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Visual Landmark Selection for Mobile Robot Navigation

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Abstract—**A large number of landmarks selection techniques has been proposed. However, finding optimal solutions requires to solve some hard problems. In this paper, we consider the ρ-minimum overlapping region decomposition problem that was proposed for landmarks selection. This problem is NP-complete. We describe an approach to solve the problem optimally. This approach is based on an explicit reduction from the problem to the satisfiability problem. Also, we consider some greedy algorithms for solution of the problem.**

Index Terms—**visual landmarks, landmarks selection, mobile robot navigation, NP-complete, satisfiability problem, greedy algorithms.**

I. Introduction

DIFFERENT problems of technical vision have been extensively studied recently (see e.g. [1] – [3]). In particular, visual navigation is received a lot of attention in contemporary robotics (see e.g. [4] – [6]). It should be noted that visual sensors can be used not only for solution of different problems of navigation. For instance, visual sensors are widely used in various systems of robot self-awareness (see e.g. [7] – [14]).

The various methods of selection, extraction and recognition of visual landmarks have been extensively applied for the mobile robot navigation (see e.g. [15] – [18]). Visual landmarks robot navigation approaches select certain features in the snapshot image as landmarks, and try to establish correspondences between these landmarks and features extracted from the current view image. Such approaches differ with respect to the strategy for selecting the landmarks. Some methods strive to extract maximally distinctive features (see e.g. [19] – [22]). Other approaches use less unique features. For instance, we can mention dark and bright sectors (see e.g. [23] – [26]), Harris corners (see e.g. [27], [28]), and colored regions (see e.g. [29], [30]). Visual landmarks can be used for external cameras robot localization (see e.g. [31] – [33]). In particular, problems of sensor placement have been extensively studied recently (see e.g. [34] – [38]).

The representation of knowledge of the surrounding world plays an important role in mobile robot navigation tasks (see e.g. [39] – [52]). It is not surprising that a huge variety of landmarks selection techniques has been proposed (see e.g. [53] – [62]). However, finding optimal solutions usually requires to solve some hard problem (see e.g. [63] – [65]). In particular, we consider the ρ-minimum overlapping region decomposition problem. The problem was proposed in [65] for landmarks selection. The ρ-minimum overlapping region decomposition problem is **NP**-complete. It should be noted that many robotic problems are computationally hard (e.g. [66] – [72]). Frequently, hard problems give us essentially better solutions. Although the ρ-minimum overlapping region decomposition problem is **NP**-complete, quality of visual navigation methods which use landmarks depends critically on the method of selection of landmarks. So, we need an optimal method for solution of the problem. In this paper, we consider an approach to solve the problem optimally. This approach is based on an explicit reduction from the problem to the satisfiability problem.

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II. Preliminaries and problem definitions

In this section, we consider basic definitions from [65].

Let a virtual grid is overlaid on the floor of the environment. We assume that vertices of this virtual grid are accessible points of the environment. The pose space is the set of positions of the virtual grid at which the robot can be at any time. We assume that images for visual landmarks were acquired from the discrete subset of the pose space. Therefore, we can use an undirected planar graph

$$G = (V, E)$$

as a model for this subset. In particular, we assume that each node $v \in V$ corresponds to a sampled pose. Also, we assume that two nodes are adjacent if and only if the corresponding poses are contiguous in 2-D space.

We assume that a set of interest-point-based features are extracted and stored in a database during collection of images. Let F be the set of computed features from all collected images. The visibility set of v is the set $F_v \subseteq F$ of all features that are visible from pose $v \in V$.

We consider a view-based localization approach. In this approach, the current pose of the robot is estimated using the locations of a small number of features in the current image, matched against their locations in the training images. This set of simultaneously visible features constitutes a landmark. Clearly, the minimum number of features necessary for this task depends on the method employed for pose estimation.

