Design of H-inf Controller with Tuning of Weights Using Particle Swarm Optimization Method

H. I. Ali, Member IAENG, S. B. Mohd Noor, M. H. Marhaban, S. M. Bashi

Abstract— In this paper a new method based on a particle swarm optimization (PSO) algorithm for tuning the weighing functions parameters to design an $H_\infty$ controller is presented. The PSO algorithm is used to minimize the infinity norm of the transfer function of the nominal closed loop system to obtain the optimal parameters of the weighting functions. This method is applied to a typical industrial pneumatic servo actuator with system uncertainty and wide range of load variation to illustrate the design procedure of the proposed method. It is shown that the proposed method can simplify the design procedure of $H_\infty$ control to obtain optimal robust controller for pneumatic servo actuator system.

Index Terms—Robust control, PSO, $H_\infty$ control, Pneumatic actuator, parametric uncertainty.

I. INTRODUCTION

$H_\infty$ is one of the best known techniques available nowadays for robust control. It is a method in control theory for optimal controller design. Basically, it is an optimization method that takes into consideration a strong definition of the mathematical way to express the ability to include both classical and robust control concepts within a single design framework. It is known that $H_\infty$ control is an effective method for attenuating disturbances and noise that appear in the system. It is one of the best techniques in linear control system. The “$H$” stands for Hardy space. “Infinity” means that it is designed to accomplish minimax restrictions in the frequency domain. The $H_\infty$ norm of a dynamic system is the maximum amplification that the system can make to the energy of the input signal [1, 2].

One of the most important parts and a key step in the design of the $H_\infty$ controller is the selection of weighting functions and weighting gains for specific design problems. This is not an easy procedure and often needs many iterations as well as fine-tuning. Furthermore, it is hard to find a general formula for the weighting functions that will work in every case. Therefore, to obtain a good control design it is necessary to use suitable selected and tuned weighting functions [3].

In this paper, a method for the position control design of a pneumatic servo actuator system using $H_\infty$ controller is presented. The PSO algorithm is used to find the optimal values of the parameters of the weighting functions that lead to obtain the optimal $H_\infty$ controller by minimizing the infinity norm of the transfer function matrix of the nominal closed loop system. The PSO method is used because of its simplicity and ease of implementation. The structured (parametric) uncertainty is considered in the design.

II. Particle Swarm Optimization Algorithm (PSO)

PSO is one of a powerful optimization method with high efficiency in comparison to other methods. It is a stochastic Evolutionary Computation technique based on the movement and intelligence of swarms. The PSO mechanism is initialized with a population of random solutions and searches for optima by updating generations. A swarm consists of N particles that are moving around in a D dimensional search space. Each particle keeps track of its coordinates in the space of the problem, which are associated with the best solution (best fitness) it has achieved so far. The best particle in the population is denoted by (global best), while the best position that has been visited by the current particle is denoted by (local best). The global best individual connects all members of the population to one another. That is, each particle is influenced by every best performance of any member in the entire population. The local best individual is seen as the ability for particles to remember past personal success. The particle swarm optimization concept involves, at each time step, changing the velocity of each particle towards its global best and local best locations. The particles are manipulated according to the following equations of motion [4,5, 6, 7]:

$$v_i^{k+1} = h \times v_i^k + c_1 \times \text{rand} \times (x_i^k - x_i^g) + c_2 \times \text{rand} \times (x_i^p - x_i^g)$$ (1)
\[ x_{i}^{k+1} = x_{i}^{k} + v_{i}^{k+1} \]  
where \( v_{i}^{k} \) is the particle velocity, \( x_{i}^{k} \) is the current particle position, \( w \) is the inertia weight, \( x_{i}^{b} \) and \( x_{i}^{g} \) are the best value and the global best value, \( rand \) is a random function between 0 and 1, \( c_{1} \) and \( c_{2} \) are learning factors.

The PSO requires only a few lines of computer code to realize PSO algorithm. Also it is a simple concept, easy to implement, and computationally efficient algorithm [8, 9].

