Constructing Two Edge-Disjoint Hamiltonian Cycles and Two-Equal Path Cover in Augmented Cubes

Ruo-Wei Hung

Abstract*—The n-dimensional hypercube network Q_n is one of the most popular interconnection networks since it has simple structure and is easy to implement. The n-dimensional augmented cube AQ_n, an important variation of the hypercube, possesses several embedding properties that hypercubes and other variations do not possess. The advantages of AQ_n are that the diameter is only about half of the diameter of Q_n and it is node-symmetric. Recently, some interesting properties of AQ_n have been investigated in the literature. The presence of edge-disjoint Hamiltonian cycles provides an advantage when implementing algorithms that require a ring structure by allowing message traffic to be spread evenly across the interconnection network. A network G contains two-equal path cover and is called two-equal path coverable if for any two distinct pairs of nodes $\langle \mu_s, \mu_t \rangle$ and $\langle \upsilon_s, \upsilon_t \rangle$ of G, there exist two node-disjoint paths P and Q satisfying that (1) P joins μ_s and μ_t, and Q joins υ_s and υ_t, (2) $|P| = |Q|$, and (3) every node of G appears in $P \cup Q$ exactly once. In this paper, we first prove that the augmented cube AQ_n contains two edge-disjoint Hamiltonian cycles for $n \geqslant 3$. We then prove that AQ_n, with $n \geqslant 2$, is two-equal path coverable. Based on the proofs of existences, we further present linear time algorithms to construct two edge-disjoint Hamiltonian cycles and two-equal path cover in an n-dimensional augmented cube AQ_n.

***Index Terms*—two edge-disjoint Hamiltonian cycles, two-equal path cover, augmented cubes, interconnection networks, parallel computing.**

I. Introduction

PARALLEL computing is important for speeding up computation. The design of an interconnection network is the first thing to be considered. Many topologies have been proposed in the literature [4], [9], [10], [11], [12], and the desirable properties of an interconnection network include symmetry, relatively small degree, small diameter, embedding capabilities, scalability, robustness, and efficient routing. Among the proposed interconnection networks, the hypercube is a popular interconnection network with many attractive properties such as regularity, symmetry, small diameter, strong connectivity, recursive construction, partition ability, and relatively low link complexity [30]. The architecture of an interconnection network is usually modeled by a graph, where the nodes represent the processing elements and the edges represent the communication links. In this paper, we will use graphs and networks interchangeably.

The n-dimensional augmented cube, denoted by AQ_n, was first proposed by Choudum et al. [7] and possesses some properties superior to the hypercube. The diameter of n-dimensional augmented cube is only about half of the diameter of n-dimensional hypercube, and augmented cubes are node-symmetric [7]. Recently, some interesting properties, such as conditional link faults, of the augmented cube AQ_n have been investigated in the literature. Choudum and Sunitha [7] proved AQ_n, with $n \geqslant 2$, is pancyclic; that is, AQ_n contains cycles of arbitrary length. Wang et al. [33] showed that AQ_n, with $n \geqslant 4$, remains pancyclic provided faulty vertices and/or edges do not exceed $2n-3$. Hsieh and Shiu [13] proved that AQ_n is node-pancyclic, in which for every node u and any integer $\ell \geqslant 3$, the graph contains a cycle of length ℓ such that u is in the cycle. Hsu et al. [16] proved that AQ_n is geodesic pancyclic and balanced pancyclic. Recently, Chan et al. [6] improved the results in [16] to obtain a stronger result for geodesic pancyclic and fault-tolerant panconnectivity of the augmented cube AQ_n. In [25], Ma et al. proved that AQ_n contains paths between any two distinct vertices of all lengths from their distance to $2^n - 1$; and that AQ_n still contains cycles of all lengths from 3 to 2^n when any $(2n-3)$ edges are removed from AQ_n. Xu et al. [34] determined the vertex and the edge forwarding indices of AQ_n as $2^n/9 + (-1)^{n+1}/9 + n2^n/3 - 2^n + 1$ and 2^{n-1}, respectively. Recently, Chan [5] computed the distinguishing number of the augmented cube AQ_n.

A *Hamiltonian cycle* in a graph is a simple cycle that passes through every node of the graph exactly once. The ring structure is important for distributed computing, and its benefits can be found in [21]. Two Hamiltonian cycles in a graph are said to be *edge-disjoint* if they do not share any common edge. The edge-disjoint Hamiltonian cycles can provide advantage for algorithms that make use of a ring structure [32]. Consider the problem of all-to-all broadcasting in which each node sends an identical message to all other nodes in the network [32]. There is a simple solution for the problem using an n-node ring that requires $n-1$ steps, i.e., at each step, every node receives a new message from its ring predecessor and passes the received message to its ring successor. If the network admits edge-disjoint rings, then messages can be divided and the parts broadcast along different rings without any edge contention. If the network can be decomposed into edge-disjoint Hamiltonian cycles, then the message traffic will be evenly distributed across all communication links (edges). Edge-disjoint Hamiltonian cycles also form the basis of an efficient all-to-all broadcasting algorithm for networks that employ wormhole or cut-through routing [23]. Further, edge-disjoint Hamiltonian cycles provide the edge-fault tolerant hamiltonicity of an interconnected network; that is, when a Hamiltonian cycle of an interconnected network contains one faulty edge, then the

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other edge-disjoint Hamiltonian cycle can be used to replace it for transmission. In addition, two edge-disjoint Hamiltonian cycles of an interconnection network can be applied to logical dual-ring topology [31]. A dual-ring topology allows traffic to flow in opposite directions, with one ring counter-rotating to the other. Normally in a dual-ring network one ring is the primary path while the secondary ring is the secondary path (or backup path). SONET is an example of a network that may use a dual-ring topology. On the other hand, if one ring experiences a failure, the other one provides operability. Thus, a dual-ring topology provides edge-fault tolerance. In this paper, we use a recursive construction to show that, for any integer $n \geqslant 3$, there are two edge-disjoint Hamiltonian cycles in the n-dimensional augmented cube AQ_n.