A world instance consists of a tuple

$$\langle G, F, \{F_v \mid v \in V\}\rangle,$$

where the graph G models a discrete set of sampled poses, F is a set of features, and $\{F_v \mid v \in V\}$ is a collection of visibility sets. A region is a set of poses $R \subseteq V$ such that for all poses $u, v \in R$, there is a path between u and v completely contained in R, i.e., for all $u, v \in R$, there is

$$\{u = v_0, \ldots, v_h\} \subseteq R$$

such that

$$(v_i, v_{i+1}) \in E, \text{for all} 0 \leq i < h.$$

A decomposition of V is a collection of regions

$$D = \{R_1, \dots, R_d\} \subseteq 2^V$$

such that $V = \cup_{i=1}^{d} R_i$. The ρ-neighborhood of a pose $v \in V$ is the set

$$N_\rho(v) = \{u \mid u \in V, \delta(u, v) \leq \rho\},$$

where $\delta(u, v)$ is the length of the shortest path between nodes u and v in G. The ρ-overlapping is a decomposition

$$D = \{R_1, \dots, R_d\}$$

of V such that for all $v \in V$, there is i such that $N_\rho(v) \subseteq R_i$.

Let k be the number of features required for reliable localization at each position, according to the localization method employed. The ρ-minimum overlapping region decomposition problem (ρ-MORDP) for a world instance $\langle G, F, \{F_v \mid v \in V\}\rangle$ consists of finding a minimum-size ρ-overlapping decomposition $D = \{R_1, \dots, R_d\}$ of V such that $|\cap_{v \in R_i} F_v| \geq k$, for all $1 \leq i \leq d$.

We assume that $N_0(v) = \{v\}$, for any $v \in V$. It should be noted that ρ-MORDP can be reduced to 0-MORDP, and that a solution to the reduced 0-MORDP can be transformed back into a solution of ρ-MORDP (see [65]). Therefore, we can consider only 0-MORDP.

Let

$$V = \{v[1], v[2], \dots, v[n]\}.$$

Let

$$D = \{R_1, \dots, R_p\}$$

is an 0-overlapping decomposition of V such that

$$|\cap_{v \in R_i} F_v| \geq k, \text{for all} 1 \leq i \leq p.$$

Since D is an 0-overlapping decomposition of V, for any $v[i]$, $1 \leq i \leq n$, there is some R_j, $1 \leq j \leq p$, such that $v[i] \in R_j$. Therefore,

$$\cap_{v \in R_j} F_v \subseteq F_{v[i]}.$$

Since $|\cap_{v \in R_j} F_v| \geq k$, it is clear that $|F_{v[i]}| \geq k$. Note that

$$\{\{v[1]\}, \{v[2]\}, \dots, \{v[n]\}\}$$

is an 0-overlapping decomposition of V. Therefore, we can assume that $p \leq n$. If $p < n$, then there are

$$i[1], i[2], \dots, i[n-p]$$

such that

$$R_j \neq \{v[i[s]]\}, \text{for any} 1 \leq j \leq p, 1 \leq s \leq n-p.$$

So, if $p < n$, then

$$D' = D \cup \left(\cup_{s=1}^{t} \{\{v[i[s]]\}\}\right)$$

is an 0-overlapping decomposition of V such that

$$|\cap_{v \in R} F_v| \geq k, \text{for all} R \in D'.$$

Therefore, the decision version of 0-MORDP can be formulated as following.

0-MORDP:

INSTANCE: *A world instance*

$$\langle G, F, \{F_v \mid v \in V\}\rangle$$

and positive integers $d \leq n$ *and* k.

QUESTION: *Is there an* 0*-overlapping decomposition*

$$D = \{R_1, \dots, R_d\}$$

of V *such that* $|\cap_{v \in R_i} F_v| \geq k$*, for any* $1 \leq i \leq d$*?*

III. An explicit reduction from 0-MORDP to the satisfiability problem

0-MORDP is **NP**-complete [65]. Encoding different hard problems as instances of variants of the satisfiability problem and solving them with very efficient satisfiability algorithms has caused considerable interest (see e.g. [73] – [79]). In this paper, we consider an explicit reduction from 0-MORDP to the satisfiability problem.

