Wavelet Transform-based Residual Modified GM(1,1) for Hemispherical Resonator Gyroscope Lifetime Prediction

Chenglong Dai, Dechang Pi, Zhen Fang, Hui Peng

Abstract—**The hemispherical resonator gyroscope (HRG) is a new vibration gyro. It has outstanding characteristics of high accuracy, great reliability, long lifespan, and no wear-out, which make it impracticable to get the lifetime with whole life test. To shorten test duration and predict HRG's lifetime, a prediction method combines grey model and grey correlation analysis is proposed in this paper. In the method, wavelet transform is used to reduce noises in original data and an improved GM(1,1) model with residuals is put forward to predict long-term drift data. At last, grey correlation analysis is utilized to evaluate HRG's failure stage and get HRG's lifetime. Applying the method to drift data of 4 different HRGs and the experimental results show the residual modified GM(1,1) model has higher performances not only on data fitting but also on forecasting than the conventional GM(1,1). The predictive lifetimes of 4 HRGs are: 8622 days for 1#, 5748 days for 2#, 7664 days for 3#, and 6707 days for 4#. Based on the 10 global oldest spacecraft (all more than 8760 days), our predictive results are reliable and our method is valid for long-term prediction.**

Index Terms—**Hemispherical Resonator Gyroscope (HRG), Lifetime prediction, Wavelet transform, GM(1,1), Residual modified model, Grey correlation analysis**

I. Introduction

With good features of high reliability, no wear-out, and long lifespan, HRGs have been widely applied in many space missions [1]. As the important unit in inertia system, HRGs affect the reliabilities of inertia systems and spacecraft. A survey [2] shows: 60% failure of inertia system is from electronic circuits and 40% from inertia platform, in which 60% is caused by gyroscope. Obviously, it becomes more and more meaningful to evaluate HRG's lifetime. But worse still is that reports on the HRGs are limited, compared with other types of gyros, like dynamically tuned gyroscope (DTG) [3]-[5], MEMS gyro [6]-[8], fiber optic gyro [9]-[11] and so on [12],[13]. For hemispherical resonator gyros, reference [14] analyzed performances of HRG with drift data and [15] researched the influences of temperature complement on HRGs' navigation accuracy. In these studies, they do not focus on the lifetime of HRG, even no prediction methods mentioned in them. To study HRG's lifespan, we bring in grey model to analyze its lifetime in the paper.

Since professor Deng put forward the grey system [16] in the early of 1980s, it has been widely used to solve problems with partial unknown parameter systems. Since the last 30 years of 20th century, grey theory has been fast developed, especially GM(1,1) model. The GM(1,1) is a famous prediction model and the accelerated generating operation (AGO) [17] is its most important operation, which is able to efficiently reduce the randomness of modeling data. Besides, not like neural networks or support vector machine which are mostly applied in image classification [18],[19] and time series forecasting [20],[21], grey model can forecast with only four data [17] which is helpful for small-sample modeling and forecasting, like HRG. Nowadays, GM(1,1) and its improved models are widely utilized in social science [22],[23], power consumption [24],[25], and so on [26]. In these studies and the others, it's seen that grey system-based methods can achieve good performance in prediction. Moreover, to get low-noise data, many researchers brought in wavelets to reduce noises in data. For example, haar wavelet was used to decompose and reconstruct the original data and then the low-noise data are used to forecast [27]. Reference [28] also utilized wavelet to analyze and predict time series data. In these studies, it's seen that wavelet transforms can reasonably reduce noises in data or in signals and provide low-noise data for analyzing and predicting.

In our method, both wavelets and grey correlation analysis which is also applied in [29] are combined to assist the improved GM(1,1) model to predict HRG's lifetime.

II. GM(1,1)

GM(1,1) model is one of the most widely used prediction models. Its differential equation has time-varying coefficients, i.e., when new data adding, GM(1,1) model will accordingly rebuild itself to fit the law of the new data sequence. The steps to build GM(1,1) model is introduced as follow.

1. Let $X^{(0)}$ be the modeling data sequence:

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n-1), x^{(0)}(n)) \tag{1}$$

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where $X^{(0)}$ is a nonnegative data sequence and $n(n \geq 4)$ is the number of $X^{(0)}$.

2. To reduce the randomness in modeling data, do accelerated generating operation one time (1-AGO) for $X^{(0)}$, then a smoother data sequence $X^{(1)}$ is generated,

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n-1), x^{(1)}(n)) \quad (2)$$

where $x^{(1)}(k)$ can be obtained by using (3),

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, \cdots, n \quad (3)$$

Theoretically speaking, for nonnegative data, they will become smoother if they amass many times. But doing 1-AGO for data is normally enough in grey model.

