

# A Comparison between SLM, PTS, and CF Schemes for the Reduction of PAPR of OFDM System with CPM Mappers

Emammer Shafter and Raveendra K. Rao

**Abstract**—In this paper, three existing reduction techniques are applied to different Orthogonal Frequency Division Multiplexing (OFDM) in Continuous Phase Modulation (CPM) mappers. particularly, selective mapping (SLM), partial transmit sequence (PTS), and clipping and filtering (CF) techniques are investigated. The analysis was performed for an OFDM system with CPM mappers with 128 and subcarriers. it is shown that PTS method performs very well in comparison to the SLM and CF schemes in terms of PAPR reduction for the same number of subcarriers. Various subclasses of CPM mappers such as single- $h$  CPFSK, multi- $h$  CPFSK, and asymmetric multi- $h$  CPFSK are considered to reduce PAPR. Next, these mappers in conjunction with SLM, PTS, and CF techniques are considered. A comparison of the PAPR reduction capability of CPM mappers relative to memoryless BPSK mappers in an OFDM system is presented. It is noted that, in general, CPM mappers offer superior PAPR performance compared to memoryless mappers in an OFDM system. A detailed analysis of schemes has also been furnished in our paper.

**Index Terms**—Orthogonal Frequency Division Multiplexing (OFDM), Continuous Phase Modulation (CPM), CPFSK, Peak-to-Average Power Ratio (PAPR), SLM, PTS, and CF.

## I. INTRODUCTION

ORTHOGONAL Frequency Division Multiplexing (OFDM) system has been widely used for high data rate transmission applications, due to many attractive features such as high spectral efficiency, robustness to channel fading, flexibility, Easy equalization. Besides alot of advantages, some drawbacks become apparent, when using CPM mappers in OFDM systems in transmission systems. A major problems are that the transmit signal exhibits high peak to average power ratio (PAPR), sensitivety to frequency errors, and Inter-carrier Interference (ICI) between the subcarriers which make it more useful for high speed data transmission over other data transmission techniques [1]. One of the major disadvantages of OFDM is high PAPR associated with the transmitted signal. Large PAPR leads to both in-band distortion and out of band radiation. It also increases the complexity of the analog-to-digital and digital-to analog converter and reduces the efficiency of the Radio Frequency (RF) power amplifier used. Therefore it is useful to reduce the PAPR of the OFDM system. Transmitting a signal with high PAPR requires highly linear power amplifiers with large back-off to avoid adjacent channel interference due to nonlinear effects [2]. To reduce the high PAPR of OFDM systems many techniques have

been proposed in recent years [3],[4], examples of these techniques are selected clipping and filtering [5], block coding [6], partial transmit sequence (PTS) technique [7], and selective mapping technique (SLM)[8], and tone reservation and injection [9]. In this paper, Here, we concentrate on three techniques for the reduction of the PAPR; namely SLM, PTS, and CF techniques by [3],[4],[5],[6],[7],[8]. introducing mappers with memory in an OFDM system with dual purpose: i) to enhance bit error probability performance of the system; and ii) to reduce PAPR of the transmitted OFDM signal. Specifically, we introduce CPM mappers in an OFDM system. The advantage of using such a mapper is that it is possible to systematically introduce memory amongst adjacent OFDM symbols through an appropriate choice of modulation parameters [10]. In this special issue, the PAPR properties of OFDM signals with CPM mapper are examined with three reduction techniques namely SLM, PTS, and CF. In particular, three subclasses of CPM mappers, single- $h$  CPFSK, multi- $h$  CPFSK, and asymmetric multi- $h$  CPFSK in an OFDM system are considered.

The reset of the paper is organized as follows. In Section II, related work is given. In Section III, a brief description of PAPR in an OFDM system is presented. In Section IV, system descriptions is briefly discussed, and the three subclasses of CPM mappers described. In Section V PAPR reduction techniques are given in Section V. In Section VI Numerical results and their discussion is provided and the paper is concluded in Section VII with suggestions for further work.

## II. RELATED WORKS

### A. OFDM SYSTEM and PAPR Reduction Techniques

The paper by Kineplex Collins, [11] in 1950 proposed the first voice band modem using a multicarrier system. In [12], Chang and Saltzberg in 1966 have described OFDM system in their research paper. The paper by, Weinstein and Ebert in 1970 suggested the use of Fast Fourier Transform (FFT) and guard interval in OFDM systems [13]. In 1980 Peled and Ruiz suggested OFDM transmission over telephone line [14]. The paper by Leonard Cimini, in 1985 suggested the use of OFDM in mobile applications [15]. Alard and Lasalle suggested the use of OFDM for digital broadcasting in [16]. The paper by Raut, Castellain, and Lefloch, in 1989 described a complete OFDM broadcasting system in [17].

The paper by Muller and Huber, [18] studied the PAPR problem for OFDM system by PTS and SLM techniques in 1997. The paper by S. Han and J. Lee, [3] describes some of

Emammer Shafter and Raveendra K. Rao are with the Innovation Centre for Information Engineering (ICIE), Department of Electrical and Computer Engineering, Faculty of Engineering, the University of Western Ontario, London, Ontario, N6A 5B9, Canada (Email:{eshafter, rrao}@uwo.ca)

the important PAPR reduction techniques for OFDM system such as amplitude clipping and filtering (CF), coding, partial transmit sequence (PTS), selected mapping (SLM), interleaving, tone reservation, tone injection, and active constellation extension. Also, they have addressed the problem of PAPR reduction in OFDMA and MIMO-OFDM. The paper by Jiang and Wu, [19] studied and analyzed different OFDM PAPR reduction schemes. In this paper, they studied the most important aspects and provided the mathematical analysis of the PAPR. The paper by Armstrong [20], introduced a technique to reduce the PAPR in OFDM system. The idea behind this technique is to clip the high amplitude peaks. It is shown that, by clipping the oversampled time domain signal followed by filtering using an FFT-based, frequency domain components. Filtering after clipping can reduce out-of-band radiation but may also cause some peak regrowth so that the signal after clipping and filtering will exceed the clipping level at some points. To reduce overall peak regrowth, a repeated clipping-and-filtering operation can be used. In [21] Thompson in 2005 introduces the concept of the constant envelope OFDM (CE-OFDM) scheme. In CE-OFDM the signal is combined with the constant envelope in order to reduce the high PAPR. Most of the CE-OFDM system is the same as the OFDM but the difference is that in CE-OFDM, the signal transformation takes place through phase modulation and angle demodulation.

