

# Working Capital Management in Single-period Production Planning for Maximisation of Shareholder Wealth

Xiao Jun Wang and Shiu Hong Choi

**Abstract**—Dynamic lot sizing is pivotal to uncertain batch production, especially under the capacitated multi-product environment. Although much progress has been made in this area, most studies are not applicable to capacitated make-to-order (MTO) manufacturing environments with stochastic interarrival orders. This paper develops a dynamic lot sizing model for the stochastic multi-product MTO production environment, aimed to realize the overall business objective of maximising the sustainable economic interests of business owners, i.e., shareholder wealth. Management of working capital is examined to explore its critical impacts on shareholder wealth. Computational studies are conducted to numerically and analytically demonstrate the significance of the shareholder wealth optimisation in stochastic multi-product manufacturing. Moreover, effective management in working capital is proven to be of huge assistance in keeping operations stability and improving shareholder wealth.

**Index Terms** — lot sizing, shareholder wealth, stochastic, working capital

## I. INTRODUCTION

IN many industrial sectors, multi-product manufacturing is universal and successful. This universality and success, to a significant extent, is attributable to the dispersion of operations risks and diversification of profit sources arising from multi-product manufacturing. As shown by [1], multi-product business was prevalent in the global market. In terms of the output share, multi-product firms made up around 70% of the total manufacturing output, while multi-industry and multi-sector firms formed 50% and 31% respectively.

The global prevalence of multi-product manufacturing fires up a great deal of enthusiasm for academic research in production planning. Reference [2], for example, studied the multi-product economic lot scheduling problem with manufacturing and remanufacturing, which were performed on the same production line and assumed to produce products of the same quality and to fulfil the same demand stream. Reference [3] dealt with the issue of investing in reduced setup times and defect rates for a multi-product manufacturer in a just-in-time (JIT) manufacturing environment.

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In the current research on production planning, batch production with lot sizing is one of the most pivotal. Impacts of lot sizing on optimisation are twofold. First, uncertain arrivals of orders and capacitated machines may give rise to a great deal of work-in-process (WIP) inventory. Second, lot sizing is closely related to work flow times. Lot sizes that are smaller would result in high utilization of machines due to excessive machine time. In contrast, lot sizes that are too much larger may cause excessive waiting times of incoming orders.

Accordingly, lot sizing for stochastic multi-product manufacturing is a very common and important problem facing manufacturers. Much literature has explored the lot sizing policy for various multi-product manufacturing circumstances. Reference [4], for instance, demonstrated the necessity of the capacity-constrained lot sizing research when taking account of economic factors. In the basis of an approximate work flow time, [5] applied the partial differentiation approach to determine optimal lot sizes for a multi-product manufacturing environment, in order to either minimise the total cost or maximise the operational profit. A more recent research conducted by [6] examined a lot sizing issue with nonlinear production rates in a multi-product single-machine manufacturing firm in Carlisle, where learning effects were allowed, with the aim to minimise the total production cost.

In this paper, we attempt to examine the lot sizing decision-making problem of production planning for stochastic multi-product MTO manufacturing. Different than other studies, our proposed model is characterized by the following three aspects.

### A. Stochastic Multi-product MTO Manufacturing

This paper fixates on stochastic MTO batch manufacturing with unexpected interarrival orders of multiple products and uncertain production machines.

To enhance the generality and preciseness of the proposed model, all random variables involved are characterized by their first and second moments, rather than making any assumptions on their distributions, for such assumptions may be misleading. Currently, it is common to assume the Poisson process for incoming orders, and the negative-exponential distribution for processing times. Reference [7], however, have argued that these factitious assumptions were extremely restrictive and thus unrealistic. This is exactly the main reason that we describe each random variable using their statistic merits.

Although such optimisation is interesting, no exact solution is available for the expected work flow time in this case. As an alternative, approximations are typically applied to deal with this type of production planning [8-10].

### B. Shareholder Wealth Maximisation

Thus far, selection of optimisation objectives seems a bit ill-considered. Most research studies focus on work flow times [3, 11, 12], costs [13, 14], or profits [15-17], with little consideration of the overall operations goal of business owners. Indeed, it is the maximisation of the shareholder wealth and sustaining a steady real cash flow that is the top priority of most enterprises [18-21]. Inappropriate choice in optimisation objectives may not necessarily benefit equity holders, and even undermine their profitability.

Efforts to tackle this problem have been attempted in some studies. Reference [22], for instance, derived a holistic model for short-term supply chain management (SCM), aimed to optimise the change in equity. An integrated lot sizing queuing model for a single-item, single-server case was built to maximise economic value added (EVA) [23], which was one of the popular financial performance metrics to measure the economic interests of business shareholders.

Indeed, benefits of selecting the shareholder wealth as the optimisation objective are huge. For a start, a series of relevant costs are considered, which can thoroughly reflect a firm's cost structure. Second, a deep concern of shareholder wealth is the real purchasing power, rather than the nominal value. Finally, shareholder wealth measures the sustainable economic profitability, adjusted for differences in business sizes.

In this paper, we choose to measure the shareholder wealth using the financial metric—cash flow return on investment (CFROI), due primarily to its economic superiority to other popular metrics, such as net present value (NPV) [24], return on investment (ROI) [25], and EVA [26-28].

### C. Working Capital Management

Another critical concern in our research is on working capital management. Working capital is defined as the difference between current assets and current liabilities. It is the source of short-term capital, and thus represents a business's short-term liquidity. A better liquidity lays the foundation for a firm's sustainable development and stable operations. Efficient management of working capital in production planning is hugely beneficial to profitability [8, 29-32].

This paper examines working capital management in the form of investing or financing operating cash flows, arising from operational activities. Dependent on operating cash flows, only the short-term financial activities are allowable. In other words, a manufacturer is allowed to invest its surplus operating cash flows in financial instruments with higher liquidity, aimed to earn more profits by taking full advantage of time value of money (TVM). When working capital is insufficient for the firm's operations, for example, resulting from business distresses, it is allowed to finance its daily production by short-term financing instruments, such as taking short-term loans from banks, issuing short-term

corporate bonds, and so forth. As a consequence, except the traditional operating activities associated with production planning, we need to take account of two additional financial activities, that is, the financing and investing activities on the operating cash flows.

This paper is organized as follows. In section 2, a stochastic lot sizing multi-product model integrated with the objective function for maximising shareholder wealth under relevant constraint conditions is derived. In section 3, several propositions related to the global optimisation are proved. In section 4, numerical experiments are conducted to validate the proposed model. Section 5 concludes this paper.

## II. MODEL FORMULATION

### A. Problem Description

The concerned manufacturing environment for stochastic MTO production planning is illustrated in Fig.1, where  $N$  types of products are being processed.