3. Smooth accumulated data: Average two adjacent data in $X^{(1)}$, and get sequence $Z^{(1)}$:

$$Z^{(1)} = (z^{(1)}(1), z^{(1)}(2), \cdots, z^{(1)}(n-1), z^{(1)}(n)) \quad (4)$$

Where, $z^{(1)}(k)$ is calculated by using (5).

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k+1), k = 1, 2, \cdots, n-1 \quad (5)$$

4. Now, grey differential function can be built as (6) shows.

$$x^{(0)}(k) + az^{(1)}(k) = b \quad (6)$$

and its whitening equation, the GM(1,1) model, is

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b \quad (7)$$

where $[a,b]^T$ are parameters, which can be calculated out by using (8).

$$[a,b]^T = (B^T B)^{-1} B^T Y \quad (8)$$

And where

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \quad \mathrm{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}.$$

Based on (7), the solution of $x^{(1)}(t)$ at moment k is also achieved:

$$\hat{x}^{(1)}(k+1) = [x^{(0)}(1) - \frac{b}{a}]e^{-ak} + \frac{b}{a} \quad (9)$$

5. 1-AGO is done before, so here do inverse accumulated generating operation one time (1-IAGO) for accumulated predictive data and then the predictive result at moment $k+1$ is achieved by using (10).

$$\hat{x}^{(0)}(k+1) = [x^{(0)}(1) - \frac{b}{a}]e^{-ak}(1 - e^{a}) \quad (10)$$

III. Lifetime Prediction Models

A. Residual modified GM(1,1)

The prediction accuracy of the conventional GM(1,1) needs improving, so residual error is used to adjust GM(1,1) model, and then a residual modified GM(1,1) model is achieved.

Based on modeling drift data and predictive data by using GM(1,1) model, the residuals $\varepsilon^{(0)}$ is gained as (11) shows.

$$\varepsilon^{(0)} = (\varepsilon^{(0)}(1), \varepsilon^{(0)}(2), \cdots, \varepsilon^{(0)}(n-1), \varepsilon^{(0)}(n)) \quad (11)$$

Where $\varepsilon^{(0)}(i)$ can be calculated by using (12).

$$\varepsilon^{(0)}(k) = x^{(0)}(k) - \hat{x}^{(0)}(k) \quad (12)$$

1. Based on residual error, the residual modeling data sequence is $(|\varepsilon^{(0)}(k_0)|, |\varepsilon^{(0)}(k_0+1)|, \cdots, |\varepsilon^{(0)}(n)|)$, also set as:

$$\varepsilon^{(0)} = (\varepsilon^{(0)}(k_0), \varepsilon^{(0)}(k_0+1), \cdots, \varepsilon^{(0)}(n)) \quad (13)$$

2. Do 1-AGO for $\varepsilon^{(0)}$ to reduce the randomness in it and then $\varepsilon^{(1)}$ is gained.

$$\varepsilon^{(1)} = (\varepsilon^{(1)}(k_0), \varepsilon^{(1)}(k_0+1), \cdots, \varepsilon^{(1)}(n)) \quad (14)$$

where $\varepsilon^{(1)}(k)$ can be obtained by using (15),

$$\varepsilon^{(1)}(k) = \sum_{i=1}^{k} \varepsilon^{(0)}(i), \quad k = 1, 2, \cdots, n \quad (15)$$

Then, the time response function of residuals is accordingly obtained as (16) shows.

$$\hat{\varepsilon}^{(0)}(k+1) = (\varepsilon^{(0)}(k_0) - \frac{b_\varepsilon}{a_\varepsilon}) \cdot \exp(-a_\varepsilon(k - k_0)) + \frac{b_\varepsilon}{a_\varepsilon}, \ (k \geq k_0) \quad (16)$$

3. Adjust $X^{(1)}$ with $\varepsilon^{(0)}$, and then the modified time response function is achieved as well.

$$\hat{x}^{(1)}(k+1) = \begin{cases} (x^{(0)}(1) - \frac{b}{a}) \cdot e^{-ak} + \frac{b}{a} & (k < k_0) \\ (x^{(0)}(1) - \frac{b}{a}) \cdot e^{-ak} + \frac{b}{a} & \\ \quad \pm (\varepsilon^{(0)}(k_0) - \frac{b_\varepsilon}{a_\varepsilon}) \cdot \exp(-a_\varepsilon(k - k_0)) & (k \geq k_0) \end{cases} \quad (17)$$