### B. Continuous Phase Modulation (CPM)

Little work has been done to reduce PAPR of the transmitted OFDM signal and enhancing bit error probability performance of the system by introducing CPM mappers in an OFDM system. In [9], Lim, Heo, and No have evaluated different PAPR reduction techniques such as clipping, SLM, PTS, TR, TI, and ACE and their modifications in order to obtain a low computational complexity. Rahmatallah and Mohan have studied the peak-to-average power ratio (PAPR) problem in OFDM systems. In this study they have described the OFDM systems and its PAPR problem. They also evaluated the metrics according to which the performance of PAPR reduction techniques [22]. The paper by Huang et al., [23] proposes a companding transform method to reduce the PAPR in OFDM systems. By looking into the performance of four typical companding schemes, namely, LST, LNST, NLST, and NLNST, important results are obtained regarding the design criteria of companding forms. It is proved through a theoretical analysis that a good tradeoff between BER performance and PAPR reduction may be achieved by appropriate selection of the companding parameters. Jiang and Zhu in 2005 have introduced a new coding scheme for the PAPR problem of OFDM systems has termed complement block coding (CBC). The modified scheme (MCBC) has been proposed and analyzed in the same article as well [24]. The performance analysis with closed form bit error probability expressions for CPFSK systems has been investigated in [25]. The theoretical and simulation results have been presented and extended to M-ary modulation for Coherent CPFSK systems. Moreover, results for coherently and noncoherently detected CPFSK are derived in [26]. In [27], Tasadduq and Rao have introduced a method of combined weighting and block coding to solve the PAPR problem of OFDM systems.

They also have investigated the interplay of unlike weighting functions to obtain great PAPR reduction. Tasadduq and Rao have proposed a new class of orthogonal frequency division multiplexing-continuous phase modulation (OFDM-CPM) signals in 2002. Moreover, they introduced various subclasses of CPM mappers such as single-h CPFSK, multi-h CPFSK, and asymmetric multi-h CPFSK are considered to reduce PAPR [10]. In [28], Tasadduq and Rao have investigated and proposed a method that employ multi-amplitude CPM signals and partial transmit sequences in order to reduce the PAPR of OFDM-CPM signals. In [29], an investigation of bit error rate (BER) over typical wireless multipath channels with AWGN is provided. Moreover, the performance of different subclasses of OFDM-CPM signals is presented and analysed. Also they proved that OFDM-CPM is the appropriate signaling technique in multipath fading channels.

### III. DEFINITION OF PAPR IN AN OFDM SYSTEM

The transmit signal in multicarrier transmission system (OFDM) can have high peak values in time domain since all the subcarriers are added during IFFT operation. So this system have high peak to average power ratio (PAPR) than single carrier system. This reduces the efficiency of power amplifier and forces the power amplifier to operate in non linear region, causes out band radiation that affect signals in adjacent frequencies and in-band distortion this affects the received signals by rotation and attenuation. The PAPR problem is more important in many wireless applications like LTE and other mobile communication systems.

A multicarrier signal is the sum of many independently modulated signals. Denoting the collection of data symbols  $C_n, n = 0, 1, \dots, N-1$ , as a vector  $C = [C_0, C_1, \dots, C_{N-1}]^T$ , the complex baseband representation of a multicarrier signal can be written as:

$$S(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C_n \exp^{j2\pi n \Delta f t}, 0 \leq t < NT, \quad (1)$$

where,  $\Delta f (= 1/NT)$  is the subcarrier spacing,  $NT$  is the data block period, and  $N$  is the number of subcarriers in the system. The Peak-to-Average Power Ratio (PAPR) of the OFDM signal can then be defined as the ratio of the maximum power to that of the average power, and is given by

$$PAPR = \frac{\max |S(t)|^2}{\frac{1}{NT} \int_0^{NT} |S(t)|^2 dt}, 0 \leq t \leq NT \quad (2)$$

For computation of PAPR in  $NL$  equidistant samples of  $S(t)$  will be considered where  $L$  is an integer greater than or equal to 1. These  $L$ -times oversampled signal samples are represented as a vector  $S = [S_0, S_1, \dots, S_{NL-1}]^T$  and can be written as

$$S_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C_n \exp^{j2\pi k n \Delta f T/L}, k = 0, 1, \dots, NL-1 \quad (3)$$

It is noted that the sequence  $S_k$  can be interpreted as the inverse discrete Fourier transform (IDFT) of data block  $C$  with  $(L-1)N$  zero padding. In fact, for an accurate measure

of the PAPR the signal is sampled with  $L = 4$ , Thus, PAPR can be defined as

$$PAPR = \frac{\max |S_k|^2}{E[|S_k|^2]}, \quad 0 \leq k \leq LN - 1 \quad (4)$$

where  $E[\cdot]$  is the average power. The sampling rate is the Nyquist rate or a multiple of it. It has been proved that using an oversampling of 4 results in discrete time PAPR that closely matches the continuous time[13].

#### IV. SYSTEM DESCRIPTIONS

The block diagram of a portion of the OFDM transmitter that employs CPM modulator/mapper is shown in Fig. 1. The data stream is fed to the S/P block to get parallel stream of data bits  $a_{p,k}$ . The CPM mapper/modulator then accepts data bits  $a_{p,k}$ ,  $p = 1, 2, 3, \dots$ , and  $k = 0, 1, \dots, N-1$ , and produces mapped symbols  $C_{p,k}$ . The suffix  $p$  denotes the OFDM symbol number and  $k$  the subcarrier number. The parallel output from the IFFT block is then converted to a serial stream by the parallel-to-serial ( $P/S$ ) block and then the cyclic prefix is added to produce  $S_p^{(CP)}$  signal for transmission. Next, we provide descriptions of three types of CPM mappers namely: single- $h$  CPFSK, multi- $h$  CPFSK, and asymmetric multi- $h$  CPFSK mappers.