Finding node-disjoint paths is one of the important issues of routing among nodes in various interconnection networks. Node-disjoint paths can be used to avoid communication congestion and provide parallel paths for an efficient data routing among nodes. Moreover, multiple node-disjoint paths can be more fault-tolerant of nodes or link failures and greatly enhance the transmission reliability. A *path cover* of a graph G is a family of node-disjoint paths that contain all nodes of G. For an embedding of linear arrays in a network, the path cover implies every node can be participated in a pipeline computation. Finding a minimum path cover and its variants of a graph have been investigated [17], [18], [26], [27], [28]. In this paper, we will study a variation of path cover, called *two-equal path cover*. A graph G contains *two-equal path cover* and is called *two-equal path coverable* if for any two distinct pairs of nodes $\langle \mu_s, \mu_t \rangle$ and $\langle \upsilon_s, \upsilon_t \rangle$ of G, there exists a path cover $\{P, Q\}$ of G such that (1) P joins μ_s and μ_t, (2) Q joins υ_s and υ_t, and (3) $|P| = |Q|$. Finding two-equal path cover in an interconnected network can be applied to the routing problem in which the network is decomposed into two disjoint sub-networks with the same number of nodes such that each sub-network contains a Hamiltonian path. In this paper, we will show that the augmented cube AQ_n, with $n \geqslant 2$, is two-equal path coverable. Based on the proof of existence, we present a recursive algorithm to construct two-equal path cover of an n-dimensional augmented cube AQ_n given any two distinct pairs of nodes $\langle \mu_s, \mu_t \rangle$ and $\langle \upsilon_s, \upsilon_t \rangle$ of AQ_n.

Related areas of investigation are summarized as follows. The edge-disjoint Hamiltonian cycles in k-ary n-cubes has been constructed in [2]. Barden et al. [3] constructed the maximum number of edge-disjoint spanning trees in a hypercube. Petrovic et al. [29] characterized the number of edge-disjoint Hamiltonian cycles in hyper-tournaments. Hsieh et al. [14] constructed edge-disjoint spanning trees in locally twisted cubes. The existence of a Hamiltonian cycle in augmented cubes has been shown in [7], [15]. However, there has been no work reported so far on edge-disjoint properties in augmented cubes. Hsu et al. [15] considered the fault hamiltonicity and the fault hamiltonian connectivity of the augmented cube AQ_n. Lee et al. [24] studied the Hamiltonian path problem on AQ_n with a required node being the end node of a Hamiltonian path. Abuelrub [1] studied the robustness capability of crossed cubes in constructing a Hamiltonian cycle despite the presence of faulty nodes or edges. Lai et al. [22] showed that crossed cubes and twisted cubes contain two-equal path cover. A preliminary version of this paper has appeared in [19]. Recently, we present a linear time algorithm to construct two edge-disjoint Hamiltonian cycles in locally twisted cubes [20].

The rest of this paper is organized as follows. In Section II, the structure of augmented cubes is introduced, and some definitions and notations used in the paper are given. Section III first shows the existence of two edge-disjoint Hamiltonian cycles in augmented cubes. We then present a recursive algorithm to construct two edge-disjoint Hamiltonian cycles of an augmented cube using the proof of existence. In Section IV, we prove that augmented cubes are two-equal path coverable. We then give a linear time algorithm for constructing two-equal path cover of an augmented cube using the proof of existence. Finally, we conclude this paper in Section V.

II. Preliminaries

We usually use a graph to represent the topology of an interconnection network. A graph $G = (V, E)$ is a pair of the node set V and the edge set E, where V is a finite set and E is a subset of $\{(u, v) | (u, v)$ is an unordered pair of $V\}$. We will use $V(G)$ and $E(G)$ to denote the node set and the edge set of G, respectively. If (u, v) is an edge in a graph G, we say that u *is adjacent to* v and u, v *are incident to* edge (u, v). A *neighbor* of a node v in a graph G is any node that is adjacent to v. Moreover, we use $N_G(v)$ to denote the set of neighbors of v in G. The subscript 'G' of $N_G(v)$ can be removed from the notation if it has no ambiguity.

Let $G = (V, E)$ be a graph with node set V and edge set E. A path P of length ℓ in G, denoted by $v_0 \to v_1 \to \cdots \to v_{\ell-1} \to v_\ell$, is a sequence $v_0, v_1, \cdots, v_{\ell-1}, v_\ell$ of nodes such that $(v_i, v_{i+1}) \in E$ for $0 \leqslant i \leqslant \ell - 1$. The first node v_0 and the last node v_ℓ visited by P are called the *path-start* and *path-end* of P, denoted by $start(P)$ and $end(P)$, respectively, and they are called the *end nodes* of P. Path $v_\ell \to v_{\ell-1} \to \cdots \to v_1 \to v_0$ is called the *reversed path*, denoted by P_{rev}, of path P. That is, path P_{rev} visits the nodes of path P from $end(P)$ to $start(P)$ sequentially. In addition, P is a cycle if $|V(P)| \geqslant 3$ and $end(P)$ is adjacent to $start(P)$. A path $P = v_0 \to v_1 \to \cdots \to v_{\ell-1} \to v_\ell$ may contain another subpath Q, denoted as $v_0 \to v_1 \to \cdots \to v_{i-1} \to Q \to v_{j+1} \to \cdots \to v_{\ell-1} \to v_\ell$, where $Q = v_i \to v_{i+1} \to \cdots \to v_j$, $start(Q) = v_i$, and $end(Q) = v_j$ for $0 \leqslant i \leqslant j \leqslant \ell$. A path (or cycle) in G is called a *Hamiltonian path* (or *Hamiltonian cycle*) if it contains every node of G exactly once. A graph G is *Hamiltonian connected* if, for any two distinct nodes u, v, there exists a Hamiltonian path with end nodes u, v. Two paths (or cycles) P_1 and P_2 connecting a node u to a node v are said to be *edge-disjoint* if and only if $E(P_1) \cap E(P_2) = \emptyset$. Two paths (or cycles) Q_1 and Q_2 of graph G are called *node-disjoint* if and only if $V(Q_1) \cap V(Q_2) = \emptyset$. Two node-disjoint paths Q_1 and Q_2 can be *concatenated* into a path, denoted by $Q_1 \Rightarrow Q_2$, if $end(Q_1)$ is adjacent to $start(Q_2)$.

Definition 1. A graph G contains *two-equal path cover* and is called *two-equal path coverable* if for any two distinct pairs of nodes $\langle \mu_s, \mu_t \rangle$ and $\langle \upsilon_s, \upsilon_t \rangle$ of G, there exist two node-disjoint paths P and Q satisfying that (1) $start(P) =$

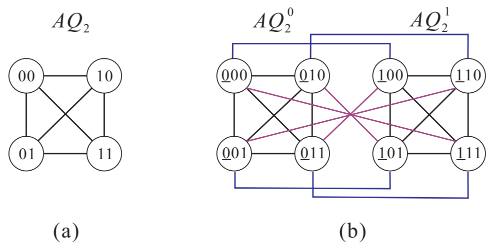

Fig. 1. (a) The 2-dimensional augmented cube AQ_2, and (b) the 3-dimensional augmented cube AQ_3 containing AQ_2^0, AQ_2^1

μ_s and $end(P) = \mu_t$, (2) $start(Q) = \upsilon_s$ and $end(Q) = \upsilon_t$, (3) $|P| = |Q|$, and (4) $V(P) \cup V(Q) = V(G)$.