4. As 1-AGO was done before, do 1-IAGO for $\hat{x}^{(1)}(k+1)$, and the inverse accelerated residual modified time response function is gained, as (18) shows.

$$\hat{x}^{(0)}(k+1) = \begin{cases} (1 - e^{a})(x^{(0)}(1) - \frac{b}{a}) \cdot e^{-ak} & (k < k_0) \\ (1 - e^{a})(x^{(0)}(1) - \frac{b}{a}) \cdot e^{-ak} & \\ \quad \pm a_\varepsilon(\varepsilon^{(0)}(k_0) - \frac{b_\varepsilon}{a_\varepsilon}) \cdot \exp(-a_\varepsilon(k - k_0)) & (k \geq k_0) \end{cases} \quad (18)$$

By the way, equation (18) is the residual modified GM(1,1) model, which is used to forecast in the following work.

B. Grey correlation analysis method

Grey correlation analysis method is able to evaluate compactness between two curves based on the likeness of them. The closer the curve fits the referential, the larger grey correlation degree the curve has.

Let $x_0 = \{x_0(1), x_0(2), \cdots, x_0(n)\}$ be the referential sequence, $x_i (i = 1, 2, \cdots, m)$ be the comparative sequence, and x_i can be also recorded as

$$\begin{matrix} x_1 = \{x_1(1), x_1(2), \cdots, x_1(n)\} \\ x_2 = \{x_2(1), x_2(2), \cdots, x_2(n)\} \\ \vdots \\ x_m = \{x_m(1), x_m(2), \cdots, x_m(n)\} \end{matrix}.$$

The grey correlation of k^{th} $x_0(k)$ and k^{th} $x_i(k)$ in sequence x_0 and x_i can be calculated by using (19).

$$\gamma(x_0(k), x_i(k)) = \frac{m + \xi M}{|x_0(k) - x_i(k)| + \xi M} \quad (19)$$

where $m = \min_i \min_k |x_0(k) - x_i(k)|$, $M = \max_i \max_k |x_0(k) - x_i(k)|$, and ξ ($\xi \in (0,1]$) is the resolution, generally $\xi = 0.5$. Finally, the grey correlation degree between sequence x_0 and x_i can be calculated with (20).

$$\gamma(x_0, x_i) = \frac{1}{n}\sum_{k=1}^{n}\gamma(x_0(k), x_i(k)) \tag{20}$$

C. Evaluation methods for prediction accuracy

In this paper, we bring in mean relative percentage error (MRPE), root mean square error (RMSE) and normalized mean square error (NMSE) to evaluate models' prediction accuracies.

(1) Mean Relative Percentage Error (MRPE) is used to evaluate the reliability of predictive data by comparing with modeling data.

$$E_{MRPE}MRPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \times 100\% \tag{21}$$

(2) Root Mean Square Error (RMSE) is used to evaluate the deviation between predictive data and modeling data.

$$E_{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n}} \tag{22}$$

(3) Normalized Mean Square Error (NMSE) is used to evaluate the oscillation of predictive data and modeling data.

$$E_{NMSE} = \frac{1}{\delta^2 n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \tag{23}$$

where,

$$\delta^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 \tag{24}$$

In all evaluation methods introduced above, y is the modeling data, $\hat{y}$ is the predictive data, $\bar{y}$ is the mean of modeling data, and n is the number of y.

IV. EXPERIMENTS

A. Data

The HRG's drift data we use in this paper are provided by China Electronics Technology Group Corporation 26th Research Institute in Chongqing, China. Four HRGs are tested and each gyro's random drifts are recorded by using (25) and (26). And the test duration is from 26, June, 2009 to 8, February, 2012; 958 days in all and 1590 data are obtained.

$$\sigma = \frac{1}{K}\sqrt{\frac{\sum_{i=1}^{n}(O_i - \bar{O})^2}{n-1}} \tag{25}$$

$$\bar{O} = \frac{1}{n}\sum_{i=1}^{n}O_i \tag{26}$$

In (25) and (26), O_i is the output sample after average handling, (Unit: V). $\bar{O}$ is mean output (Unit: V); K is gyroscope scale factor, (Unit: V/(°/s)); σ is drift data, (Unit: °/h).

B. Data preprocessing

For four HRGs are in the same situation, we just take HRG 1# as the example in the paper to verify the long-term predicting abilities of GM(1,1), residual GM(1,1), BPNN, and SVM. By respectively using GM(1,1), residual GM(1,1), BPNN, and SVM, we first predict 1590 data (actually the 1590 data are the simulation data) for HRG 1# with the 1590 original drift data which contain high-noise data. The simulation results without data preprocessing are shown in Fig.1. In addition, we partially enlarge Fig.1 and its details are displayed in Fig.2.

