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Performance Analysis of an Adaptive Algorithm for DOA Estimation

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***Abstract*—This paper presents an adaptive approach to the problem of estimating the direction of arrival angles of narrowband signals emitted from multiple sources. We reformulate the problem in state-space, and employ a multi-model partitioning algorithm, combined with extended Kalman filters, for combined identification of the number of sources and estimation of the angles of arrival. The proposed algorithm's performance is assessed by simulation in several operational scenaria. The results presented demonstrate that the algorithm is capable of tracking changes in the angles of arrival, and of detecting variations in the number of sources present.**

***Index Terms*—State estimation, system identification, Kalman filtering, direction of arrival estimation.**

I. Introduction

In this paper we address the problem of estimating the direction of arrival (DOA) angles of narrowband signals emitted from multiple sources, based on measurements obtained by a linear sensor array. This problem has been the focus of substantial research effort during the last two decades due to its importance in several application areas, including sonar and radar signal processing [1], mobile communications [2], acoustics and speech processing [3], and structural health monitoring [4].

Most of the proposed solutions employ the Maximum Likelihood (ML) approach in some form, either stochastic or deterministic [5], [6]. Suboptimal techniques with reduced computational requirements include the Minimum Variance (MV), MUSIC, Minimum Norm, ESPRIT, and weighted subspace fitting (WSF) algorithms. A key assumption made by all the above methods is that the number of sources that contribute to the received signal is known. In many problems, however, this prior knowledge is not available. For example, in sonar applications, the number of targets tracked is both unknown and time-varying, as new contacts are occasionally detected. Since the number of sources directly affects the signal model, the DOA estimation problem is in fact a combined estimation and system identification problem. The identification of the number of sources is typically carried our separately by information theoretic methods, such as the Final Prediction Error (FPE) criterion, the Akaike Information Criterion (AIC) [7] and the Minimum Description Length (MDL) Criterion [8]. The techniques that result from the above criteria do not guarantee convergence to the correct model; in fact, they typically exhibit model overfit or underfit. They also require large sets of measurements, which inhibits real-time operation, while they cannot address adequately the case of a variable number of sources.

The problem of DOA estimation is mathematically presented in the following section. The algorithm presented in Section 3 addresses the combined problem of DOA estimation and model identification. The estimation part is performed by a bank of Extended Kalman Filters (EKFs), each of which is implemented assuming a particular constant number of sources being present. A Multi-Model Partitioning Algorithm (MMPA) [9] is then used to select the correct model and corresponding EKF. The algorithm's performance is assessed by simulation experiments presented in Section 4. Finally, Section 5 summarizes our conclusions.

II. Problem Statement

We assume a linear array comprising m isotropic sensors, which receive the signals emitted from n far-field point sources. Let ϕ_i be the direction of the i-th source, and $s_i(t)$ the complex envelope of the corresponding received signal. We assume for the moment that the sources are stationary, i.e. the ϕ_i angles are constant. We further assume that the signal characteristics are invariant in time and

$$s_i(t) = e^{j\omega_i(t)}, \qquad i = 1, \ldots, n \tag{1}$$

We define the column vector $a(\phi_i)$ of dimension m to be the complex array response to a unit waveform from a source at direction ϕ_i. Assuming that n signals are simultaneously intercepted, under the narrowband assumption, the array output $z(t)$ is a column vector comprising the output signals $z_i(t)$ at individual sensors $i = 1, \ldots, m$, which is given by the following equation:

$$z(t) = A(\phi)s(t) + v(t) \tag{2}$$

where

$$\begin{aligned} z(t) &= \begin{bmatrix} z_1(t) & z_2(t) & \ldots & z_m(t) \end{bmatrix}^T \\ A(\phi) &= \begin{bmatrix} a(\phi_1) & a(\phi_2) & \ldots & a(\phi_n) \end{bmatrix}^T \\ \phi &= \begin{bmatrix} \phi_1 & \phi_2 & \ldots & \phi_n \end{bmatrix}^T \\ s(t) &= \begin{bmatrix} s_1(t) & s_2(t) & \ldots & s_n(t) \end{bmatrix}^T \end{aligned}$$

The vector process $\{v(t)\}$ of dimension m represents additive measurement noise, which is assumed to be white, Gaussian, with zero mean and covariance matrix R. The columns of the $m \times n$ matrix $A(\phi)$ are the array propagation vectors $a(\phi_i),\ i = 1, \ldots, n$. These vectors can be determined by considering the geometry of the problem, as depicted in Fig. 1.