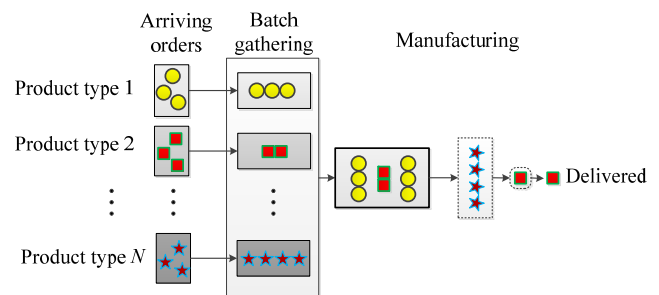


Fig. 1. Multi-product MTO manufacturing.

For all  $N$  types of products, orders randomly arrive at the manufacturer on an individual basis. When each type of these orders accumulate to a batch of lot size  $Q_i$ , where the subscript  $i \in [1, N]$  denotes the specific type of products, they are collected and immediately transferred in batches to queue for the batch setup.

The batch setup is incurred before each batch is processed. Setup times are merely dependent on the product type, without any relationship with either the incoming sequence of batches or lot sizes. Moreover, the setup times are mutually independent.

Subsequently, these partially completed orders are moved to the processing stage for further work on an item-by-item basis, where they are converted into finished products for immediate delivery to end customers one by one, without having to wait until the whole batch is completed.

The term “stochastic” refers to that the interarrival times of orders, setup times, and processing times cannot be predicted with certainty. All working stages are assumed mutually independent. In the event of competition for capacitated resources, orders are assumed to be served in accordance with the first-come-first-served principle. We further assume that each order contains only one product item, and that the prices of all types of products are exogenous.

### B. Shareholder Wealth Formulation

Before examining the effective management of working

capital through short-term financial activities on operating cash flows, we need to consider two additional sources of cash flow—cash flows from either investing or financing activities. The total cash flows are the sum of these three types of cash flows, as in

$$CF_{N,t} = OCF_t + ICF_t + FCF_t, \quad (1)$$

where  $CF_{N,t}$  denotes the total nominal cash flow produced in the  $t^{\text{th}}$  period;  $OCF_t$  is the operating cash flows, arising from operational activities in the  $t^{\text{th}}$  period;  $ICF_t$  and  $FCF_t$  respectively represent the investing and financing cash flows in the  $t^{\text{th}}$  period.

As recommended by [33],  $OCF_t$  equals net income  $NI_t$  plus noncash expenses  $NC_t$ .  $NI_t$  can be computed by subtracting the fixed and variable costs, denoted respectively by  $C_{F,t}$  and  $C_{V,t}$ , from the sales revenue  $R_t$ , that is,

$$OCF_t = NI_t + NC_t = (R_t - C_{F,t} - C_{V,t}) + NC_t \quad (2)$$

where

$$R_t = \sum_{i \in [1,N]} \lambda_{i,t} p_{i,t} \quad (3)$$

$$C_{V,t} = \sum_{i \in [1,N]} \lambda_{i,t} \left( \frac{s_{i,t}}{Q_i} + E(W_{i,t}) h_{i,t}^{WIP} + \omega_{i,t} \right). \quad (4)$$

Here  $\lambda_{i,t}$  represents the interarrival rate for product type  $i$  in the  $t^{\text{th}}$  period and  $p_{i,t}$  is its unit sales price. The symbol  $s_{i,t}$  denotes the unit setup cost, whereas  $h_{i,t}^{WIP}$  is the unit inventory cost of holding WIPs in the  $t^{\text{th}}$  period for product type  $i$ . Other variable costs, such as sales costs, procuring costs, and the like, are independent of lot sizes and thus aggregated as  $\omega_{i,t}$ , referred to as other aggregate unit variable.  $E(W_{i,t})$  is the mean work flow time for product type  $i$ .

The non-cash cost expense  $NC_t$  in the proposed model is merely composed of the depreciation expense of the long-term asset investments. By the straight-line depreciating approach, we have

$$NC_t = \frac{A_D}{L} \quad (5)$$

where  $A_D$  is the initial outlay invested in the long-term assets, which is assumed to be depreciated straight-line to the salvage value of zero during their estimated life  $L$ .

As stated previously, the manufacturer is allowed to adopt a policy of rolling over the surplus operating cash flows through the short-term investing instruments, leading to the following short-term investing return:

$$ICF_t = IR_t \sum_{k=1}^t \max[\max(OCF_k, 0) - \min(OCF_k, 0), 0] \quad (6)$$

where  $\max(OCF_t, 0)$  denotes the positive cash flow from the operating activities in the  $t^{\text{th}}$  period.  $IR_t$  stands for the investing rate of return in the  $t^{\text{th}}$  period.

In a similar fashion, we can estimate the financing cost expense as

$$FCF_t = FR_t \sum_{k=1}^t \min[\max(OCF_k, 0) - \min(OCF_k, 0), 0] \quad (7)$$

where  $\min(OCF_t, 0)$  represents the amount of working capital in shortage aiming at sustaining daily operation activities.  $FR_t$  is the financing cost of capital in the  $t^{\text{th}}$  period.

$CF_{N,t}$  is now a nominal term and has yet to be adjusted for inflation to be eligible for a qualifying input to CFROI. The inflation level for a period is typically estimated as that period's GDP deflator index divided by the GDP deflator index at the beginning of the planning horizon. Then, the real cash flow in this period equals its nominal counterpart divided by its estimated inflation level, as in:

$$CF_{R,t} = \frac{CF_{N,t}}{r_t} = \frac{CF_{N,t}}{GDP_t / GDP_0} \quad (8)$$

where  $GDP_t$  represents the GDP deflator index in the  $t^{\text{th}}$  period and  $r_t$  denotes the inflation rate in this period.

Then, based on the financial definitions of internal rate of return (IRR) and discounted cash flow (DCF), we relate all the relevant input parameters together to derive the shareholder wealth in terms of CFROI [18, 19, 33]:

$$A = \sum_{i=1}^T \frac{CF_{R,i}}{(1+Z)^i} + \frac{A_N}{(1+Z)^T} \quad (9)$$

where  $Z$  denotes the shareholder wealth in terms of CFROI.  $A$  and  $A_N$  respectively refers to the initial investments in the total assets and the non-depreciating assets with the relationship  $A = A_D + A_N$ .  $T$  refers to the time length of the planning horizon.

### III. PROPERTIES OF GLOBAL OPTIMISATION SOLUTION

In the above section, all parameters have been clearly stated except for the expected work flow time  $E(W_{i,t})$ . It is defined in our paper as the time that elapses after an order arrives at the manufacturer and before delivery to customers.