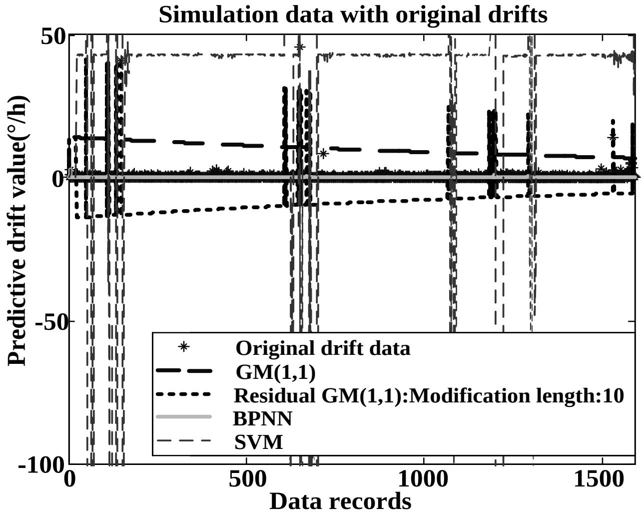


Fig.1 Simulation data with original drift data of 1#

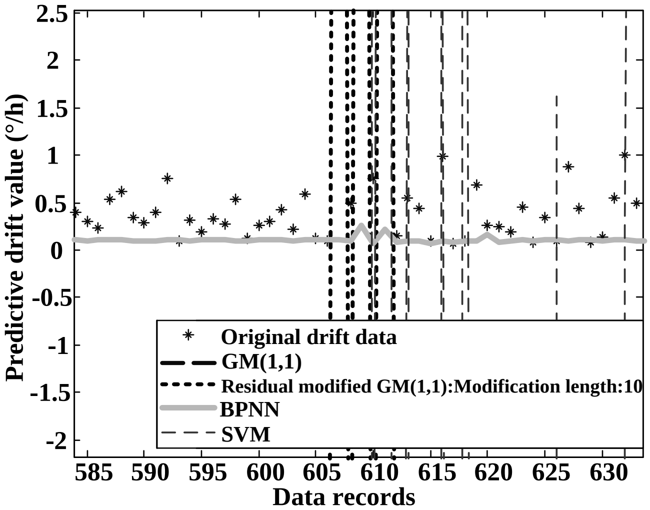


Fig.2 Partial zoomed figure of Fig.1

See Fig.1 and Fig.2, it's known that all the models do not get a good simulation result with the original drift data, for the noises exist in original data affect the models' self-training and make the results of BPNN, SVM, GM(1,1) and residual modified GM(1,1) deviate from original data a lot. Taking Fig.2 as the example, the original data disperse a lot, and it makes not only GM(1,1), residual modified GM(1,1) model but also SVM (the red dashed line) and BPNN (the green solid line) unsatisfactorily fit the original data. Worse still, there are large differences between the simulation results and the original data. In a word, all the prediction models cannot achieve reasonable simulation results by using original drift data. The simulation results of 2#, 3#, and 4# are as same as 1#. So in order to obtain reasonable and high-accuracy simulation, we use wavelets in different scales to reduce the noises in original data, which is also necessary. And then we can use the processed data to do long-term prediction for 4 HRGs.

