Multiple Particle Swarm Optimizers with Inertia Weight with Diversive Curiosity and Its Performance Test *

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Abstract— **This paper presents a new method of curiosity-driven multi-swarm search, called multiple particle swarm optimizers with inertia weight with diversive curiosity (MPSOIWα/DC). Compared to a plain MPSOIW, it has the following outstanding features: (1) Decentralization in multi-swarm exploration with hybrid search, (2) Concentration in evaluation and behavior control with diversive curiosity, (3) Practical use of the results of evolutionary PSOIW, and (4) Their effective combination. This achievement expands the applied object of cooperative PSO with the multi-swarm's decision-making. To demonstrate the effectiveness of the proposal, computer experiments on a suite of multidimensional benchmark problems are carried out. We examine the intrinsic characteristics of the proposal, and compare the search performance with other methods. The obtained experimental results clearly indicate that the search performance of the MPSOIWα/DC is superior to that by the EPSOIW, PSOIW, OPSO, RGA/E, and MPSOα/DC for the given benchmark problems.**

Keywords: ***cooperative particle swarm optimization, evolutionary particle swarm optimizer with inertia weight, hybrid search, localized random search, stagnation, exploitation and exploration, diversive and specific curiosity, swarm intelligence***

1 Introduction

In a few words, optimization is to find the most suitable value of a function within a given domain. In contrast to traditional optimization methods such as steepest descent method, conjugate gradient method, and quasi-newton method, which may be good at solution accuracy and exact computation but have brittle operations and necessary information to search environment, in general, the methods of genetic and evolutionary computation (GEC)[1] provide a more robust, efficient, and expandable approach in treating with high-grade nonlinear, multimodal optimization problems, and complex practical problems in the real-world without prior knowledge [13, 18, 28].

As a new member of GEC, particle swarm optimization (PSO) [8, 15] has been successfully applied in different fields of science, technology, engineering, and applications [22]. This is because the technique has distinctive features, i.e. information exchange, intrinsic memory, and directional search in the mechanism and composition compared to the other members such as genetic algorithms [12] and evolutionary programming [11].

For improving the convergence, solution accuracy, and search efficiency of a plain particle swarm optimizer (the PSO), many basic variants of the PSO such as a particle swarm optimizer with inertia weight [24], a canonical particle swarm optimizer [5, 6], fully informed particle swarm [16] etc. were proposed. The principal objective of these optimizers (algorithms) was to put in enforcing the search strategy and information transfer in the interior of a particle swarm to increase diversification for realizing efficient search. Especially, in recent years, the method of multi-swarm search is rapidly developing from the basis of the method of single swarm search. A large number of studies and investigations on cooperative PSO[2] in relation to symbiosis, interaction, and synergy are in the researcher's spotlight. Various kinds of algorithms on cooperative PSO, for example, hybrid PSO, multi-population PSO, multiple PSO with decision-making strategy etc., were published [1, 10, 20, 32] for attaining high search ability and solution accuracy more by deepening on group searching with cooperative actions.

In comparison with those methods that only operate a single particle swarm, it is an indisputable fact that different attempts and strategies to reinforcement of multi-

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[1] GEC usually refers to genetic algorithms (GAs), genetic programming (GP), evolutionary programming (EP), and evolution strategies (ES).

[2] Cooperative PSO is generally considered as multiple swarms (or sub-swarms) searching for a solution (serially or in parallel) and exchanging some information during the search according to some communication strategy.

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swarm search can be perfected well, which mainly focus on the rationality of information propagation, cooperation, optimization, hierarchical expression, and intelligent control within these particle swarms for efficiently finding an optimal solution in a wide search space. With regard to the effect of them, a lot of publications and reports have been shown that the methods of cooperative PSO have better adaptability and extensibility than ones of uncooperative PSO for dealing with different optimization problems [4, 14, 20, 36].

Due to great requests to enlarge search performance, utilizing the techniques of group searching, parallel processing with intelligent strategy has become one of extremely important approaches to optimization. For magnifying cooperative PSO research and improving the search performance of a plain multiple particle swarm optimizers with inertia weight (MPSOIW), this paper presents multiple particle swarm optimizers with inertia weight with diversive curiosity (MPSOIWα/DC), which is a new method of cooperative PSO.

In comparison with the plain MPSOIW, the proposal has the following outstanding features: (1) Decentralization in multi-swarm exploration with hybrid search[3] (MPSOIWα), (2) Concentration in evaluation and behavior control with diversive curiosity (DC), (3) Practical use of the results of evolutionary PSOIW (PSOIW*), and (4) Their effective combination. According to these features, the MPSOIWα/DC could be expected to alleviate stagnation in the optimization, and to enhance search ability and solution accuracy by enforcement of group decision-making and managing the trade-off between exploitation and exploration in the multi-swarm's heuristics.

From the viewpoint of methodology, the MPSOIWα/DC is an analogue of the method of multiple particle swarm optimization with diversive curiosity [35], which has been successfully applied to the plain multiple particle swarm optimizers (MPSO) and multiple canonical particle swarm optimizers (MCPSO) [34]. Nevertheless, the creation and actualization of the proposal are not only to improve the search performance of the plain MPSOIW, but also to expand the applied object and area of the curiosity-driven multi-swarm search. This is just our motivation, and study purpose further to deepen the approach of cooperative PSO with an integration way, i.e. reinforcement of hybrid search, parameter selection, and swarm intelligence.

The rest of the paper is arranged as follows. In Section 2, the algorithms of the PSO and PSOIW are briefly described. Section 3 introduces the structure and features of the MPSOIWα/DC, and the adopted LRS and internal indicator, respectively. Section 4 analyzes and discusses the experimental results for a suite of the multi-dimensional benchmark problems to verify the effectiveness of the proposed method. Finally, the concluding remarks appear in Section 5.

[3]The hybrid search, here, is compose of the particle swarm optimizer with inertia weight and the localed random search.

2 Basic Algorithms

For convenience to the following description of the PSO and PSOIW, let the search space be N-dimensional, $\mathbf{\Omega} \in \Re^N$, the number of particles in a swarm be P, the position and velocity of the i-th particle be $\vec{x}^i = (x^i_1, x^i_2, \cdots, x^i_N)^T$ and $\vec{v}^i = (v^i_1, v^i_2, \cdots, v^i_N)^T$, respectively.