**Proposition 1.**  $E(W_{i,t})$  can be approximated using

$$E(W_{i,t}) = \frac{Q_i - 1}{2\lambda_{i,t}} + \frac{\text{Var}(X_t^b) + \text{Var}(T_t^b)}{2[E(X_t^b) - E(T_t^b)]} + \tau_{i,t} + \frac{Q_i + 1}{2\mu_{i,t}}, \quad (10)$$

subject to

$$\begin{cases} Q_i \geq 1 \\ \rho_i < 100\% \end{cases} \quad \forall i \in [1, N], t \in [1, T]. \quad (11)$$

*Proof.* For orders of each product type  $i, i \in [1, N]$ , once placed, they immediately enter the gathering stage for the gathering service without any delay, leading to

$$E(W_{i,t}^{qs}) = 0, \quad (12)$$

where  $W_{i,t}^{qs}$  represents the queuing time that an individual order of product type  $i$  has to take for the gathering service in the  $t^{\text{th}}$  period.

Using  $X_{i,j,t}$  to denote the interarrival time of the  $j^{\text{th}}$  order of product type  $i$  in the  $t^{\text{th}}$  period, then the gathering time

$W_{i,j,t}^g$  that this order spends in the gathering stage is

$$W_{i,j,t}^g = \sum_{m=j+1}^{Q_i} X_{i,m,t}. \quad (13)$$

Taking expectations on both sides of (13) results in

$$E(W_{i,j,t}^g) = \sum_{m=j+1}^{Q_i} E(X_{i,m,t}) = \frac{Q_i - j}{\lambda_{i,t}}, \quad (14)$$

and then

$$\begin{aligned} E(W_{i,t}^g) &= \sum_{j=1}^{Q_i} E(W_{i,j,t}^g) P\left\{E(W_{i,j,t}^g) = \frac{Q_i - j}{\lambda_{i,t}}\right\} \\ &= \sum_{j=1}^{Q_i} \frac{1}{Q_i} \cdot \frac{Q_i - j}{\lambda_{i,t}} = \frac{Q_i - 1}{2\lambda_{i,t}}. \end{aligned} \quad (15)$$

The symbol  $P(\square)$  represents the probability that the random event encompassed in the brace occurs.  $W_{i,t}^g$  denotes the gathering time that orders of product type  $i$  take to be gathered to a batch of a given lot size  $Q_i$ .

Next, instead of treating setup and processing separately, we perceive them as one integral part, named the batch service stage. Further suppose another situation where completed orders would not individually leave the processing stage to be delivered to customers until the whole batch where it resides has been gathered, as illustrated in Fig.2.

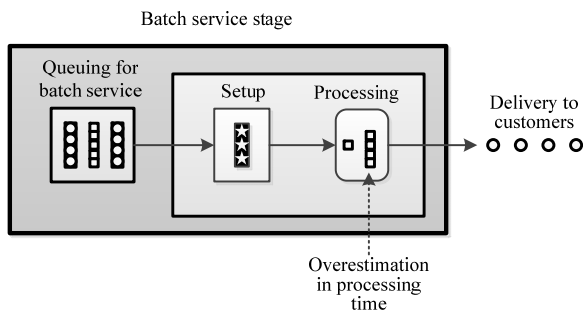


Fig. 2. The supposed batch service stage for multi-product manufacturing.

This artificially supposed scenario is almost identical to the original manufacturing environment except for overestimation of processing times for all orders. Hence, simply subtracting the overestimated time from the work flow time spent in the batch service stage, we can arrive at the setup and processing times specific for our original production environment.

For the batch service stage, the expected batch work flow time  $E(W_{i,t}^{bf})$  can be estimated as the sum of the weighted expected queuing time  $E(W_t^{bq})$  for all  $N$  types of product orders and their expected batch service time  $E(W_{i,t}^{bs})$  [5], as in

$$E(W_{i,t}^{bf}) = E(W_t^{bq}) + E(W_{i,t}^{bs}), \quad (16)$$

where  $E(W_t^{bq})$  can be estimated using the approximation suggested by [10], that is,

$$E(W_t^{bq}) = \frac{Var(X_t^b) + Var(T_t^b)}{2[E(X_t^b) - E(T_t^b)]}, \quad (17)$$

In (17),  $E(X_t^b)$  represents the weighted expected batch interarrival times considering all types of orders, and  $E(T_t^b)$  is the weighted expected batch service time considering all types of product orders.  $Var(X_t^b)$  and  $Var(T_t^b)$  respectively denote their variances.

Since the batch interarrival time  $X_{i,j,t}^b$  of the  $j^{\text{th}}$  batch of product type  $i$  is the time that all orders in this batch need to take to be gathered into a batch of size  $Q_i$ , then

$$X_{i,j,t}^b = \sum_{k=1}^{Q_i} X_{i,(j-1)Q_i+k,t}. \quad (18)$$

As all  $X_{i,j,t}$ 's are independent and identically distributed (*i.i.d.*), where  $X_{i,j,t}$  refers to the interarrival time of the  $j^{\text{th}}$  order of product type  $i$  in the  $t^{\text{th}}$  period, we have

$$\begin{cases} E(X_{i,j,t}^b) = \sum_{k=1}^{Q_i} E(X_{i,(j-1)Q_i+k,t}) = \frac{Q_i}{\lambda_{i,t}} \\ Var(X_{i,j,t}^b) = \sum_{k=1}^{Q_i} Var(X_{i,(j-1)Q_i+k,t}) = Q_i \sigma_{X_{i,t}}^2 \end{cases}. \quad (19)$$

Similarly, the batch service time  $T_{i,j,t}^b$  of the  $j^{\text{th}}$  batch of product type  $i$  is

$$T_{i,j,t}^b = Y_{i,j,t} + \sum_{k=1}^{Q_i} Z_{i,(j-1)Q_i+k,t}. \quad (20)$$

Since  $Y_{i,j,t}$ 's and  $Z_{i,j,t}$ 's are respectively *i.i.d.* for any  $j$  in the  $t^{\text{th}}$  period, it follows

$$\begin{cases} E(T_{i,j,t}^b) = \tau_{i,t} + \frac{Q_i}{\mu_{i,t}} \\ Var(T_{i,j,t}^b) = \sigma_{Y_{i,t}}^2 + Q_i \sigma_{Z_{i,t}}^2 \end{cases} \quad (21)$$

where  $Y_{i,j,t}$  represents the batch setup time of the  $j^{\text{th}}$  batch of orders for product type  $i$ , and  $Z_{i,j,t}$  denotes the processing time of the  $j^{\text{th}}$  order of product type  $i$ .

