Robust Parameter Design Methodology for Microwave Circuits Considering the Manufacturing Variations

Takafumi Nakagawa and Tasuku Kirikoshi

Abstract—Our target is to achieve a higher first run rate in the quantity production of microwave circuits. For this purpose, we propose a useful robust parameter design methodology in which the multi-objective problem is treated as a single optimization problem under limiting conditions. A set of controllable factors, which provide an acceptable production, is calculated by considering such noise factors as manufacturing variations. We used the iterative technique with the Monte Carlo method to search for these values. The noise factors are assigned to Taguchi’s orthogonal array to reduce the CPU time. Our proposed method is applied to the design of a microwave amplifier. This method’s performance is compared with four optimization methods in the microwave circuit simulation, and its effectiveness is experimentally confirmed. The calculated controllable factors are not unique among these optimization methods to minimize the variations of the gain in manufactures. Our method is more efficient to find many candidates than the other optimization methods. The produced amplifiers have achieved a first run rate of 97% in its manufacture.

Index Terms—robust design, microwave circuit, multi-objective optimization, SN ratio

I. INTRODUCTION

Robust design is an important technology that provides an acceptable product for variability in a first run and upgrades product quality at low cost. Computer aided engineering (CAE) can be used as an alternative to assist product design in many cases of microwave circuit designs. Conventional techniques using statistical or worst-case modeling have been usually used by many designers [1], [2], [3], [4]. In these works, they ascertain the degree of the performance variability by the Monte Carlo approach or an experimental design method after deciding the parameters. The conventional design tries to find the values of the controllable factors for the allowance of manufacturing variations. But it is unknown whether it gives smaller variability until the manufacturing is completed. Moreover, considering the tradeoff among frequency response, gain, noise figure, power consumption, VSWR, and cost, the design leads to a multi-objective problem. At present, the simulated annealing algorithm (SA) and a stochastic algorithm based on evolution theory such as genetic algorithms (GA) are usually used to solve the multi-objective problem [5], [6], [7]. Using these traditional methods, it generally takes much CPU time to determine the optimal values. Therefore, many approaches have reduced the CPU time using optimization methods based on orthogonal design [8], [9] or techniques finding the pareto front of tradeoff functions [10], [11], [12]. However, these works did not consider the effects of noises, which are an important part of the philosophy of robust design. The conventional approach by Taguchi is well known as the quality control to improve the performance of products at low cost [13], [14], [15]. Taguchi employed an orthogonal array (OA) to arrange the experiments and used signal-to-noise ratios (SN ratio) to evaluate the variability of response in an experimental run. But Taguchi’s method has a limitation because it is an additive linear model and is incompatible with the multi-objective problem. Several approaches have been applied to multiple-objective problems [16], [17], [18], [19], [20]. However, these works cannot prevent trapping in a local minimum without reaching global optimization. Other techniques such as response-surface methodology have been studied for the designs of microwave circuits [21], [22], [23]. In these works, regression techniques are used to fit the recorded response values to a user-defined model. As a result, computing time is greatly required to decide the fitting function when the number of designable factors and objectives becomes large. A method using GA combined with Taguchi’s method was also proposed to consider the effects of noises [24], [25]. In these works, the quality loss function is minimized with OA assigned noise factors. Another effective interactive technique for solving multi-objective problems has been proposed [26], [27], where the tradeoff between objective functions is analyzed with a newly defined tradeoff matrix, and the interactive multi-objective design optimization based on the Satisficing Trade-Off Method is used.

In this paper, we propose a useful robust design methodology for microwave circuit design and apply it to the design of a microwave amplifier. The validity of this method is studied with computer simulations and experiments.
II. ROBUST PARAMETER DESIGN METHODOLOGY

We treat the multi-objective problem as a single optimization problem [28]. We calculate the sets of design values under the limiting conditions based on the specifications. The Taguchi’s SN ratio is used to evaluate the robustness of a circuit’s ideal performance. The multi-objective problem is evaluated by the following formulation (1):

Maximize $\eta(\hat{x})$

Subject to $\hat{x} \in X = \{\hat{x} \in R^n \mid g_i(\hat{x}) \leq 0, (i = 1, \cdots, m)\}$,  

(1)

where $\eta(\hat{x})$ is the SN ratio, which describes the variability of the performance. The performance is described by a function of $f(\hat{x}, M)$. $\hat{x} = (x_1, \cdots, x_n)$ is a set of controllable factors, and $M$ is the input signal. Suffix $s$ refers to the number of controllable factors. $R^n$ is the feasible region, and $g_i(\hat{x})$ denotes the limiting condition. $m$ is the number of objective functions which refer to the specifications. $\eta$ is calculated by Eq. (2) [20]:

$$\eta = 10^{-\log(\beta^2 / \sigma^2)}.$$  

(2)