2.1 The PSO

In the beginning of the PSO search, the position and velocity of the i-th particle are generated in random, then they are updated by

$$\begin{cases} \vec{x}^i_{k+1} = \vec{x}^i_k + \vec{v}^i_{k+1} \\ \vec{v}^i_{k+1} = w_0\vec{v}^i_k + w_1\vec{r}_1 \otimes (\vec{p}^i_k - \vec{x}^i_k) + w_2\vec{r}_2 \otimes (\vec{q}_k - \vec{x}^i_k) \end{cases} \tag{1}$$

where w_0 is an inertial coefficient, w_1 is a coefficient for individual confidence, w_2 is a coefficient for swarm confidence. $\vec{r}_1, \vec{r}_2 \in \Re^N$ are two random vectors in which each element is uniformly distributed over $[0, 1]$, and the symbol $\otimes$ is an element-wise operator for vector multiplication. $\vec{p}^i_k (= arg \max_{j=1,\cdots,k} \{g(\vec{x}^i_j)\}$, where $g(\cdot)$ is the criterion value of the i-th particle at time-step k) is the local best position of the i-th particle up to now, and $\vec{q}_k (= arg \max_{i=1,2,\cdots} \{g(\vec{p}^i_k)\})$ is the global best position among the whole swarm. In the original PSO, the parameter values, $w_0 = 1.0$ and $w_1 = w_2 = 2.0$, are used [15].

For preventing particles spread out to infinity in search, a boundary value, v_{max}, is introduced into the above mentioned update rule to limit the biggest velocity of each particle by

$$\begin{cases} v^{ij}_k = v_{max}, & if\ \ v^{ij}_k > v_{max} \\ v^{ij}_k = -v_{max}, & if\ \ v^{ij}_k < -v_{max} \end{cases} \tag{2}$$

where v^{ij}_k is the j-th element of the i-th particle's velocity $\vec{v}^i_k$ at time-step k.

2.2 The PSOIW

As is commonly known, the weak convergence is a disadvantage of the PSO [2, 5]. This is because the value of the inertial coefficient, $w_0 = 1.0$, is not the best one to manage the trade-off between exploitation and exploration [30, 31, 32]. For improving the convergence of the PSO by achieving a search shift which smoothly changes from exploratory mode to exploitative mode in the optimization, Shi et al. modified the update rule of the particle's velocity in Eq. (1) by constant reduction of the inertia coefficient over time-step [9, 24] as follows.

$$\vec{v}^i_{k+1} = w(k)\,\vec{v}^i_k + w_1\vec{r}_1 \otimes (\vec{p}^i_k - \vec{x}^i_k) + w_2\vec{r}_2 \otimes (\vec{q}_k - \vec{x}^i_k) \tag{3}$$

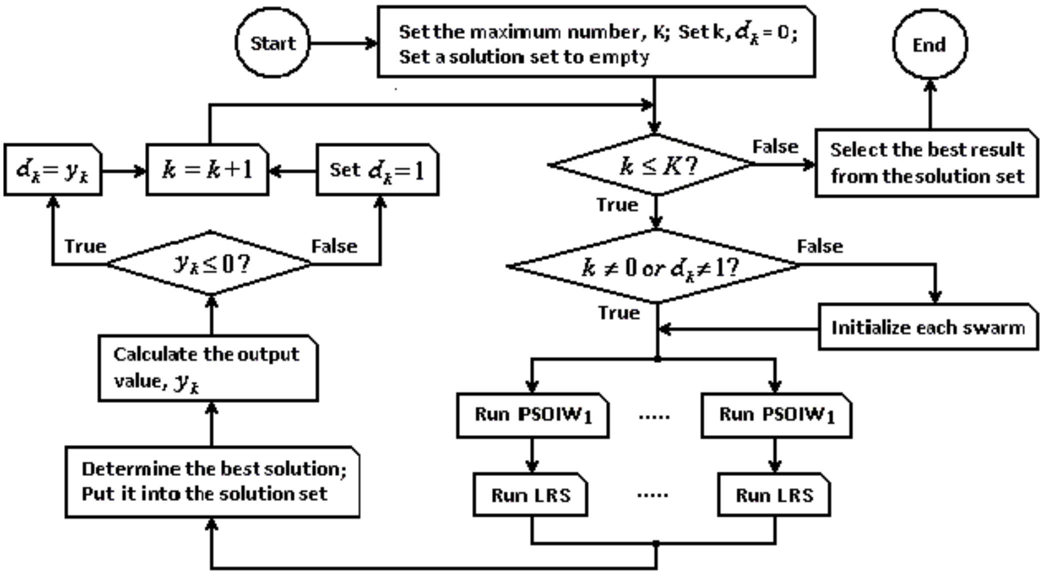


Figure 1: A flowchart of the MPSOIWα/DC

where $w(k)$ is a variable inertia weight which is linearly reduced from a starting value, w_s, to a terminal value, w_e, with the increment of time-step k as follows.

$$w(k) = w_s + \frac{w_e - w_s}{K} \times k \tag{4}$$

where K is the maximum number of iteration for the PSOIW search. In the original PSOIW, the boundary values are adopted to $w_s = 0.9$ and $w_e = 0.4$, respectively, and $w_1 = w_2 = 2.0$ are still set as in the original PSO. Since the terminal value, w_e, is smaller than 1.0, it is perfectly obvious that the PSOIW has good convergence than the PSO.

Based on the effect of the linearly damped inertia weight, the weakness of the PSO is overcome, and besides the solution accuracy is also improved by the PSOIW run. However, the search results often converge to local solutions in treating with multimodal problems. Moreover, the phenomenon called stagnation in optimization is easily caused as much as the terminal value w_e becomes small. Consequently, this fault is fatal for the use of the PSOIW alone to treat with complex optimization problems.

3 The MPSOIWα/DC

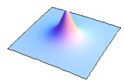
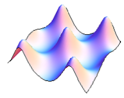
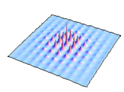
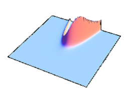
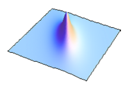
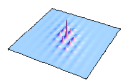
In order to thoroughly conquer the above mentioned shortcoming of the PSOIW, we propose to use the multiple particle swarm optimizers with diversive curiosity, called MPSOIWα/DC. This is a powerful method of integration of different approaches, which includes the practical use of parameter selection, and intelligent, hybrid and multi-swarm search to comprehensively managing the trade-off between exploitation and exploration in the multi-swarm's heuristics and to alleviate stagnation by group decision-making.