Equations (19)-(21) holds for all  $j \geq 1$ , implying that the expected times and variances of the interarrival batch times and batch service times are independent of  $j$ . Thus, we can ignore the subscript  $j$  without giving rise to any ambiguity. Then, considering all  $N$  types of customer orders, we get

$$E(X_t^b) = 1 / \sum_{i=1}^N \frac{\lambda_{i,t}}{Q_i}, \quad (22)$$

$$E(T_t^b) = \frac{\sum_{i=1}^N \frac{\lambda_{i,t}}{Q_i} E(T_{i,t}^b)}{\sum_{i=1}^N \frac{\lambda_{i,t}}{Q_i}} = \frac{\sum_{i=1}^N \frac{\lambda_{i,t}}{Q_i} \left( \tau_{i,t} + \frac{Q_i}{\mu_{i,t}} \right)}{\sum_{i=1}^N \frac{\lambda_{i,t}}{Q_i}}, \quad (23)$$

$$Var(X_{i,t}^b) = \frac{\sum_{i=1}^N \lambda_{i,t} Var(X_{i,t}^b)}{\sum_{i=1}^N \lambda_{i,t}} = \frac{\sum_{i=1}^N \lambda_{i,t} \sigma_{X_{i,t}^b}^2}{\sum_{i=1}^N \lambda_{i,t}}, \quad (24)$$

$$Var(T_t^b) = \frac{\sum_{i=1}^N \lambda_{i,t} Var(T_{i,t}^b)}{\sum_{i=1}^N \lambda_{i,t}} = \frac{\sum_{i=1}^N \lambda_{i,t} (\sigma_{T_{i,t}^b}^2 + Q_i \sigma_{Z_{i,t}}^2)}{\sum_{i=1}^N \lambda_{i,t}}. \quad (25)$$

Based on (22) and (23), we get the traffic intensity or the resource utilization rate as

$$\rho_t = \frac{E(T_t^b)}{E(X_t^b)} = \sum_{i=1}^N \lambda_{i,t} \left( \tau_{i,t} + \frac{Q_{i,t}}{\mu_{i,t}} \right). \quad (26)$$

For orders of product type  $i$ , we readily get the batch service time

$$E(T_{i,t}^b) = \tau_{i,t} + \frac{Q_i}{\mu_{i,t}}. \quad (27)$$

$E(T_i^b)$  is not for our original manufacturing environment.

We have to subtract the overestimated time from it to arrive at the batch service time specific for our proposed model.

Under the batch service scenario, after the processing service is finished, the  $j^{\text{th}}$  order of product type  $i$  has to wait until the remaining  $Q_i - j$  orders are processed one by one, and thus its processing time is overestimated by

$$O_{i,j,t} = \sum_{k=j+1}^{Q_i} Z_{i,k,t}, \quad (28)$$

where  $O_{i,j,t}$  represents the overestimated processing time for the  $j^{\text{th}}$  order of product type  $i$ . Thus,

$$E(O_{i,j,t}) = \sum_{k=j+1}^{Q_i} E(Z_{i,k,t}) = \frac{Q_i - j}{\mu_{i,t}}, \quad (29)$$

and hence,

$$\begin{aligned} E(O_{i,t}) &= \sum_{j \in [1, Q_i]} E(O_{i,j,t}) P \left( E(O_{i,j,t}) = \frac{Q_i - j}{\mu_{i,t}} \right) \\ &= \sum_{j \in [1, Q_i]} \frac{Q_i - j}{\mu_{i,t}} \frac{1}{Q_i} = \frac{Q_i - 1}{2\mu_{i,t}}. \end{aligned} \quad (30)$$

As a consequence, the expected batch service time specific for our proposed manufacturing environment is

$$\begin{aligned} E(W_{i,t}^{bs}) &= E(T_{i,t}^b) - E(O_{i,t}) \\ &= \left( \tau_{i,t} + \frac{Q_i}{\mu_{i,t}} \right) - \frac{Q_i - 1}{2\mu_{i,t}} \\ &= \tau_{i,t} + \frac{Q_i + 1}{2\mu_{i,t}}. \end{aligned} \quad (31)$$

Substituting the above equations into (16), we can figure out the expected batch work flow time, as in

$$E(W_{i,t}^{bf}) = \frac{Var(X_{i,t}^b) + Var(T_{i,t}^b)}{2[E(X_{i,t}^b) - E(T_{i,t}^b)]} + \tau_{i,t} + \frac{Q_i + 1}{2\mu_{i,t}}. \quad (32)$$

Hence,

$$\begin{aligned} E(W_{i,t}) &= E(W_{i,t}^{qs}) + E(W_{i,t}^g) + E(W_{i,t}^{bf}) \\ &= \frac{Q_i - 1}{2\lambda_{i,t}} + \frac{Var(X_{i,t}^b) + Var(T_{i,t}^b)}{2[E(X_{i,t}^b) - E(T_{i,t}^b)]} + \tau_{i,t} + \frac{Q_i + 1}{2\mu_{i,t}}. \end{aligned} \quad (33)$$

Further considering the constraints on the lot size and utilization rate, that is

$$\begin{cases} Q_i \geq 1 \\ \rho_t < 100\% \end{cases} \quad \forall i \in [1, N], t \in [1, T], \quad (34)$$

for, in any case, lot sizes should not be less than one, and a realistic queuing system must have the traffic intensity less than 100%. □

Then we attempt to demonstrate the effectiveness of our proposed approach in achieving the global solution under the single-period, single-product production environment, that is,  $T = 1$  and  $N = 1$ . In this case, (10) can be transformed to

$$E(W) = \frac{Q-1}{2\lambda} + \frac{Q(\sigma_x^2 + \sigma_z^2) + \sigma_y^2}{2[(\frac{1}{\lambda} - \frac{1}{\mu})Q - \tau]} + \tau + \frac{Q+1}{2\mu}, \quad (35)$$

where the subscripts  $i$  and  $t$  are intentionally ignored, for their omissions in this case can facilitate our derivation but giving rise to no ambiguity.

**Proposition 2.** Under the single-period, single-product environment,  $E(W)$  is convex for  $Q \in \text{dom } E(W)$ , where  $\text{dom } E(W)$  defines the domain of  $E(W)$ .

*Proof.* Divide  $E(W)$  into two components  $f$  and  $g$  with  $E(W) = f + g$ , where

$$\begin{cases} f = [Q(\sigma_x^2 + \sigma_z^2) + \sigma_y^2] / 2 \left[ Q \left( \frac{1}{\lambda} - \frac{1}{\mu} \right) - \tau \right] \\ \phi = \frac{Q-1}{2\lambda} + \frac{Q+1}{2\mu} + \tau \end{cases} \quad (36)$$

$\forall Q^{(1)}, Q^{(2)} \in \text{dom } E(W)$ , we can get

$$\begin{aligned} &f(Q^{(2)}) - f(Q^{(1)}) - \nabla f(Q^{(1)})^T (Q^{(2)} - Q^{(1)}) \\ &= \frac{\left[ \sigma_y^2 \left( \frac{1}{\lambda} - \frac{1}{\mu} \right) + \tau(\sigma_x^2 + \sigma_z^2) \right] (Q^{(1)} - Q^{(2)})^2}{2 \left[ Q^{(1)} \left( \frac{1}{\lambda} - \frac{1}{\mu} \right) - \tau \right]^2 \left[ Q^{(2)} \left( \frac{1}{\lambda} - \frac{1}{\mu} \right) - \tau \right]}. \end{aligned} \quad (37)$$

From  $\rho < 100\%$  it follows that  $\forall Q \in E(W)$ ,  $\frac{1}{\lambda} - \frac{1}{\mu} > 0$

and  $\left( \frac{1}{\lambda} - \frac{1}{\mu} \right) Q - \tau > 0$ . As a consequence, the right-hand side of (37) has to be larger than or equal to zero, that is,

$$f(Q^{(2)}) \geq f(Q^{(1)}) + \nabla f(Q^{(1)})^T (Q^{(2)} - Q^{(1)}), \quad (38)$$

satisfying the first-order convexity condition [34].