Let

$$\varphi[1] = \wedge_{\substack{1 \leq i \leq d, \\ 1 \leq j \leq n}} \vee_{1 \leq s \leq n} x[i, j, s],$$

$$\varphi[2] = \wedge_{\substack{1 \leq i \leq d, \\ 1 \leq j \leq n, \\ 1 \leq s[1] < s[2] \leq n}} (\neg x[i, j, s[1]] \vee \neg x[i, j, s[2]]),$$

$$\varphi[3] = \wedge_{1 \leq i \leq n} \vee_{\substack{1 \leq j \leq d, \\ 1 \leq s \leq n}} y[i, j, s],$$

$$\varphi[4] = \wedge_{\substack{1 \leq i \leq n, \\ 1 \leq j \leq d, \\ 1 \leq s \leq n}} (\neg y[i, j, s] \vee x[j, s, i]),$$

$$\varphi[5] = \wedge_{\substack{1 \leq i \leq d, \\ 1 \leq j[1] < j[2] \leq n, \\ 1 \leq s \leq n}} \vee_{1 \leq t \leq n} z[1, i, j[1], j[2], s, t],$$

$$\varphi[6] = \wedge_{\substack{1 \leq i \leq d, \\ 1 \leq j[1] < j[2] \leq n, \\ 1 \leq s < n}} (z[2, i, j[1], j[2], s] \vee \neg z[2, i, j[1], j[2], s+1]),$$

$$\varphi[7] = \wedge_{\substack{1 \leq i \leq d, \\ 1 \leq j[1] < j[2] \leq n}} z[2, i, j[1], j[2], 1],$$

$$\varphi[8] = \wedge_{\substack{1 \leq i \leq d, \\ 1 \leq j[1] < j[2] \leq n, \\ 1 \leq s < n, \\ 1 \leq t[1] \leq n, \\ 1 \leq t[2] \leq n, \\ 1 \leq t[3] \leq n, \\ 1 \leq t[4] \leq n, \\ (v[t[3]], v[t[4]]) \notin E}} (\neg z[2, i, j[1], j[2], s] \vee$$

$$\neg z[1,i,j[1],j[2],s,t[1]]\vee$$

$$\neg z[1,i,j[1],j[2],s+1,t[2]]\vee$$

$$\neg x[i,t[1],t[3]]\vee\neg x[i,t[2],t[4]]),$$

$$\varphi[9]=\wedge_{\substack{1\le i\le d,\\ 1\le j[1]<j[2]\le n,\\ 1\le t[1]\le n,\\ 1\le t[2]\le n,\\ 1\le t[3]\le n,\\ 1\le t[4]\le n,\\ t[3]\ne j[1]}}(\neg z[1,i,j[1],j[2],1,t[1]]\vee$$

$$\neg z[1,i,j[1],j[2],2,t[2]]\vee$$

$$\neg x[i,t[1],t[3]]\vee\neg x[i,t[2],t[4]]),$$

$$\varphi[10]=\wedge_{\substack{1\le i\le d,\\ 1\le j[1]<j[2]\le n,\\ 1\le s<n,\\ 1\le t[1]\le n,\\ 1\le t[2]\le n,\\ 1\le t[3]\le n,\\ 1\le t[4]\le n,\\ t[4]\ne j[2]}}(\neg z[2,i,j[1],j[2],s]\vee$$

$$z[2,i,j[1],j[2],s+1]\vee$$

$$\neg z[1,i,j[1],j[2],s,t[1]]\vee$$

$$\neg z[1,i,j[1],j[2],s+1,t[2]]\vee$$

$$\neg x[i,t[1],t[3]]\vee\neg x[i,t[2],t[4]]),$$

$$\varphi=\wedge_{i=1}^{10}\varphi[i],$$

$$\psi[1]=\wedge_{\substack{1\le i\le d,\\ 1\le j\le k}}\vee_{1\le s\le n}w[i,j,s],$$

$$\psi[2]=\wedge_{\substack{1\le i\le d,\\ 1\le j\le k,\\ 1\le s[1]<s[2]\le n}}(\neg w[i,j,s[1]]\vee\neg w[i,j,s[2]]),$$

$$\psi[3]=\wedge_{\substack{1\le i\le d,\\ 1\le j[1]<j[2]\le k,\\ 1\le s\le n}}(\neg w[i,j[1],s]\vee\neg w[i,j[2],s]),$$