Concretely, Figure 1 illustrates a flowchart of the MPSOIWα/DC, which shows the data processing and information control in the method. It is the same as the MPSOα/DC [35] in instruction just instead of the part of implementing the PSO. The detailed characteristics on it are described below.

In the PSOIWα/DC, the plural PSOIWs are executed in parallel, and the localized random search (LRS) [35] is implemented to find the most suitable solution from a limited space for the solution found by each PSOIW. The continuous action of the PSOIW and LRS, here, constitutes a hybrid search (i.e. memetic algorithm [19]). Then the best solution, $\vec{q}_k^{\,b}$, is determined with maximum selection from the whole solutions found by the multi-swarm search (i.e. redundant search). Subsequently, it is put in a solution set being a storage memory for information processing.

As an internal indicator in the multi-swarm, its role is to monitor whether the status of the best solution $\vec{q}_k^{\,b}$ continues to change or not at all time-step for making up the concentration in evaluation and behavior control. Concretely, while the value of the output y_k is zero, this means that the multi-swarm concentrates on exploring the surroundings of the solution $\vec{q}_k^{\,b}$ for "cognition". If once the value of the output y_k become positive, it indicates that the multi-swarm has lost interest, i.e. feeling of boredom, to search the region around the solution $\vec{q}_k^{\,b}$ for "motivation". The concepts of psychology, "cognition"

Table 1: Functions and criteria to the given suite of the multi-benchmark problems. The search space for each problem is limited to the search space $\mathbf{\Omega} \in (-5.12, 5.12)^N$.

Problem	Function	Criterion	N=2
Sphere	$f_{Sp}(\vec{x}) = \sum_{d=1}^{N} x_d^2$	$g_{Sp}(\vec{x}) = \frac{1}{f_{Sp}(\vec{x})+1}$	
Griewank	$f_{Gr}(\vec{x}) = \frac{1}{4000}\sum_{d=1}^{N} x_d^2 - \prod_{d=1}^{N} cos\Big(\frac{x_d}{\sqrt{d}}\Big) + 1$	$g_{Gr}(\vec{x}) = \frac{1}{f_{Gr}(\vec{x})+1}$	
Rastrigin	$f_{Ra}(\vec{x}) = \sum_{d=1}^{N} \Big(x_d^2 - 10\,cos\,(2\pi x_d) + 10\Big)$	$g_{Ra}(\vec{x}) = \frac{1}{f_{Ra}(\vec{x})+1}$	
Rosenbrock	$f_{Ro}(\vec{x}) = \sum_{d=1}^{N-1} \Big[100\big(x_{d+1} - x_d^2\big)^2 + \big(1 - x_d\big)^2\Big]$	$g_{Ro}(\vec{x}) = \frac{1}{f_{Ro}(\vec{x})+1}$	
Schwefel	$f_{Sw}(\vec{x}) = \sum_{d=1}^{N}\Big(\sum_{j=1}^{d} x_j\Big)^2$	$g_{Sw}(\vec{x}) = \frac{1}{f_{Sw}(\vec{x})+1}$	
Hybrid	$f_{Hy}(\vec{x}) = f_{Ra}(\vec{x}) + 2f_{Sw}(\vec{x}) + \frac{1}{12}f_{Gr}(\vec{x}) + \frac{1}{20}f_{Sp}(\vec{x})$	$g_{Hy}(\vec{x}) = \frac{1}{f_{Hy}(\vec{x})+1}$	

and "motivation", are further explained in formulation later.

Due to the big reduction of boredom behavior in whole multi-swarm search, the search efficiency finding an optimal solution or near-optimal solutions will be drastically improved. Here, it is to be noted that the repeat of initialization decided by the signal $d_k = 1$ in Figure 1 is a mere expression style which introduces the mechanism of diversive curiosity for realizing a positive search. Of course, the implementation style is not an isolated one, it also can be performed by other operation ways in practice.

For convenience to understand the details of two important parts, i.e. the LRS and internal indicator, their mechanisms are minutely described in the next subsections.

3.1 The LRS

As be generally known, random search methods are the simplest ones of stochastic optimization with undirectional search, and are effective and robust in handling many complex optimization problems [25]. To efficiently obtain an optimal solution by using a hybrid search method, we introduce the LRS [34] into the MPSOIWα/DC to find the most suitable solution from a limited space surrounding the solution found by the PSOIW. According to the additional operation, the probability of escaping from a local solution will be steeply raised for efficient searching.

Concretely, the procedure of the LRS is implemented as follows.

step-**1:** Let $\vec{q}_k^{\,s}$ be a solution found by the s-th particle swarm at time-step k, and set $\vec{q}_{now}^{\,s} = \vec{q}_k^{\,s}$. Give the terminating condition, J (the total number of the LRS run), and set $j = 1$.

step-**2:** Generate a random data, $\vec{d}_j \in \Re^N \sim N(0, \sigma_N^2)$ (where σ_N is a small positive value given by user, which determines the small limited space). Check whether $\vec{q}_k^{\,s} + \vec{d}_j \in \mathbf{\Omega}$ is satisfied or not. If $\vec{q}_k^{\,s} + \vec{d}_j \notin \mathbf{\Omega}$ then adjust $\vec{d}_j$ for moving $\vec{q}_k^{\,s} + \vec{d}_j$ to the nearest valid point within $\mathbf{\Omega}$. Set $\vec{q}_{new} = \vec{q}_k^{\,s} + \vec{d}_j$.

step-**3:** If $g(\vec{q}_{new}) > g(\vec{q}_{now}^{\,s})$ then set $\vec{q}_{now}^{\,s} = \vec{q}_{new}$.

step-**4:** Set $j = j + 1$. If $j \leq J$ then go to the *step*-2.

step-**5:** Set $\vec{q}_k^{\,s} = \vec{q}_{now}^{\,s}$ to correct the solution found by the s-th particle swarm at time-step k. Stop the search.

Due to the complementary feature of the used hybrid search, the correctional function seems to be close to the HGAPSO [14] in search effect, which implements a plain GA and the PSO with the mixed operations for improving the adaptation to treat with various blended distribution problems.