In contrast,  $g$  is a linear function in terms of  $Q$ , and thus we can readily prove that for all  $Q^{(1)}, Q^{(2)} \in \text{dom } g = \text{dom } E(W)$  and  $\theta$  with  $0 \leq \theta \leq 1$ ,

$$g(\theta Q^{(1)} + (1-\theta)Q^{(2)}) \leq \theta g(Q^{(1)}) + (1-\theta)g(Q^{(2)}). \quad (39)$$

Thus,  $\phi$  is also a convex function in its domain, for the above equation is the definition of convexity [34].

The sum of convex functions remains convex. Hence we can get the conclusion summarized in Proposition 2.  $\square$

**Proposition 3.** Under the single-period, single-product environment, the global optimisation result to minimise the expected work flow time can be achieved if and only if

$$Q^* = \frac{1}{1/\lambda - 1/\mu} \left[ \sqrt{\frac{[(\sigma_x^2 + \sigma_z^2)\tau + \sigma_y^2(1/\lambda - 1/\mu)]}{1/\lambda + 1/\mu}} + \tau \right]. \quad (40)$$

*Proof.* The continuity and differentiability of  $E(W)$  in terms of  $Q$  imply that its global optimisation result has to be one of its stationary points, i.e. meeting the following first-order condition:

$$\frac{dE(W)}{dQ} = -\frac{(\sigma_x^2 + \sigma_z^2)\tau + \sigma_y^2 \left(\frac{1}{\lambda} - \frac{1}{\mu}\right)}{2 \left[ Q \left(\frac{1}{\lambda} - \frac{1}{\mu}\right) - \tau \right]^2} + \left(\frac{1}{2\mu} + \frac{1}{2\lambda}\right) = 0, \quad (41)$$

which can be further transformed into

$$\left[ Q \left(\frac{1}{\lambda} - \frac{1}{\mu}\right) - \tau \right]^2 = \frac{(\sigma_x^2 + \sigma_z^2)\tau + \sigma_y^2 \left(\frac{1}{\lambda} - \frac{1}{\mu}\right)}{\frac{1}{\lambda} + \frac{1}{\mu}}. \quad (42)$$

In proof of Proposition 2, we have demonstrated that  $\forall Q \in E(W)$ ,  $Q \left(\frac{1}{\lambda} - \frac{1}{\mu}\right) - \tau > 0$ , thus

$$Q = \frac{1}{1/\lambda - 1/\mu} \left[ \sqrt{\frac{(\sigma_x^2 + \sigma_z^2)\tau + \sigma_y^2(1/\lambda - 1/\mu)}{1/\lambda + 1/\mu}} + \tau \right]. \quad (43)$$

As a consequence, here exists only one stationary point for  $E(W)$ . Further considering its convexity, we can infer that it has to be the optimal lot size to globally minimise  $E(W)$ .  $\square$

Similar to the work flow time, for the single-period, single-product production environment, the shareholder wealth can be summarized as

$$Z = \frac{CF_R + A_N}{A} - 1. \quad (44)$$

The following two propositions states the characteristics related to its global optimisation solution.

**Proposition 4.** Under the single-period, single-product environment, the shareholder wealth is a concave function in terms of  $Q$  for all  $Q \in \text{dom } Z$ , where  $\text{dom } Z$  defines its domain.

*Proof.* Substituting relevant equations into (44) yields

$$Z = \frac{GDP_0}{A \times GDP_1} (OCF + ICF + FCF) + \frac{A_N}{A} - 1. \quad (45)$$

Let  $\varphi_1 = \frac{GDP_0}{A \times GDP_1} OCF$ ,  $\varphi_2 = \frac{GDP_0}{A \times GDP_1} ICF$ , and

$\varphi_3 = \frac{GDP_0}{A \times GDP_1} FCF$ . Then, in order to prove the concavity of  $Z$ , we only need to prove that  $\varphi_1$  is concave. The reasoning is as follows. Based on the proposed management strategy of working capital,  $\varphi_2$  is adjusted for a multiplier factor when  $\varphi_1 > 0$ , and otherwise zero. In contrast,  $\varphi_3$  is the result adjusted for the financing cost when  $\varphi_1 < 0$ , and otherwise zero. So if  $\varphi_1$  is a concave function, both  $\varphi_2$  and  $\varphi_3$  have to be concave. The sum of three concave functions remains concave.

The concavity proof of  $\varphi_1$  constitutes to prove its negative is convex. Denote its negative by  $\bar{\varphi}_1 = -\varphi_1 = \psi_1 + \psi_2 + \psi_3$ , where

$$\begin{cases} \psi_1 = \frac{\lambda h^{WIP} GDP_0}{AGDP_1} \frac{Q(\sigma_x^2 + \sigma_z^2) + \sigma_y^2}{2[(1/\lambda - 1/\mu) - \tau]} \\ \psi_2 = \frac{\lambda s GDP_0}{AGDP_1} \frac{1}{Q} \\ \psi_3 = \left[ \frac{h^{WIP}(Q-1)}{2} + \frac{\lambda h^{WIP}(Q+1)}{2\mu} \right] \frac{GDP_0}{AGDP_1} \end{cases} \quad (46)$$

We can easily test that  $\psi_1$  and  $\psi_2$  satisfy the first-order convexity condition [34], i.e.,  $\forall Q^{(1)}, Q^{(2)} \in \text{dom } Z$ ,

$$\begin{cases} \psi_1(Q^{(2)}) \geq \psi_1(Q^{(1)}) + \nabla \psi_1(Q^{(1)})^T (Q^{(2)} - Q^{(1)}) \\ \psi_2(Q^{(2)}) \geq \psi_2(Q^{(1)}) + \nabla \psi_2(Q^{(1)})^T (Q^{(2)} - Q^{(1)}) \end{cases} \quad (47)$$

Moreover, for all  $\forall Q^{(1)}, Q^{(2)} \in \text{dom } Z$  and  $\theta \in [0,1]$ , it is easy to prove

$$\psi_3(\theta Q^{(1)} + (1-\theta)Q^{(2)}) \leq \theta \psi_3(Q^{(1)}) + (1-\theta)\psi_3(Q^{(2)}), \quad (48)$$

meaning  $\psi_3$  is also convex in its domain.