##### A. OFDM systems with single- $h$ CPFSK Mapper

The parameter  $h$  defines the CPFSK mapper and takes values between  $0 < h < 1$  and is a ratio of two integers numbers  $P$  and  $Q$ , i.e.,  $h = \frac{P}{Q}$ . The quantity  $h$  is referred to as the modulation index. The choice of  $h$  determines the number of phase states in the mapper. As an example, consider the bits along the  $k$ th subcarrier,  $a_{1,k}, a_{2,k}, \dots$ , where  $a_{i,k} = \pm 1$  for  $i = 1, 2, \dots$  of a single- $h$  CPFSK mapper. Then the number of possible phase states,  $\theta_{p,k}$  for  $h = \frac{1}{2}, \frac{2}{3}$ , and  $\frac{1}{4}$  would be 4, 3, and 8, respectively. Table I shows the possible phases for  $h = \frac{1}{2}, \frac{2}{3}$  and  $\frac{1}{4}$ . [10] In single- $h$  CPFSK mapper, the value of  $h$  is fixed for all OFDM symbols [10]. The expression for  $C_{p,k}$  is given by

$$C_{p,k} = \cos(\theta_{p,k}) + j \sin(\theta_{p,k}) \quad (5)$$

where

$$\theta_{p,k} = a_{p,k} \pi h + \pi h \sum_{q=0}^{p-1} a_{q,k} + \phi \quad (6)$$

and  $\phi$  is the initial phase set equal to zero without lose of generality for a coherent system.

TABLE I: Possible phases states for single- $h$  CPFSK mapper for  $h = \frac{1}{2}$ ,  $h = \frac{2}{3}$ , and  $h = \frac{1}{4}$

| $h = \frac{P}{Q}$ | $\theta_{p,k}$                                                                                         |
|-------------------|--------------------------------------------------------------------------------------------------------|
| $\frac{1}{2}$     | $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$                                                                |
| $\frac{2}{3}$     | $0, \frac{2\pi}{3}, \frac{4\pi}{3}$                                                                    |
| $\frac{1}{4}$     | $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$ |

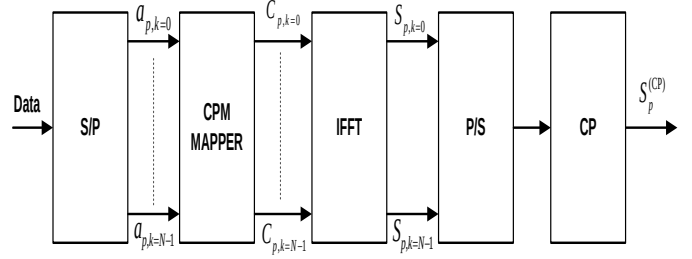


Fig. 1: Portion of OFDM transmitter with CPM mapper

##### B. OFDM systems with multi- $h$ CPFSK Mapper

In the multi- $h$  CPFSK mapper, we vary the value of  $h$  from symbol to symbol. The parameter  $h$  is cyclically chosen from a set  $H_K$  of  $K$  values,  $\{h_1, h_2, \dots, h_K\}$ .

The expression for  $\theta_{p,k}$  for this mapper is given by

$$\theta_{p,k} = \begin{cases} a_{p,k} \pi h_{[k]} + \sum_{q=0}^{p-1} a_{q,k} \pi h_{[q]} + \phi, & k > 1 \\ a_{1,k} \pi h_{[1]} + \phi, & k = 1 \end{cases}$$

For illustration, we take the first four symbols for an arbitrary  $k$ th subcarrier with  $H_2 = \{\frac{2}{3}, \frac{1}{4}\}$  and data sequence  $a_{p,k} = [+1, +1, -1, +1]$ . Assuming the initial phase to be zero, then the number of possible phase states,  $\theta_{p,k}$  for  $h = \frac{2}{3}$  and  $\frac{1}{4}$  would be 4 and 3 respectively.[10]

##### C. OFDM systems with Asymmetric multi- $h$ CPFSK

While in multi- $h$  CPFSK,  $h$  values are chosen independently of data bits  $a_{p,k} = (\pm 1)$ , in this case we choose  $h$  a function of  $a_{p,k}$ . That is, the value of  $h$  during the  $i$ th symbol interval is chosen  $h_{+i}$  or  $h_{-i}$  accordingly as data is a  $+1$  or  $-1$  respectively. For this mapper, the expression for  $\theta_{p,k}$  is given by

$$\theta_{p,k} = \begin{cases} a_{p,k} \pi h_{\pm[k]} + \sum_{q=0}^{p-1} a_{q,k} \pi h_{\pm[q]} + \phi, & k > 1 \\ a_{1,k} \pi h_{\pm[1]} + \phi, & k = 1 \end{cases}$$

This gives additional flexibility to the designers to enhance system performance. Let the  $h$  values employed for data a  $\pm 1$  be  $H_{+i} = \{\frac{2}{3}, \frac{1}{4}\}$  and the ones for data  $-1$  be  $H_{-i} = \{\frac{1}{4}, \frac{2}{3}\}$ . Then the number of possible phase states,  $\theta_{p,k}$  for  $H_{+i} = \{\frac{2}{3}, \frac{1}{4}\}$  and  $H_{-i} = \{\frac{1}{4}, \frac{2}{3}\}$  would be 4 and 3 respectively [10].