$$\psi[4]=\wedge_{\substack{1\le i\le d,\\ 1\le j\le k,\\ 1\le s\le n,\\ 1\le t[1]\le n,\\ 1\le t[2]\le n,\\ v[s]\notin F_{v[t[2]]}}}(\neg w[i,j,s]\vee\neg x[i,t[1],t[2]]),$$

$$\psi=\wedge_{i=1}^{4}\psi[i],$$

$$\xi=\varphi\wedge\psi.$$

Theorem. *There is an* 0*-overlapping decomposition*

$$D=\{R_1,\ldots,R_d\}$$

of V *such that for all* $1\le i\le d$, $|\cap_{v\in R_i}F_v|\ge k$ *if and only if* ξ *is satisfiable.*

Proof. Let ξ is satisfiable. In this case, there are some values

$$x_0[i[1],i[2],i[3]],y_0[i[1],i[2],i[3]],$$

$$z_0[i[1],i[2],i[3],i[4],i[5]],w_0[i[1],i[2],i[3]]$$

of variables

$$x[i[1],i[2],i[3]],y[i[1],i[2],i[3]],$$

$$z[i[1],i[2],i[3],i[4],i[5]],w[i[1],i[2],i[3]]$$

such that $\xi=1$. Since $\xi=1$, it is clear that $\varphi[i]=1$, for all $1\le i\le 10$. In view of $\varphi[1]=1$, it is easy to see that there is $s_0(i,j)$, $1\le i\le d$ and $1\le j\le n$, such that

$$x[i,j,s_0(i,j)]=1.$$

Since $\varphi[2]=1$, $s_0(i,j)$ is a function. Let

$$R_i=\{v[t]\mid t=s_0(i,j),1\le j\le n\}.$$

Since $\varphi[3]=1$, for any $1\le i\le n$, there are $j_0\in\{1,\ldots,d\}$ and $s_0\in\{1,\ldots,n\}$ such that $y[i,j_0,s_0]=1$. Whereas $\varphi[4]=1$ and $y[i,j_0,s_0]=1$, for any $t\in\{1,\ldots,n\}$, there are i and j such that $t=s_0(i,j)$. Therefore, $V=\cup_{i=1}^{d}R_i$. It is easy to see that $\varphi[5]=1$ if and only if for all $1\le i\le d$, $1\le j[1]<j[2]\le n$, and $1\le s\le n$, there is

$$t_0(i,j[1],j[2],s)\in\{1,\ldots,n\}$$

such that

$$z[1,i,j[1],j[2],s,t_0(i,j[1],j[2],s)]=1.$$

Clearly, $\varphi[6]=1$ and $\varphi[7]=1$ if and only if

$$z[2,i,j[1],j[2],1]\ldots z[2,i,j[1],j[2],n]=1^{\alpha}0^{\beta},$$

for some α, β such that $\alpha+\beta=n$, $\alpha>0$, and for all $1\le i\le d$, $1\le j[1]<j[2]\le n$. If $s\le\alpha$, then $\varphi[8]=1$ if and only if

$$(v[s_0(i,t_0(i,j[1],j[2],s))],$$

$$v[s_0(i,t_0(i,j[1],j[2],s+1))])\in E.$$

Since $\varphi[9]=1$, it is easy to see that

$$s_0(i,t_0(i,j[1],j[2],1))=j[1].$$

Similarly, in view of $\varphi[10] = 1$, if $s = \alpha$, then

$$s_0(i, t_0(i, j[1], j[2], s + 1)) = j[2].$$

Since $\varphi[5] = \varphi[8] = \varphi[9] = \varphi[10] = 1$, it is easy to check that R_i is a region, for any i. Therefore, $D = \{R_1, \ldots, R_d\}$ is an 0-overlapping decomposition of V.

Since $\xi = 1$, it is clear that $\psi[i] = 1$, for all $1 \le i \le 4$. In view of $\psi[1] = 1$, it is easy to see that there is $u_0(i, j)$, $1 \le i \le d$ and $1 \le j \le k$, such that $w[i, j, u_0(i, j)] = 1$. Since $\psi[2] = 1$, $u_0(i, j)$ is a function. It is easy to see that $\psi[3] = 1$ if and only if

$$u_0(i, j[1]) \ne u_0(i, j[2]),$$

for $j[1] \ne j[2]$. Since $\psi[4] = 1$, it is easy to check that $| \cap_{v \in R_i} F_v| \ge k$, for all i.