Now, we introduce augmented cubes. The node set of the n-dimensional augmented cube AQ_n is the set of binary strings of length n. A binary string b of length n is denoted by $b_{n-1}b_{n-2}\cdots b_1b_0$, where b_{n-1} is the most significant bit. We denote the complement of bit b_i by $\overline{b}_i = 1 - b_i$ and the leftmost bit complement of binary string b by $\overline{b} = \overline{b}_{n-1}b_{n-2}\cdots b_1b_0$. We then give the recursive definition of the n-dimensional augmented cube AQ_n, with integer $n \geqslant 1$, as follows.

Definition 2. [7] Let $n \geqslant 1$. The n-dimensional augmented cube, denoted by AQ_n, is defined recursively as follows.
(1) AQ_1 is a complete graph with the node set $\{0, 1\}$.
(2) For $n \geqslant 2$, AQ_n is built from two disjoint copies AQ_{n-1} according to the following steps. Let AQ_{n-1}^0 denote the graph obtained by prefixing the label of each node of one copy of AQ_{n-1} with 0, let AQ_{n-1}^1 denote the graph obtained by prefixing the label of each node of the other copy of AQ_{n-1} with 1. Then, adding 2^n edges between AQ_{n-1}^0 and AQ_{n-1}^1 by the following rule. A node $b = 0b_{n-2}b_{n-3}\cdots b_1b_0$ of AQ_{n-1}^0 is adjacent to a node $a = 1a_{n-2}a_{n-3}\cdots a_1a_0$ of AQ_{n-1}^1 if and only if either
(i) $a_i = b_i$ for all $n-2 \geqslant i \geqslant 0$ (in this case, (b, a) is called a *hypercube edge*), or
(ii)$a_i = \overline{b}_i$ for all $n-2 \geqslant i \geqslant 0$ (in this case, (b, a) is called a *complement edge*).

It was proved in [7] that AQ_n is node transitive, $(2n-1)$-regular, and has diameter $\lceil \frac{n}{2} \rceil$. By Definition 2, AQ_n contains 2^n nodes and $(2n-1) \cdot 2^{n-1}$ edges. In addition, AQ_n can be decomposed into two sub-augmented cubes AQ_{n-1}^0 and AQ_{n-1}^1, where for each $i \in \{0, 1\}$, AQ_{n-1}^i consists of those nodes $b = b_{n-1}b_{n-2}\cdots b_1b_0$ with *leading bit* $b_{n-1} = i$. For each $i \in \{0, 1\}$, AQ_{n-1}^i is isomorphic to AQ_{n-1}. For example, Fig. 1(a) shows AQ_2 and Fig. 1(b) depicts AQ_3 consisting of two sub-augmented cubes AQ_2^0, AQ_2^1. The following proposition can be easily verified from Definition 2.

Proposition 1. *Let AQ_n be the augmented cube decomposed into AQ_{n-1}^0 and AQ_{n-1}^1. For any $b \in V(AQ_{n-1}^i)$ and $i \in \{0, 1\}$, $\overline{b} \in V(AQ_{n-1}^{1-i})$ and $\overline{b} \in N(b)$.*

Let b is a binary string $b_{\ell-1}b_{\ell-2}\cdots b_1b_0$ of length ℓ. We denote b^τ the new binary string obtained by repeating b string τ times. For instance, $(10)^2 = 1010$ and $0^3 = 000$.

The following Hamiltonian connected property of the augmented cube can be proved by induction.

Lemma 2. *For any integer $n \geqslant 2$, AQ_n is Hamiltonian connected.*

Proof: We prove this lemma by induction on n, the dimension of the augmented cube AQ_n. Obviously, AQ_2 is Hamiltonian connected since it is a complete graph with 4 nodes. Assume that AQ_k, with $k \geqslant 2$, is Hamiltonian connected. We will prove that AQ_{k+1} is Hamiltonian connected. We first decompose AQ_{k+1} into two sub-augmented cubes AQ_k^0 and AQ_k^1. Let u, v be any two distinct nodes of AQ_{k+1}. There are two cases:

Case 1: $u, v \in V(AQ_k^i)$, for $i \in \{0, 1\}$. By inductive hypothesis, there is a Hamiltonian path P in AQ_k^i with end nodes u, v. Let $P = u \to P'$ and let $start(P') = w$. By inductive hypothesis, there is a Hamiltonian path Q in AQ_k^{1-i} such that $start(Q) = \overline{u}$ and $end(Q) = \overline{w}$. By Proposition 1, $\overline{u} \in N(u)$ and $\overline{w} \in N(w)$. Then, $u \Rightarrow Q \Rightarrow P'$ is a Hamiltonian path of AQ_{k+1} with end nodes u, v.

Case 2: $u \in V(AQ_k^i)$ and $v \in V(AQ_k^{1-i})$, for $i \in \{0, 1\}$. Let w be a node in AQ_k^i such that $w \neq u$ and $\overline{w} \neq v$. By inductive hypothesis, there is a Hamiltonian path P in AQ_k^i such that $start(P) = u$ and $end(P) = w$. In addition, there is a Hamiltonian path Q in AQ_k^{1-i} such that $start(Q) = \overline{w}$ and $end(Q) = v$. By Proposition 1, $\overline{w} \in N(w)$. Then, $P \Rightarrow Q$ is a Hamiltonian path of AQ_{k+1} with end nodes u, v.

In either case, AQ_{k+1} is Hamiltonian connected. By induction, AQ_n, with $n \geqslant 2$, is Hamiltonian connected. ■

III. Two Edge-Disjoint Hamiltonian Cycles

In this section, we first show the existence of two edge-disjoint Hamiltonian cycles in augmented cubes. Based on the proof of existence, we design a recursive algorithm to construct two edge-disjoint Hamiltonian cycle of an n-dimensional augmented cube.

Obviously, AQ_2 contains no two edge-disjoint Hamiltonian cycles since each node is incident to only three edges. For any integer $n \geqslant 3$, we will show that there exist two edge-disjoint Hamiltonian paths, P and Q, in AQ_n such that $start(P) = 0(0)^{n-3}00$, $end(P) = 1(0)^{n-3}00$, $start(Q) = 0(0)^{n-3}10$, and $end(Q) = 1(0)^{n-3}10$. By Proposition 1, $start(P) \in N(end(P))$ and $start(Q) \in N(end(Q))$. Thus, AQ_n, $n \geqslant 3$, contains two edge-disjoint Hamiltonian cycles. In the following, we will show how to construct two such edge-disjoint Hamiltonian cycles. We first show that AQ_3 contains two such edge-disjoint Hamiltonian paths as follows.

