Designing Mamdani-Type Fuzzy Reasoning for Visualizing Prediction Problems Based on Collaborative Fuzzy Clustering

Mukesh Prasad, Dong-Lin Li, Chin-Teng Lin, Shiv Prakash, Jagendra Singh, Sudhanshu Joshi

Abstract—In this paper a collaborative fuzzy c-means (CFCM) is used to generate fuzzy rules for fuzzy inference systems to evaluate the time series model. CFCM helps system to integrate two or more different datasets having similar features which are collected at the different environment with the different time period and it integrates these datasets together in order to visualize some common patterns among the datasets. In order to do any mode of integration between datasets, there is a necessity to define the common features between datasets by using some kind of collaborative process and also need to preserve the privacy and security at higher levels. This collaboration process gives a common structure between datasets which helps to define an appropriate number of rules for structural learning and also improve the accuracy of the system modeling.

Index Terms—Privacy and security, Fuzzy inference system, Collaborative fuzzy c-mean, Structure learning, Fuzzy c-means.

I. INTRODUCTION

In recent years, fuzzy rule based modeling systems are extensively used in various fields such as biological, geophysical and engineering systems. Fuzzy expert system is a combination of rules and membership function, which is generated by fuzzy c-means (FCM) clustering or some other clustering methodology. In this study collaborative fuzzy clustering technique is used to generate a number of rules and calculate membership function. Fuzzy c-means clustering is proposed by Benzek [1, 2] and it is modified by time to time and has been used in different application in the different real world problems. There are numerous modifications and several appealing clustering methods have been proposed in recent years for time series prediction models [23, 25] and fuzzy inference systems [22, 24]. The main objective of fuzzy clustering is to assure that it operates not only on data but takes full advantage of various sources of knowledge which comes from different patterns of available data when dealing with the problem at hand.

A group of patterns in different fields has a different variety of information. To get a comprehensive study of these patterns, knowledge based clustering [3] is recommended and collaborative clustering [4] between datasets is introduced by Pedrycz. Because of data confidentiality and some security problem, we cannot access information of data directly from the dataset. To deal with this kind of problem Pedrycz introduced collaborative clustering [4, 5]. In this clustering algorithm, several subsets of patterns can be processed together with an objective of finding a structure that is common to all of them. The main objective of this study is to introduce and explore various scenarios where knowledge could be seamlessly included in the algorithmic architecture of fuzzy clustering for generating a collaborative rule based system, which is very useful to decide an accurate model for the system and give an efficient tool for data analysis for understanding and visualizing data for structural modeling.

The proposed model divides the given datasets into two equal parts and applies FCM on each of dataset in order to calculate the centroid and partition matrix. Further, CFCM uses this information (centroid and partition matrix) to interact within two different datasets in order to extract the common features among them. The extracted common feature of one of the dataset is used to evaluate the performance of the proposed model. So, the proposed model uses only half of the patterns of given dataset to find out the accurate and stable system model, while Genfis3 model [7] uses all patterns of the dataset. Genfis3 generates a fuzzy inference system (FIS) using FCM clustering on given separate sets of input and output dataset. Genfis3 accomplishes this by extracting a set of rules that model the data behavior. The rule extraction method first uses the FCM function to determine the number of rules and membership functions for the antecedents and consequents. The experimental results show that the proposed model is superior to Genfis3 on given sets of problems.

The rest of the paper is organized as follows: Section II gives a simple introduction of FCM, collaborative fuzzy clustering along and flow diagram of the proposed model. Section III shows the experimental results on Mackey glass time series data [13, 14, 22] and finally the conclusions are
covered in Section IV.

II. PROPOSED SYSTEM

A. Fuzzy C-Means

Fuzzy c-means (FCM) was originally introduced by Bezdek in 1981. FCM is a data clustering technique which allows each data point belongs to one or more clusters that is specified by a membership function. FCM performs the clustering which is based on minimization of the objective function given by Eq. (1).

\[
J_m = \sum_{i=1}^{N} \sum_{j=1}^{C} u_{ij}^m \| x_i - c_j \|^2 
\]

where \( m \) is any real number greater than 1, \( u_{ij} \) is the degree of membership of \( x_i \) in the cluster \( j \), \( x_i \) is the \( i^{th} \) of \( d \)-dimensional data, \( c_j \) is the \( d \)-dimension of the cluster and \( || \cdot || \) is any norm expressing the similarity between any measured data and the center.