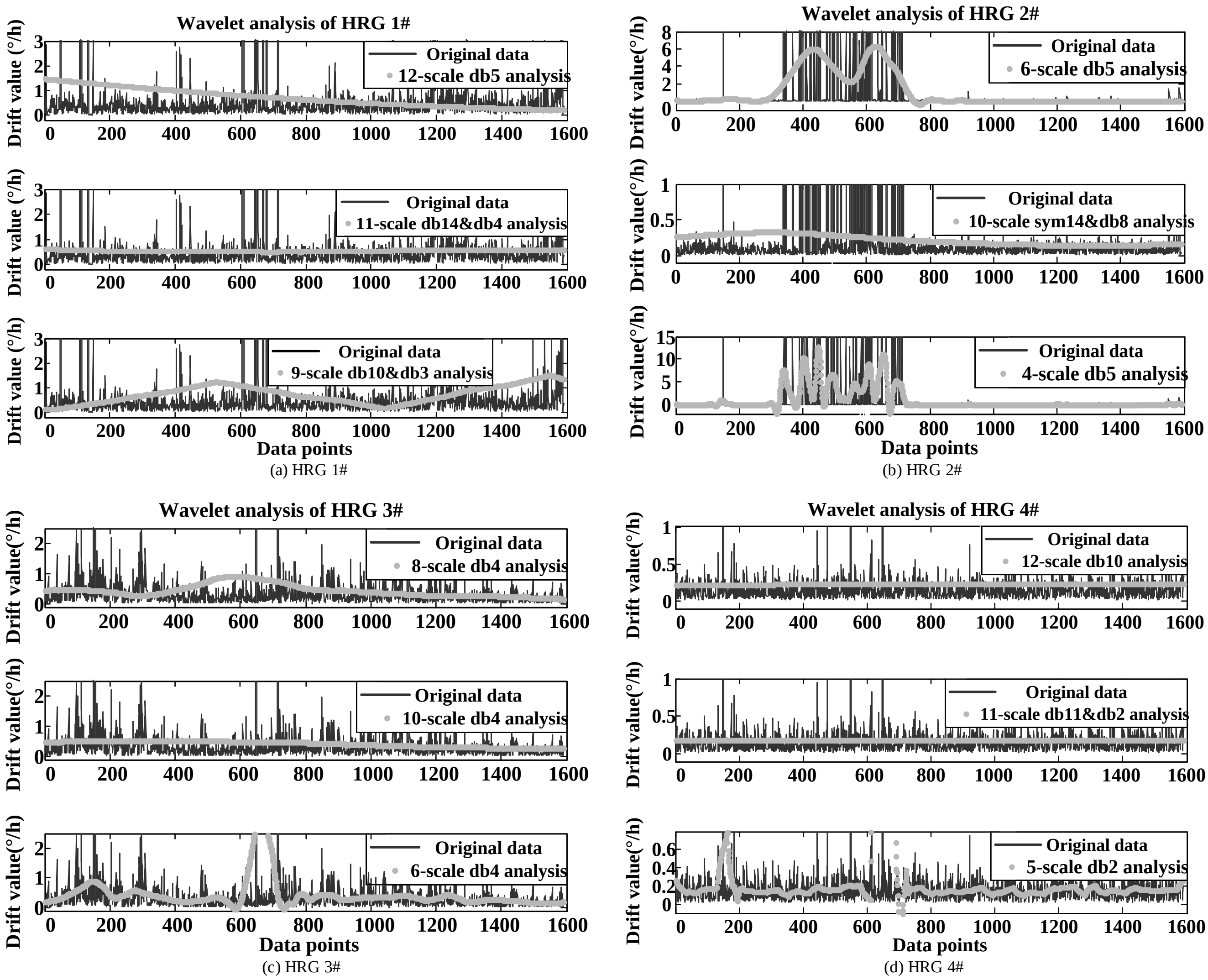
Wavelets db (daubechies) and sym (symlets) in different scales or in different combinations are applied to decompose and reconstruct test data of 4 HRGs to reduce their noises and achieve low-noise data for forecasting. And the 4 HRGs' processed results are shown in Fig.3.

See figure (a), (b), (c), and (d) in Fig.3, the processed data in second subfigures are close to the original data and the wavelets used in them reduce noises better than others. For example, in figure (c), the processed results in first subfigure do not perfectly satisfy the law of original data and the third one's does not denoise the original data well because its processed data are unsmooth or over fitting. On the contrary, the results in second subfigure learn the law of original drift data better and the wavelets used in the second reduce noises better. Namely, the second subfigure has better decomposing and remodeling performance than others. Moreover, we've tried different wavelets with different scale analyses to deal with original drift data, but their results are not as good as the wavelets used in the second subfigure. In addition, some other wavelets are over fitting (like the third subfigures in (a), (b), (c), and (d) of Fig.3) or under fitting (like the first subfigures in all figures of Fig.3). Based on the analysis before, we use preprocessed data in second subfigure to predict and evaluate in the following work.