Table 2: The major parameters used in the MPSOIWα/DC.

Parameter	Value	Parameter	Value
the number of individuals, M	10	the number of iterations, K	400
the number of generation, G	20	the maximum velocity, v_{max}	5.12
the number of particles, P	10	the range of LRS, σ_N^2	0.05
the number of particle swarms, S	3	the number of LRS run, J	10
the duration of judgment, L	10∼90	the tolerance coefficient, ε	$10^{-6} \sim 10^{-2}$

Table 3: The resulting appropriate values of parameters in the PSOIW to each given 5D benchmark problem.

Problem	Parameters			
	$\hat{w}_s$	$\hat{w}_e$	$\hat{w}_1$	$\hat{w}_2$
Sphere	0.7267±0.0521	0.1449±0.1574	1.2116±0.6069	1.9266±0.0651
Griewank	0.7761±0.0822	0.2379±0.2008	1.2661±0.6332	0.2827±0.0708
Rastrigin	2.0716±0.9143	0.8816±0.6678	12.942±7.9204	5.0663±1.5421
Rosenbrock	0.7702±0.1660	0.5776±0.2137	1.9274±0.3406	1.9333±0.4541
Schwefel	0.8552±0.3210	0.1253±0.2236	1.6106±1.3345	1.9610±1.3754
Hybrid	1.4767±0.2669	0.6101±0.5335	5.0348±1.7687	9.2134±5.0915

3.2 Internal Indicator

Curiosity, as a general concept in psychology, is an emotion related to natural inquisitive behavior for humans and animals, and its importance and effect in motivating search cannot be ignored [7, 21]. Berlyne categorized it into two types: diversive curiosity (DC) [4] and specific curiosity (SC) [5] [3]. In the matter of the mechanism of the former, Loewenstein insisted that "diversive curiosity occupies a critical position at the crossroad of cognition and motivation" in [17].

Based on the assumption of the "cognition" is the act of exploitation, and the "motivation" is the intention to exploration, Zhang et al. proposed the following internal indicator for distinguishing and detecting the above two behavior patterns in the multi-swarm search [?, 32, 33].

$$y_k(L,\varepsilon) = max\Big(\varepsilon - \sum_{l=1}^{L} \frac{\big|g(\vec{q}_k^{\,b}) - g(\vec{q}_{k-l}^{\,b})\big|}{L},\ 0\Big) \tag{5}$$

where $\vec{q}_k^{\,b}(= arg \max_{s=1,\cdots,S} \{g(\vec{q}_k^{\,s})\}$, where S is the number of plural particle swarms) denotes the best solution found by the whole particle swarms at time-step k. As two adjustable parameters of the internal indicator, L is duration of judgment, and ε is the positive tolerance coefficient (sensitivity).

It is obvious that the bigger the value of the coefficient ε is, the higher the probability of $\frac{1}{L}\sum_{l=1}^{L}\big|g(\vec{q}_k^{\,b}) - g(\vec{q}_{k-l}^{\,b})\big| < \varepsilon$ is, and vice verse. The change of the output y_k reflects the result of group decision-making generated by the whole particle swarms about the present search. Since ineffective behavior and useless attempt of the multi-swarm search in the optimization is overcome by the reliable way of alleviating stagnation, the search efficiency could be greatly enhanced.

[4]Diversive curiosity signifies instinct to seek novelty, to take risks, and to search for adventure.

[5]Specific curiosity signifies instinct to investigate a specific object for its full understanding.

4 Computer Experiments

To facilitate comparison and analysis of the performance indexes of the proposed method, we use a suite of the multi-benchmark problems [27] and the corresponding criteria in Table 1.

It is obviously displayed from the fitness functions of the given benchmark problems with two dimensions in Table 1 that the distribution characteristics of these problems, i.e. the *Sphere* problem is unimodal with axes-symmetry, the *Rosenbrock* problem is unimodal with asymmetry, the *Schwefel* problem is unimodal with line symmetry, the *Griewank* and *Rastrigin* problems are multimodal with different distribution density and axes-symmetry, and the *Hybrid composition* problem is multimodal with different distribution density and line symmetry.

For dealing with these problems, Table 2 gives the major parameters in the MPSOIWα/DC employed in the next computer experiments.

4.1 Preliminaries

In order to ensure higher search performance of the MPSOIWα/DC, the optimized PSOIW is applied for accomplishment of the efficient multi-swarm search. For doing this, we adopt the method of meta-optimization of the PSOIW heuristics, called evolutionary particle swarm