Fuzzy partitioning is carried out through an iterative optimization of the objective function given in Eq. (1) with updating of membership \( u_{ij} \) and the cluster center \( c_j \) is given by Eq. (2) and (3) respectively.

\[
u_{ij} = \frac{1}{\sum_{k=1}^{C} \left( \| x_i - c_j \|^2 \right)^{\beta^{-1}}} 
\]

\[
c = \frac{\sum_{i=1}^{N} u_{ij}^m x_i}{\sum_{i=1}^{N} u_{ij}^m} 
\]

This iteration containing Eq. (2) and (3) will stop when

\[
\max_{ij} | u_{ij}^{(k+1)} - u_{ij}^{(k)} | < \varepsilon
\]

where \( \varepsilon \) is a stopping criterion between 0 and 1, \( k \) is the iteration steps. This process converges to a local minima or a saddle point of \( J_m \). The Procedure of FCM is shown in Table I.

B. Collaborative Fuzzy Clustering

Collaborative fuzzy clustering is originally proposed by Pedrycz. Collaborative clustering has its two typical forms called horizontal collaborative clustering and vertical collaborative clustering. We used vertical collaborative clustering in our previous work [6] for analysis of EEG data. In this paper, we used horizontal collaborative clustering. To accommodate the collaboration technique in the optimization procedure, the objective function is given by Eq. (5).

\[
Q(l) = \sum_{k=1}^{C} \sum_{i=1}^{N} u_{ik}^2 [l] d_{ik}^2 [l] 
+ \sum_{m=1}^{P} \beta[l,m] \sum_{k=1}^{N} \sum_{i=1}^{n} \{ u_{ik}[l] - u_{ik}[m] \}^2 d_{ik}^2 [l] 
\]

where \( l = 1, 2, \ldots, P \), \( P \) is number of datasets, \( N \) is number of patterns in the dataset, \( c \) is number of cluster, \( n \) is number of features, \( \beta[l,m] \) denotes the collaborative coefficient with collaborative effect on dataset \( l \) through \( m \). \( \beta \) is a user define parameter based on database \( (\beta > 0) \), \( u \) represents the partition matrix and \( d \) is a Euclidean distance between patterns and prototypes.

![Fig. 1. Collaborative clustering Scheme](image1)

![Fig. 3. A General scheme of vertical clustering](image2)

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The collaboration between the subsets of database is accomplished by a matrix of connections as shown in fig. 1. In horizontal collaborative clustering as shown in fig. 2, it deals with the same patterns but different feature spaces, probably disjoint subsets of feature space. The communication policy is based on the partition matrix. In vertical collaborative clustering as shown in fig. 3, it is concerned with different patterns usually disjoint subsets of patterns but the same feature space. Hence the communication platform is based on the level of the prototypes.

The optimization of $Q[l]$ involves the determination of the partition matrix $u[l]$ and the prototypes $v[l]$. First we solve the problem for each data set separately and allow the results to interact, forming thereby collaboration between the sets. The minimization of the objective function with respect to the partition matrix requires the use of Lagrange multipliers because of the existence of the standard constraints imposed on the partition matrix. This paper, proposed an augmented, objective function $V$ incorporates the Lagrange multiplier $\lambda$ and deal with each individual pattern ($t=1, 2, ..., N[l]$).

Collaborative fuzzy partitioning is carried out through an iterative optimization of the objective function given in Eq. (5) with an update of partition matrix $u[l]$ and the prototype $v[l]$ and it carries to solve the new objective function $\tilde{Q}[l]$ as follows:

$$\tilde{Q}[l] = \sum_{i=1}^{s} \sum_{j=1}^{c} u_{ij}^2 Q[i][j] + \sum_{n=1}^{N[l]} \beta[l,m] \sum_{i=1}^{s} \left[ u_{ij}[l] - u_{ij}[m] \right] d_{ij}[l] - \lambda \left( \sum_{i=1}^{s} u_{ij}[l] - 1 \right)$$

(6)

where $\lambda$ represents a Lagrange multiplier. The important conditions lead us to the local minimum of $\tilde{Q}[l]$ are as follows:

$$\frac{\partial \tilde{Q}[l]}{\partial u_{ij}[l]} = 0; \frac{\partial \tilde{Q}[l]}{\partial \lambda} = 0$$

