of the system by a high rate on both the buy and sell sides. Table III shows that on the buying side, the system, on average, outperforms the standard system by more than four basis points. On an industrial scale, this means a large amount of savings for financial institutions that employ such systems to vary the participation rates. Other descriptive statistics such as the standard deviation, skewness and kurtosis are also included.
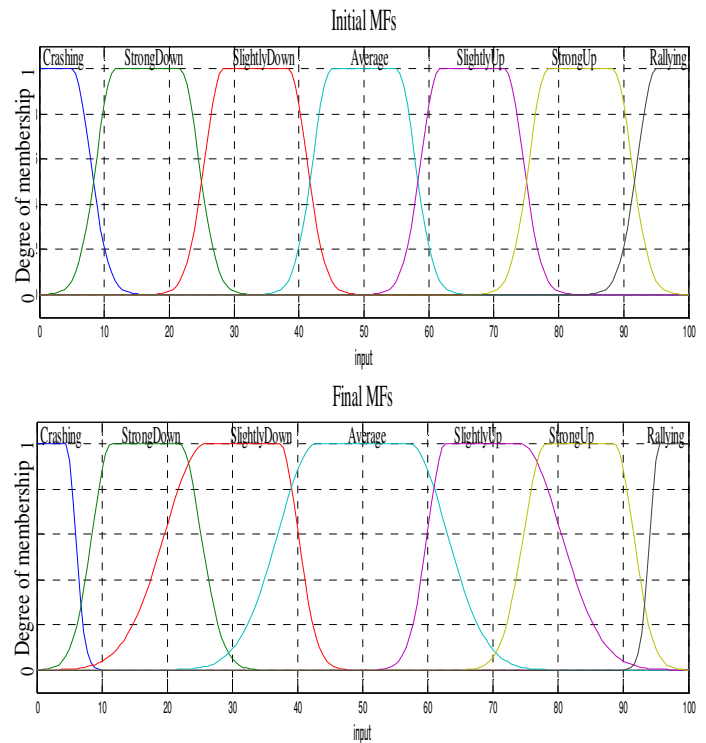

Figure 6: Bell-shaped membership functions optimised with ANFIS
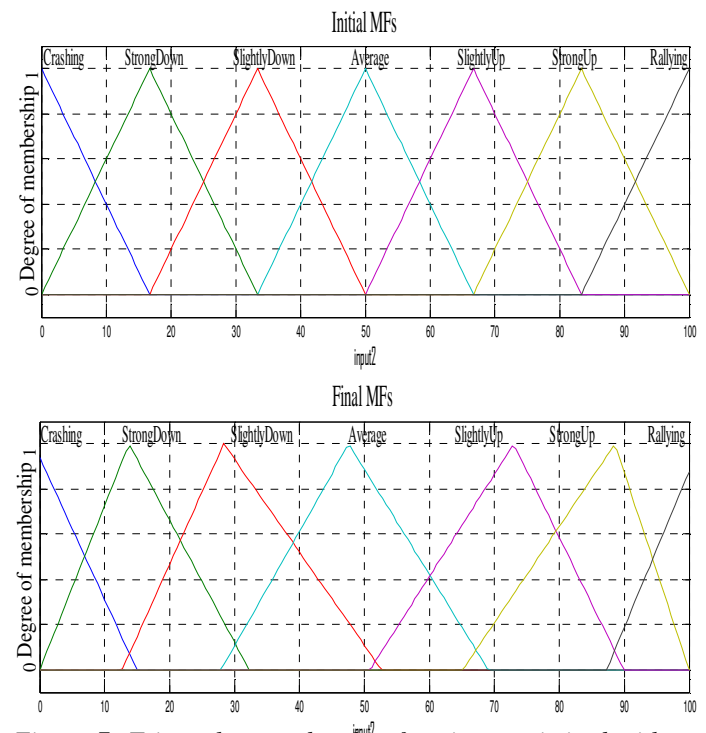

Figure 7: Triangular membership functions optimised with ANFIS

TABLE III.   ANALYSIS OF *OPTIMISED* RESULTS OF BUYING AND SELLING 1 MILLION SHARES OF NOK AND VOD WITH THE DESCRIPTIVE STATISTICS FOR THE IMPROVEMENT INDICATORS (IN BPS PER TRADE)