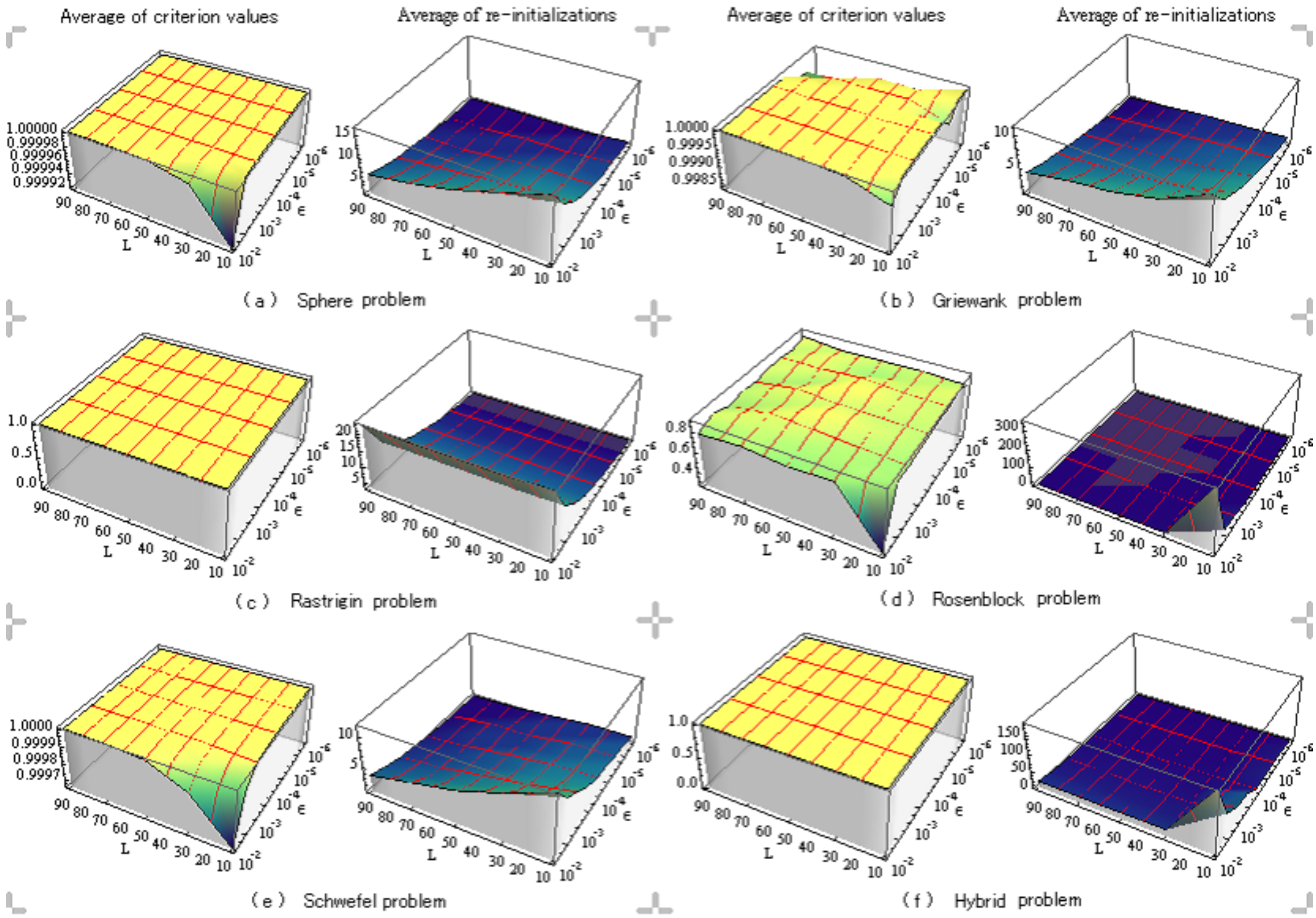


Figure 2: The distributions of average of criterion values and average of re-initialization frequencies with tuning the parameters, L and ε, for each problem.

optimizer with inertia weight (EPSOIW) [38] estimates appropriate values of parameters in the PSOIW corresponding to each given problem. The method uses a real-coded genetic algorithm and a cumulative fitness function to stable estimation.

As the results of the meta-optimization run, Table 3 shows the obtained values of parameters in the estimated PSOIW corresponding to each given 5D benchmark problem with 20 trials. We can see that the obtained average of the parameter values of each PSOIW is quite different from that of the original PSOIW. This fact indicates that the parameter values in the original PSOIW are not the best ones to these given problems, and different problem should be solved by different proper values of parameters in the PSOIW, and implementing the EPSOIW to find a rational PSOIW with high performance is necessary.

Then the average of the parameter values, $\hat{w}_e$, are less than 1 for each problem, it suggests that the search behaviors of particles finally converge at an optimal solution and a near-optimal solution. In contrast to this, the average of the parameter values, $\hat{w}_1$ and $\hat{w}_2$, drastically exceeds 1, this result indicates that the global search have more randomization without restriction for efficiently dealing with the *Rastrigin* and *Hybrid* problems by escaping from local minimum since both of them are all complex multimodals.

Consequently, these estimated PSOIW in Table 3 as the optimized optimizers, PSOIW*, are used in the MPSOIWα/DC for improving the convergence and search accuracy, and certainly improving the search performance of the proposed method to the given benchmark problems.

4.2 Performance of the MPSOIW$^*\alpha$/DC

For clarifying the characteristics of the proposed method, Figure 2 gives the search performance of the MPSOIW$^*\alpha$/DC for each benchmark problem with 20 trials by tuning the parameters of the internal indicator, L and ε. The following outstanding features of the MPSOIW$^*\alpha$/DC are observed.

- The average of re-initialization frequencies monotonously increases with increment of the tolerance parameter, ε, and decrement of the