C. Prediction and analysis

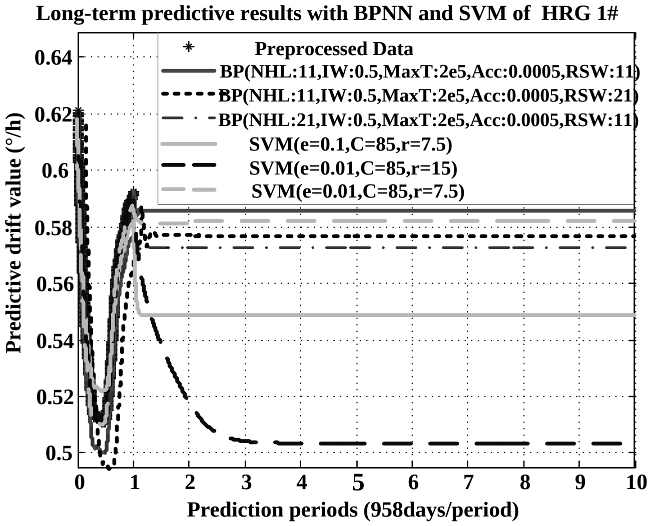
Such methods as grey system, support vector machine, and back propagation neural network are well-known in time series prediction in recent decades. So we firstly studied back propagation neural network (BPNN) and support vector machine (SVM) for long-term prediction with preprocessed data of HRG 1# and we tried several groups of different parameters for both BPNN and SVM, but all the predictive results are not receivable. The results are shown in Fig.4.

Both BPNN and SVM have good performances on data fitting and short-term prediction, i.e., the self-training curves of BPNN and SVM are very close to the modeling data, see region [0-1] in Fig.4. But when predicting more data, both BPNN and SVM with different parameters get the constant, as the lines in region [2-10] display. The predictive results of 2#, 3# and 4# are also as same as 1#. In a word, BPNN and SVM have poor performance on long-term predicting, and although they have excellent self-training abilities, they are just suitable for short-term forecasting.



(a) HRG 1# (b) HRG 2# (c) HRG 3# (d) HRG 4#

Fig.3 Wavelet Analysis for original data of 4 HRGs

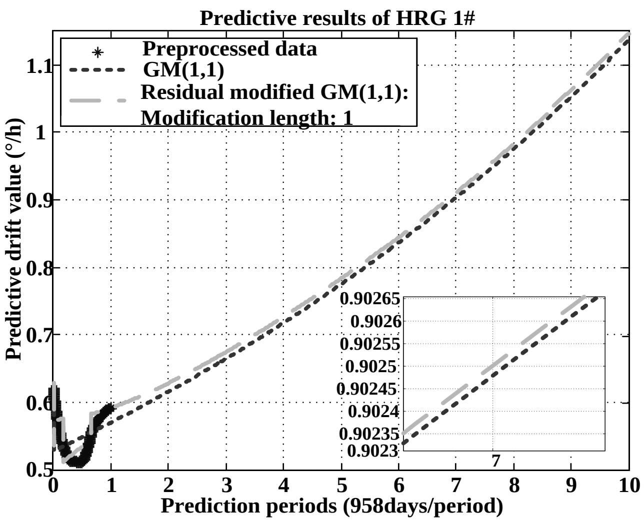


* NHL: Nodes in hidden layer; IW: Inertia weight; MaxT: Maximum training time; Acc: Training accuracy; RSW: Regressive step width.

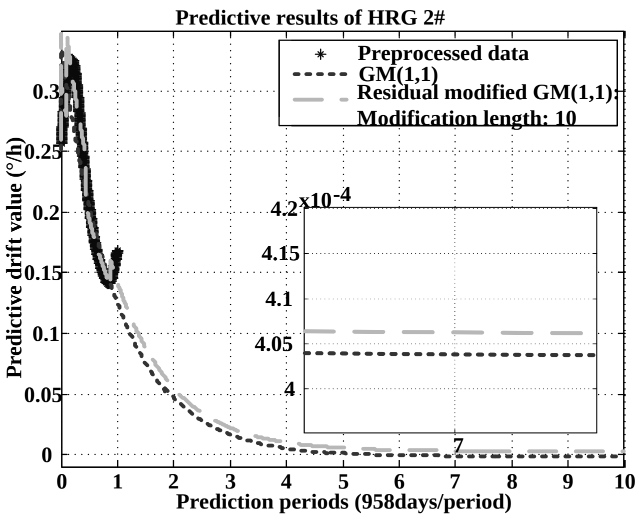
Fig.4 BPNN's and SVM's long-term prediction for HRG 1#

Due to HRG having long lifespan, use BPNN and SVM to predict its lifetime is impossible, for BPNN and SVM are not suitable for long-term prediction. In the paper, we researched grey model and propose an improved GM(1,1) with residuals. Here, we predict simulation sequence and nine-time predictive sequences for 4 different HRGs with low-noise data by respectively using the conventional GM(1,1) and the residual modified GM(1,1) model. By the way, every prediction period is 958 days, and one period contains 1590 data. Nine-time prediction means that $1590*9=14310$ data will be predicted after simulation sequence. The predictive results of HRG 1#, 2#, 3#, and 4# are respectively shown in subfigure (a), (b), (c), and (d) in Fig.5.