Now, we assume that there is an 0-overlapping decomposition $D = \{R_1, \ldots, R_d\}$ of V such that $| \cap_{v \in R_i} F_v| \ge k$, for all i. Let

$$R_i = \{v[t(i, 1)], v[t(i, 2)], \ldots, v[t(i, p_i)]\}.$$

For any $1 \le i \le d$ and $1 \le j \le p_i$, we assume

$$x[i, j, s] = 1$$

if and only if

$$s = t(i, j).$$

For any $1 \le i \le d$ and $p_i + 1 \le j \le n$, we assume

$$x[i, j, s] = 1$$

if and only if

$$s = t(i, 1).$$

Let

$$y[i, j, s] = 1$$

if and only if

$$x[j, s, i] = 1.$$

Since R_i is a region, for any $1 \le r[1] < r[2] \le p_i$, there is a path

$$v[t(i, q_i[1])], v[t(i, q_i[2])], \ldots, v[t(i, q_i[m_i])]$$

such that $m_i \le n$, $q_i[1] = r[1]$, $q_i[m] = r[2]$. Let

$$z[2, i, j[1], j[2], s] = 1$$

if and only if

$$x[i, j[1], r[1]]x[i, j[2], r[2]]+$$

$$x[i, j[1], r[2]]x[i, j[2], r[1]] > 0,$$

$s < m_i$. For any $s < m_i$, we assume that

$$z[1, i, j[1], j[2], 1, t[1]] = z[1, i, j[1], j[2], 2, t[2]] = 1$$

if and only if

$$x[i, t[1], t[3]] = x[i, t[2], t[4]] = 1.$$

It is easy to check that $\varphi = 1$.

Since $| \cap_{v \in R_i} F_v| \ge k$, for all i, there is

$$\{v[t(i, a_i[1])], v[t(i, a_i[2])], \ldots, v[t(i, a_i[k])]\} \subseteq \cap_{v \in R_i} F_v.$$

Let $w[i, j, s] = 1$ if and only if $s = t(i, a_i[j])$. It is easy to check that $\psi = 1$. $\square$

Clearly, ξ is a CNF. Therefore, in view of the Theorem, ξ gives us an explicit reduction from 0-MORDP to SAT. Using standard transformations (see e.g. [80]) we can obtain an explicit transformation ξ into ζ such that $\xi \Leftrightarrow \zeta$ and ζ is a 3-CNF. It is easy to see that ζ gives us an explicit reduction from 0-MORDP to 3SAT.

IV. Growing regions from seeds

A number of greedy algorithms for solution of 0-MORDP was proposed in [65]. In this section, we consider algorithms A.1, A.2, and A.3 from [65]. Those three algorithms differ only criteria for selecting the pose.

We assume that $|F_v| \ge k$, for all $v \in V$. Now, we consider a general schema of algorithms A.1, A.2, and A.3. Let U be a current set of nodes. Let Δ be a partial decomposition. At first, we assume that $U = V$, $\Delta = \emptyset$. While $U \ne \emptyset$, we repeat the algorithm from Figure 1.

```
Select v ∈ U
R := {v}
repeat
    W := {u | u ∈ {v | v ∈ V\R,
                      ∃w ∈ R(w, v) ∈ E},
             |F_u ∩ (∩_{v∈R} F_v) | ≥ k}
    if W ≠ ∅ then
        if U ∩ W ≠ ∅ then W := U ∩ W
        u := arg max_{w∈W} |F_w ∩ (∩_{v∈R} F_v) |
        R := R ∪ {u}
    end
until W = ∅
U := U\R
D := D ∪ {R}
```

Fig. 1. A general schema of algorithms A.1, A.2, and A.3 (see [65]).

Algorithms A.1, A.2, and A.3 use the following criteria for pose selection (see [65]).