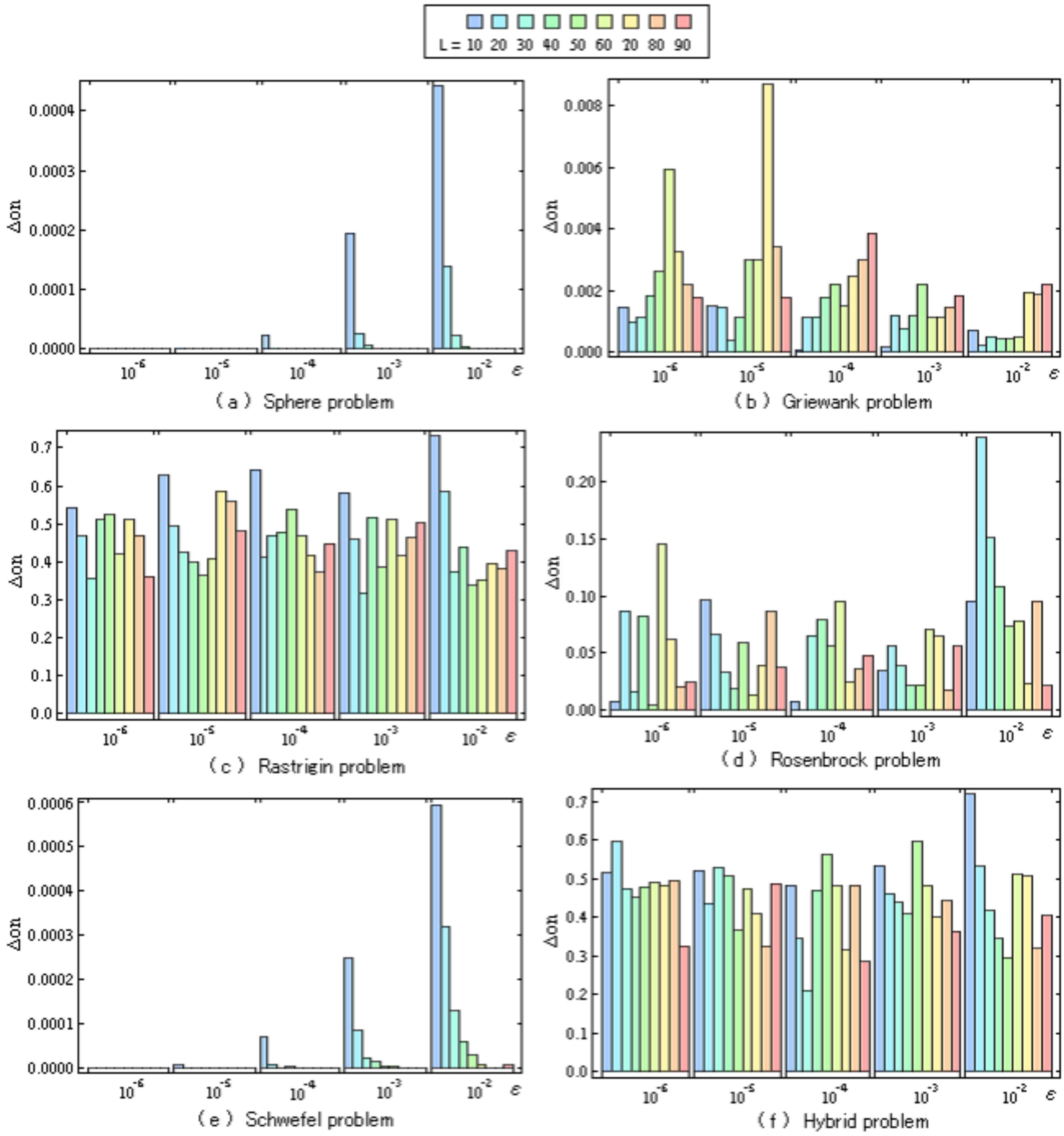


Figure 3: The performance comparison between the MPSOIW$^*\alpha$/DC and MPSOIWα/DC.

duration of judgment, L, for each benchmark problem.

- The average of criterion values do not change at all with tuning the parameters, L and ε, for the *Rastrigin* and *Hybrid* problems.

- For obtaining better search performance, the recommended range of parameters of the MPSOIW$^*\alpha$/DC: $L^*_{Sp} \in (10 \sim 90)$ and $\varepsilon^*_{Sp} \in (10^{-6} \sim 10^{-2})$ for the *Sphere* problem; $L^*_{Gr} \in (10 \sim 90)$ and $\varepsilon^*_{Gr} \in (10^{-5} \sim 10^{-3})$ for the *Griewank* problem; $L^*_{Ra} \in (10 \sim 90)$ and $\varepsilon^*_{Ra} \in (10^{-6} \sim 10^{-2})$ for the *Rastrigin* problem; $L^*_{Ro} \in (10 \sim 90)$ and $\varepsilon^*_{Ro} \in (10^{-5} \sim 10^{-3})$ for the *Rosenbrock* problem; $L^*_{Ri} \in (10 \sim 90)$ and $\varepsilon^*_{Sw} \in (10^{-6} \sim 10^{-2})$ for the *Schwefel* problem; and $L^*_{Hy} \in (10 \sim 90)$ and $\varepsilon^*_{Hy} \in (10^{-6} \sim 10^{-2})$ for the *Hybrid* problem are available.

As to the results of the *Rastrigin* and *Hybrid* problems, the obtained average of the criterion values in Figure 2(c) and Figure 2(f) are mostly unchanged with tuning the parameters, L and ε. This phenomenon suggests that the optimized PSOIW* has powerful search ability to deal with the two multimodal problems.

On the other hand, due to stochastic factor in the PSOIW search and complexity of the *Rosenbrock* problem, some irregular change of the experimental results can be discovered in Figure 2(d). Moreover, because of the effect of the used hybrid search, the fundamental finding, “the zone of curiosity,” in psychology [7] is not distinguished