III. PNEUMATIC SERVO ACTUATOR MODEL

Consider the pneumatic servo actuator system given in [10, 11]. This system is widely used in industrial applications because it is cheap, clean, lightweight, easy to maintain and it provides a high degree of compliance. On the other hand, it is difficult to achieve precise position for such systems because of the friction forces and the nonlinearity due to the compressibility of the air. Further, the variation in thermodynamic conditions causes an uncertainty in a number of model’s parameters. Therefore, it is need to apply the robust control techniques to control such as systems. Fig. 1 shows the schematic diagram of the pneumatic servo actuator. The source of power used in this type of actuator is compressed air supplied to the jet pipe, and distributed between the two ways pneumatic cylinder as type of actuator is compressed air supplied to the jet pipe, pneumatic servo actuator. The source of power used in this applications because it is cheap, clean, lightweight, easy to implement, and computationally efficient algorithm [8, 9].

Consider the pneumatic servo actuator system given in [10, 11]. This system is widely used in industrial applications because it is cheap, clean, lightweight, easy to maintain and it provides a high degree of compliance. On the other hand, it is difficult to achieve precise position for such systems because of the friction forces and the nonlinearity due to the compressibility of the air. Further, the variation in thermodynamic conditions causes an uncertainty in a number of model’s parameters. Therefore, it is need to apply the robust control techniques to control such as systems. Fig. 1 shows the schematic diagram of the pneumatic servo actuator. The source of power used in this type of actuator is compressed air supplied to the jet pipe, and distributed between the two ways pneumatic cylinder as type of actuator is compressed air supplied to the jet pipe, pneumatic servo actuator. The source of power used in this applications because it is cheap, clean, lightweight, easy to implement, and computationally efficient algorithm [8, 9].

The coefficient \( \gamma \) is typically very small compared to other terms of the system and has a very small effect on the system performance. In particular, when under chocked flow conditions, \( L_{a} = 0 \). The maximum mass flow rate occurs under chocked flow conditions and the unchecked flow rate is bounded by the chocked flow rate. Also, since the dynamics of the control valve are much faster than the required response of the servo actuator, the relationship between control signal and valve can be approximated by a proportional gain as mentioned in [14, 15]. Fig. 2 shows the block diagram of the pneumatic servo actuator system.

The frequency characteristic of the pneumatic actuator with all parameters uncertainty is shown in Fig. 3. These characteristics show that the system bandwidth decreases when the load increases, until the system becomes slower. Also the phase margin decreases when the load increases and this makes the system to oscillate and be unstable system.

IV. \( H_{\infty} \) CONTROLLER DESIGN WITH PARAMETRIC UNCERTAINTY

The pneumatic servo actuator system can be represented as shown in Fig. 4. The parameters \( a, b \) and \( c \) can be assumed to be:

\[ a = \frac{MV_{o}}{2KP_{a}KA_{p}\alpha RT}, \quad b = \frac{fV_{o}}{2KP_{a}KA_{p}\alpha RT}, \quad c = \frac{AP_{o}}{K_{p}KR_{T}} \]  

(6)

Fig. 1. Schematic diagram of pneumatic servo actuator system.

Fig. 2. Block diagram of pneumatic servo actuator system.
The three physical parameters \( a, b \) and \( c \) are unknown exactly, therefore, they can be assumed to be within the range of the system parameters in Table I, That is:

\[
\begin{align*}
  a & = (a_{\text{ap}} + a_{\delta a})_p a_{\delta a} \quad \text{and} \quad b = (b_{\text{bp}} + b_{\delta b})_p b_{\delta b} \\
  c & = (c_{\text{cp}} + c_{\delta c})_p c_{\delta c}
\end{align*}
\]

(7)

where \( a_{\text{ap}} = 0.0664 \times 10^{-5} \) and \( b_{\text{bp}} = 0.1327 \times 10^{-4} \) for small and wide ranges of load variation, respectively, and \( \varepsilon = 0.01585 \) are the nominal values of \( a, b \) and \( c \). \( a_{\delta a}, b_{\delta b} \) and \( c_{\delta c} \) represent the possible perturbations on these parameters. In this work we let \( a_{\text{ap}} = 0.99999991 \) and \( b_{\text{bp}} = 0.999246 \) for the two ranges of load variation, respectively, \( p_a = 0.64 \), \( p_b = 0.174 \) and \(-1 \leq a_{\delta a}, b_{\delta b}, c_{\delta c} \leq 1 \). Note that this represents up to 99.999991% and 99.9246% uncertainty in the parameter \( a \), 64% uncertainty in the parameter \( b \) and 17.4% uncertainty in the parameter \( c \).