Lemma 3. *There are two edge-disjoint Hamiltonian paths P and Q in AQ_3 such that $start(P) = 000$, $end(P) = 100$, $start(Q) = 010$, and $end(Q) = 110$.*

Proof: We prove this lemma by constructing two such paths. Let
$P = 000 \to 010 \to 011 \to 001 \to 101 \to 111 \to 110 \to 100$, and let
$Q = 010 \to 001 \to 000 \to 011 \to 111 \to 100 \to 101 \to 110$.
Fig. 2 depicts the construction of P and Q. Clearly, P and Q are edge-disjoint Hamiltonian paths in AQ_3. ■

Using Lemma 3, we prove the following lemma by induction.

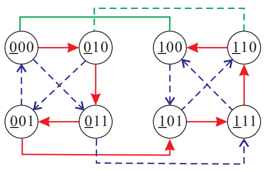

Fig. 2. Two edge-disjoint Hamiltonian paths (cycles) in AQ_3, where the solid arrow lines indicate a Hamiltonian path P and the dashed arrow lines indicate the other edge-disjoint Hamiltonian path Q

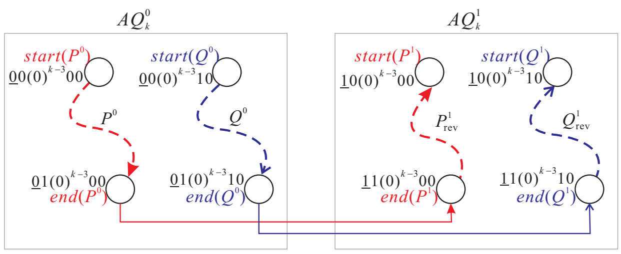

Fig. 3. The construction of two edge-disjoint Hamiltonian paths in AQ_{k+1}, with $k \geqslant 3$, where the dashed arrow lines indicate the paths and the solid arrow lines indicate concatenated edges

Lemma 4. *For any integer $n \geqslant 3$, there are two edge-disjoint Hamiltonian paths P and Q in AQ_n such that $start(P) = 0(0)^{n-3}00$, $end(P) = 1(0)^{n-3}00$, $start(Q) = 0(0)^{n-3}10$, and $end(Q) = 1(0)^{n-3}10$.*

Proof: We prove this lemma by induction on n, the dimension of the augmented cube. It follows from Lemma 3 that the lemma holds true when $n = 3$. Assume that the lemma is true for the case of $n = k \geqslant 3$. Consider AQ_{k+1}. We first partition AQ_{k+1} into two sub-augmented cubes AQ^0_k and AQ^1_k. By the induction hypothesis, there are two edge-disjoint Hamiltonian paths P^i and Q^i, for $i \in \{0,1\}$, in AQ^i_k such that $start(P^i) = i0(0)^{k-3}00$, $end(P^i) = i1(0)^{k-3}00$, $start(Q^i) = i0(0)^{k-3}10$, and $end(Q^i) = i1(0)^{k-3}10$. By Proposition 1, we have that

$$end(P^0) \in N(end(P^1)) \text{ and } end(Q^0) \in N(end(Q^1)).$$

Let $P = P^0 \Rightarrow P^1_{\text{rev}}$ and let $Q = Q^0 \Rightarrow Q^1_{\text{rev}}$, where P^1_{rev} and Q^1_{rev} are the reversed paths of P^1 and Q^1, respectively. Then, P and Q are two edge-disjoint Hamiltonian paths in AQ_{k+1} such that $start(P) = 0(0)^{k-2}00$, $end(P) = 1(0)^{k-2}00$, $start(Q) = 0(0)^{k-2}10$, and $end(Q) = 1(0)^{k-2}10$. Fig. 3 depicts the construction of two such edge-disjoint Hamiltonian paths in AQ_{k+1}. Thus, the lemma holds true when $n = k+1$. By induction, the lemma holds true. ■

By Proposition 1, nodes $start(P) = 0(0)^{n-3}00$ and $end(P) = 1(0)^{n-3}00$ are adjacent, nodes $start(Q) = 0(0)^{n-3}10$ and $end(Q) = 1(0)^{n-3}10$ are adjacent, and the two edges $(start(P), end(P))$ and $(start(Q), end(Q))$ are distinct. Thus the following two theorems hold true.

Theorem 5. *There exist two edge-disjoint Hamiltonian paths in AQ_n for any integer $n \geqslant 3$.*

Theorem 6. *There exist two edge-disjoint Hamiltonian cycles in AQ_n for any integer $n \geqslant 3$.*

Based on the proofs of Lemmas 3 and 4, we design a recursive algorithm to construct two edge-disjoint Hamiltonian paths of an n-dimensional augmented cube. The algorithm typically follows a divide-and-conquer approach [8] and is sketched as follows. It is given by an n-dimensional augmented cube AQ_n with $n \geqslant 3$. If $n = 3$, then the algorithm constructs two edge-disjoint Hamiltonian paths according to the proof of Lemma 3. Suppose that $n > 3$. It first decomposes AQ_n into two sub-augmented cubes AQ^0_{n-1} and AQ^1_{n-1}, where for each $i \in \{0,1\}$, AQ^i_{n-1} consists of those nodes $b = b_{n-1}b_{n-2}\cdots b_1b_0$ with leading bit $b_{n-1} = i$. Then, the algorithm computes two edge-disjoint Hamiltonian paths of AQ^0_{n-1} and AQ^1_{n-1} recursively. Finally, it concatenates these computed four cycles into two edge-disjoint Hamiltonian paths of AQ_n according to the proof of Lemma 4, and outputs two such concatenated paths. The algorithm is formally presented as follows.