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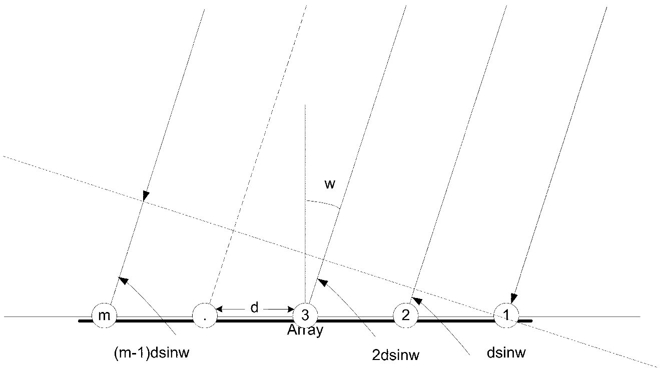


Fig. 1. Arrival delays of wavefront across the array

Let d be the spacing between sensors. When the signal wavefront from the i-th source reaches a sensor, the distance of the wavefront to the next sensor down the array is $d \sin \phi_i$. Thus the time delay between two consecutive sensors is $\tau_i = (d \sin \phi_i)/c$, where c is the propagation velocity. The corresponding phase difference is $\omega_i \tau_i = (2\pi d \sin \phi_i)/\lambda_i$, where λ_i is the signal wavelength and ω_i is the angular frequency. Therefore the array response $a(\phi_i)$ for a source at direction ϕ_i and wavelength λ_i becomes:

$$a(\phi_i) = \begin{bmatrix} 1 & e^{-j\omega_i \tau_i} & e^{-j\omega_i 2\tau_i} & \dots & e^{-j\omega_i (m-1)\tau_i} \end{bmatrix}^T \quad (3)$$

The output of the l-th sensor, where $l = 1, \dots, m$, can be written using (2) and (3) as follows:

$$z_l(t) = \sum_{i=1}^{n} e^{j2\pi[ct-(l-1)d \sin \phi_i]/\lambda_i} \quad (4)$$

In order to formalize the problem in the state space, we choose the state vector to consist of the angles of arrival ϕ_i of the n received signals, and the corresponding signal wavelengths λ_i:

$$x = \begin{bmatrix} \lambda_1 & \lambda_2 & \dots & \lambda_n & \phi_1 & \phi_2 & \dots & \phi_n \end{bmatrix}^T$$

Under the assumption of stationary sources and observer, the system state x is constant. However, in the general case of moving sources (e.g. in tracking applications), the state equation will be nonlinear and complex:

$$x(k+1) = f\big(x(k)\big) + w(k) \qquad k = 0, 1, \dots \quad (5)$$

The array output is sampled at distinct time instants, producing the following sequence of measurements:

$$z(k) = h\big(x(k)\big) + v(k) \qquad k = 1, 2, \dots \quad (6)$$

The function h in (6) is the nonlinear function of λ_i and ϕ_i given in (4). Equations (5) and (6) define a nonlinear discrete-time state-space model. The objective is, at each time step k, to obtain an estimate $\hat{x}(k|k)$ of the state $x(k)$, given the set of measurements $z(k)$ up to and including time k.

III. A Multi-Model Partitioning Algorithm for DOA Estimation

Assuming that the number of sources n is a known constant, the state space model given by (5)–(6) is completely specified. An Extended Kalman Filter (EKF) can therefore be employed to process the measurements $z(k)$ and obtain recursively the state estimates $\hat{x}(k|k)$. The EKF is a suboptimal estimator, since it approximates the nonlinear equation (6) by a linear one. However, its main advantage is that it computes the estimates in real time, without requiring a large batch of data. When the actual number of sources n differs from the one assumed by the EKF, the algorithm will exhibit large errors, due to the model mismatch.