The three constant blocks in Fig. 4 have been replaced by block diagrams in terms of \( a_{\text{ap}} a_{\delta a}, b_{\text{bp}} b_{\delta b}, \) etc., in a unified approach. The quantity \( \frac{1}{a} \) can be represented as an upper linear fractional transformation (LFT) in \( a_{\delta a} \) as:

\[
M_a = \begin{bmatrix} -p_a & \frac{1}{a} \\ -p_a & \frac{1}{a} \end{bmatrix}
\]

(9)

Similarly, the parameters \( b \) and \( c \) can be represented as an upper LFT in \( b_{\delta b} \) and \( c_{\delta c} \) as:

\[
b = F_u(M_b, b_{\delta b})
\]

(10) with

\[
M_b = \begin{bmatrix} 0 & \frac{\bar{b}}{p_b} \\ \bar{b} & \bar{b} \end{bmatrix}
\]

(11) and

\[
c = F_u(M_c, c_{\delta c})
\]

(12) with

\[
M_c = \begin{bmatrix} 0 & \frac{\bar{c}}{p_c} \\ \bar{c} & \bar{c} \end{bmatrix}
\]

(13)

Table I. The nominal system model parameters and their range

| Uncertain Parameter | \( A_p \) \( \text{m}^2 \) | \( R \) \( \frac{J}{\text{kg.K}} \times 10^4 \) | \( V_o \) \( \text{m}^3 \) | \( P_i \) bar | \( M \) kg | \( f \) \( \text{N.sec/m} \) | \( K \) \( \text{kg.s/V} \times 10^3 \) | \( T_s \) \( \text{K} \) | \( \gamma \) | \( K_p \) \( V/\text{m} \) |
|---------------------|----------------|-----------------|-----------------|----------|--------|----------------|-----------------|--------|----------------|
| Minimum value       |                |                 |                 |          |        |                |                  |        |                |
| Nominal value       | 0.005          | 287             | 2.5             | 3        | 60     | 3.4            | 293.15          | 1.4    | 400             |
| Maximum value       | 4              | 4               | 5, 100          | 80       |        |                |                  |        |                |
Fig. 5 shows the representation of uncertain parameters as LFTs. The relationship between the input signals, \( w \) the output signals, \( z \) can be expressed as [1]:

\[
z = F_y(N, \Delta) = [N_{22} + N_{21}\Delta(1-N_{11}\Delta)^{-1}N_{12}]w
\]  

(14)

where \( N \) may represent \( M_a \) or \( M_b \) or \( M_c \) and \( \Delta \) may represent \( \delta_a \) or \( \delta_b \) or \( \delta_c \). The inputs and outputs of \( \delta_a, \delta_b \) and \( \delta_c \) are denoted as \( y_a, y_b, y_c \) and \( u_a, u_b, u_c \), respectively, as shown in Fig. 6.

The equations relating all inputs to corresponding outputs around the uncertain parameters can be obtained as:

\[
\begin{bmatrix}
y_a \\
v_b \\
y_c
\end{bmatrix}
= M_a
\begin{bmatrix}
u_a \\
u-b-v_c
\end{bmatrix}
\]  

(15)

\[
\begin{bmatrix}
y_b \\
v_c
\end{bmatrix}
= M_b
\begin{bmatrix}
u_b \\
y_p
\end{bmatrix}
\]  

(16)

\[
\begin{bmatrix}
y_c
\end{bmatrix}
= M_c
\begin{bmatrix}
u_{ac} \\
y_p
\end{bmatrix}
\]  

(17)

where

\[
u_a = \delta_a y_a, \quad u_b = \delta_b y_b, \quad u_c = \delta_c y_c
\]  

(18)

The system state space representation can be expressed as:

\[
x_1 = y_p, \quad x_2 = y_p = \dot{x}_1 \quad \text{such that} \quad \dot{x}_2 = \dot{y}_p = \ddot{x}_1
\]  

(19)