Algorithm CONSTRUCTING-2EDHP
Input: AQ_n, an n-dimensional augmented cube with $n \geqslant 3$.
Output: Two edge-disjoint Hamiltonian paths P and Q in AQ_n such that $start(P) = 0(0)^{n-3}00$, $end(P) = 1(0)^{n-3}00$, $start(Q) = 0(0)^{n-3}10$, and $end(Q) = 1(0)^{n-3}10$.
Method:

1. **if** $n = 3$, **then**
2. let $P = 000 \to 010 \to 011 \to 001 \to 101 \to 111 \to 110 \to 100$;
3. let $Q = 010 \to 001 \to 000 \to 011 \to 111 \to 100 \to 101 \to 110$;
4. **output** "P and Q" as two edge-disjoint Hamiltonian paths of AQ_3;
5. decompose AQ_n into two sub-augmented cubes AQ^0_{n-1} and AQ^1_{n-1} such that AQ^i_{n-1}, $i \in \{0,1\}$, consists of those nodes $b = b_{n-1}b_{n-2}\cdots b_1b_0$ with leading bit $b_{n-1} = i$;
6. call Algorithm CONSTRUCTING-2EDHP given AQ^0_{n-1} to compute two edge-disjoint Hamiltonian paths P^0 and Q^0 of AQ^0_{n-1}, where $start(P^0) = \underline{0}0(0)^{n-4}00$, $end(P^0) = \underline{0}1(0)^{n-4}00$, $start(Q^0) = \underline{0}0(0)^{n-4}10$, $end(Q^0) = \underline{0}1(0)^{n-4}10$;
7. call Algorithm CONSTRUCTING-2EDHP given AQ^1_{n-1} to compute two edge-disjoint Hamiltonian paths P^1 and Q^1 of AQ^1_{n-1}, where $start(P^1) = \underline{1}0(0)^{n-4}00$, $end(P^1) = \underline{1}1(0)^{n-4}00$, $start(Q^1) = \underline{1}0(0)^{n-4}10$, $end(Q^1) = \underline{1}1(0)^{n-4}10$;
8. compute $P = P^0 \Rightarrow P^1_{\text{rev}}$ and $Q = Q^0 \Rightarrow Q^1_{\text{rev}}$, where P^1_{rev} and Q^1_{rev} are the reversed paths of P^1 and Q^1, respectively;
9. **output** "P and Q" as two edge-disjoint Hamiltonian paths of AQ_n.

For example, Fig. 4 shows two edge-disjoint Hamiltonian paths of AQ_4 consisting of two sub-augmented cubes AQ^0_3 and AQ^1_3, constructed by Algorithm CONSTRUCTING-2EDHP. The correctness of Algorithm CONSTRUCTING-2EDHP immediately follows from Lemmas 3 and 4. Now, we analyze its time complexity. Let m be the number of the nodes in AQ_n. Then, $m = 2^n$. Let $T(m)$ be the running time of Algorithm CONSTRUCTING-2EDHP given AQ_n. It is easy to verify from lines 2 and 3 that $T(m) = O(1)$ if $n = 3$. Suppose that $n > 3$. By visiting every node of AQ_n once, decomposing AQ_n into AQ^0_{n-1} and AQ^1_{n-1} can be

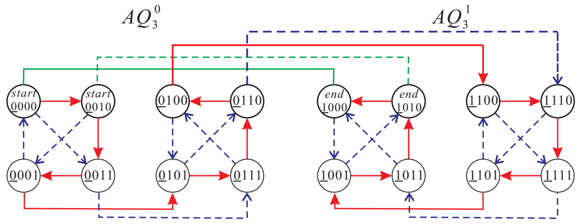

Fig. 4. Two edge-disjoint Hamiltonian paths (cycles) in AQ_4, where the solid arrow lines indicate a Hamiltonian path P and the dashed arrow lines indicate the other edge-disjoint Hamiltonian path Q

done in $O(m)$ time, where each node in AQ^i_{n-1}, $i \in \{0, 1\}$, is labeled with leading bit i. Thus, line 5 of the algorithm can be done in $O(m)$ time. Then, the decomposition of the problem yields two subproblems, each of which is 1/2 the size of the original. It takes time $T(m/2)$ to solve one subproblem, and so it takes time $2 \cdot T(m/2)$ to solve the two subproblems. In addition, concatenating four paths into two paths (line 8) can be easily done in $O(m)$ time. Thus, we obtain the following recurrence equation:

$$T(m) = \begin{cases} O(1) & \text{, if } n = 3; \\ 2 \cdot T(m/2) + O(m) & \text{, if } n > 3. \end{cases}$$

The solution of the above recurrence is $T(m) = O(m \log m) = O(n2^n)$. Thus, the running time of Algorithm CONSTRUCTING-2EDHP given AQ_n is $O(n2^n)$. Since an n-dimensional augmented cube AQ_n contains 2^n nodes and $(2n-1)\cdot 2^{n-1}$ edges, the algorithm is a linear time algorithm.

Let P and Q be two edge-disjoint Hamiltonian paths output by Algorithm CONSTRUCTING-2EDHP given AQ_n. By Definition 2, $start(P) \in N(end(P))$ and $start(Q) \in N(end(Q))$. In addition, the edge connecting $start(P)$ with $end(P)$ is different from the edge connecting $start(Q)$ with $end(Q)$. Thus, P and Q are two edge-disjoint Hamiltonian cycles of AQ_n. We hence conclude the following theorem.

Theorem 7. *Algorithm* CONSTRUCTING-2EDHP *correctly constructs two edge-disjoint Hamiltonian cycles (paths) of an n-dimensional augmented cube AQ_n, with $n \geqslant 3$, in $O(n2^n)$-linear time.*

IV. TWO-EQUAL PATH COVER

In this section, we first show that, for any $n \geqslant 2$, the n-dimensional augmented cube AQ_n is two-equal path coverable. That is, for any two distinct pairs of nodes $\langle \mu_s, \mu_t \rangle$ and $\langle \upsilon_s, \upsilon_t \rangle$ of AQ_n, there exist two node-disjoint paths P and Q of AQ_n satisfying that (1) $start(P) = \mu_s$ and $end(P) = \mu_t$, (2) $start(Q) = \upsilon_s$ and $end(Q) = \upsilon_t$, (3) $|P| = |Q|$, and (4) $V(P) \cup V(Q) = V(AQ_n)$. Using the proof of existence, we design a recursive algorithm to construct two-equal path cover of AQ_n given any two pairs of nodes $\langle \mu_s, \mu_t \rangle$ and $\langle \upsilon_s, \upsilon_t \rangle$ of AQ_n.

We will prove the existence of two-equal path cover by induction on n, the dimension of AQ_n. Initially, AQ_2 clearly contains two-equal path cover since it is a complete graph with four nodes.