- A.1 selects the pose $v \in U$ at which the least number of features is visible, i.e., $v := \arg\min_{u \in U} |F_u|$.
- A.2 selects the pose $v \in U$ at which the greatest number of features is visible, i.e., $v := \arg\max_{u \in U} |F_u|$.
- A.3 randomly selects a pose $v \in U$.

Note that there is no sufficiently clear evidence for selection of those criteria. We consider a genetic algorithm GAS for pose selection. Let

$$H[i] = h[i, 1]h[i, 2] \ldots h[i, p]$$

where

$$h[i,j] \in \{0,1\}$$

and $p > n$, for all $1 \leq i \leq r$, $1 \leq j \leq p$, and for any possible G. We consider

$$\mathcal{H} = \{H[1], H[2], \ldots, H[r]\}$$

as a population of chromosomes of GAS. We can consider $H[i]$ as a sequence of choices. We can use this sequence as a criterion for A.x. In particular, if $h[i,j] = 1$, then

$$v := \arg\min_{u \in U} |F_u|$$

at jth step. If $h[i,j] = 0$, then

$$v := \arg\max_{u \in U} |F_u|$$

at jth step. For given $H[i]$, we can calculate $|D|$. We can use the value of $|D|$ as the value of the fitness function.

Let A and B are sets. For given A, B, and $a \in A$, the elementary operation $M(A, B, a)$ can be defined as follows:

$$A := A \backslash \{a\},$$

$$B := B \cup \{a\}.$$

Using SAT-solvers we can obtain an optimal decomposition D. Let D_i is a decomposition that is obtained using $H[i]$. Let $h(D_i, D)$ be the minimal number of elementary operations that needed to obtain D from D_i. Let GAS_SAT is a genetic algorithm that uses $h(D_i, D)$ as the fitness function.

In our computational experiments, we consider A.1, A.2, A.3, and two additional algorithms, A.4, A.5. Those algorithms use the general schema of A.1, A.2, and A.3.

A.4 use GAS as the criterion for pose selection.
A.5 use GAS_SAT as the criterion for pose selection.

V. Artificial physics optimization algorithms for 0-MORDP

Note that different nature-inspired heuristics have proven very effective for solving different global optimization problems (see e.g. [81]). In particular, we can mention an artificial physics optimization algorithm. In this section, we consider an artificial physics optimization algorithm for the solution of 0-MORDP.

Note that in an artificial physics optimization algorithm, a swarm of individuals is sampled randomly from a problem space in the initialization. Masses of individuals of the swarm should be calculated in the procedure of calculation force. The procedure of motion uses the total force to calculate the velocity of individuals. It should be noted that a felicitous design of force law can drive individuals search problem space intelligently and efficiently. Therefore, the main advantage of artificial physics optimization algorithms consists in the proper design of force law. Note that different virtual forces are considered (see e.g. [81]). In particular, we can mention negative exponential force law, unimodal force law, linear force law [81]. It is well-known that Runge Kutta neural networks can be used for the prediction of different nonlinear systems [82]. Therefore, we use Runge Kutta neural networks for the design of a general force law. In this paper, we consider 4-order Runge Kutta neural networks with multilayer perceptron networks. We use a gradient learning algorithm for 4-order Runge Kutta neural networks. Note that SAT-solvers allow us to obtain optimal decompositions. We use SAT-solvers to create a training set for 4-order Runge Kutta neural networks.

In our computational experiments, we consider an artificial physics optimization algorithm with negative exponential force law (A.6) and an artificial physics optimization algorithm with Runge Kutta neural network force law (A.7).

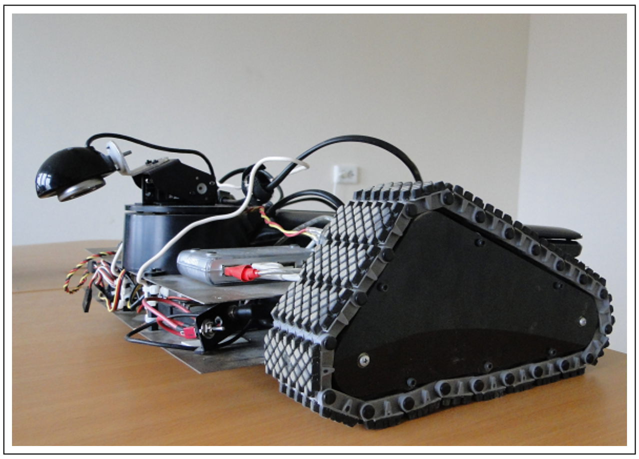

Fig. 2. Robot Kuzma-II.