In practical applications, n is an unknown parameter, although we may be able to set an upper bound $n \leq n_{\max}$. In such cases a viable approach is to employ a bank of EKFs, operating in parallel and independent of each other [10]. Each filter is implemented based on the same set of equations (5)–(6), but assuming a particular value of n. All filters operate concurrently on the same measurements $z(k)$; however, since they assume different system models, each filter produces its own model-conditional estimate $\hat{x}(k|k;n)$. It must be noted that the conditional system models, and therefore the conditional estimates, are of different dimensions.

In order to select the correct model and corresponding filter among the $n_{\max}$ candidate models, we employ a multi-model partitioning algorithm (MMPA). The general form of the algorithm [9], [11] can produce an unconditional state estimate from a (possibly infinite) set of model-conditional estimates; this can be either a minimum-variance, or a maximum a posteriori (MAP) estimate, depending on the formulation. The algorithm is based on calculation of the a posteriori probability of each model being the correct one. In the problem at hand, the a posteriori probability $p(n|k)$ of the parameter n can be recursively calculated as follows:

$$p(n|k) = \frac{L(k|k;n)}{\displaystyle\sum_{i=1}^{n_{\max}} L(k|k;i)p(i|k-1)} p(n|k-1) \quad (7)$$

where $L(k|k;n)$ is a likelihood function given by

$$L(k|k;n) = |P_{\tilde{z}}(k|k-1;n)|^{-1/2} \times \exp\left[-\tfrac{1}{2}\tilde{z}^T(k|k-1;n)P_{\tilde{z}}^{-1}(k|k-1;n)\tilde{z}(k|k-1;n)\right] \quad (8)$$

The quantities $\tilde{z}(k|k-1;n) = z(k) - H(k;n)\hat{x}(k|k-1;n)$ and $P_{\tilde{z}}(k|k-1;n)$ are the conditional innovation sequences and corresponding covariance matrices produced by the conditional EKFs, where $H(k;n)$ are the observation matrices produced by the filters during linearization of the nonlinear function h in (6).

At each step k, the algorithm selects as the number of sources the value of n that maximizes the a posteriori probability $p(n|k)$, for $n = 1, \dots, n_{\max}$. The DOA estimates are then given by the conditional estimate $\hat{x}(k|k;n)$ of the corresponding n-th EKF. This approach has, among others, the advantage of producing estimates of both the number of sources and their DOA in real time; i.e. there is no need to collect a large set of measurements. In addition, if the number of sources changes, we would expect the a posteriori probabilities to reflect this change and select the correct filter, again in real time.

IV. Simulation Results

The proposed algorithm is a suboptimal estimator of a nonlinear system. Its performance can therefore be assessed only by experimentation. Here we present results obtained by simulation of the algorithm in several typical operational scenaria. In all cases, the array consists of 15 isotropic sensors with equal spacing of $d = 0.45\lambda$.

The algorithm employs a parallel bank of six EKFs, each of which is implemented assuming a different number of sources n being present in the received signal, from $n = 1$ to $n_{\max} = 6$. The a posteriori probabilities corresponding to each elemental filter were calculated from (7) and (8).

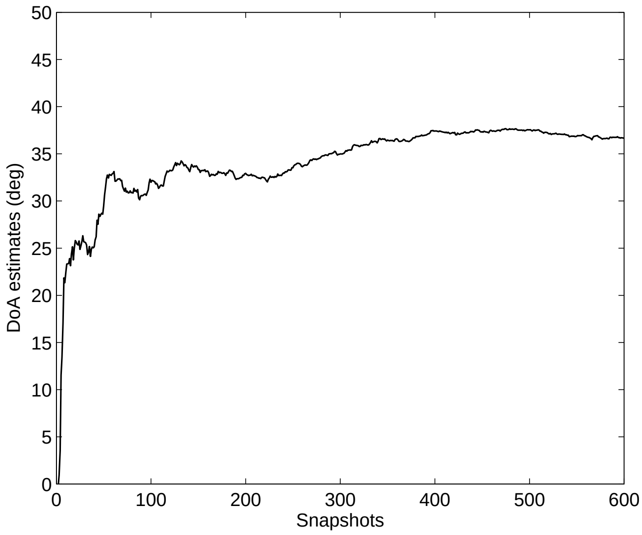
Experiment 1: A single source emitting from a constant direction $\phi = 40°$ was simulated. In this case, the signal received by the array is given by (4) for $n = 1$. The received signal was corrupted by additive Gaussian noise of equal power to that of the signal (SNR $= 0$ dB). The results of this experiment are shown in Fig. 2.