Lemma 8. *AQ_2 is two-equal path coverable.*

Lemma 9. *For any integer $n \geqslant 2$, AQ_n is two-equal path coverable.*

Proof: We prove this lemma by induction on n, the dimension of the augmented cube. It follows from Lemma 8 that the lemma holds true for the case of $n = 2$. Now, assume that AQ_k, with $k \geqslant 2$, contains two-equal path cover. We will prove that AQ_{k+1} contains two-equal path cover. First, we decompose AQ_{k+1} into two sub-augmented cubes AQ^0_k and AQ^1_k. Let $\langle \mu_s, \mu_t \rangle$ and $\langle \upsilon_s, \upsilon_t \rangle$ be any two pairs of distinct nodes in AQ_{k+1}. We will construct two node-disjoint paths P and Q of AQ_{k+1} such that P joins μ_s and μ_t, Q joins υ_s and υ_t, and $|P| = |Q| = 2^k$. There are the following four cases:

Case 1: $\mu_s, \mu_t, \upsilon_s, \upsilon_t$ are in the same sub-augmented cube. Without loss of generality, assume that $\mu_s, \mu_t, \upsilon_s, \upsilon_t$ are in AQ^0_k. By inductive hypothesis, there is a path cover $\{P^0, Q^0\}$ of AQ^0_k such that $|P^0| = |Q^0|$, $start(P^0) = \mu_s$, $end(P^0) = \mu_t$, $start(Q^0) = \upsilon_s$, and $end(Q^0) = \upsilon_t$. Let $P^0 = \mu_s \to P'$ and $Q^0 = \upsilon_s \to Q'$. Let $w_P = start(P')$ and let $w_Q = start(Q')$. Let $\langle \overline{\mu}_s, \overline{w}_P \rangle$ and $\langle \overline{\upsilon}_s, \overline{w}_Q \rangle$ be two pairs of distinct nodes in AQ^1_k. By inductive hypothesis, there are two node-disjoint paths P^1 and Q^1 of AQ^1_k such that $|P^1| = |Q^1| = 2^{k-1}$, $start(P^1) = \overline{\mu}_s$, $end(P^1) = \overline{w}_P$, $start(Q^1) = \overline{\upsilon}_s$, and $end(Q^1) = \overline{w}_Q$. By Proposition 1, $\overline{\mu}_s \in N(\mu_s)$, $\overline{w}_P \in N(w_P)$, $\overline{\upsilon}_s \in N(\upsilon_s)$, and $\overline{w}_Q \in N(w_Q)$. Let $P = \mu_s \Rightarrow P^1 \Rightarrow P'$ and let $Q = \upsilon_s \Rightarrow Q^1 \Rightarrow Q'$. Then, $\{P, Q\}$ is a path cover of AQ_{k+1} such that P joins μ_s and μ_t, Q joins υ_s and υ_t, and $|P| = |Q| = 2^k$. The construction of two such paths in this case is shown in Fig. 5(a).

Case 2: μ_s, μ_t, υ_s are in the same sub-augmented cube, and υ_t is in another sub-augmented cube. Without loss of generality, assume that μ_s, μ_t, υ_s are in AQ^0_k. Let x be a node in AQ^0_k such that $x \in V(AQ^0_k) - \{\mu_s, \mu_t, \upsilon_s\}$ and $\overline{x} \neq \upsilon_t$. By inductive hypothesis, there is a path cover $\{P^0, Q^0\}$ of AQ^0_k such that $|P^0| = |Q^0|$, $start(P^0) = \mu_s$, $end(P^0) = \mu_t$, $start(Q^0) = \upsilon_s$, and $end(Q^0) = x$. Let $P^0 = \mu_s \to P'$ and let $w_P = start(P')$. Consider that $\overline{w}_P \notin \{\overline{x}, \upsilon_t\}$. Let $\langle \overline{\mu}_s, \overline{w}_P \rangle$ and $\langle \overline{x}, \upsilon_t \rangle$ be two pairs of distinct nodes in AQ^1_k. By inductive hypothesis, there are two node-disjoint paths P^1 and Q^1 of AQ^1_k such that $|P^1| = |Q^1| = 2^{k-1}$, $start(P^1) = \overline{\mu}_s$, $end(P^1) = \overline{w}_P$, $start(Q^1) = \overline{x}$, and $end(Q^1) = \upsilon_t$. By Proposition 1, $\overline{\mu}_s \in N(\mu_s)$, $\overline{w}_P \in N(w_P)$, and $\overline{x} \in N(x)$. Let $P = \mu_s \Rightarrow P^1 \Rightarrow P'$ and let $Q = Q^0 \Rightarrow Q^1$. Then, $\{P, Q\}$ is a path cover of AQ_{k+1} such that P joins μ_s and μ_t, Q joins υ_s and υ_t, and $|P| = |Q| = 2^k$. The construction of two such paths in this case is shown in Fig. 5(b). On the other hand, consider that $\overline{w}_P \in \{\overline{x}, \upsilon_t\}$. Since $|V(AQ^0_k)| = |V(AQ^1_k)| = 2^k \geqslant 4$, we can easily choose w_P and x such that $\overline{w}_P \notin \{\overline{x}, \upsilon_t\}$. Then, we can build two node-disjoint paths P and Q of AQ_{k+1} by the same construction.

Case 3: μ_s, μ_t are in the same sub-augmented cube, and υ_s, υ_t are in another sub-augmented cube. Without loss of generality, assume that μ_s, μ_t are in AQ^0_k. By Lemma 2, there are Hamiltonian paths P and Q of AQ^0_k and AQ^1_k, respectively, such that P joins μ_s, μ_t and Q joins υ_s, υ_t. Thus, $\{P, Q\}$ is a path cover of AQ_{k+1} with $|P| = |Q| = 2^k$. Fig. 5(c) depicts the construction of two such paths in this case.

Case 4: μ_s, υ_s are in the same sub-augmented cube, and μ_t, υ_t are in another sub-augmented cube. Without loss of generality, assume that μ_s, υ_s are in AQ^0_k. Let x, y be two distinct nodes of AQ^0_k such that $x, y \in V(AQ^0_k) - \{\mu_s, \upsilon_s\}$

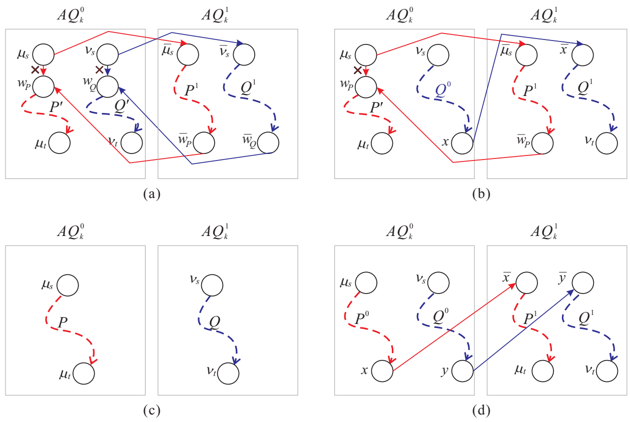

Fig. 5. The constructions of two node-disjoint paths in AQ_{k+1}, with $k \geqslant 2$, for (a) $\mu_s, \mu_t, \upsilon_s, \upsilon_t \in AQ_k^0$, (b) $\mu_s, \mu_t, \upsilon_s \in AQ_k^0$ and $\upsilon_t \in AQ_k^1$, (c) $\mu_s, \mu_t \in AQ_k^0$ and $\upsilon_s, \upsilon_t \in AQ_k^1$, and (d) $\mu_s, \upsilon_s \in AQ_k^0$ and $\mu_t, \upsilon_t \in AQ_k^1$, where the dashed arrow lines indicate the paths, the solid arrow lines indicate concatenated edges, and the symbol '×' denotes the destruction to an edge in a path