As a consequence,  $\bar{\varphi}_1$  is proven to be convex, and thus its negative  $\varphi_1$  is concave in its domain, which completes the proof of the proposition.  $\square$

**Proposition 5.** Under the single-period, single-product environment, the global optimisation result for maximisation of the shareholder wealth has to meet the following quartic relationship with one unknown

$$A^2 B h^{WIP} Q^4 - 2 A B \tau h^{WIP} Q^3 + C Q^2 + 4 A s \tau Q - 2 s \tau^2 = 0, \quad (49)$$

where

$$A = 1/\lambda - 1/\mu,$$

$$B = 1/\lambda + 1/\mu,$$

$$C = B \tau^2 h^{WIP} - 2 s A^2 - [(\sigma_x^2 + \sigma_z^2)\tau + \sigma_y^2 A] h^{WIP}.$$

*Proof.* The shareholder wealth  $Z$  is continuous and differentiable in terms of lot sizes in its domain. Thus, its optimal lot size has to meet the first-order condition:

$$\frac{dZ}{dQ} = \frac{GDP_0}{A \times GDP_1} \frac{d}{dQ} (OCF + ICF + FCF) = 0. \quad (50)$$

Simplifying (50) in the case of either  $OCF \geq 0$  or  $OCF < 0$  can yield the same result, as in

$$\frac{dOCF}{dQ} = -\lambda \left( -\frac{s}{Q^2} + \frac{dE(W)}{dQ} h^{WIP} \right) = 0. \quad (51)$$

Then, substitute (41) into the above relationship, we can get the necessary condition for the global optimisation result of the shareholder wealth maximisation, as presented in this proposition.  $\square$

Now we turn to the multi-product case for the single-period planning horizon. By differentiating the work flow time and the shareholder wealth with respect to all types of lot sizes and then setting them equal to 0, we can obtain the necessary conditions of the global optimisation solutions in this case, respectively for the work flow time minimisation and the shareholder wealth.

**Proposition 6.** Under the multi-product, single-period environment, the optimal lot sizes to minimise the total expected work flow time has to meet the following condition, i.e.,  $\forall j \in [1, N]$ ,

$$\frac{1}{N} \left( \frac{1}{\lambda_j} + \frac{1}{\mu_j} \right) = \frac{\lambda_j \sigma_{Y_j}^2 \left[ \left( 1 - \sum_{i=1}^N \frac{\lambda_i}{\mu_i} - \sum_{i=1}^N \frac{\lambda_i \tau_i}{Q_i} \right) + \lambda_j \tau_j \left[ \sum_{i=1}^N \lambda_i (\sigma_{X_i}^2 + \sigma_{Z_i}^2) + \sum_{i=1}^N \frac{\lambda_i \sigma_{Y_i}^2}{Q_i} \right] \right]}{Q_j^2 \left[ \left( 1 - \sum_{i=1}^N \frac{\lambda_i}{\mu_i} - \sum_{i=1}^N \frac{\lambda_i \tau_i}{Q_i} \right)^2 \right]} \quad (52)$$

*Proof.* For the multi-product production case, we define the total expected work flow time as the average of the expected lead times for each product type, i.e.

$$T(\bar{Q}) = \frac{1}{N} \sum_{i=1}^N E(W_i). \quad (53)$$

Partially differentiating (53) with respect  $\bar{Q} = (Q_1, Q_2, \dots, Q_N)$  results in

$$\frac{\partial T(\bar{Q})}{\partial Q_j} = \frac{1}{2} \left( \frac{1}{\lambda_j} + \frac{1}{\mu_j} \right) - \frac{N \lambda_j \sigma_{Y_j}^2 \left[ \left( 1 - \sum_{i=1}^N \frac{\lambda_i}{\mu_i} - \sum_{i=1}^N \frac{\lambda_i \tau_i}{Q_i} \right) + \lambda_j \tau_j \left[ \sum_{i=1}^N \lambda_i (\sigma_{X_i}^2 + \sigma_{Z_i}^2) + \sum_{i=1}^N \frac{\lambda_i \sigma_{Y_i}^2}{Q_i} \right] \right]}{2 Q_j^2 \left[ \left( 1 - \sum_{i=1}^N \frac{\lambda_i}{\mu_i} - \sum_{i=1}^N \frac{\lambda_i \tau_i}{Q_i} \right)^2 \right]} \quad (54)$$

The proposition can be proved by setting (54) to zero.  $\square$

**Proposition 7.** Under the multi-product, single-period environment, the optimal lot sizes to maximise the shareholder wealth has to satisfying the following condition, i.e.,  $\forall j \in [1, N]$ ,

$$-\frac{s_j}{Q_j^2} + \frac{\partial E(W_j)}{\partial Q_j} h_j^{WIP} = 0 \quad (55)$$

where

$$\frac{\partial E(W_j)}{\partial Q_j} = \frac{1}{2\lambda_j} + \frac{1}{2\mu_j} - \frac{\lambda_j \sigma_{Y_j}^2 \left[ \left( 1 - \sum_{i=1}^N \frac{\lambda_i}{\mu_i} - \sum_{i=1}^N \frac{\lambda_i \tau_i}{Q_i} \right) + \lambda_j \tau_j \left[ \sum_{i=1}^N \lambda_i (\sigma_{X_i}^2 + \sigma_{Z_i}^2) + \sum_{i=1}^N \frac{\lambda_i \sigma_{Y_i}^2}{Q_i} \right] \right]}{Q_j^2 \left[ \left( 1 - \sum_{i=1}^N \frac{\lambda_i}{\mu_i} - \sum_{i=1}^N \frac{\lambda_i \tau_i}{Q_i} \right)^2 \right]} \quad (56)$$

*Proof.* First, partially differentiating the shareholder wealth with respect to  $\bar{Q} = (Q_1, Q_2, \dots, Q_N)$  and let it equal zero, as in

$$\frac{\partial Z}{\partial Q_j} = 0. \quad (57)$$

As illustrated in Proposition 5, it follows that  $\frac{\partial OCF}{\partial Q_j} = 0$ .

The proposition can be proven by rearranging this equation.  $\square$

#### IV. NUMERICAL STUDIES

This section applies the above propositions to validate the proposed model, and then analytically compares the optimisation results.

##### A. Single-product Production Planning

We start with a simple example where only one type of product is produced. The operational data can be achieved from a pioneering research study conducted by [21]. The remaining economic parameters can be obtained from a firm's managerial accounting system (Garrison and Noreen 2000). Their values are as follows  $T = 1$  year,  $N = 2$ ,  $L = 5$  year,  $\lambda = 1$ ,  $\tau = 10$  minutes,  $\mu = 2$ ,  $\sigma_X^2 = 0.5$ ,  $\sigma_Y^2 = 10$ ,  $\sigma_Z^2 = 0.0625$ ,  $p = \$200$ ,  $s = \$1500$ ,  $h^{WIP} = \$1$ ,  $\omega = \$2.5$ ,  $A = \$10$  million,  $A_N = \$2$  million,  $C_F = \$1.5$  million.