As a result, the following equations can be obtained:

\[
\begin{align*}
x_1 &= x_2 \\
x_2 &= x_3 \\
\dot{x}_3 &= -p_a u_a + \frac{1}{a} (u - v_b - v_c)
\end{align*}
\]  

(20) (21) (22)

\[
y_a = p_b u_b + b x_3
\]  

(23)

\[
v_c = p_c u_c + \bar{c} x_3
\]  

(24)

The equations governing the pneumatic servo actuator system dynamic behaviour can be obtained as:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
y_a \\
y_b \\
y_c \\
y_p
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & \frac{-c}{a} & 0 & -p_a & \frac{-p_b}{a} & \frac{-p_c}{a} & \frac{1}{a} \\
0 & \frac{-c}{a} & 0 & -p_a & \frac{-p_b}{a} & \frac{-p_c}{a} & \frac{1}{a} \\
0 & \frac{-c}{a} & 0 & -p_a & \frac{-p_b}{a} & \frac{-p_c}{a} & \frac{1}{a} \\
0 & \frac{-c}{a} & 0 & -p_a & \frac{-p_b}{a} & \frac{-p_c}{a} & \frac{1}{a} \\
0 & \frac{-c}{a} & 0 & -p_a & \frac{-p_b}{a} & \frac{-p_c}{a} & \frac{1}{a}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
y_a \\
y_b \\
y_c \\
y_p
\end{bmatrix}
\]  

(25)

and

\[
\begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix}
= \begin{bmatrix}
\delta_a & 0 & 0 \\
0 & \delta_b & 0 \\
0 & 0 & \delta_c
\end{bmatrix}
\begin{bmatrix}
y_a \\
y_b \\
y_c
\end{bmatrix}
\]  

(26)

Let \( G_p \) denotes the input/output dynamics of the pneumatic system, which takes into account the uncertainty of parameters. \( G_p \) has four inputs \( (u_a, u_b, u_c, u) \), four outputs \( (y_a, y_b, y_c, y_p) \) and three states \( (x_1, x_2, x_3) \).

The state space representation of \( G_p \) is [2]:

\[
G_p = \begin{bmatrix}
A & B_1 & B_2 \\
C_1 & D_{11} & D_{12} \\
C_2 & D_{21} & D_{22}
\end{bmatrix}
\]  

(27)

where
A. Bilinear Transform and Weighting Functions Selection

Since the proposed system has \( jw \)-axis pole, the \( H_\infty \) controller, if it is reliably computed, would have a marginally stable closed loop pole at the corresponding \( jw \)-axis location. This problem leads to singularities in the equations that determine the state space realization of \( H_\infty \) control law. Therefore, a simple bilinear transform has been found to be extremely useful when used with robust control synthesis. This transformation can be formulated as a \( jw \)-axis pole shifting transformation [16]:

\[
s = \frac{\hat{s} + p_{b1}}{\hat{s} + 1} \quad \frac{p_{b2}}{p_{b2}}
\]

where \( p_{b1} < 0 \) and selected to be 0.1, \( p_{b2} \) is selected to be infinity. This is equivalent to simply shifting the \( jw \)-axis by \( p_{b1} \) units to the left. The \( H_\infty \) controller was obtained for the shifted system then it was shifted back to the right with the same units.

The design requirements and objectives for pneumatic servo actuator system in this work is to find a linear, output feedback control \( u(s) = K(s)y_p(s) \), which ensures that the closed loop system will be internally stable. Also, the required closed loop system performance should be achieved for the nominal plant \( G_p \).

To obtain a good control design, it is necessary to select suitable weighting functions. The performance and control weighting functions formulas that have been used in this work are [16]:

\[
W_p(s) = \frac{\beta(\alpha^2 + 2\zeta_1 w_c \sqrt{\alpha s + w_c^2})}{(\beta s^2 + 2\zeta_2 w_c \sqrt{\beta s + w_c^2})}
\]

\[
W_u(s) = \frac{s^2 + 2w_{bc}^2 + w_{bc}^2}{\sqrt{M_u}}
\]

where \( \beta \) is the d.c. gain of the function which controls the disturbance rejection, \( \alpha \) is the high frequency gain which controls the response peak overshoot, \( w_c \) is the function crossover frequency, \( \zeta_1 \) and \( \zeta_2 \) are the damping ratios of crossover frequency, \( w_{bc} \) is the controller bandwidth, \( M_u \) is the magnitude of \( KS \), and \( \varepsilon \) is a small value.