Despite the high noise level, the algorithm was able to identify correctly the number of sources within the first three steps. As shown in Fig. 2(c), the a posteriori probability $p(n = 1|k)$, corresponding to the correct hypothesis of a single source present, converged rapidly to 1 from time step $k = 3$ onwards, while the rest probabilities $p(n|k)$ converged to 0 for $n = 2, \ldots, 6$. This convergence behavior is due to the exponential in (8), which is used to calculate the a posteriori probabilities. The estimates produced by the EKF conditioned on $n = 1$ is shown in Fig. 2(a) as a function of time (snapshots taken). It can be seen that the response time of the EKF is higher than that of the a posteriori probabilities, since the filter requires about 100 measurements to estimate the source direction within a 10% margin of error. However, the filter converged in all our simulation trials, displaying a low steady-state error.

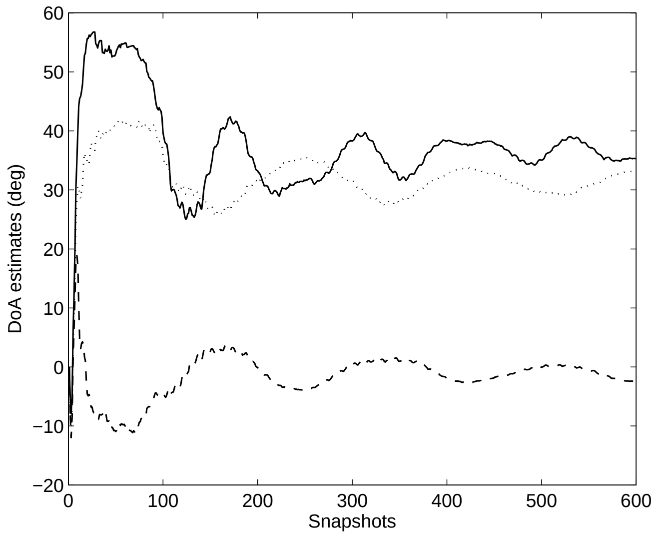
The rest EKFs (i.e. the ones assuming $n = 2, \ldots, 6$ sources present) gave erratic or divergent results; however these were discarded by the algorithm, since $p(n|k) \approx 0$. The estimates produced by the EKF conditioned on $n = 3$ are depicted in Fig. 2(b) for comparison purposes. The three curves shown are the estimates corresponding to the three sources assumed by the filter. All estimates display an oscillatory behavior.

Experiment 2: Three completely coherent signals arriving from directions $\phi = 10°$, $20°$ and $30°$ were simulated. Again, measurement noise was added with SNR $= 0$ dB. This is an unfavorable situation for DOA estimation because of the narrow spacing between the sources. As shown in Fig. 3, after a transient period of about 10 samples, the algorithm identified correctly the number of sources, as indicated by $p(n = 3|k) \approx 1$. The EKF corresponding to $n = 3$ was able to differentiate between the three sources after 30 samples, and produced steady-state estimates of the DOAs after about 150 samples.

Experiment 3: Here we assess the capability of the algorithm to identify a variable number of sources, while simultaneously estimating their DOAs. The noise level and all other parameters remain as previously described, but the



(a)