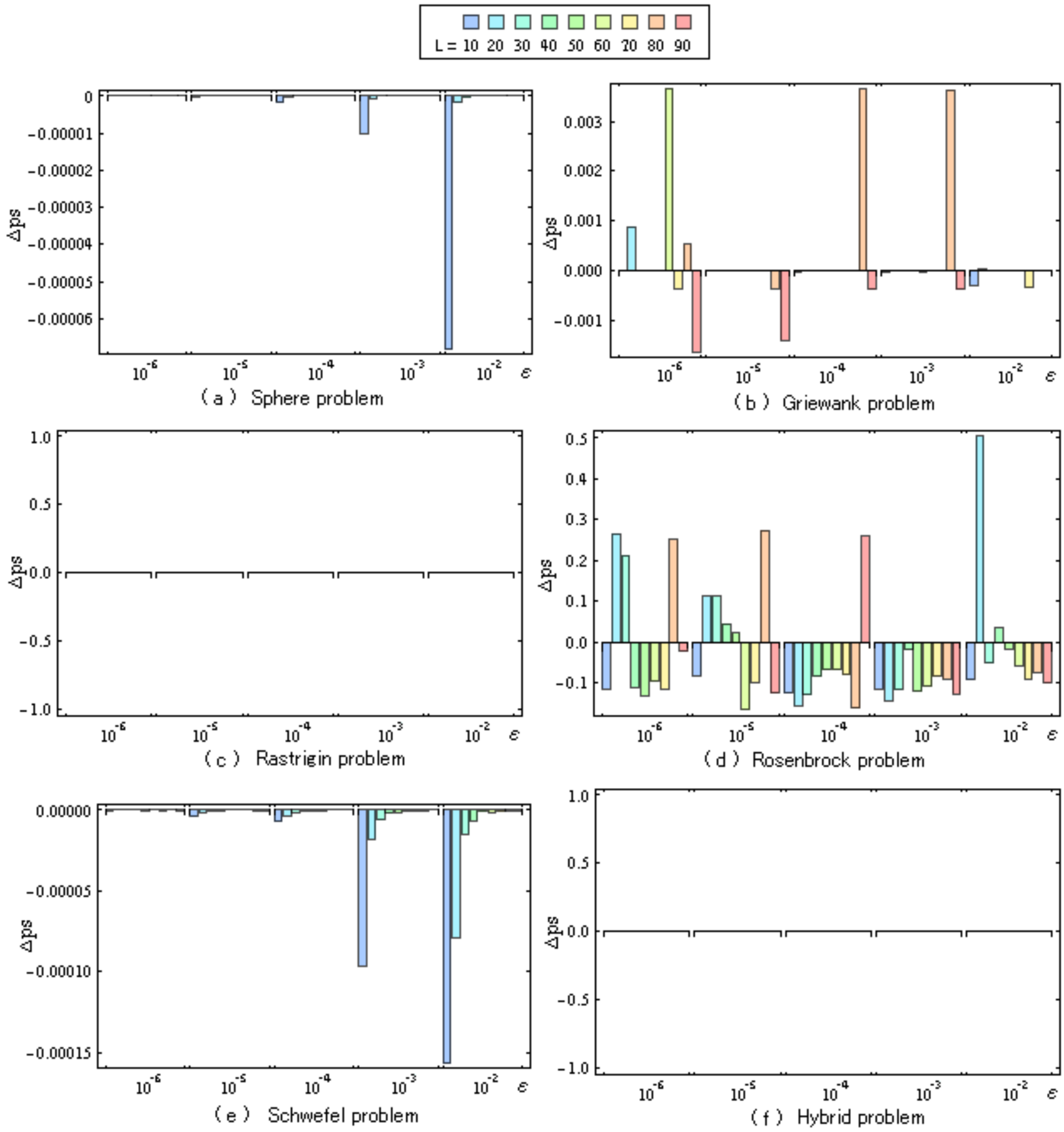


Figure 4: The performance comparison between the MPSOIW$^*\alpha$/DC and PSOIW$^*\alpha$/DC.

except for the *Rosenbrock* problem. Hence, the surface of "the average of criterion values" seems to be a plane without the change of the parameters L and ε. This means that the MPSOIW$^*\alpha$/DC has good adaptability to efficiently deal with the given benchmark problems under the setting range of parameters, L and ε.

We also observe that the average of re-initialization frequencies is over 300 times in the case of the parameters, i.e. L=10 and $\varepsilon = 10^{-2}$, for the *Rosenbrock* problem in Figure 2(d). Since the average of the criterion values is the lowest than that in the other cases, this result shows that the search behavior of the multi-swarm seems to have entered "the zone of anxiety," [7] which leads the search performance of the MPSOIWα/DC to be lower. However, the average of re-initialization frequencies is close to 150 times in the same case for the *Hybrid* problem in Figure 2(f), the situation of anxiety does not appear.

4.3 Performance Comparison

To declare the intrinsic characteristics and the respective effect of the multi-swarm and hybrid search in the MPSOIWα/DC, the following experiments are carried out.

4.3.1 Optimized Swarms vs. Non-optimized Swarms

For confirming the effectiveness of the optimized PSOIW used in the MPSOIWα/DC, Figure 3 shows the resulting difference, $\Delta_{ON} = \bar{g}^*_O - \bar{g}^*_N$ ($\bar{g}^*_O$: the average of

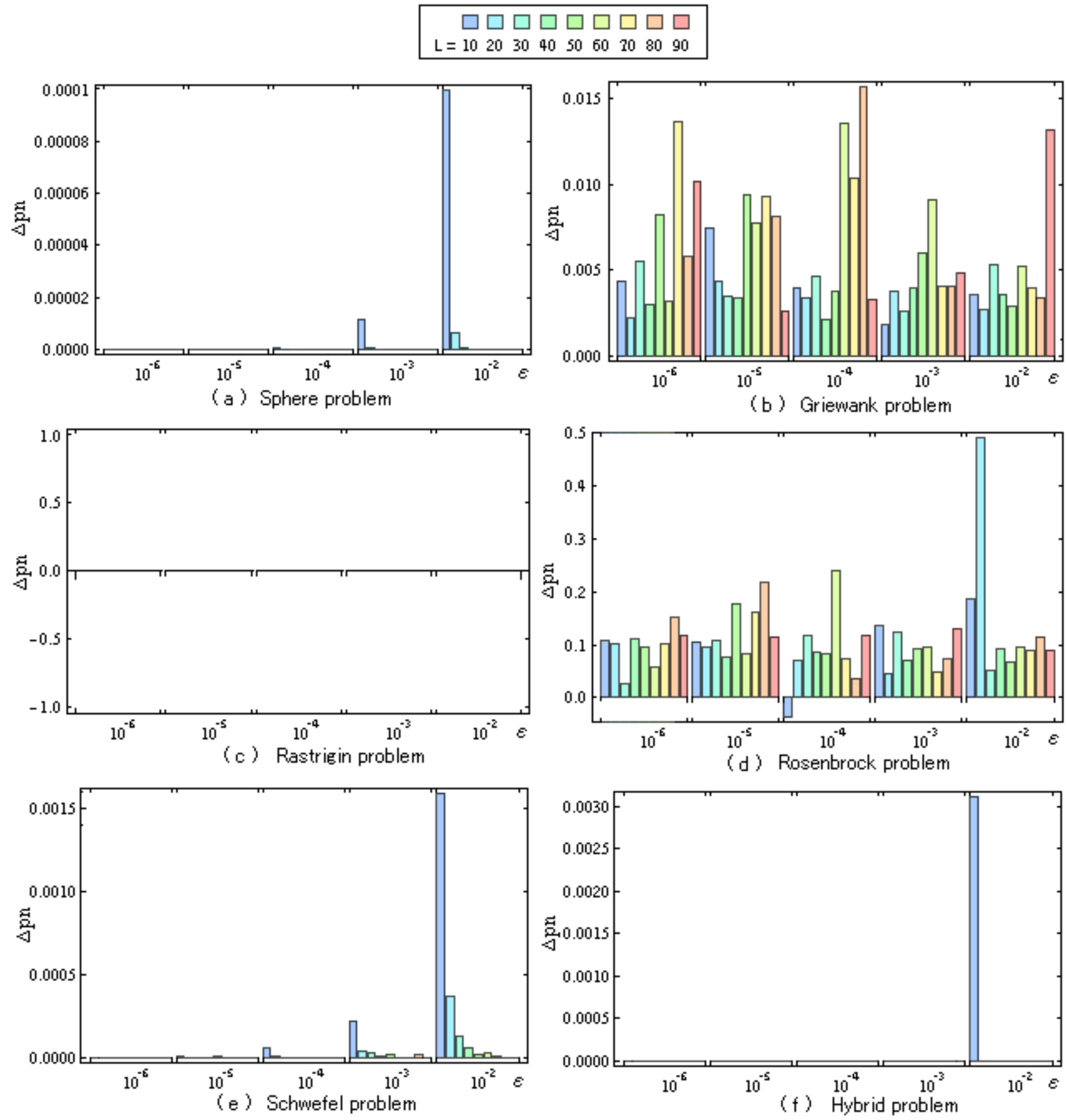


Figure 5: The performance comparison between the MPSOIW$^*\alpha$/DC and MPSOIW*/DC.

criterion values of the MPSOIW$^*\alpha$/DC, $\bar{g}_N^*$: the average of criterion values of the MPSOIWα/DC). The obtained results indicate that the search performance of the MPSOIWα/DC are greatly improved by introduction of the optimized PSOIW especially for the *Rastrigin*, *Rosenbrock*, and *Hybrid* problems. Since all of the differences are positive, the effect of the optimized PSOIW in the optimization is remarkable.