Fig. 3. Robot Neato XV-11 with an onboard computer and a camera.

VI. Real world data for experiments

We use three different types of robots to perform experiments on real world data. For our experiments, we use autonomous mobile robots Kuzma-II (see e.g. [83], see also Figure 2; design of the robot Kuzma-II based on the well-known Johnny 5 Robot [84]) and Neato XV-11 [85]. Also, we use humanoid robot Nao [86].

This three types of robots allow us to obtain essentially different data for experiments. In particular, we have obtained five different data sets, RW1 (Kuzma-II), RW2 (Neato XV-11), RW3 (Nao), RW4 (Neato XV-11 and Kuzma-II), RW5 (Neato XV-11 with an onboard camera).

We consider Neato XV-11 with an onboard computer and a camera (see Figure 3). For such configuration of Neato, we consider a model of navigation with two sensors, a laser sensor and a camera.

Kuzma-II is equipped with a 2 DOF robotic camera. This robotic camera allows Kuzma-II to have a large visibility sector. However, due to the low position of the camera, Kuzma-II has relatively small visibility radius. In particular, in most cases, the robot does not see the environment behind obstacles.

Nao has a larger visibility radius than the radius of Kuzma-II. However, the visibility sector of Nao is much smaller than the visibility sector of Kuzma-II. Frequently, F_v is not connected.

For Neato XV-11, we consider a model of external stationary cameras robot localization. Note that Neato is equipped with a laser sensor. Therefore, Neato can solve localization problem without cameras. However, cameras can be used to identify dangerous areas (e.g. wet floor), dusty places, and difficult to navigate sites. If Neato can easily use the laser navigation in some area of a flat, then we can consider all points of this area as a same node. Such unification allows us to reduce $|V|$. But, in this case, we obtain a very large visibility radius.

Also, we consider a model of external mobile camera robot localization for Neato XV-11. In this case, Neato uses an information from Kuzma-II.

VII. Synthetic data for experiments

We use a model of synthetic data generation from [65] to perform experiments on synthetic data. The synthetic data for experiments was produced using a simulator that randomly generates worlds, given a mixture of probability distributions for each of the defining parameters of the world.

There are three main components of synthetic data, the value of the perimeter, obstacles, and features. The value of the perimeter depends from the sides count and the vertex radius. Obstacles depend from the total obstacles count, the sides count, and the vertex radius. Features depend from the total features count, the visibility angular extent, and the visibility range.

A synthetic world consists of a 2-D top view of the pose space defined by a polygon, with internal polygonal obstacles and a collection of features on the polygons. For such synthetic worlds, we use four different world settings from [65]. In particular, we have obtained four different data sets, SW1, SW2, SW3, SW4.

VIII. Computational experiments

To obtain optimal solutions of 0-MORDP, we consider our explicit reduction from 0-MORDP to the satisfiability problem and use our own genetic algorithms OA[1] (see [87]), OA[2] (see [88]), and OA[3] (see [89]) for the satisfiability problem. We have used a heterogeneous cluster based on three clusters (Cluster USU, umt, um64) [90]. Each test was runned on a cluster of at least 100 nodes. Note that due to restrictions on computation time (20 hours) we have used savepoints. Selected experimental results are given in Tables I – III.