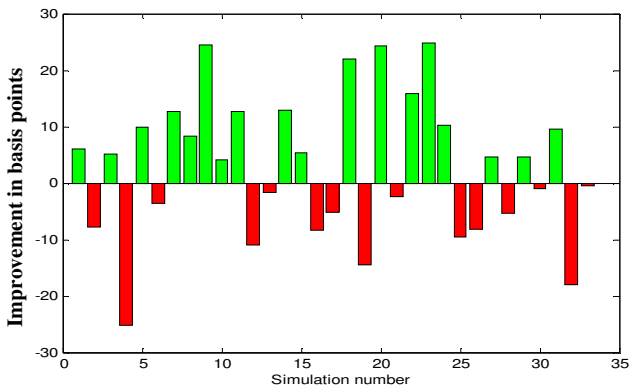|	Mean	Median	Std Dev	Skewness	Kurtosis
Buying NOK	6.94	6.57	12.99	0.15	2.53
Buying VOD	14.48	4.33	2.95	-0.74	3.28
Selling NOK	9.36	5.79	9.18	-0.52	2.61
Selling VOD	7.71	6.91	28.23	0.86	9.38

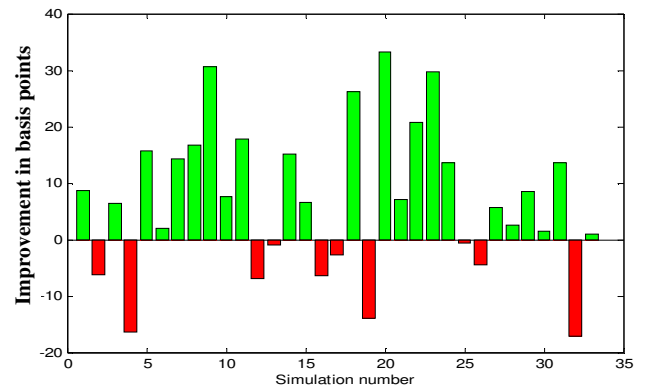*Figure 8: FULMAS outperformance to SVS system when buying NOK*

*Figure 12: **Optimised** FULMAS outperformance to SVS system when buying NOK*
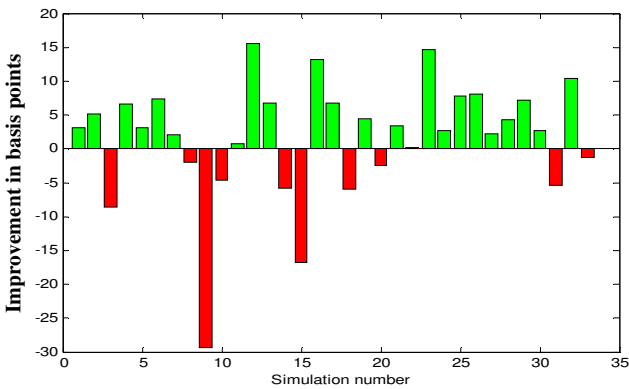

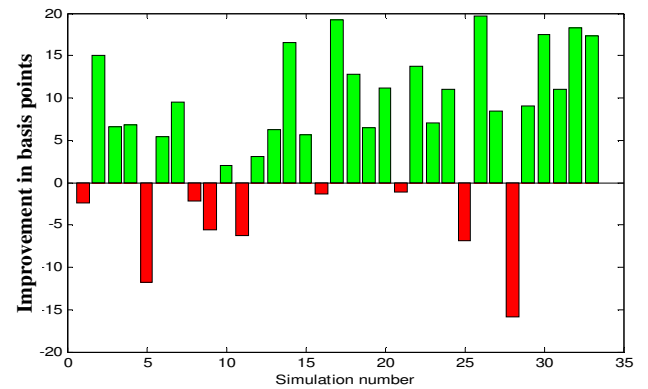*Figure 9: FULMAS outperformance to SVS system when selling NOK*

*Figure 13: **Optimised** FULMAS outperformance to SVS system when selling NOK*
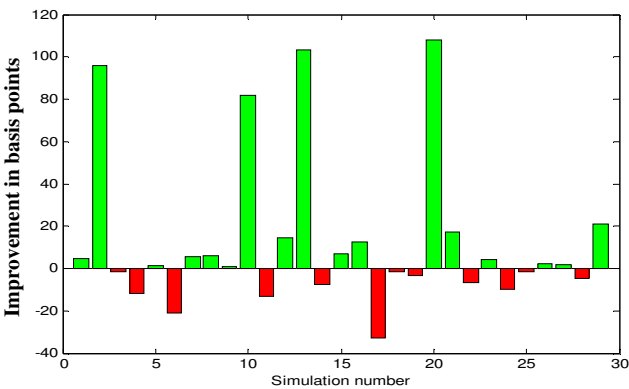