and $\overline{x}, \overline{y} \notin \{\mu_t, \upsilon_t\}$. Let $\langle \mu_s, x\rangle$ and $\langle \upsilon_s, y\rangle$ be two pairs of distinct nodes in AQ_k^0, and let $\langle \overline{x}, \mu_t\rangle$ and $\langle \overline{y}, \upsilon_t\rangle$ be two pairs of distinct nodes in AQ_k^1. By inductive hypothesis, there are two node-disjoint paths P^0 and Q^0 of AQ_k^0 such that $|P^0| = |Q^0| = 2^{k-1}$, $start(P^0) = \mu_s$, $end(P^0) = x$, $start(Q^0) = \upsilon_s$, and $end(Q^0) = y$. In addition, there are two node-disjoint paths P^1 and Q^1 of AQ_k^1 such that $|P^1| = |Q^1| = 2^{k-1}$, $start(P^1) = \overline{x}$, $end(P^1) = \mu_t$, $start(Q^1) = \overline{y}$, and $end(Q^1) = \upsilon_t$. By Proposition 1, $\overline{x} \in N(x)$ and $\overline{y} \in N(y)$. Let $P = P^0 \Rightarrow P^1$ and let $Q = Q^0 \Rightarrow Q^1$. Then, $\{P, Q\}$ forms a path cover of AQ_{k+1} such that P joins μ_s and μ_t, Q joins υ_s and υ_t, and $|P| = |Q| = 2^k$. The construction of two such paths in this case is shown in Fig. 5(d).

It follows from the above cases that AQ_{k+1} contains two-equal path cover. By induction, AQ_n, with $n \geqslant 2$, contains two-equal path cover, and, hence, AQ_n is two-equal path coverable. Thus, the lemma holds true. ∎

Given any two nodes u, v of AQ_n, we can use the proof of Lemma 2 to obtain an algorithm, called Algorithm CONSTRUCTING-HP, for constructing a Hamiltonian path of AQ_n with end nodes u, v. Using the proof of Lemma 9 and Algorithm CONSTRUCTING-HP, we design a recursive algorithm to construct two-equal path cover of an n-dimensional augmented cube. The algorithm also uses a divide-and-conquer approach [8] and is sketched as follows. It is given by an n-dimensional augmented cube AQ_n, with $n \geqslant 2$, and any two distinct pairs of nodes $\langle \mu_s, \mu_t\rangle$ and $\langle \upsilon_s, \upsilon_t\rangle$. If $n = 2$, then the algorithm constructs two paths such that one path consists of one edge connecting μ_s and μ_t, and the other path consists of one edge connecting υ_s and υ_t. Suppose that $n > 2$. It first decomposes AQ_n into two sub-augmented cubes AQ_{n-1}^0 and AQ_{n-1}^1, where for each $i \in \{0, 1\}$, AQ_{n-1}^i consists of nodes $b = b_{n-1}b_{n-2}\cdots b_1b_0$ with leading bit $b_{n-1} = i$. Consider the possible cases of $\mu_s, \mu_t, \upsilon_s, \upsilon_t$ appeared in the divided sub-augmented cubes (in the proof of Lemma 9). The algorithm then computes two-equal path covers of AQ_{n-1}^0 and AQ_{n-1}^1 recursively. It finally concatenates the paths in the computed two-equal path covers to form two equal path cover of AQ_n according to the proof of Lemma 9. The algorithm is formally presented as follows.

Algorithm CONSTRUCTING-2EPC
Input: AQ_n, an n-dimensional augmented cube with $n \geqslant 2$, and two distinct pairs of nodes $\langle \mu_s, \mu_t\rangle$ and $\langle \mu_s, \upsilon_t\rangle$.
Output: Two-equal path cover $\{P, Q\}$.
Method:

1. **if** $n = 2$, **then**
2. let $P = \mu_s \to \mu_t$;
3. let $Q = \upsilon_s \to \upsilon_t$;
4. **output** "$\{P, Q\}$" as two-equal path cover of AQ_2;
5. decompose AQ_n into two sub-augmented cubes AQ_{n-1}^0 and AQ_{n-1}^1 such that AQ_{n-1}^i, $i \in \{0, 1\}$, consists of nodes $b = b_{n-1}b_{n-2}\cdots b_1b_0$ with leading bit $b_{n-1} = i$;
6. Consider the following four cases:
7. ***Case 1:*** $\mu_s, \mu_t, \upsilon_s, \upsilon_t$ are in the same sub-augmented cube AQ_{n-1}^i, $i \in \{0, 1\}$.
8. call Algorithm CONSTRUCTING-2EPC given AQ_{n-1}^i and two pairs of nodes $\langle \mu_s, \mu_t\rangle$ and $\langle \upsilon_s, \upsilon_t\rangle$ to compute two equal path cover $\{P^i, Q^i\}$, where $start(P^i) = \mu_s$, $end(P^i) = \mu_t$, $start(Q^i) = \upsilon_s$, $end(Q^i) = \upsilon_t$;
9. let $P^i = \mu_s \to P'$ and $Q^i = \upsilon_s \to Q'$, where $w_P = start(P')$ and $w_Q = start(Q')$;
10. call Algorithm CONSTRUCTING-2EPC given AQ_{n-1}^{1-i} and two pairs of nodes $\langle \overline{\mu}_s, \overline{w}_P\rangle$ and $\langle \overline{\upsilon}_s, \overline{w}_Q\rangle$ to compute two-equal path cover $\{P^{1-i}, Q^{1-i}\}$, where $start(P^{1-i}) = \overline{\mu}_s$, $end(P^{1-i}) = \overline{w}_P$, $start(Q^{1-i}) = \overline{\upsilon}_s$, $end(Q^{1-i}) = \overline{w}_Q$;
11. compute $P = \mu_s \Rightarrow P^{1-i} \Rightarrow P'$ and $Q = \upsilon_s \Rightarrow Q^{1-i} \Rightarrow Q'$;
12. **output** "$\{P, Q\}$" as two-equal path cover of AQ_n;
13. ***Case 2:*** μ_s, μ_t, υ_s are in the same sub-augmented cube AQ_{n-1}^i, $i \in \{0, 1\}$, and υ_t is in another sub-augmented cube.
14. let $x \in V(AQ_{n-1}^i) - \{\mu_s, \mu_t, \upsilon_s\}$ such that $\overline{x} \neq \upsilon_t$;
15. call Algorithm CONSTRUCTING-2EPC given AQ_{n-1}^i and two pairs of nodes $\langle \mu_s, \mu_t\rangle$ and $\langle \upsilon_s, x\rangle$ to compute two equal path cover $\{P^i, Q^i\}$, where $start(P^i) = \mu_s$, $end(P^i) = \mu_t$, $start(Q^i) = \upsilon_s$, $end(Q^i) = x$;
16. let $P^i = \mu_s \to P'$, where $w_P = start(P')$ and $\overline{w}_P \neq \upsilon_t$;
17. call Algorithm CONSTRUCTING-2EPC given AQ_{n-1}^{1-i} and two pairs of nodes $\langle \overline{\mu}_s, \overline{w}_P\rangle$ and $\langle \overline{x}, \upsilon_t\rangle$ to compute two-equal path cover $\{P^{1-i}, Q^{1-i}\}$, where $start(P^{1-i}) = \overline{\mu}_s$, $end(P^{1-i}) = \overline{w}_P$, $start(Q^{1-i}) = \overline{x}$, $end(Q^{1-i}) = \upsilon_t$;
18. compute $P = \mu_s \Rightarrow P^{1-i} \Rightarrow P'$ and $Q = Q^i \Rightarrow Q^{1-i}$;
19. **output** "$\{P, Q\}$" as two-equal path cover of AQ_n;
20. ***Case 3:*** μ_s, μ_t are in the same sub-augmented cube AQ_{n-1}^i, $i \in \{0, 1\}$, and υ_s, υ_t are in another sub-augmented cube.
21. call Algorithm CONSTRUCTING-HP given AQ_{n-1}^i and nodes μ_s, μ_t to compute a Hamiltonian path P of AQ_{n-1}^i with $start(P) = \mu_s$ and $end(P) = \mu_t$;
22. call Algorithm CONSTRUCTING-HP given AQ_{n-1}^{1-i}