Let $N(X, Y)$ be the average number of regions for given data set X and algorithm Y. Let $N_{opt}(X)$ be the average optimal number of regions for given data set X. We have calculated values of

$$\frac{N(X,Y)}{N_{opt}(X)}$$

TABLE I
Experimental results for OA[1]

time	average	max	best
RW1	16.32 min	6.23 h	4.63 sec
RW2	2.97 h	36.44 h	21.34 min
RW3	34.25 min	18.62 h	0.28 sec
RW4	1.19 h	13.28 h	8.17 min
RW5	11.8 min	4.18 h	0.43 sec
SW1	1.77 h	21.83 h	12.3 sec
SW2	3.18 h	20.76 h	14.21 sec
SW3	2.57 h	24.37 h	15.39 sec
SW4	2.89 h	25.16 h	3.73 min

TABLE II
Experimental results for OA[2]

time	average	max	best
RW1	3.06 h	24.19 h	12.6 min
RW2	4.19 h	67.33 h	17.5 min
RW3	2.22 h	54.11 h	19.8 min
RW4	3.72 h	34.3 h	17.2 min
RW5	2.66 h	22.7 h	23.1 min
SW1	9.53 h	36.48 h	37.4 min
SW2	8.12 h	38.12 h	42.1 min
SW3	10.9 h	45.6 h	35.4 min
SW4	11.2 h	42.4 h	39.2 min

for different greedy algorithms and data sets. Selected experimental results are given in Tables IV – VII.

IX. Conclusion

In this paper, we have considered an approach to create solvers for the ρ-minimum overlapping region decomposition problem. In particular, explicit polynomial reductions from the problem to 3SAT is constructed. We have considered some greedy algorithms for solution of the problem. Also, we have considered computational experiments for different data sets.

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TABLE III
EXPERIMENTAL RESULTS FOR OA[3]

time	average	max	best
RW1	1.89 min	3.11 h	18.2 sec
RW2	37.15 min	3.28 h	39.51 sec
RW3	7.12 min	3.12 h	2.6 sec
RW4	8.46 min	4.53 h	5.14 sec
RW5	4.32 min	2.96 h	6.12 sec
SW1	37.5 min	19.62 h	15.42 sec
SW2	48.21 min	24.31 h	11.18 sec
SW3	1.64 h	26.1 h	6.15 sec
SW4	1.23 h	18.16 h	8.2 sec

TABLE IV
EXPERIMENTAL RESULTS FOR A.1, A.2, AND A.3.

time	A.1	A.2	A.3
RW1	231 %	256 %	655 %
RW2	673 %	721 %	1657 %
RW3	433 %	484 %	912 %
RW4	222 %	278 %	783 %
RW5	219 %	215 %	476 %
SW1	216 %	187 %	499 %
SW2	223 %	194 %	536 %
SW3	372 %	391 %	597 %
SW4	411 %	417 %	643 %

TABLE V
EXPERIMENTAL RESULTS FOR A.4 WITH DIFFERENT NUMBER OF GENERATIONS OF GAS.

time	10^3	10^4	10^5	10^6
RW1	184 %	153 %	146 %	137 %
RW2	531 %	472 %	423 %	395 %
RW3	327 %	297 %	266 %	243 %
RW4	174 %	169 %	163 %	152 %
RW5	168 %	157 %	151 %	139 %
SW1	185 %	161 %	139 %	124 %
SW2	188 %	173 %	165 %	163 %
SW3	311 %	267 %	232 %	229 %
SW4	338 %	315 %	280 %	276 %

TABLE VI
EXPERIMENTAL RESULTS FOR A.5 WITH DIFFERENT NUMBER OF GENERATIONS OF GAS_SAT.

time	10^3	10^4	10^5	10^6
RW1	166 %	141 %	109 %	108 %
RW2	473 %	162 %	110 %	109 %
RW3	252 %	133 %	108 %	108 %
RW4	128 %	117 %	103 %	102 %
RW5	163 %	144 %	101 %	101 %
SW1	181 %	155 %	108 %	107 %
SW2	182 %	169 %	114 %	112 %
SW3	274 %	112 %	105 %	105 %
SW4	319 %	163 %	117 %	116 %

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TABLE VII
EXPERIMENTAL RESULTS FOR A.6 AND A.7.

time	A.6	A.7
RW1	482 %	102.14 %
RW2	1038.2 %	104.63 %
RW3	627.43 %	103.52 %
RW4	273.1 %	100.96 %
RW5	311.8 %	100.29 %
SW1	417.91 %	102.33 %
SW2	398.3 %	105.4 %
SW3	543.52 %	101.26 %
SW4	613.7 %	104.21 %

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