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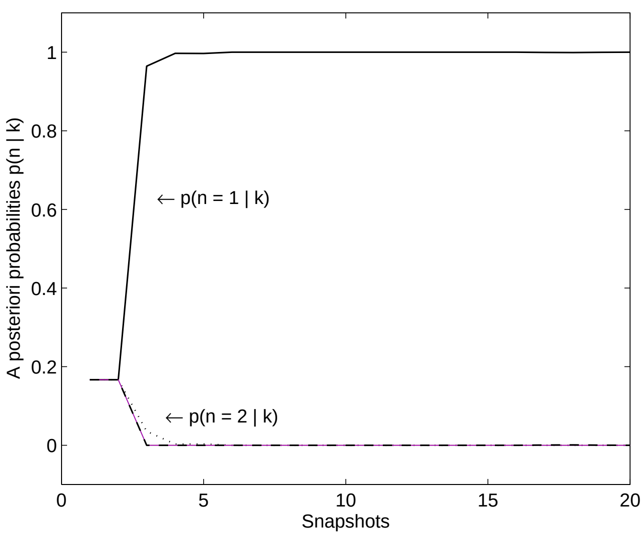

(c)

Fig. 2. Results of experiment 1: (a) DOA estimates by the EKF conditioned on the correct hypothesis of $n = 1$ (single source). (b) Erratic behaviour of the EKF conditioned on the hypothesis of $n = 3$ sources. (c) A posteriori probabilities $p(n|k)$ for $n = 1, \ldots, 6$, calculated by the MMPA.

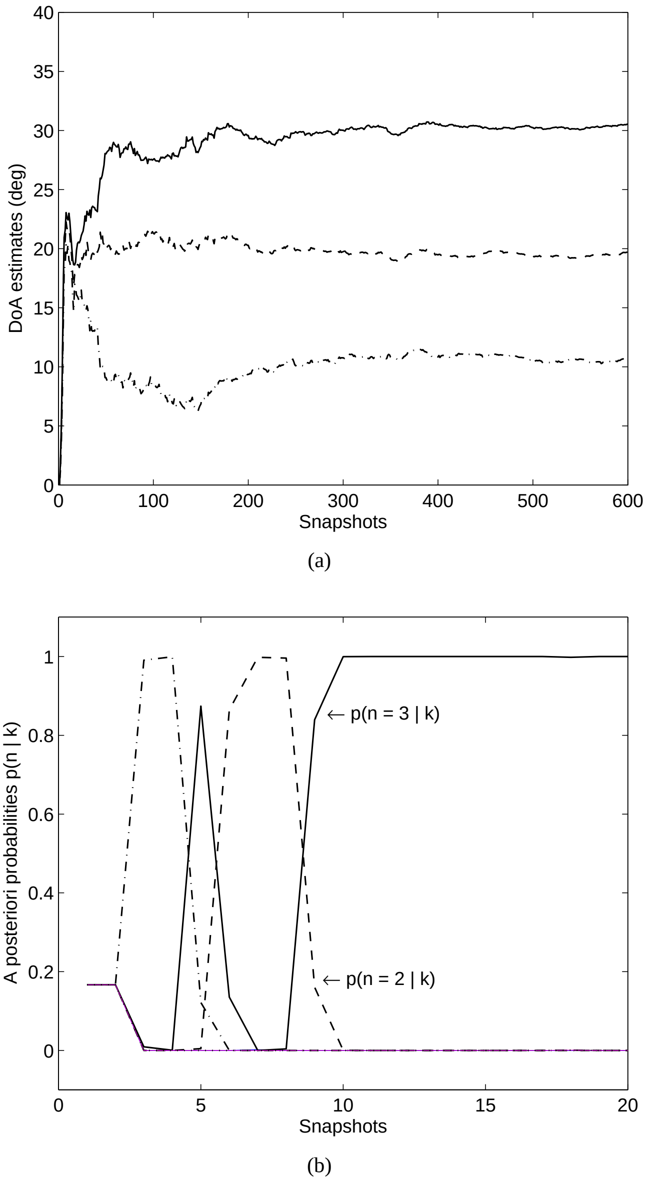

Fig. 3. Results of experiment 2: (a) Estimates of the EKF conditioned on the correct hypothesis of $n = 3$ sources present. (b) A posteriori probabilities $p(n|k)$ for $n = 1, \ldots, 6$, calculated by the MMPA.

number of sources varies as follows: during the first 100 steps of the simulation, a single source is present at direction $\phi_1 = 10°$. At time step $k = 100$, a second source appears at direction $\phi_2 = 30°$. The two sources are present in the signal received until $k = 400$, when the second source stops transmitting.