B. CONTROLLER DESIGN

The \( H_\infty \) controller was designed so that \( H_\infty \)-norm from input \( w = F_d \) to output \( z = \begin{bmatrix} e_p \\ e_u \end{bmatrix} \) is minimized.

\[
\begin{align*}
W_u(s) &= &\frac{F_d}{G_d(s)} \\
G_p(s) &= &\frac{e_p}{e_u} \\
K(s) &= &\frac{Y_p(s)}{Y_u(s)}
\end{align*}
\]
Where \( F_d \) is the input disturbance signal, \( e_p, e_u \) are the weighted error and control signals. Fig. 8 shows the standard feedback diagram of the system with weights. The generalized plant \( P \) is expressed by:

\[
\begin{bmatrix}
    e_p \\
    e_u \\
    e
\end{bmatrix} = \begin{bmatrix}
    W_p G_d - W_p G_p \\
    0 & W_u \\
    G_d & - G_p
\end{bmatrix} \begin{bmatrix}
    F_d \\
    u
\end{bmatrix}
\]

(33)

where \( u \) is the control signal.

The lower linear fractional transformation of the generalized plant \( P \) and controller \( K(s) \) can be described by:

\[
F_I(P, K) = \begin{bmatrix}
    W_p G_d S \\
    W_u G_d KS
\end{bmatrix}
\]

(34)

The objective of \( H_\infty \) control is to find the controller \( K(s) \) that internally stabilizes the system such that \( \|T_{zw}(s)\|_\infty \) is minimized [3]. Where \( \|T_{zw}\|_\infty \) is the transfer function of the system from input \( w \) to output \( z \) and can be expressed as:

\[
\|T_{zw}\|_\infty = \begin{bmatrix}
    W_p G_d S \\
    W_u G_d KS
\end{bmatrix}_\infty
\]

(35)

The \( H_\infty \) control minimizes the cost function in equation (35) using \( \gamma \) iteration [17] to find the stabilizing controller such that \( \|T_{zw}(s)\|_\infty < \gamma \). To find the optimal value of \( \gamma \), the PSO algorithm was used to tune the parameters of the selected weighting functions. The weighting functions have a significant effect on the overall design of \( H_\infty \) control technique.

The optimal value of \( \gamma \) is the infimum overall \( \gamma \) such that the \( H_\infty \) control conditions are satisfied. A suboptimal \( H_\infty \) controller was obtained using the following Matlab command:

\[
>> [K, T_{zw}\gamma_{suboptimal}] = \text{h inf syn}(P, n_g, n_u, \gamma_{\text{min}}, \gamma_{\text{max}}, \text{tol})
\]

(36)

where \( n_g \) and \( n_u \) are the dimensions of \( y_p \) and \( u \), \( \gamma_{\text{min}} \) and \( \gamma_{\text{max}} \) are the lower and upper bound for \( \gamma_{\text{optimal}} \), and \( \text{tol} \) is the tolerance to the optimal value.

The fitness function used in PSO algorithm is the performance criteria stated in equation (35). The algorithm obtains the minimum value of the infinity norm of the performance criteria from the search space that minimizes the objective function in equation (35). The following parameters have been used for carrying out the QFT controller design using PSO:

i) The members of each individual in the PSO algorithm are \( \beta, \alpha, \omega, \xi_1, \xi_2, w_hc, M_u \).

ii) Population size equal to 100.

iii) Inertia weight factor \( h = 2 \).

iv) \( c_1 = 2 \) and \( c_2 = 2 \).

v) Maximum iteration is set to 100.

The PSO steps for obtaining the optimal parameters of the proposed controller can be summarized as:

1. Define the system model, \( G_p(s) \).
2. Define the structure of \( W_p, W_u \) according to equations (31) and (32).
3. Initialize the individuals of the population randomly in the search space.
4. For each initial of the population (vector of the parameters to be optimised), determine the fitness function in equation (35).
5. Compare each value of equation (35) with its personal best \( x_i \). The best value among the \( x_i \) is denoted as \( x^p_i \).
6. Update the velocity of each individual according to (2).
7. Update the position of each individual according to (1).
8. If the number of iterations reaches the maximum, then go to step 9, otherwise, go to step 4.
9. The latest \( x^p_i \) is the optimal parameters of the weighting functions.