(7)

where $s=1, 2, ..., c$ and $t=1, 2, ..., N$. In Eq. (8), the derivative with respect to the partition matrix is computed.

$$\frac{\partial \tilde{Q}[l]}{\partial u_{ij}[l]} = 2u_{ij}[l]d_{ij}[l]$$

(8)

$$+ 2 \sum_{n=1}^{N[l]} \beta[l,m] (u_{ij}[l] - u_{ij}[m]) d_{ij}[l] - \lambda = 0$$

In other words, the partition matrix is calculated as:

$$u_{ij}[l] = \frac{\lambda + 2d_{ij}[l]\sum_{m=1}^{c} \beta[l,m]u_{ij}[m]}{2 \left( d_{ij}[l] + d_{ij}[l]\sum_{m=1}^{c} \beta[l,m]\right)}$$

(9)

Now, we can shorten the Eq. (10) by introducing the following shorthand notation:

$$\varphi[l] = \sum_{m=1}^{c} \beta[l,m]u_{ij}[m]$$

(10)

$$\varphi[l] = \sum_{m=1}^{c} \beta[l,m]$$

(11)

Eq. (10) is expressed in the form of the expression as shown in Eq. (13) by the constraint imposed on the membership values $\sum_{i=1}^{c} u_{ij}[l] = 1$.

$$\sum_{i=1}^{c} \frac{\lambda + 2d_{ij}[l]\varphi[l]}{2d_{ij}[l](1 + \varphi[l])} = 1$$

(12)

Next, the Lagrange multiplier determines in the form as follows:

$$\lambda = 2 \frac{1 - \frac{1}{1 + \varphi[l]} \sum_{i=1}^{c} \varphi[l]}{\sum_{i=1}^{c} \frac{1}{d_{ij}[l]}\left(1 - \frac{\varphi[l]}{1 + \varphi[l]}\right)}$$

(13)

Putting this multiplier into the partition matrix formula as shown in Eq. (10), this gives the final expression as follows:

$$u_{ij}[l] = \frac{\varphi[l]}{1 + \varphi[l]} + \frac{1}{\sum_{i=1}^{c} \frac{1}{d_{ij}[l]}\left(1 - \frac{\varphi[l]}{1 + \varphi[l]}\right)}$$

(14)

The calculations for the prototypes in form of the Euclidean distance are considered between the patterns and the prototypes. The necessary condition for minimizing the objective function is of the form $\Delta Q[l]Q[l]=0$. Now the objective function as shown in Eq. (5) with distance function is expressed as follow:

$$Q[l] = \sum_{n=1}^{N[l]} \sum_{i=1}^{s} \sum_{j=1}^{c} (x_{ij} - v_{ij}[l])^2$$

(15)

The patterns in this expression as shown in Eq. (16) come from the $t^{th}$ data set. Computing the derivative of $Q[l]$ with respect to $v_{ij}[l]$ and setting it to 0, we obtain Eq. (17).
The final expression for the prototypes after some grouping of the terms is as follows:

\[
\beta[l,m] = -\sum_{i=1}^{N} \left| u_{i}^{l} - u_{i}^{m} \right| (x_{i} - v_{i}) = 0
\]

The termination criterion relies on the changes to the partition matrices obtained in successive iterations of the clustering method, for instance a Tchebyshev distance could serve as a sound measure of changes in the partition matrices. Subsequently, when this distance is lower than an assumed threshold value (\(\varepsilon > 0\)), the optimization is terminated. The Procedure of CFCM is shown in Table II.

C. Fuzzy Inference System (Mamdani)

A fuzzy inference system (FIS) is a system that uses fuzzy set theory to map inputs (features) to outputs (classes). There are mainly two FIS named as the Mamdani [8] and the Sugeno [9]. In this study we discussed about Mamdani inference system as shown in fig. 4.

To compute the output of this FIS given the inputs, we need to follow these six steps as shown in Table III.

TABLE II
Procedure of CFCM

Given: subsets of patterns \(X_1, X_2, X_3, \ldots, X_p\).

Select: distance function, number of clusters (\(c\)), termination condition, and collaboration matrix \(\beta[l,m]\).