Slope $\beta$ is determined by the least-squares method of $y_\hat{x}$:

$$\beta = \frac{\sum_{i=1}^{q} \sum_{j=1}^{p} y_\hat{x} / q}{\sum_{i=1}^{q} \sum_{j=1}^{p} e_\hat{x}^2_i / q},$$  

(3)

$$y_\hat{x} = \beta M^* + e_\hat{x}, i = 1, \cdots, p; j = 1, \cdots, q,$$  

(4)

$$M^* = \sum_{j=1}^{q} y_\hat{x} / q,$$  

(5)

where $M^*$ is the average of calculations for all noise, and $e_\hat{x}$ is the regression error. $j$ refers to the experimental runs in OA. $i$ refers to the number of input signals. The total square error from regression line $\sigma^2$ is given by

$$\sigma^2 = \frac{\sum_{i=1}^{q} \sum_{j=1}^{p} (y_\hat{x} - \beta M^*)^2 + \sum_{i=1}^{q} \sum_{j=1}^{p} e_\hat{x}^2_i / q}{(pq-1)qM^*},$$  

(6)

When each $y_\hat{x}$ coincides with $M^*$, $\beta_\hat{x}$ equals one.

The objective functions are calculated against the controllable factors decided with the Monte Carlo method. Taguchi’s OA is used to consider the noise factors to reduce the CPU time. The proposed method searches for the optimal values in the direction that increases the SN ratio. A bigger SN ratio gives smaller variability from Eq. (2). The procedure runs in the following steps, and its details are shown in Fig. 1.

Step 1: Assign the noise factors to Taguchi’s OA.

Step 2: In the first step, set of controllable factors $x_i$ is randomly searched with the Monte Carlo method in the range of $[x_{i,\text{high}}, x_{i,\text{low}}]$:

$$x_i = \text{rand}(0,1) \times (x_{i,\text{high}} - x_{i,\text{low}}).$$

After the second step, $x_i$ is randomly selected in the range of $[-\Delta, \Delta]$:

$$x_i = w_i \times \text{rand}(0,1) \times [-\Delta, \Delta],$$

where $w_i$ is the previous values of $x_i$ and $\Delta$ is a search strip width.

Step 3: Calculate objective functions $f(\hat{x}, M)$ for each experimental number of OA and set of controllable factors $\hat{x}$.

Step 4: When $\max[\eta(\hat{x})]$ is larger than the previous one under the satisfaction of the limiting conditions, $w_i$ is replaced by $x_i$. If there was no desirable result, the width of $\Delta\%$ is reduced by half of the previous one.

Step 5: A set of controllable parameters, which gives the maximum SN ratio, is selected among the calculated results.

III. APPLICATION TO MICROWAVE AMPLIFIER

We designed an input matching circuit for a microwave amplifier with our proposed method. Fig. 2 shows the layout of an amplifier with FETs. The schematic drawing of a CAE model is illustrated in Fig. 3. The input signal is divided into four circuits through the input matching circuit and amplified by four FETs in parallel. All signals are combined with the output matching circuit. It is important to reduce the variance of gain for stable performance. Commercial CAE code [29] is used to calculate the performance of the microwave circuit.
A. Identifying Noise and Controllable Factors

Ten kinds of noise factors and nine kinds of controllable factors are illustrated in Fig. 3. The noise factors are tabulated in Table I, where “A” and “B” are the manufacturing variations of the thickness and the permittivity of the base plate. “C” and “D” are the manufacturing tolerances about the inductance of the lines connected to FETs. “E” and “F” are related to the variations of inductance on the DC cut-off circuit, and “H” and “I” are the manufacturing tolerances of the capacitors of both input and output DC cut-off circuits. “G” is the variation of the inductance of the microstrip line connecting the capacitor, and “J” is the manufacturing tolerance of the bypass capacitors. Ten kinds of noise factors are assigned to OA of L12 (2¹¹) in Table II, where the number denotes the noise levels described in Table I.

In Fig. 3, nine kinds of controllable factors are described as symbols enclosed in squares. The controllable factors are lengths L1, L2, and L3 and widths W1, W2, and W3 on the microstrip lines and gate wire inductance L5 connected to each FET. In addition, two kinds of resistance, R1 and R2, are optimized. The FET is modeled by measured S parameters. The calculation is done by a linear computation.