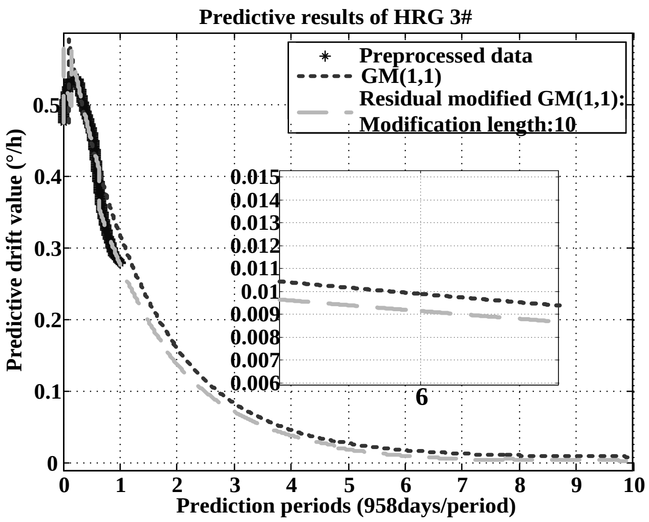
With the predictive results of two methods, it's seen that simulation sequence of residual modified GM(1,1) fits modeling sequence better than the conventional GM(1,1) model. Taking subfigure (c) as the example, the blue stellate line stands for modeling data, the red dotted one is simulation sequence of GM(1,1) and the green dashed line is that of residual modified GM(1,1). Just in region [0-1] in subfigure (c), the green dashed line is closer to the stellate line than the red dotted one, so the dashed curve indicates the modified model trains modeling data better. Besides, it's also shown that the more periodic data two methods predict, the closer the predictive results of two methods go to. But actually there exists big differences between them, see the zoomed little figure in Fig.5.



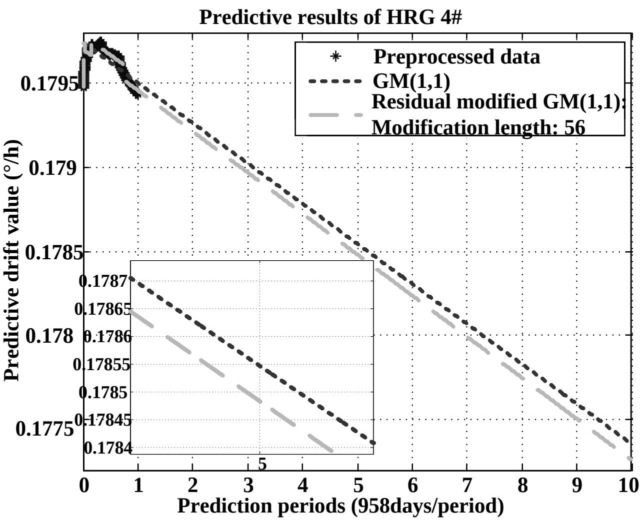
(a) HRG 1#



(b) HRG 2#



(c) HRG 3#



(d) HRG 4#

Fig.5 Drift predictive results of GM(1,1) and residual modified GM(1,1)

Moreover, residual sum of squares (RSS), MRPE, RMSE, and NMSE between modeling data and simulation data are respectively calculated out to evaluate the accuracies of two methods, which are shown in Table I.

Table I
The Simulation Prediction Accuracies of Residual Modified GM(1,1) and GM(1,1)

HRGs	Prediction Models	RSS	MRPE (%)	RMSE	NMSE
1#	GM(1,1)	1.3030	4.38	0.0286	0.8494
	Residual GM(1,1)	0.3148	1.89	0.0141	0.2052
2#	GM(1,1)	1.1259	9.70	0.0266	0.1565
	Residual GM(1,1)	0.5796	5.26	0.0191	0.0806
3#	GM(1,1)	3.6658	14.03	0.0480	0.9836
	Residual GM(1,1)	0.8508	6.48	0.0231	0.2283
4#	GM(1,1)	6.0216	0.03	6.1540	0.4403
	Residual GM(1,1)	3.0153	0.01	4.3548	0.2205

According to Table I, it's seen that RSSs, MRPEs, RMSEs, and NMSEs of residual modified GM(1,1) are much smaller than those of conventional GM(1,1). That is, residual modified GM(1,1) model has higher prediction accuracy than GM(1,1).

According to the predictive results of residual modified GM(1,1), we calculate out grey correlation degrees for simulation sequence as well as for each time periodic prediction of 4 HRGs, as Table II shows.