4.3.2 Single Swarm vs. Multiple Swarms

For equal treatment in search, the number of particles used in a single swarm is the same to the total number of particles used in the multi-swarms. Figure 4 shows the resulting difference, $\Delta_{PS} = \bar{g}_P^* - \bar{g}_S^*$ ($\bar{g}_P^*$: the average of criterion values of the MPSOIW$^*\alpha$/DC, $\bar{g}_S^*$: the average of criterion values of the PSOIW$^*\alpha$/DC). We can see that the search performance of both the MPSOIW$^*\alpha$/DC and PSOIW$^*\alpha$/DC seems to be the same for the *Rastrigin* and *Hybrid* problems. This is because the effect of the EPSOIW, i.e. the optimized PSOIW are suitable to efficiently deal with the *Rastrigin* and *Hybrid* problems.

In comparison with the differences between two methods for the *Sphere*, *Rastrigin*, *Rosenbrock*, and *Hybrid* problems, we observe that the search performance of the MPSOIW$^*\alpha$/DC is better than that by the PSOIW$^*\alpha$/DC under the low-sensitivity condition of $\varepsilon \leq 10^{-5}$. This fact suggests that the superior search performance can be obtained by operating singular swarm under the high sensitivity condition of $\varepsilon \geq 10^{-4}$.

Table 4: The mean and standard deviation of criterion values in each method for each 5D benchmark problem with 20 trials. The values in bold signify the best result for each problem.

Problem	MPSOIW*α/DC	EPSOIW	PSOIW	OPSO	RGA/E	MPSOIW*α/DC
Sphere	**1.0000**±0.000	**1.0000**±0.000	**1.0000**±0.000	**1.0000**±0.000	0.9990±0.000	**1.0000**±0.000
Griewank	**1.0000**±0.000	0.9847±0.006	0.8505±0.119	0.9448±0.043	0.9452±0.078	**1.0000**±0.000
Rastrigin	**1.0000**±0.000	**1.0000**±0.000	0.2325±0.159	0.2652±0.118	0.9064±0.225	**1.0000**±0.000
Rosenblock	**0.9959**±0.006	0.6070±0.217	0.5650±0.179	0.3926±0.197	0.3898±0.227	0.9893±0.012
Schwefel	**1.0000**±0.000	**1.0000**±0.000	**1.0000**±0.000	0.7677±0.412	0.9875±0.214	**1.0000**±0.000
Hybrid	**1.0000**±0.000	0.8025±0.405	0.3905±0.374	0.3061±0.359	0.1531±0.133	**1.0000**±0.000

4.3.3 Effect of the LRS

For investigating the performance difference between the MPSOIW*α/DC and MPSOIW*/DC, Figure 5 shows the obtained experimental results corresponding to same problems. Note that the difference of them is defined by $\Delta_{PN} = \bar{g}^*_P - \bar{g}^*_N$ ($\bar{g}^*_N$: the average of criterion values of the MPSOIW*/DC). Dissimilar to the preceding results, the search performance of the MPSOIW*α/DC is better than that of the MPSOIW*/DC in the most cases for each given benchmark problem except the *Rastrigin* problem. This fact clearly indicates that the LRS plays an important role in drastically improving the search performance of the MPSOIW*/DC.

On the other hand, the effect of the LRS is not remarkable for the *Sphere*, *Schwefel*, and *Hybrid* problems in any case. These results fit in with "no free lunch" (NFL) theorem [29]. They suggest that the effect of the LRS closely depends on the object of search, which related to how to set the parameter values for the running number, J, and the search range, σ^2_N, and the inherent feature of the given benchmark problems. This is also a hot topic regarding how to rationally manage the trade-off between computational cost and search performance [26]. The details on discussion for the issue are omitted here.

4.3.4 Comparison with Other Methods

For further illuminating the effectiveness of the proposed method, we compare the search performance with the other methods such as the EPSOIW, PSOIW, OPSO (optimized particle swarm optimization) [18], and RGA/E.

Table 4 gives the obtained experimental results of implementing these methods with 20 trials. It is well shown that the search performance of the MPSOIW*α/DC is better than that by the EPSOIW, PSOIW, OPSO, and RGA/E. The results sufficiently reflect that the merging of both multiple hybrid search and the mechanism of diversive curiosity takes the active role in handling these benchmark problems. In particular, A big increase, i.e. the average of criterion values by implementing the MPSOIW*α/DC steeply rises from 0.5650 to 0.9959, in search performance is achieved well for the *Rosenbrock* problem.

5 Conclusion

A new method of cooperative PSO – multiple particle swarm optimizers with inertia weight with diversive curiosity, MPSOIWα/DC, has been proposed in this paper. Owing to the essential strategies of decentralization in search and concentration in evaluation and behavior control, the combination of the adopted hybrid search and the mechanism of diversive curiosity, theoretically, has good capability to greatly improve search efficiency and to alleviate stagnation in handling complex optimization problems.

Applications of the MPSOIWα/DC to a suite of the 5D benchmark problems well demonstrated its effectiveness. The obtained experimental results verified that unifying the both characteristics of multi-swarm search and the LRS is successful and effective. In comparison with the search performance of the EPSOIW, PSOIW, OPSO, and RGA/E, the proposed method has an enormous latent capability in treating with different benchmark problems and the outstanding powers of multi-swarm search. Accordingly, the basis of the development study of cooperative PSO research in swarm intelligence and optimization is further expanded and consolidated.

It is left for further study to apply the MPSOIWα/DC to data mining, system identification, multi-objective optimization, practical problems in the real-world, and dynamic environments.

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