#### V. CPM MAPPER WITH SLM TECHNIQUE

The block diagram of the CPM mapper with SLM technique is shown in Fig. 2. In SLM technique a whole set of candidate signals is generated representing the same information, and then the most favorable signal as regards to PAPR is chosen and transmitted. The CPM mapper output  $[C_{p,k=0}, \dots, C_{p,k=N-1}]^T$  is multiplied with different phase sequences and fed to the IFFT block to produce OFDM symbols as shown in Fig. 2. One of these OFDM symbols will have minimum PAPR which is selected and transmitted. Suppose the CPM mapper output is a vector  $[C_{p,k=0}, \dots, C_{p,k=N-1}]^T$ , then this vector is multiplied by  $U$  different phase sequences, each of length  $N$ ,  $B^{(u)} = [b_{u,0}, b_{u,1}, \dots, b_{u,N-1}]^T$ ,  $u = 1, 2, \dots, U$ , resulting in  $U$  modified data blocks. The modified data block for the  $u$ th phase sequence is represented as  $[C_0 b_{u,0}, C_1 b_{u,1}, \dots, C_{N-1} b_{u,N-1}]^T$ ,  $u = 1, 2, \dots, U$ . The

output of the IFFT block for this modified data with an oversampling factor of  $L$  is given by

$$S_k^{(u)} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C_n b_{u,n} e^{j2\pi kn\Delta f T/L}, u = 1, 2, \dots, U. \quad (7)$$

The PAPR for each  $\{S_k^{(u)}, k = 0, 1, \dots, NL - 1\}, u = 1, 2, \dots, U$ , block is computed and then the minimum of these is chosen for transmission. That is,

$$\min_{1 \leq u \leq U} \{S_k^{(u)}, k = 0, 1, \dots, NL - 1\} \quad (8)$$

It is noted that when using SLM the transmitter needs to convey to the receiver the value of  $u$ .

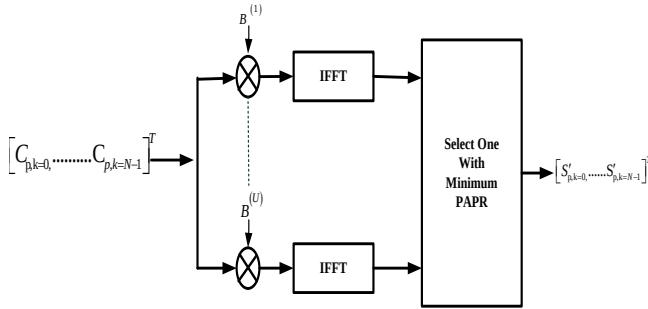


Fig. 2: A block diagram of CPM mapper with SLM technique

#### VI. CPM MAPPER WITH PTS TECHNIQUE

The main idea behind the PTS technique, is that, the data block which is generated from the CPM mapper is partitioned into non-overlapping subblocks and each subblock is rotated with a statistically independent rotation factor [7]. The rotation factor, which generates the time domain data with the lowest peak amplitude, is also transmitted to the receiver as side information. IFFT is then applied to each subblock subsequence and the resulting signal subsequences are summed after being multiplied by a set of distinct rotating vectors. Next the PAPR is computed for each resulting sequence and then the signal sequence with the minimum PAPR is transmitted. The output of the CPM mapper  $[C_{p,k=0}, \dots, C_{p,k=N-1}]^T$  is fed to the IFFT block to produce OFDM symbols as shown in Fig. 3. The output of the IFFT block for this data with an oversampling factor of  $L$  is given by

$$X_k^{(u)} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C_n e^{j2\pi kn\Delta f T/L}, u = 1, 2, \dots, U. \quad (9)$$

The output of the IFFT then is multiplied by  $U$  different phase sequences, each of length  $N$ ,  $B^{(u)} = [b_{u,0}, b_{u,1}, \dots, b_{u,N-1}]^T, u = 1, 2, \dots, U$ , resulting in  $U$  modified data blocks. The modified data block for the  $u$ th phase sequence is represented as  $[C_0 b_{u,0}, C_1 b_{u,1}, \dots, C_{N-1} b_{u,N-1}]^T, u = 1, 2, \dots, U$ .

The PAPR for each  $\{X_k^{(u)}, k = 0, 1, \dots, NL - 1\}, u = 1, 2, \dots, U$ , block is computed and then combined for transmission. That is

$$\hat{C}_n = \sum_{u=1}^U X_k^{(u)}, k = 0, 1, \dots, NL - 1 \quad (10)$$

It is noted that when using PTS the transmitter needs to convey to the receiver the value of  $u$ .

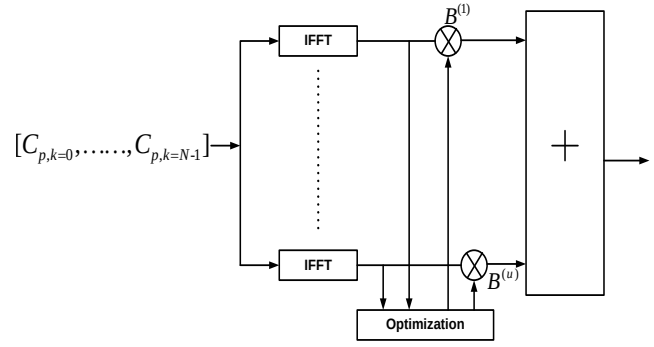


Fig. 3: A block diagram of CPM mapper with PTS technique

#### VII. CPM MAPPER WITH CF TECHNIQUE

Clipping and filtering technique is the simplest PAPR reduction technique, which limits the transmit signal to a pre-specified level. However, clipping results in amplitude distortion, which called as clipping noise and expands the transmitted signal spectrum. Clipping is a nonlinear process and causes in-band distortion, which causes degradation in the performance of bit BER and introduces out-of-band noise, thereby decreases the spectral efficiency [30]. Clipping and Filtering technique is effective in removing components of the expanded spectrum. Although filtering can decrease the spectrum growth, filtering after clipping can reduce the out-of-band radiation at the cost of peak re-growth[31]. The technique of Iterative Clipping and Filtering reduces the PAPR without spectrum expansion. However, the iterative signal takes long time and it will increase the computational complexity of an OFDM transmitter [32]. But without performing interpolation before clipping causes it out-of-band. To avoid out-of-band, signal should be clipped after interpolation. However, this causes significant peak re-growth. So, it can use Iterative Clipping and Frequency domain filtering to avoid peak re-growth. A block diagram of repeated clipping and frequency filtering technique is shown in Fig. 4.