The proposed PSO algorithm for obtaining the optimal values of the weighting functions parameters is described by the flowchart shown in Fig. 9. The overall block diagram of the system with PSO tuning algorithm is shown in Fig. 10.

The interval for \( \gamma \) iteration was selected between 0.1 and 10. The obtained controllers for the two cases of load variation range, respectively, are:

\[
K(s) = \frac{2.797a^2 + 598.6s + 3.203 \times 10^5}{s^3 + 7035s^2 + 2.422 \times 10^5 s + 3.222 \times 10^6}
\]

(37)

\[
K(s) = \frac{2.021s^4 + 90.13s^3 + 656s^2 + 6.191 \times 10^4 s}{s^5 + 2383s^4 + 5.195 \times 10^5 s^3 + 4.236 \times 10^7 s^2 + 1.444 \times 10^9 s + 5.588 \times 10^7}
\]

(38)

The optimal weighting functions parameters obtained with \( \gamma = 0.8559 \) using PSO algorithm are shown in Table II.

V. RESULTS AND DISCUSSION

Fig. 11 shows the singular values of the closed loop system with the controller \( K(s) \). As is seen, the maximum value of the closed loop system is less than one, that is, the condition \( \left\|W_p(1 + G_pK)^{-1}\right\|_\infty < 1 \) has been satisfied. This can be checked by computing the sensitivity function of the closed loop system and comparing it with the inverse of the performance weighting function as shown in Fig. 12. It is clear that the sensitivity function lies below the inverse of \( W_p \), which means that the performance criterion was satisfied. Fig. 13 shows the frequency response of the open loop uncertain system with the controller. From this plot, it can be seen that the minimum gain and phase margins that have been satisfied for the system with small and wide
ranges of load variation are 14.1 dB, 60.6° and 12.6 dB, 55.9°, respectively. This means that the system is stable with all parameters uncertainty, that is, the robust stability has been satisfied. The time response characteristics of the closed loop nominal and uncertain systems are shown in Figs. 14 and 15, respectively. From these figures it can be seen that the time response specifications that have been achieved for the two cases of load variation range are: rise time=0.239 s, settling time (2%)=0.498 s, maximum overshoot=10% for the case of small range of load variation and rise time=0.573 s, settling time (2%)=1.18 s, maximum overshoot =11% for the case of wide range of load variation.

The time response characteristic of the system subjected to disturbance is shown in Fig. 16.

It shows that the disturbance attenuation specifications have been met. For practical requirements, it is required that the control signal be small to avoid the problem of saturation. Fig. 17 shows the frequency characteristics of the control signal where a small magnitude maximum value has been obtained. However, for discretizing the system and the obtained controllers, the Zero-Order-Hold and Bilinear Transformation methods were used, respectively. With a sampling time, \( T_s = 0.02 \) s, the following discrete controllers for small and wide ranges of load variation were obtained, respectively:

\[
K(z) = \frac{0.004131(z + 1)(z^2 + 1.433z + 0.706)}{(z + 0.9187)(z^2 - 1.371z + 0.5036)}
\]

(39)

\[
K(z) = \frac{0.0011952(z + 1)(z - 0.8083)(z^2 - 0.873z + 0.5826)}{(z + 0.9187)(z - 0.814)(z - 0.9992)(z^2 - 1.771z + 0.7925)}
\]

(40)

Fig. 18 shows the discrete time response specifications of the controlled system.