##### Global Optimisation Solution

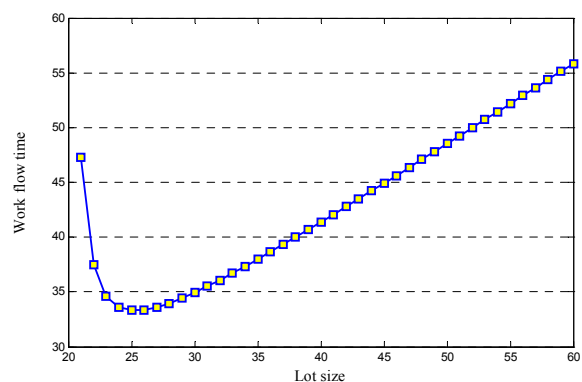


Fig. 3. Changes of work flow time with respect to lot size.

To further provide a clear picture of the convexity of the work flow time, which has been proven in proposition 1, its changes with respect to the lot size is graphed in Fig.3.

Then, substituting the above parameter values into proposition 3, we can figure out the optimal lot size to globally minimise the work flow time, as in:

$$Q^* = \frac{1}{(1-0.5)} \left( \sqrt{\frac{(0.5+0.0625) \times 10 + (1-0.5) \times 10}{(1+0.5)}} + 10 \right) \quad (58)$$

$$= 25.3229$$

with the minimum work flow time of 33.2969 minutes, corresponding to a shareholder wealth of  $Z(Q^*) = 39.41\%$ .

In a similar fashion, the concavity of CFROI with respect to  $Q$ , which proof has been given in proposition 4, is represented in Fig.4.

Now we demonstrate how to obtain the optimal optimisation result for the shareholder wealth maximisation using propositions 4 and 5. First substituting the parameter values into the necessary condition (49) yields

$$0.3750Q^4 - 15Q^3 - 610.6250Q^2 + 30000Q - 300000 = 0 \quad (59)$$

with four solutions to this equation, that is, -44.8727, 45.7117, 19.5805+2.5727i, and 19.5805-2.5727i. Considering the constraint condition, we see that only 45.7117 lies in the feasible region  $\{Q | Q > 20\}$ , that is, it is the only one stationary point of CFROI with respect to  $Q$ . Since CFROI is concave in its domain, we can infer that 45.7117 is the optimal lot size to maximise the shareholder wealth, with the maximum CFROI of 57.78%.

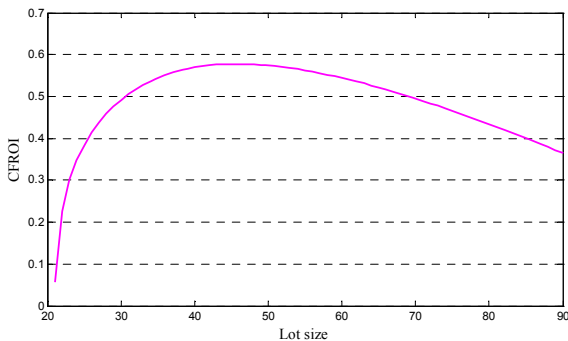


Fig. 4. Changes of shareholder wealth with respect to lot size.

**Effects of Working Capital Management**

We have demonstrated that those proven propositions can effectively figure out the global optimisation problems for both the work flow time minimisation and the shareholder wealth maximisation.

This section applies these approaches to explore how the management of working capital can affect the interests of a firm’s shareholders.

Table I lists the optimisation results for a series of different financial environments. It can be seen that irrespective of work flow time minimisation or shareholder wealth maximisation, working capital management imposes no impact on the optimal lot sizes, which remains unchanged respectively at 25.3229 and 45.7117 for these two optimisation cases when the financing environment changes.

Moreover, as illustrated in Proposition 1, the work flow time depends exclusively on the operational parameters and lot size. Thus, we can conclude that the work flow times under the global optimisation conditions also remains constant for these two optimisation cases, as illustrated in the third and the sixth columns of Table I.

TABLE I  
IMPACTS OF WORKING CAPITAL MANAGEMENT ON OPTIMISATION UNDER SINGLE-PRODUCT PRODUCTION

IR, FR	Work flow time minimisation			Shareholder wealth maximisation		
	Q	E(W)	Z	Q	E(W)	Z
0	25.3229	33.2969	33.73%	45.7117	45.4227	51.22%
1%	25.3229	33.2969	34.86%	45.7117	45.4227	52.53%
2%	25.3229	33.2969	36.00%	45.7117	45.4227	53.84%
5%	25.3229	33.2969	39.41%	45.7117	45.4227	57.78%
10%	25.3229	33.2969	45.10%	45.7117	45.4227	64.34%
15%	25.3229	33.2969	50.78%	45.7117	45.4227	70.90%

Although impacts of working capital on the optimal lot sizes and the work flow times are ineffectual, its impacts on the shareholder wealth are significant, as represented in Fig.5. It can be observed that the higher the investing rate and the financing cost, the more evident the impacts of effective management of working capital on shareholder wealth

Also, in contrast with the work flow time minimisation, the optimal lot size under the shareholder wealth optimisation shifts up from 33.3229 to 45.7117, leading to an apparent increase of shareholder wealth for each level (for example a rise from 45.10% to 64.34% when  $IR = FR = 10\%$ ) in spite of a larger increase in the work flow time. This result unveils the misalignment between the work flow time optimisation and the overall business objective of maximising the shareholder wealth. It highlights the significance of the overall business optimisation objective to production planning.

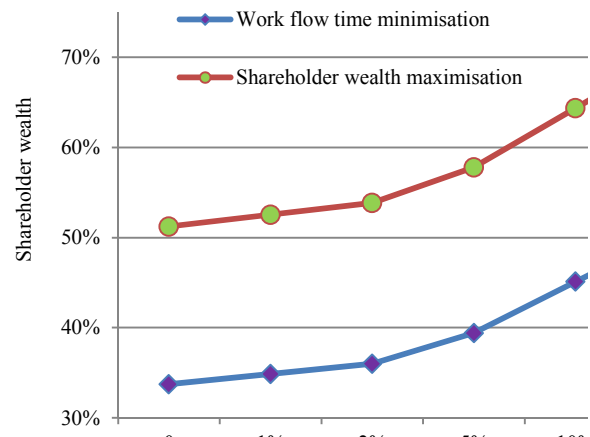


Fig. 5. Impacts of working capital management on CFROI under various financial environments.

Such optimisation results reflect some interesting managerial logics in production planning. First, corporate management determines its optimal lot size based on market demand and its capacity without considering the financing and investing circumstance at that time. This is the main reason that the optimal lot sizes for both optimisation cases are independent on the investing rate and financing cost. Then, based on cash flows arising from operations, executive effectively manages these current assets and liabilities through short-term financial instruments, leading to the increase of shareholder wealth. This exactly explains why the working capital management plays a critical role in shareholder wealth creation but has no effect on the optimal production plans.



**B. Multi-product Production Planning**

This section is intended to validate the efficiency of the proposed model in dealing with multi-product production planning.