Compute: initiate randomly all partition matrices \(U[1], U[2], \ldots, U[P]\)

**Phase I**

For each data
Repeat

Compute: prototypes \(V_i[l], i=1,2,\ldots,C\) and partition matrices \(U[l]\) for all subsets of patterns
Until: a termination condition has been satisfied

**Phase II**

Repeat

For the matrix of collaborative links \(\beta[l,m]\)

Compute: prototypes \(V[l]\) and partition matrices \(U[l]\) by using (14) and (17)

Until a termination condition is satisfied.

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Fig. 4. A two input, two rule Mamdani inference system
TABLE II  
Steps of fuzzy Inference system  

1. Determine a set of fuzzy rules by using CFCM.  
2. Fuzzifying the inputs using the input membership functions.  
3. Combining the fuzzified inputs according to the fuzzy rules to establish rule strength.  
4. Finding the consequence of the rule by combining the rule strength and the output membership function.  
5. Combining the consequences to get an output distribution.  
6. Defuzzifying the output distribution (this step is only if a crisp output (class) is needed).  

D. Proposed Collaborative Model  
The proposed model as shown in fig. 5 is mainly divided into two parts; first part consists of CFCM procedure and second part consists of Mamdani based fuzzy inference system. The proposed model combines the reasoning strengths of Mamdani type fuzzy inference system with knowledge representation ability of CFCM and gives a robust and reliable modeling system. The given input data; firstly divides in two or more equally sub groups of dataset then apply FCM on each sub-groups of dataset separately and calculate prototypes and partition matrix for each datasets. Secondly, CFCM updates all partition matrix and prototype by collaborating each of them and gets common features among them and provides these features to the knowledge based sub system of fuzzy inference system. The rule base and the database are jointly referred to as the knowledge base. A rule base containing a number of fuzzy IF–THEN rules and a database which defines the membership functions of the fuzzy sets used in the fuzzy. Further the fuzzy knowledge base passes its information to inference engine which using If-Then type fuzzy rules convert the fuzzy input to the fuzzy output. Fuzzifier converts the crisp input to a linguistic variable using the membership functions stored in the fuzzy knowledge base. Inference engine uses If-Then type fuzzy rules converts the fuzzy input to the fuzzy output. Defuzzifier converts the fuzzy output of the inference engine to crisp using membership functions analogous to the ones used by the fuzzifier.

![Fig. 5. A flow diagram of the proposed model](image)

III. EXPERIMENT RESULTS  
This paper applies a Matlab function called generating fuzzy inference system (Genfis3) [7] in order to compare with the proposed method. Given separate sets of input and output data, Genfis3 generates a fuzzy inference system (FIS) using FCM clustering. Genfis3 accomplishes this by extracting a set of rules that models the data behavior. The rule extraction method first uses the FCM function to determine the number of rules and membership functions for the antecedents and consequents.

Mackey glass considers a prediction problem that applied to the proposed system model. This time series has been commonly used in [10-12, 15-20]. The time series is generated by differential Eq. (18).

\[
\frac{dx(t)}{dt} = \frac{0.2x(t - \tau)}{1 + x(t - \tau)^3} - 0.1x(t) \tag{18}
\]

For \( \tau = 17\), the system response is chaotic and simulation data are obtained using the initial conditions \( x(0) = 1.2 \) and \( \tau = 17\), is used. A set of 1000 input-output data points are generated from \( t = 124 \) to 1123 with the first 500 patterns are used for training and the rest 500 patterns for testing. Four past values are used to predict \( x(t) \) and the input-output data format is \([x(t-24), x(t-18), x(t-12), x(t-6); x(t)]\).

In this paper, we replace FCM with CFCM and used MG data to do a simulation for the proposed method, MG data contains 1000 patterns and 5 features with 7 classes i.e. (1000X5X7). We used 1000 patterns and divided 500 patterns for training and 500 patterns for testing i.e. we choose 500X5X7 dimension for training and 500X5X7 dimension for testing. Further, we divided training data into two different equal training datasets in order to follow the property of CFCM. Each dataset contains 250X5X7 dimensions. After finishing CFCM procedure, there will be

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some changes in partition matrix as well as centers of clusters and then use the updated values of partition matrix and centers of one training dataset for further modeling of systems. We compare our result with the current existing Matlab function called Genfis3.

We calculated the root mean square errors for training and testing for the proposed method as well as for Matlab based Genfis3 function. Because Genfis3 is based on FCM, so we used the TrnRMSE_FCM and TstRMSE_FCM for short notation and TrnRMSE_CFCM and TstRMSE_CFCM for the proposed method.