---

![Fig. 9. Flowchart for tuning the weighting functions using PSO](image)

---

Table II. Optimal parameters of weighting functions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Small range of load variation</th>
<th>Wide range of load variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>90.5</td>
<td>60.5</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>( \omega_c )</td>
<td>5</td>
<td>4.93</td>
</tr>
<tr>
<td>( \zeta_1 )</td>
<td>1.38</td>
<td>0.38</td>
</tr>
<tr>
<td>( \zeta_2 )</td>
<td>8.12</td>
<td>8.1242</td>
</tr>
<tr>
<td>( \omega_{bc} )</td>
<td>10.7</td>
<td>1.7</td>
</tr>
<tr>
<td>( M_u )</td>
<td>1</td>
<td>1.00112</td>
</tr>
</tbody>
</table>

---

(Advance online publication: 25 May 2011)
Fig. 11. The largest singular Value of the closed loop controlled system
a) in case of small range of load variation b) in case of wide range of load variation

Fig. 12. Frequency characteristics of sensitivity function $S$ and the inverse of the weighting function $W_p^{-1}$
(a) in case of small range of load variation b) in case of wide range of load variation

Fig. 13. Frequency response characteristics of the uncertain controlled system
a) in case of small range of load variation b) in case of wide range of load variation

Fig. 14. Closed loop time response characteristics of the controlled system
a) in case of small range of load variation b) uncertain plant

(Advance online publication: 25 May 2011)
Fig. 15. Closed loop time response characteristics of the controlled system in case of wide range of load variation with structured uncertainty a) nominal plant b) uncertain plant

Fig. 16. Time response characteristics of the closed Loop controlled system subjected to disturbance a) in case of small range of load variation b) in case of wide range of load variation

Fig. 17. Frequency response characteristics of the control signal a) in case of small range of load variation b) in case of wide range of load variation

Fig. 18. Discrete closed loop time response characteristics of the controlled system a) in case of small range of load variation b) in case of wide range of load variation
VI. CONCLUSION

In this paper, an $H_{\infty}$ controller was designed to assure robust stability and robust performance of the uncertain pneumatic servo actuator system with small and wide ranges of load variation. The $H_{\infty}$ controller was designed using structured (parametric) uncertainty to achieve robust stability and performance of the system. The two cases of load variation range have been considered in the design.

Suitable formulas for performance and control weighting functions have been selected for controller design requirements. The particle swarm optimization algorithm (PSO) was used to tune the performance and control weighting functions by minimizing the infinity norm of the transfer function matrix of the nominal closed loop system. The use of the PSO method simplified the design procedure to obtain the optimal robust controller, which achieves the position control of the pneumatic servo actuator system.

Further, it can be concluded that the $H_{\infty}$ optimal control is a powerful technique to design a robust control for the pneumatic servo actuator system with structured uncertainty and disturbances.

REFERENCES


Samsul Bahari bin Mohd Noor, graduated from University of Warwick in Electronic Engineering in 1991, and obtained his MSc and PhD in Control Engineering at Sheffield University, UK, in 1992 and 1996 respectively. He is a Senior Lecturer at the Department of Electrical and Electronic Engineering, Faculty of Engineering, UPM. He was the Head of IT unit and the Head of the Electrical and Electronic Engineering Department. Currently, he is a Deputy Dean (Development and Finance) of the Engineering Faculty and also Head of Control System and Signal Processing research area. He has been invited by the Ministry of Human Resources to develop the National Occupational Skill Standard for Process Control. He has led a few researches on Satellite Control, and Optimal and Intelligent Control of Chemical Processes. He has been a member of research groups related to instrumentation and control. Senior Member of IEEE, Member of Technical Committee (SIRIM) (2006-date). Areas of Expertise: Control engineering, process modelling and control and instrumentation, Model Predictive Control and Intelligent Control.

Mohammad Hamiruce Marhaban, received B.Eng. in Electrical and Electronic Engineering, Universiti of Salford, UK, from 1996 to 1998. He received Ph.D degree in Electronic Engineering, Universiti of Surrey, UK, from 1999 to 2003. He is a Lecturer in Department of Electrical and Electronic Engineering, Faculty of Engineering, UPM. (May 2003 to Present), and also he is a Member of IEEE. Area of Interest: Intelligent Control System and Computer Vision.

Hazem I. Ali was born in Baghdad, Iraq. He received the BSc and MSc degrees in Control and Systems Engineering from University of Technology, Baghdad, Iraq, in 1997 and 2000 respectively. From November 2007, he was a lecturer in the Department of Control and Systems Engineering, University of Technology, Baghdad, Iraq. Since 2008 he is a Ph.D student in the Department of Electrical and Electronic Engineering, Control and Automation Field, University Putra Malaysia, Malaysia. His current research interests include Robust control, Intelligent control, Process control.