In Fig.5, we see that every HRG's simulation sequence curve is the closest one to modeling curve and according to grey correlation analysis method, simulation sequence has the highest grey correlation degree in all prediction periods and other prediction sequences from onetime to nine-time period have lower degrees to modeling sequence. As a result, the grey correlation degree larger than the first one is the threshold which can be regarded as the failure stage of the HRG. And with the threshold, it is able to evaluate how long the HRG can normally work.

With Table II, the 8th grey correlation degree of HRG 1# is 0.8463 and it is the first one exceeds simulation sequence's 0.8357 and the following predictions' grey correlation degrees are all larger than the simulation's. Based on the grey correlation analysis method, the 8th period is the failure stage, and the lifetime of 1# is 9 times the prediction period, namely, $958*9=8622$ days (That is, eight-time prediction periods + onetime test period = nine-time periods, and 958 days per period), about 23.62 years. In the same way, the lifetime of 2# can reach $958*6=5748$ days, about 15.74 years; 3# can normally run $958*8=7664$ days, about 20.99 years and 4# can work $958*7=6706$ day s, about 18.37 years.

V. CONCLUSIONS

With excellent performances, the HRG is gaining much popularity around the world. As the important unit, evaluating HRG's lifetime is worthwhile to inertia systems and spacecraft. However, testing HRG's lifetime in its whole life cycle is impracticable, for HRG has features of long lifespan, high cost and small sample. Eventually a mixed method which combines residual modified GM(1,1), wavelet analysis and grey correlation analysis is proposed in the paper to predict HRG's long lifetime without whole life test. With studies in the paper, several conclusions are made as follows:

1. Because of the uncontrolled testing environment, there are some noises in original drift data which result in low-accuracy prediction and make models difficult to forecast in long term. But with reasonable wavelets for original data, noises in original drift data are well reduced and the law of original data is also obtained well, too. That is, it is necessary to use wavelets to denoise original data. And with low-noise data, prediction model can achieve high-accuracy results.

2. Back propagation neural networks and support vector machine have better performances on data fitting and short-term prediction, but for HRG which has long lifespan, they are poorer in long-term prediction than residual modified GM(1,1), that is, BPNN and SVM are not suitable for long-term prediction, so they are impractical to evaluate HRGs' lifetime in the paper.

3. The predictive curves in the later periods of two methods are similar with each other, because two models' time response functions are familiar with each other and their parameter sensitivities to time become poor as prediction period accelerates. But the modified model always has higher parameter sensitivity than the conventional one. As a consequence, the residual modified GM(1,1) has higher accuracy than the conventional GM(1,1), not only in data fitting, but also in forecasting. Therefore, residual modified GM(1,1) is more reliable than the conventional one.

Table II
Simulation Sequences' and 9 Period Prediction Sequences' Grey Correlation Degrees

HRGs	Simulation Sequences	1st period	2nd period	3rd period	4th period	5th period	6th period	7th period	8th period	9th period
1#	0.8357	0.4864	0.4294	0.5879	0.6858	0.7483	0.7913	0.8226	0.8463	0.8647
2#	0.8157	0.8054	0.8130	0.8149	0.8156	0.8158	0.8159	0.8159	0.8159	0.8159
3#	0.7868	0.6093	0.6742	0.6869	0.7010	0.7450	0.7778	0.8037	0.8249	0.8426
4#	0.8776	0.4219	0.6732	0.7718	0.8247	0.8576	0.8802	0.8965	0.9089	0.9187

4. With grey correlation analysis method getting each HRG's failure stage, 4 HRGs' lifetimes are predicted out. Their predictive lifetimes are 23.62 years for HRG 1#, 15.74 years for 2#, 20.99 years for 3#, and 18.37 years for 4#. Because no references on HRG's lifespan can be looked up, 10 global oldest spacecraft's lifespans are used to help evaluate the predictive results, for gyroscopes are their important units and it means that gyros are able to normally work as long as them. The 10 oldest spacecraft are Voyager 2 (1977.8-), Voyager 1 (1977.9-), GOES 3 (1978.6-), ATS-3 (1967.11-2001), Mirasat F2 (1976.6-2008.10), Landsat 5 (1984.3-2012.12), TDRS-1 (1983.4-2009), GOES 7 (1987.2-2012.4), TDRS-3 (1988.9-), and GOES 2 (1977.6-2001) and their lifespans are more than 20 years. According to these 10 spacecraft's lifespans, the predictive results achieved by residual modified GM(1,1) are receivable and reliable. Besides, the prediction method for hemispherical resonator gyros and the predictive results enriches the field on HRG's lifetime prediction and they can assist researchers or institutes to evaluate gyros' long lifetime or reliability as well.

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