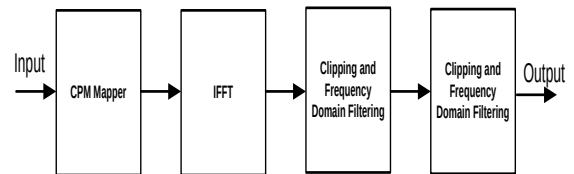


Fig. 4: A block diagram of repeated clipping and frequency filtering technique

#### VIII. SIMULATION RESULTS FOR THE PAPR-REDUCTION TECHNIQUES

The performance of the SLM, PTS, and CF methods are presented in the following figures. As the figures show, there is nearly 13.7 and 11.2 dB PAPR at 0.1 percent of data block for PSBSK and CPFSK signal respectively when no PAPR reduction method is used. This number will be reduced by using SLM, PTS, and CF methods for different CPM subclasses named (single- $h$  CPFSK, multi- $h$  CPFSK, and

assymmetric multi- $h$  CPFSK). The performance of the SLM method is presented in Figs. 5-7. As it is presented in these figures, the reduction is improved by applying SLM with BPSK and single- $h$  CPFSK ( $h = \frac{1}{2}$ ) mappers for an OFDM system with 128 sub-carriers. These plots have been arrived at by examining 10,000 random OFDM symbols.

Fig.6, shows PAPR performance of multi- $h$  CPFSK mapper for 128 subcarrier OFDM system. The set of modulation parameters used is  $(\frac{2}{3}, \frac{1}{4})$ . It is noted that this specific multi- $h$  CPFSK mapper performs nearly same as that of BPSK. However, when SLM is used with these systems multi- $h$  CPFSK outperforms BPSK by nearly more than 1dB.

Fig.7, shows CCDFs for BPSK, BPSK with SLM, asymmetric multi- $h$  CPFSK, and asymmetric multi- $h$  CPFSK with SLM systems. These CCDFs show PAPR performances for 128 subcarrier OFDM system. The modulation parameters used in the asymmetric multi- $h$  CPFSK mapper are  $H_{+i} = \{\frac{2}{3}, \frac{1}{4}\}$  and  $H_{-i} = \{\frac{1}{4}, \frac{2}{3}\}$ . It is noted that the difference in PAPR performance between asymmetric multi- $h$  CPFSK with and without SLM is nearly 3.25 dB. The difference in PAPR performance between multi- $h$  CPFSK and asymmetric multi- $h$  CPFSK mappers is approximately 1.4 dB, for an OFDM system with 128 subcarriers.

From Fig. 8, there is about 5.3 and 3.8 dB reduction for single- $h$  CPFSK and BPSK mappers, respectively when PTS method is applied. The reduction when SLM was applied is about 3.3 and 5.2 dB from Fig. 5. Comparing the CCDF of these two methods shows that PTS method has the better performance [33]-[34].

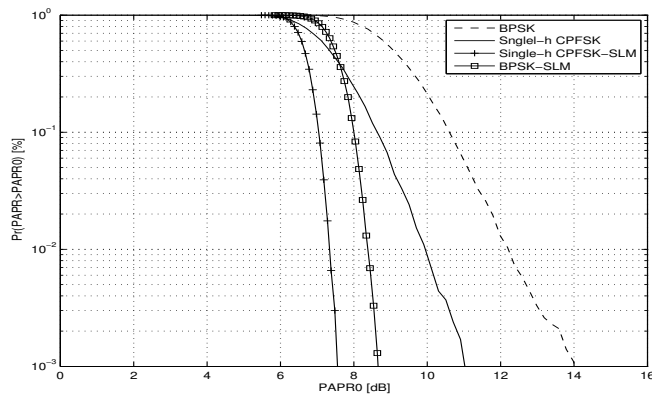


Fig. 5: PAPR Performance of 128 subcarrier OFDM system with single- $h$  CPFSK mapper ( $h = \frac{1}{2}$ ) and SLM

We assume an CPM mapper in OFDM system with 128 subcarriers ( $N = 128$ ). Also assume that the number of allowed phase factors is  $L=4$  with  $P = \{\pm 1, \pm j\}$ . We divide the 128 subcarriers into 8 subblocks with 16 contiguous subcarriers. The transmitted signal is oversampled by a factor of 4 ( $L = 4$ ). 10000 random OFDM blocks were generated to obtain the complementary cumulative density functions (CCDFs) of PAPR. In Fig. 9, shows PAPR performance of multi- $h$  CPFSK mapper for 128 subcarrier OFDM system. The set of modulation parameters used is  $(\frac{2}{3}, \frac{1}{4})$ . It is noted that this specific multi- $h$  CPFSK mapper performs nearly same as that of BPSK. However, when PTS is used with these systems multi- $h$  CPFSK outperforms BPSK by nearly more than 3 dB.

Fig. 10, shows CCDFs for BPSK, BPSK with PTS,

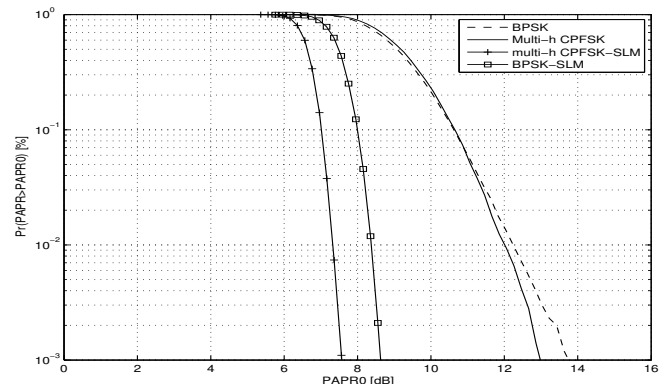


Fig. 6: PAPR Performance of 128 subcarrier OFDM system with  $\{\frac{2}{3}, \frac{1}{4}\}$  multi- $h$  CPFSK mapper and SLM

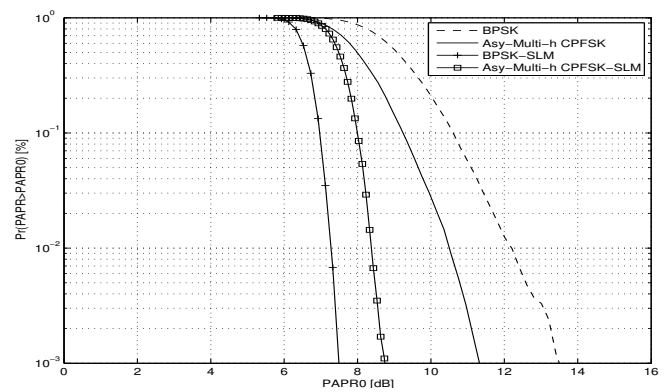


Fig. 7: PAPR Performance of 128 subcarrier OFDM system with  $H_{+i} = \{2/3, 1/4\}$  and  $H_{-i} = \{1/4, 2/3\}$  asymmetric multi- $h$  CPFSK mapper and SLM