The a posteriori probabilities calculated by the algorithm are depicted in Fig. 4(c). It can be seen that the algorithm successfully detects the transitions from one to two sources at $k = 100$ and back to one source at $k = 400$. In both cases 5-10 snapshots were sufficient to determine the correct number of sources. The estimates produced by the EKF conditioned on $n = 1$, shown in Fig. 4(a), are valid for the intervals (0, 100) and (400, 600). During these intervals we had $p(n = 1|k) \approx 1$, and the filter estimates approached the actual DOA value $\phi_1 = 10°$, as seen in Fig. 4(b). On the contrary, outside these intervals the filter exhibited erratic and oscillatory behaviour. Similarly, the second EKF yielded valid results only within the interval (100, 400), corresponding with the fact that $p(n = 2|k) \approx 1$.

Experiment 4: A slowly moving source is simulated. The noise level and all other parameters are as described in the previous experiments. The source direction now changes linearly from an initial value of $\phi = 10°$ to a final value of $\phi = 30°$ over the observation period of 600 time steps. In this situation there exists a model mismatch, since our algorithm has been designed based on a model of a constant state vector (i.e. constant DOAs). In order to provide the algorithm with a minimal tracking capability without altering the basic system model, we introduce a fictitious plant noise in the EKFs. In particular, we implement the filters assuming that the system state evolves according to

$$x(k + 1) = x(k) + w(k) \tag{9}$$

The sequence $\{w(t)\}$ in (9) is a white Gaussian noise vector with zero mean and covariance Q; in our experiments we used $Q = 10^{-4}$ for the case of a single source. On the other hand, the actual system model used to produce the DOA measurements in this experiment was of the form

$$x(k + 1) = Ax(k) \tag{10}$$

It must be noted that both (9) and (10) are special cases of (5). The model mismatch lies in the different value for the system matrix A assumed by each model.

Again the algorithm identified correctly the presence of a single source. The estimates of the EKF corresponding to $n = 1$ are presented in Fig. 5. It can be seen that the filter is able to track the moving source, although it underestimates the rate of change of the DOA angles. However, in order for the algorithm to be capable of following a faster moving source, a higher order state model is required, incorporating at least the derivatives of the arrival angles ϕ_i.

V. Conclusions

In this work we have presented an algorithm that addresses two aspects of the DOA estimation problem: the identification of the number of sources, and the estimation of the corresponding directions of arrival. Results obtained from simulation experiments show that the algorithm is capable to identify correctly the number of sources present. Typically the identification is carried out using 5 to 10 measurements in most cases, which is a lot faster than information-theoretic approaches used in the literature [7], [8]. A key advantage of the algorithm is also its capacity to detect a change in the number of sources in real time. Finally, it must also be noted that the algorithm exhibits a high degree of parallelism; all EKFs can be implemented in parallel for real-time applications.

The algorithm, as presented herein, is based on a simplified model assuming stationary sources. A natural extension would be to consider moving sources by incorporating into the system model information on the source location, velocity, and possibly acceleration. This is the subject of current investigation.