Let  $T = 1$ ,  $N = 2$ ,  $L = 5$ ,  $\lambda_1 = 2.5$ ,  $\lambda_2 = 2$ ,  $\tau_1 = 4$  minutes,  $\tau_2 = 3$  minutes,  $\mu_1 = 10$ ,  $\mu_2 = 5$ ,  $\sigma_{x_1}^2 = 1$ ,  $\sigma_{x_2}^2 = 2$ ,  $\sigma_{y_1}^2 = 7$ ,  $\sigma_{y_2}^2 = 3$ ,  $\sigma_{z_1}^2 = 0.6$ ,  $\sigma_{z_2}^2 = 1$ ,  $p_1 = \$150$ ,  $p_2 = \$160$ ,  $s_1 = \$2k$ ,  $s_2 = \$2.5k$ ,  $h_1^{WIP} = \$2.5$ ,  $h_2^{WIP} = \$3.5$ ,  $\omega_1 = \$5$ ,  $\omega_2 = \$4$ ,  $A = \$10\text{million}$ ,  $A_N = \$2\text{million}$ , and  $C_F = \$5\text{million}$ .

Using the pattern search algorithm, we can estimate the optimisation results for both work flow time minimisation and shareholder wealth maximisation under a series of different financial market environments, as listed in Table II.

It can be proved that all these optimal lot sizes meet the necessary conditions for the global optimisation solutions stated in Propositions 6 and 7, validating the critical importance of these two propositions in solving for optimisation results.

We can observe that, irrespective of the work flow time minimisation or shareholder wealth maximisation, efficient management in working capital can create more shareholder wealth or corporate value.

Moreover, the higher investing rates of return and the financing cost, the more effective the working capital management with much higher shareholder wealth. This result is clearly reflected in Fig.6.

Despite the positive role of the working capital management in shareholder wealth creation, its impacts on the optimal lot sizes and work flow times is insignificant.

This works for both the work flow time minimisation and shareholder wealth maximisation, as represented in the columns 2, 3, 5, and 6 of Table II.

Under the work flow time minimisation, for example, the optimal lot sizes remain unchanged at  $\bar{Q} = (32.4331, 23.7122)$  with a constant minimum work flow time of 47.2183 minutes when  $IR$  and  $FR$  increases from 0% to 15%. Similarly, the optimal lot sizes stay constant at  $\bar{Q} = (36.4297, 29.2086)$  with an unchanged work flow time of 47.4834 minutes for maximisation of shareholder wealth when  $IR$  and  $FR$  change.

As illustrated in Table II and Fig. 6, comparison of these two optimisation approaches further highlights the significant importance of shareholder wealth maximisation to equity holders.

The work flow time minimisation weakens the corporate capability of creating shareholder wealth with shareholder wealth dropping from 94.03% to 86.92% with  $IR = FR = 10\%$ , by focusing exclusively on the local optimisation objective—the minimisation of work flow time.

This observation suggests that corporate executives should concentrate on the overall business objective of maximising the shareholder wealth in production planning, rather than merely on local optimisation objectives, so as to maximise the full interests of a firm’s equity holders.

TABLE II  
IMPACTS OF WORKING CAPITAL MANAGEMENT ON OPTIMISATIONS UNDER MULTI-PRODUCT PRODUCTION

IR, FR	Work flow time minimisation			Shareholder maximisation		
	$\bar{Q}$	$E(W)$	Z	$\bar{Q}$	$E(W)$	Z
0	(32.4331, 23.7122)	47.2183	71.75%	(36.4297, 29.2086)	47.4834	78.20%
1%	(32.4331, 23.7122)	47.2183	73.27%	(36.4297, 29.2086)	47.4834	79.79%
2%	(32.4331, 23.7122)	47.2183	74.78%	(36.4297, 29.2086)	47.4834	81.37%
5%	(32.4331, 23.7122)	47.2183	79.34%	(36.4297, 29.2086)	47.4834	86.12%
10%	(32.4331, 23.7122)	47.2183	86.92%	(36.4297, 29.2086)	47.4834	94.03%
15%	(32.4331, 23.7122)	47.2183	94.51%	(36.4297, 29.2086)	47.4834	101.94%

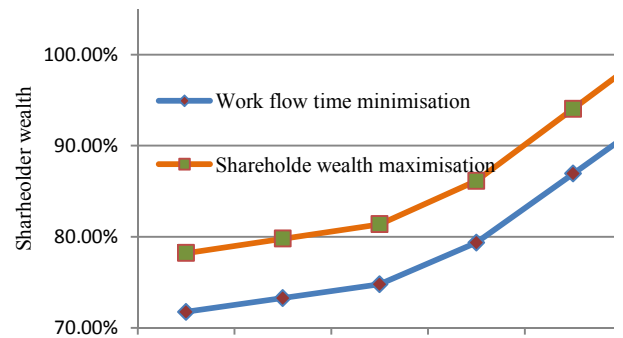


Fig.6. Impacts of working capital management on CFROI under multi-product production environment.

**V. CONCLUSION**

In this paper, we propose a single-period multi-product stochastic lot sizing optimisation model for enhancing the sustainable long-term performance of a MTO production firm under uncertainties.

The proposed model adopts general distributions for all stochastic variables involved, instead of the traditional theoretical distribution assumptions such as the Poisson process for the interarrival of customer orders, to improve its generality and extensibility for dealing with multi-product lot sizing in more realistic manufacturing scenarios. Most importantly, the model choose to optimise the sustainable long-term profitability of a firm in terms of CFROI, which is considered a relevant financial metric that can better reflect the firm’s overall business goal and hence the full interest of its equity holders. Moreover, impacts of working capital management on shareholder wealth are carefully examined to explore its significant role in production planning.

We prove some relevant propositions pertinent to the convexity or concavity of the optimisation objectives, and give their global optimisation results. These propositions provide theoretical solutions to our proposed production model, compared with the analytical optimisation results in numerical experiments.

Numerical experiments reveal the considerable spread of optimisation between the traditional operation optimisation and the proposed shareholder wealth maximisation model. This highlights the importance of taking financial and economic factors into account for production optimisation. It is also found that the effective management of working

capital is very necessary, even as important as production optimisation of operational procedures, in promoting the shareholder wealth, although its effects on the optimal lot sizes and work flow time are indifferent. Hence, in addition to the traditional short-term operational objectives, a firm should pay more attention to the full interest of its equity holders—the global long-term business goal, as well as its effectiveness in working capital management. This provides a practical guidance on the use of cash flows from operations, and highlights the importance of cash reinvestment in advancing corporate performances.

Now we are considering several possible extensions to the proposed model. For example, the model may be extended to cope with a multi-product, multi-period manufacturing environment. A multi-stage stochastic programming may be adopted as a more practical tool in line with periodic accounting purposes. Also, we are trying to examine the influences of the carbon footprint management on the shareholder wealth.

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