B. Calculations

We applied the proposed method to a design of microwave amplifier. A performance example and its target specification are shown in Fig. 4. FL and FH indicate the low and high frequency within the range of use. The dotted line shows the lower limit of a target value. The deviation caused by the noise factors is shown in Fig. 5. The vertical line denotes the normalized gain, which is expressed by Eq. (6):

\[
\text{Gain} = 10 \log \left( \frac{\text{gain}}{\text{Target}} \right) / 10
\]

where \( \text{gain} \) is the averaged gain at each frequency. In Fig. 5, the solid line indicates the averaged value and the vertical dotted symbols show the deviation produced by the noise factors. When the gain is equal to the target value, the normalized gain agrees with one. The SN ratio is calculated with a linearized function. Fig. 6 shows an example of an evaluation. The calculated data for the experimental run of No. 5 in Table II are plotted by dotted circles, which are expressed with regression line \( \gamma_5 = \beta_5 \cdot M_5 \). A straight dotted line refers to the averaged gain calculated with the experimental runs. If data of No. 5 agree with the averaged values, slope \( \beta_5 \) equals 1.0. These procedures are done for all experimental runs in Table II, and the SN ratio is calculated from (2). In this design, RF stabilization coefficient k of a power amplifier is also considered as a limiting condition.

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**TABLE I**

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<th>Noise factors</th>
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**TABLE II**

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Ten noise factors are assigned to OA. Number in matrix denotes noise levels described in Table I.

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![Fig. 3 Schematic drawing of CAE model. Ten kinds of noise factors and nine kinds of controllable factors are illustrated. Controllable factors are expressed with symbols enclosed in squares.](image)

![Fig. 4 Example of performance and its target in a microwave amplifier. Dotted line is a minimum of a target value.](image)

![Fig. 5](image)
IV. RESULTS AND DISCUSSION

Figure 7 shows the calculated results by the iterative technique with the Monte Carlo method. The horizontal line gives SN Ratio $\eta$, and the vertical line shows the minimum value of Gain. 100 points are plotted for each of ten iterations. Plotted symbols $\bullet$ are the calculated results with SA. At the initial step, search strip width $\Delta$ is set as half of the nominal values. In the following steps, $\Delta$ decreases as shown in Fig. 8, which shows the relationship between SN ratio $\eta$ and $\Delta$ for each of the iterations. From Fig. 7, the calculated results gradually converge to the pareto front calculated by SA. This shows the effectiveness of our method. In Fig. 8, $\eta$ increases from 35.8 to 43.1. This means that the coefficient of variation decreased to 43% of the initial one. The calculation is almost converged by ten iterations, and its CPU time is 360 sec with Intel Core i5-2500 processor in a Windows PC.

We also compared the calculated results with gradient search (GR), SA and GA in the commercial CAE code [29], [30]. Fig. 9 compares the optimal values, which are tabulated in Table III. Initial design shows the values without considering the effects of the noise factors. The optimal values are different among the optimization methods; the calculated result is not unique. This means that many combinations of parameters can reduce the effect of manufacturing variations. Therefore, we must find many candidates for the change of specifications, and for this purpose the random search method with the Monte Carlo method is more efficient than the other optimization algorithms.

**Table III**

<table>
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<tr>
<th>Controllable Factors</th>
<th>SN</th>
<th>W1</th>
<th>l1</th>
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<th>l2</th>
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<td>Initial Design</td>
<td>39.6</td>
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<td>Proposed Method</td>
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<td>1.75</td>
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<td>GR</td>
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CONTROLLABLE FACTORS ARE DESCRIBED IN FIG. 3.
Figure 10 shows the frequency response of gain at design points “A” ~ “D” in Fig. 7. “A” and “D” give the minimum and the maximum of the SN ratio. “B” gives the maximum gain, and “C” shows the final design. From Fig. 10, the larger SN ratio gives smaller variance of gain. Design “C” is selected to achieve the target value even if the worst case production occurred.

![Fig. 10 Frequency response of gain for design points on Fig. 7](image)

The measured gain in the mass production is plotted in Fig. 12. The maximum and minimum values correspond to the minimum and maximum gain in all measurements, respectively. The variability of average gain was within ±0.6 dB, and the standard deviation was 0.2 dB. These microwave amplifiers have achieved a first run rate of 97% in the manufactures.

![Fig. 11 Comparison between calculation and experiment. Bold line is measurement and flux of brown thin lines are calculations](image)

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