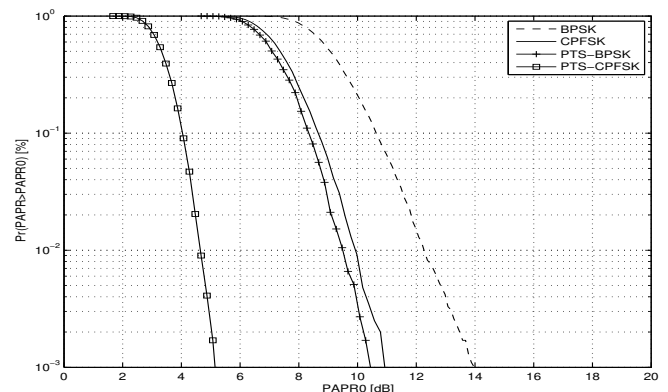


Fig. 8: PAPR Performance of 128 subcarrier OFDM system with single- $h$  CPFSK mapper ( $h = \frac{1}{2}$ ) and PTS

asymmetric multi- $h$  CPFSK, and asymmetric multi- $h$  CPFSK with SLM systems. These CCDFs show PAPR performances for 128 subcarrier OFDM system. The modulation parameters used in the asymmetric multi- $h$  CPFSK mapper are  $H_{+i} = \{\frac{2}{3}, \frac{1}{4}\}$  and  $H_{-i} = \{\frac{1}{4}, \frac{2}{3}\}$ . It is noted that the difference in PAPR performance between asymmetric multi- $h$  CPFSK with and without PTS is nearly 6 dB. The difference in PAPR performance between multi- $h$  CPFSK and asymmetric multi- $h$  CPFSK mappers is approximately 1.4 dB, for an OFDM system with 128 subcarriers. The results described above show that the PTS technique achieves significant improvement in PAPR performance when we use

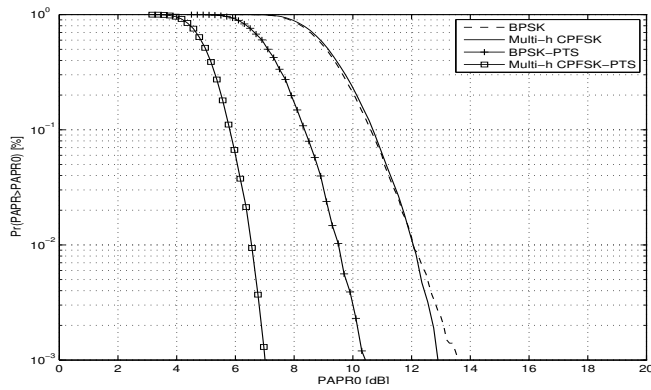


Fig. 9: PAPR Performance of 128 subcarrier OFDM system with  $\{\frac{2}{3}, \frac{1}{4}\}$  multi- $h$  CPFSK mapper and SLM

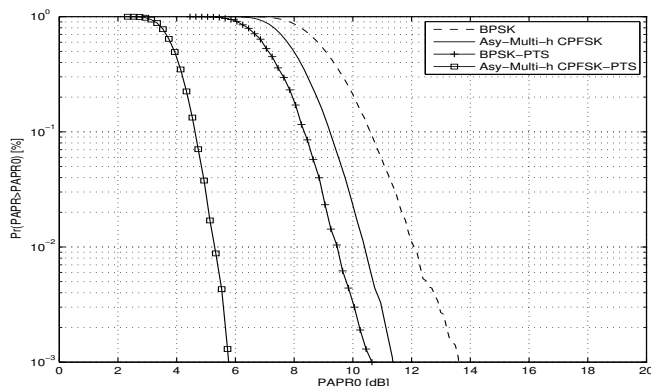


Fig. 10: PAPR Performance of 128 subcarrier OFDM system with  $H_{+i} = \{2/3, 1/4\}$  and  $H_{-i} = \{1/4, 2/3\}$  asymmetric multi- $h$  CPFSK mapper and SLM

CPM mappers in conjunction with PTS technique [34].

We also consider a 128-subcarrier with mappers with memory that introduced in an OFDM system,  $L = 4$ , and clipping factor ( $C_f = 0.25$ ). In Fig. 11, the complementary cumulative distribution function (CCDF) has been plotted to show PAPR performances for PBSK and single- $h$  CPFSK ( $h = \frac{1}{2}$ ) mappers for an OFDM system with 128 sub-carriers. Also, in the same figure are shown the performances of these two mappers with CF. These plots have been arrived at by examining 10,000 random OFDM symbols. It is observed that the OFDM system with single- $h$  CPFSK ( $h = \frac{1}{2}$ ) mapper has a PAPR that exceeds 11.1 dB for less than 0.1 percent of data blocks and for PBSK mapper it is 13.8 dB. However, when CF is used with these mappers the PAPR reduces to 4.2 dB and 8.8 dB for single- $h$  CPFSK and BPSK mappers, respectively. Thus, it is noted that single- $h$  CPFSK mapper with CF can offer an improvement in PAPR of nearly 9.6 dB relative to corresponding system without CF. Also, it is noted that the improvement in PAPR by using CF in these two systems are 6.8 dB and 5 dB for single- $h$  CPFSK and BPSK mappers, respectively. Fig. 12, shows PAPR performance of multi- $h$  CPFSK mapper for 128 subcarrier OFDM system. The set of modulation parameters used is  $(\frac{2}{3}, \frac{1}{4})$ . It is noted that this specific multi- $h$  CPFSK mapper performs nearly same as that of BPSK. However, when CF is used with these systems multi- $h$  CPFSK outperforms BPSK by more than 2 dB.