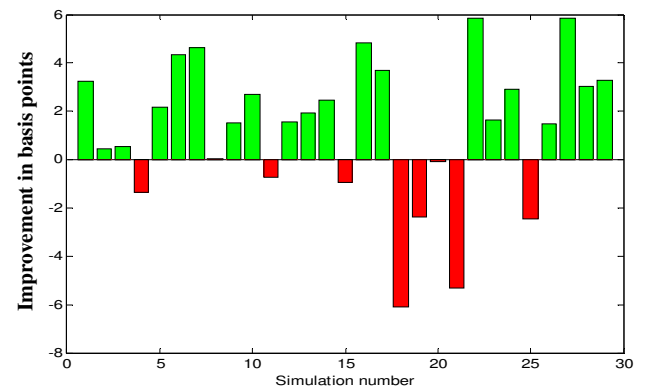*Figure 10: FULMAS outperformance to SVS system when buying VOD*

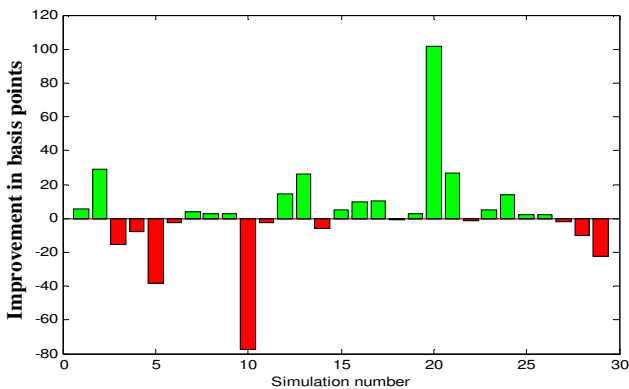*Figure 14: **Optimised** FULMAS outperformance to SVS system when buying VOD*

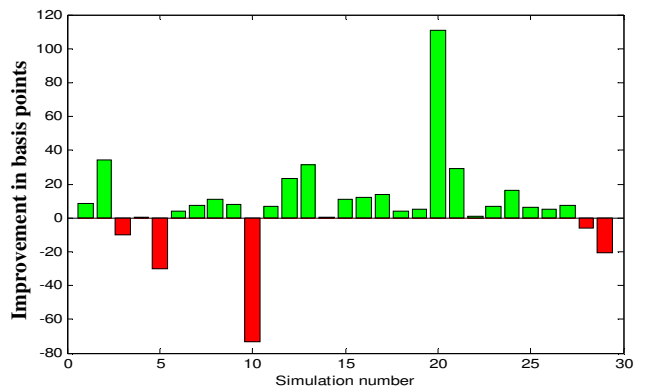*Figure 11: FULMAS outperformance to SVS system when selling VOD*

*Figure 15: **Optimised** FULMAS outperformance to SVS system when selling VOD*

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VII. SUMMARY AND DISCUSSION

It is well known that a main inadequacy of much economic theory is that it postulates exact functional relationships between variables. On the other hand in financial time series analysis, data points rarely lie exactly on straight lines or smooth functions. Attempting to accommodate these nonlinear phenomena will introduce an unacceptable level of instability in models [19]. As a result of this intractability, researchers and investors are turning to artificial intelligence techniques to better inform their models, creating decision support systems that can help a human user better understand complex financial systems such as stock markets [3]. Artificial intelligence systems in portfolio selection have been shown to have a performance edge over the human portfolio manager and recent research suggests that approaches that incorporate artificial intelligence techniques are also likely to outperform classical financial models [6].

There are still relatively few studies comparing artificial intelligence approaches and classical models such as time series theory. However, recent research continues to suggest that adopting artificial intelligence techniques for the technical analysis of financial systems will yield positive results [11]. With several researchers now questioning the Efficient Market Hypothesis [12], particularly in light of the recent global financial crisis, artificial intelligence approaches such as time series that can improve analysis will likely continue to hold the interest of researchers and investors alike. The problem of order execution is a very complicated one. To be able to provide the best price, an execution system must dynamically change the participation rates at each instance in order to cater to price changes that are driven by momentum and volatility.

This study proposes a new framework for high frequency trading using a fuzzy logic based momentum analysis system, to analyse the 'momentum' in the time series and to identify the current market condition which will then be used to decide the participation rate at that instance given the current traded volume. The system was applied to trading of financial stocks, and tested against the standard volume based trading system. The results demonstrate how the proposed Fuzzy Logic Momentum Analysis System outperforms the traditional and standard systems that are widely used in the financial industry. Additionally, FULMAS has been improved further by using ANFIS as an optimisation tool and the new results have shown a significant improvement over both the original FULMAS system and the SVS system.

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