and nodes υ_s, υ_t to compute a Hamiltonian path Q of AQ_{n-1}^{1-i} with $start(Q) = \upsilon_s$ and $end(Q) = \upsilon_t$;

23. **output** "$\{P, Q\}$" as two-equal path cover of AQ_n;
24. ***Case 4:*** μ_s, υ_s are in the same sub-augmented cube AQ_{n-1}^i, $i \in \{0, 1\}$, and μ_t, υ_t are in another sub-augmented cube.
25. let $x, y \in V(AQ_{n-1}^i) - \{\mu_s, \upsilon_s\}$ such that $\overline{x}, \overline{y} \in V(AQ_{n-1}^{1-i}) - \{\mu_t, \upsilon_t\}$;
26. call Algorithm CONSTRUCTING-2EPC given AQ_{n-1}^i and two pairs of nodes $\langle\mu_s, x\rangle$ and $\langle\upsilon_s, y\rangle$ to compute two equal path cover $\{P^i, Q^i\}$, where $start(P^i) = \mu_s$, $end(P^i) = x$, $start(Q^i) = \upsilon_s$, $end(Q^i) = y$;
27. call Algorithm CONSTRUCTING-2EPC given AQ_{n-1}^{1-i} and two pairs of nodes $\langle\overline{x}, \mu_t\rangle$ and $\langle\overline{y}, \upsilon_t\rangle$ to compute two equal path cover $\{P^{1-i}, Q^{1-i}\}$, where $start(P^{1-i}) = \overline{x}$, $end(P^{1-i}) = \mu_t$, $start(Q^{1-i}) = \overline{y}$, $end(Q^{1-i}) = \upsilon_t$;
28. compute $P = P^i \Rightarrow P^{1-i}$ and $Q = Q^i \Rightarrow Q^{1-i}$;
29. **output** "$\{P, Q\}$" as two-equal path cover of AQ_n.

The correctness of Algorithm CONSTRUCTING-2EPC follows from the proofs of Lemmas 2, 8 and 9. Now, we analyze its time complexity. Let m be the number of the nodes in AQ_n. Then, $m = 2^n$. Let $T(m)$ be the running time of Algorithm CONSTRUCTING-2EPC given AQ_n and two pairs of nodes $\langle\mu_s, \mu_t\rangle$ and $\langle\mu_s, \upsilon_t\rangle$. It is easy to verify from lines 2 and 3 that $T(m) = O(1)$ if $n = 2$. Suppose that $n > 2$. By visiting every node of AQ_n once, decomposing AQ_n into AQ_{n-1}^0 and AQ_{n-1}^1 can be done in $O(m)$ time, where each node in AQ_{n-1}^i, $i \in \{0, 1\}$, is labeled with leading bit i. Thus, line 5 of the algorithm can be done in $O(m)$ time. Then, the decomposition of the problem yields two subproblems, each of which is $1/2$ the size of the original. For each case in the algorithm, it takes time $T(m/2)$ to solve one subproblem, and so it takes time $2 \cdot T(m/2)$ to solve the two subproblems. It is not difficult to see that the other lines in each case can be easily done in $O(m)$ time. Thus, we get the following recurrence equation:

$$T(m) = \begin{cases} O(1) & \text{, if } n = 2; \\ 2 \cdot T(m/2) + O(m) & \text{, if } n > 2. \end{cases}$$

The solution of the above recurrence is $T(m) = O(m \log m) = O(n2^n)$. Thus, the running time of Algorithm CONSTRUCTING-2EPC given AQ_n is $O(n2^n)$. Since an n-dimensional augmented cube AQ_n contains 2^n nodes and $(2n-1) \cdot 2^{n-1}$ edges, the algorithm is a linear time algorithm. Thus, we conclude the following theorem.

Theorem 10. *Given an n-dimensional augmented cube AQ_n, with $n \geqslant 2$, and two pairs of nodes $\langle\mu_s, \mu_t\rangle$ and $\langle\mu_s, \upsilon_t\rangle$ in AQ_n, Algorithm* CONSTRUCTING-2EPC *correctly constructs two-equal path cover of AQ_n in $O(n2^n)$-linear time.*

V. CONCLUDING REMARKS

In this paper, we present a linear time algorithm to construct two edge-disjoint Hamiltonian cycles (paths) of an n-dimensional augmented cube AQ_n, for any integer $n \geqslant 3$. We then show that there exists two-equal path cover of AQ_n with $n \geqslant 2$. Using the proof of existence, we propose a linear time algorithm to construct two-equal path cover of AQ_n given two pairs of nodes $\langle\mu_s, \mu_t\rangle$ and $\langle\mu_s, \upsilon_t\rangle$ in AQ_n. It is interesting to see if the proposed technique can be applied to the other popular interconnection networks.

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