Fig. 13, shows CCDFs for BPSK, BPSK with CF, asym-

metric multi- $h$  CPFSK, and asymmetric multi- $h$  CPFSK with CF systems. These CCDFs show PAPR performances for 128 subcarrier OFDM system. The modulation parameters used in the asymmetric multi- $h$  CPFSK mapper are  $H_{+i} = \{\frac{2}{3}, \frac{1}{4}\}$  and  $H_{-i} = \{\frac{1}{4}, \frac{2}{3}\}$ . It is noted that the difference in PAPR performance between asymmetric multi- $h$  CPFSK with and without CF is more than 6 dB. The difference in PAPR performance between multi- $h$  CPFSK and asymmetric multi- $h$  CPFSK mappers is approximately 2 dB, for an OFDM system with 128 subcarriers.

Fig. 14, shows the conventional BPSK with three iterations. The first iteration has reduced the PAPR by 5.5 dB. In second and third iterations PAPR reduces further by 6 dB and 6.2 dB respectively. In Fig. 15, PAPR performance for the single- $h$  CPFSK ( $h = \frac{1}{2}$ ) mapper in OFDM system with three iterations are plotted. The first iteration has reduced the PAPR by 4.3 dB. In second and third iterations PAPR is reduced by 3.7 dB and 3.5 dB respectively.

The PAPR values of the three CPM mappers before clipping, after clipping, and clipping filtering are summarized in Table II to IV, as a function of number of subcarriers  $N$  and clipping factor  $C_f$ . Also in these Tables performances of corresponding BPSK systems are given.

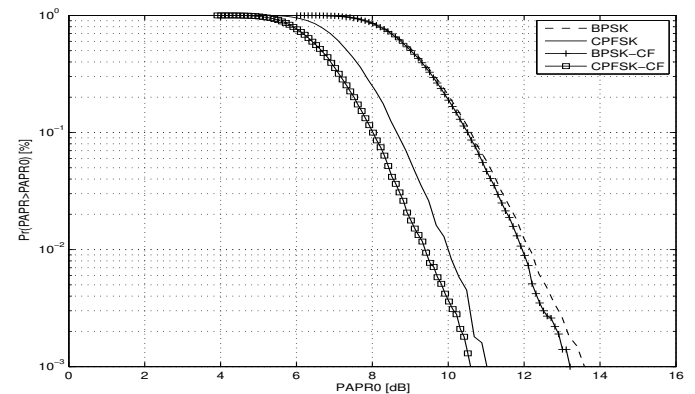


Fig. 11: PAPR Performance of 128 subcarrier OFDM system with single- $h$  CPFSK mapper ( $h = \frac{1}{2}$ ) and CF

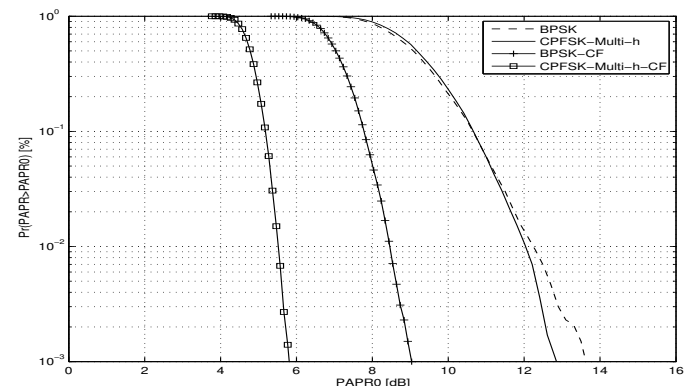


Fig. 12: PAPR Performance of 128 subcarrier OFDM system with  $\{\frac{2}{3}, \frac{1}{4}\}$  multi- $h$  CPFSK mapper and CF

In the second part a performance comparison between different methods of the PAPR reductions is done to distinguish which technique conserve the reduction found in the transmission Fig. 16, shows the performance in terms of CCDF of PAPR of the single- $h$  CPFSK signal received, a

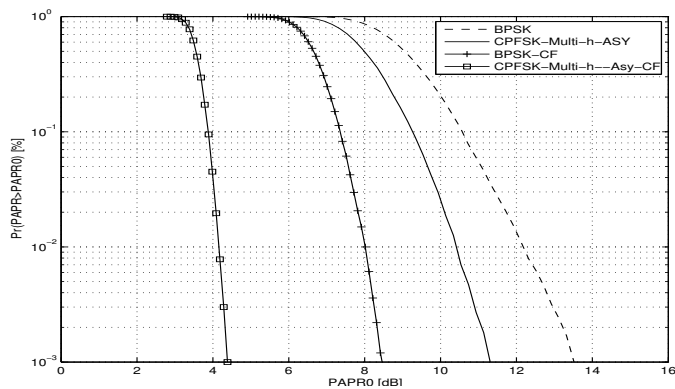


Fig. 13: PAPR Performance of 128 subcarrier OFDM system with  $H_{+i} = \{2/3, 1/4\}$  and  $H_{-i} = \{1/4, 2/3\}$  asymmetric multi- $h$  CPFSK mapper and CF

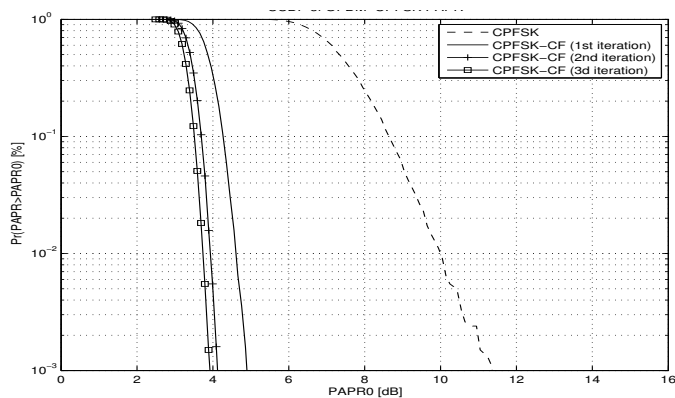


Fig. 14: PAPR Performance of 128 subcarrier OFDM system with different iterations of BPSK mapper and CF