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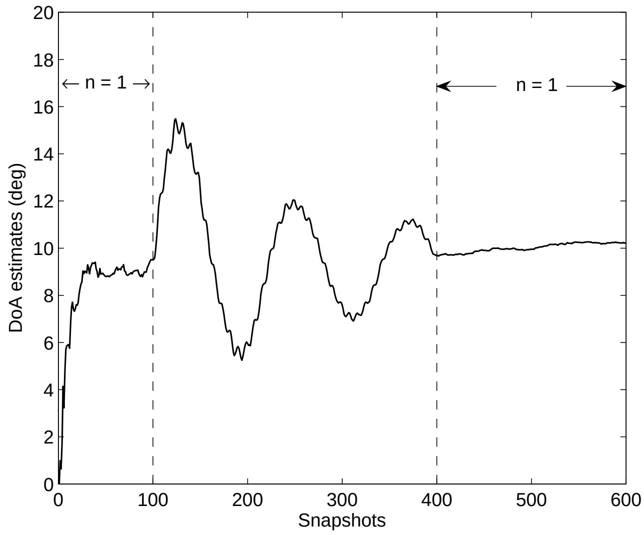
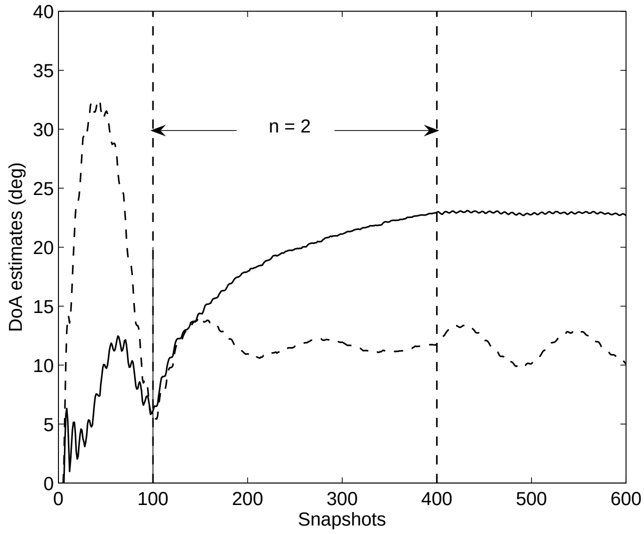


(a)



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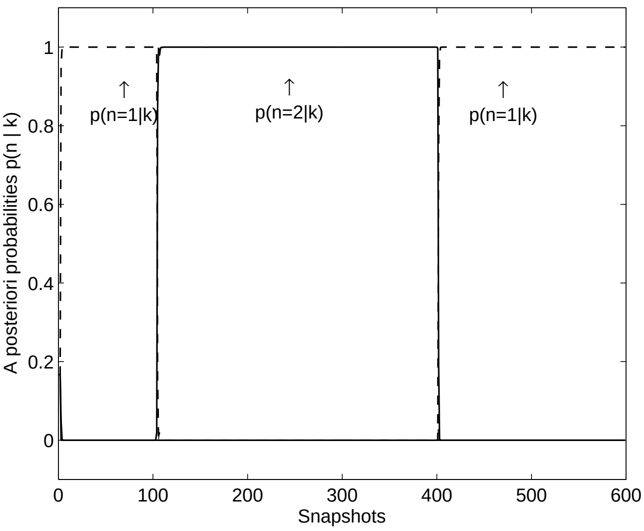

(c)

Fig. 4. Results of experiment 3 involving a variable number of sources: (a) Estimates of the EKF conditioned on the hypothesis of $n = 1$ sources present. The EKF estimates are valid only within the intervals indicated. (b) Estimates of the EKF conditioned on $n = 3$. (c) A posteriori probabilities $p(n|k)$. Note the transitions from one to two sources at snapshot $k = 100$ and back to a single source at $k = 400$.

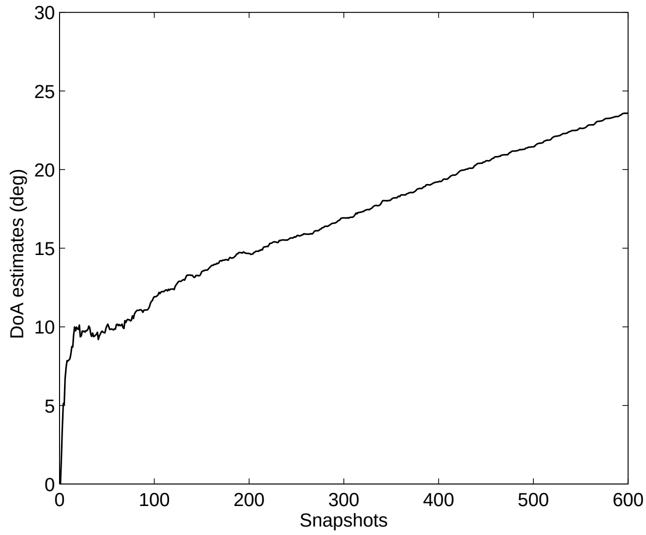

Fig. 5. Results of experiment 4: DOA estimates from a moving source.

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