Table IV, V and VI show that comparison between the proposed method and Matlab based Genfis3 method. This result is based on simulation with different values of collaborative coefficient ($\beta$), the values lies between 0.0009 $\leq$ $\beta$ $\leq$ 0.0001, 0.009 $\leq$ $\beta$ $\leq$ 0.001, and 0.09 $\leq$ $\beta$ $\leq$ 0.01 for Table IV, V and VI, respectively. The best result has been found at $\beta$=0.002 out of all simulation results as shown in all three tables. The proposed method has the best performance in terms of root mean square errors (RMSE) in both cases (training and testing). In Table IV, V and VI, we can easily see the proposed method performs better than the Genfis3 (10.9% less error in case of training and 11.8% less error in case of testing at $\beta$=0.002) due to its collaborative mechanism.

### TABLE IV

<table>
<thead>
<tr>
<th>$\beta$</th>
<th><em>M1</em></th>
<th><em>M2</em></th>
<th><em>M3</em></th>
<th><em>M4</em></th>
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*M1*= TrnRMSE_FCM
*M2*= TstRMSE_FCM
*M3*= TrnRMSE_CFCM
*M4*= TstRMSE_CFCM

We choose the best result from Table IV, V and VI then calculate their performances by showing that how centroids are affected by collaboration process as shown in Table VII (a)-(c), Table VIII (a)-(c) and Table IX (a)-(c) at $\beta$=0.0001, $\beta$=0.002 and $\beta$=0.05, respectively.

### TABLE V

<table>
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<tr>
<th>$\beta$</th>
<th><em>M1</em></th>
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<th><em>M3</em></th>
<th><em>M4</em></th>
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### TABLE VI

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### IV. CONCLUSION

In this paper, we proposed a new modeling strategy called collaborative fuzzy c-means concepts. The proposed method gives the better performance and reduces the root mean square error value for training and testing in comparable to existing Matlab function Genfis3 which uses the FCM concept to decide the system structure. For future work, we want to extend our work and compare with some other existing modeling systems with real world datasets and find a solution for big data issues. Recently big data issues (such as social media, sensor, twitter, ATM transaction and so on.) are the main research focus of researchers, which brings many challenges and difficulties to deal with it.

### TABLE VII

| Centroid (Prototype) of training datasets based on FCM
| Centroid (Prototype) of training dataset1 after collaboration based on FCM
| Centroid (Prototype) of training dataset2 after collaboration based on FCM |
|-----------------|-----------------|-----------------|
| 0.5899 0.5683 0.8447 1.0098 1.0399 | 1.1688 0.9641 0.7342 0.5896 0.7075 | 1.0883 1.1640 1.1792 1.0047 0.7715 |
| 0.8716 0.6662 0.5903 0.8255 1.0459 | 0.6960 0.5908 0.7576 1.0058 1.0850 | 0.5905 0.5417 0.8204 0.9998 1.0203 |
| 1.1245 0.8794 0.6659 0.5757 0.8094 | 1.1590 1.1649 0.9921 0.7598 0.6143 | 1.2032 1.1898 0.9503 0.6996 0.5443 |
| 1.1965 1.1571 0.9124 0.6808 0.5607 | 0.9062 1.0469 1.1207 1.1771 1.0693 | 0.9299 1.0183 1.0983 1.1523 0.1078 |
| 1.1042 1.1645 1.1570 0.9666 0.7424 | 0.9232 0.7321 0.6016 0.7318 1.0049 | 0.8922 0.6641 0.5544 0.7777 1.0160 |
| 0.6646 0.8621 1.1043 1.1329 1.2098 | 1.0679 1.1480 1.1644 1.0357 0.8048 | 1.1618 0.9081 0.6721 0.5431 0.7525 |
| 0.9392 1.0518 1.1238 1.1612 1.0405 | 0.6392 0.8248 1.0343 1.1260 1.2076 | 0.6490 0.8528 1.0164 1.1032 1.1938 |

(a) (b) (c)

(Advance online publication: 21 November 2015)
TABLE VIII
CENTROID (PROTOTYPE) OF TRAINING DATASETS AT $\beta=0.002$

<table>
<thead>
<tr>
<th>Centroid (Prototype) of training dataset based on FCM</th>
<th>Centroid (Prototype) of training dataset1 after collaboration based on CFCM</th>
<th>Centroid (Prototype) of training dataset2 after collaboration based on CFCM</th>
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<td>1.1168 0.8754 0.6756 0.5927 0.8239</td>
<td>0.9376 1.0222 1.0120 1.1512 1.0681</td>
</tr>
<tr>
<td>1.1938 1.1611 0.9208 0.6881 0.5626</td>
<td>1.1103 1.1582 1.1465 0.9533 0.7353</td>
<td>0.5865 0.5502 0.8342 1.0012 1.0224</td>
</tr>
<tr>
<td>0.5943 0.5636 0.8368 1.0085 1.0382</td>
<td>1.1748 1.1375 0.9029 0.6909 0.5875</td>
<td>0.6585 0.8583 1.0192 1.1074 1.1961</td>
</tr>
<tr>
<td>1.1309 0.8864 0.6705 0.5728 0.7991</td>
<td>0.9396 1.0733 1.1356 1.1682 1.0211</td>
<td>0.8841 0.6542 0.5561 0.7928 1.0202</td>
</tr>
<tr>
<td>0.8787 0.6717 0.5891 0.8170 1.0432</td>
<td>0.6636 0.8771 1.0627 1.1463 1.2153</td>
<td>1.1259 0.8956 0.6646 0.5492 0.7724</td>
</tr>
<tr>
<td>0.9358 1.0486 1.1214 1.1619 1.0458</td>
<td>0.8699 0.6751 0.6026 0.8315 1.0539</td>
<td>1.0964 1.1663 1.1744 0.9927 0.7605</td>
</tr>
</tbody>
</table>

(a) Based on Genfis3  
(b) Based on the proposed model

Fig. 6. The actual output (denoted as the line) and the desired output (denoted as the circle) during testing for $\beta=0.0001$.

TABLE IX
CENTROID (PROTOTYPE) OF TRAINING DATASETS AT $\beta=0.05$

<table>
<thead>
<tr>
<th>Centroid (Prototype) of training dataset based on FCM</th>
<th>Centroid (Prototype) of training dataset1 after collaboration based on CFCM</th>
<th>Centroid (Prototype) of training dataset2 after collaboration based on CFCM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9392 1.0518 1.1238 1.1612 1.0405</td>
<td>1.1162 0.8906 0.6999 0.6124 0.8226</td>
<td>1.0957 1.1594 1.1551 0.9718 0.7480</td>
</tr>
<tr>
<td>0.8715 0.6662 0.5903 0.8256 1.0460</td>
<td>0.6731 0.6370 0.8542 1.0301 1.0984</td>
<td>1.2936 1.1601 0.9193 0.6839 0.5348</td>
</tr>
<tr>
<td>0.6646 0.8621 1.0403 1.1329 1.2098</td>
<td>1.1014 1.1548 1.1467 0.9629 0.7360</td>
<td>1.0095 1.0323 1.1040 1.1392 1.0594</td>
</tr>
<tr>
<td>0.5899 0.5684 0.8448 1.0098 1.0399</td>
<td>0.9314 1.0666 1.1331 1.1669 1.0217</td>
<td>0.7427 0.8780 1.0317 1.1124 1.1983</td>
</tr>
<tr>
<td>1.1965 1.1570 0.9124 0.6808 0.5607</td>
<td>0.6767 0.8792 1.0539 1.1315 1.2153</td>
<td>1.2287 0.8849 0.6755 0.5841 0.7933</td>
</tr>
<tr>
<td>1.1042 1.1645 1.1570 0.9666 0.7423</td>
<td>1.1691 1.1280 0.8968 0.6932 0.5875</td>
<td>0.9558 0.6578 0.5677 0.8019 1.0239</td>
</tr>
<tr>
<td>1.1244 0.8794 0.6659 0.5757 0.8094</td>
<td>0.8684 0.6741 0.6042 0.8333 1.0535</td>
<td>0.6039 0.5614 0.8445 1.0003 1.0257</td>
</tr>
</tbody>
</table>

(a) Based on Genfis3  
(b) Based on the proposed model

Fig. 7. The actual output (denoted as the line) and the desired output (denoted as the circle) during testing for $\beta=0.002$.

(Advance online publication: 21 November 2015)
Fig. 8. The actual output (denoted as the line) and the desired output (denoted as the circle) during testing for $\beta=0.05$.

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