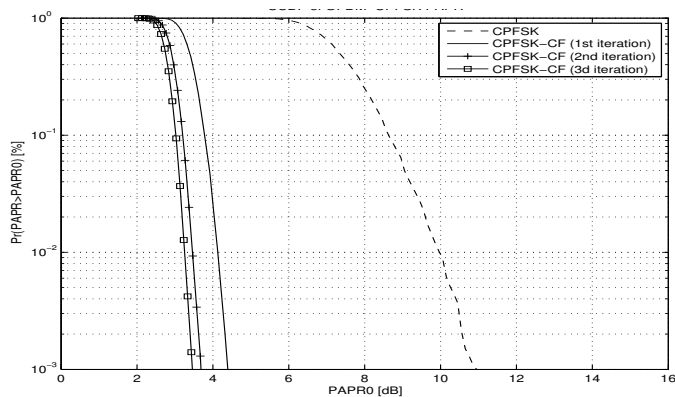


Fig. 15: PAPR Performance of 128 subcarrier OFDM system with different iterations of single- $h$  CPFSK mapper ( $h = \frac{1}{2}$ ) and CF

comparison between different schemes of the PAPR reductions is done for single- $h$  CPFSK. While the clipping and filtering scheme shows a slight decrease in PAPR, PTS gives results which diverges from CPFSK curve but with SLM technique PAPR reduction result exceed 3 dB it is equivalent to reduction PAPR calculate in the transmission part. SLM not only reduces the complexity at the reception, but it also reduces the PAPR of the OFDM signal.

We consider a different CPM mappers in OFDM system with 128 subcarriers (namely  $N=128$ ) and the oversampling rate of ( $L=4$ ) are used to analyze PAPR reduction based

TABLE II: PAPR values for before clipping, after clipping and clipping and filtering for single- $h$  CPFSK and BPSK mappers as a function of number of subcarriers and clipping factor

|                             | PAPR (dB)               |                                   |
|-----------------------------|-------------------------|-----------------------------------|
| Description                 | BPSK<br>Cf=0.25 & N=128 | Single-h-CPFSK<br>Cf=0.25 & N=128 |
| Before Clipping & Filtering | 13.8                    | 11                                |
| After Clipping & Filtering  | 8.8                     | 4.2                               |

TABLE III: PAPR values for before clipping, after clipping and clipping and filtering for multi- $h$  CPFSK and BPSK mappers as a function of number of subcarriers and clipping factor

|                             | PAPR (dB)               |                                  |
|-----------------------------|-------------------------|----------------------------------|
| Description                 | BPSK<br>Cf=0.25 & N=128 | Multi-h-CPFSK<br>Cf=0.25 & N=128 |
| Before Clipping & Filtering | 13.8                    | 13                               |
| After Clipping & Filtering  | 8.3                     | 5.8                              |

TABLE IV: PAPR values for before clipping, after clipping and clipping and filtering for multi- $h$  CPFSK asymmetric and BPSK mappers as a function of number of subcarriers and clipping factor

|                             | PAPR (dB)               |                                      |
|-----------------------------|-------------------------|--------------------------------------|
| Description                 | BPSK<br>Cf=0.25 & N=128 | Multi-h-CPFSK-Asy<br>Cf=0.25 & N=128 |
| Before Clipping & Filtering | 13.8                    | 11.2                                 |
| After Clipping & Filtering  | 8.8                     | 4.7                                  |

on different schemes clipping and filtering, partial transmit and select mapping respectively for CPM mappers in OFDM system. as shown in the following figures. These plots have been arrived at by examining 10000 random OFDM symbols. The results of the simulations are presented in this section as the Complementary Cumulative Density Function (CCDF) of the PAPR of CPM mappers in OFDM system. From Fig. 5-16, it is very clear that all schemes can reduce the PAPR in CPM mappers in OFDM system. However, their performances of the PAPR reduction are different.

The Figures 6,7,8 shows the magnitude of the peak reduction symbol of length 128 with different schemes select map-

ping, partial transmit and clipping and filtering respectively for a different subclasses of CPFSK mappers. We see that the reduction in PTS is nearly 6 dB, while in some literatures art clipping and filtering achieved only a PAPR reduction of 3dB. This confirms that the PTS help in ways beneficial to reduce fluctuations in the envelope of the CPFSK mappers. In the others methods SLM and CF reduction of PAPR exceed 3dB and 1dB respectively

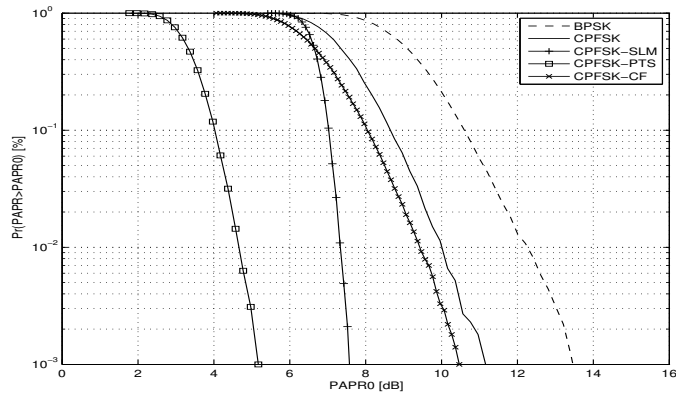


Fig. 16: PAPR Performance of 128 subcarrier for SLM, PTS and CF of single- $h$  CPFSK mapper ( $h = \frac{1}{2}$ )

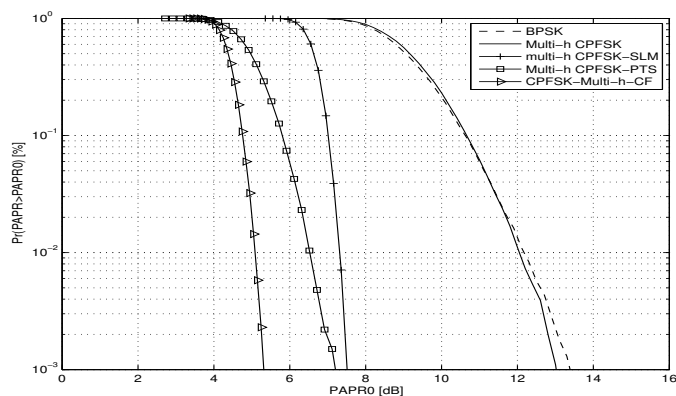


Fig. 17: PAPR Performance of 128 subcarrier for SLM, PTS and CF of multi- $h$  CPFSK mapper  $\{\frac{2}{3}, \frac{1}{4}\}$

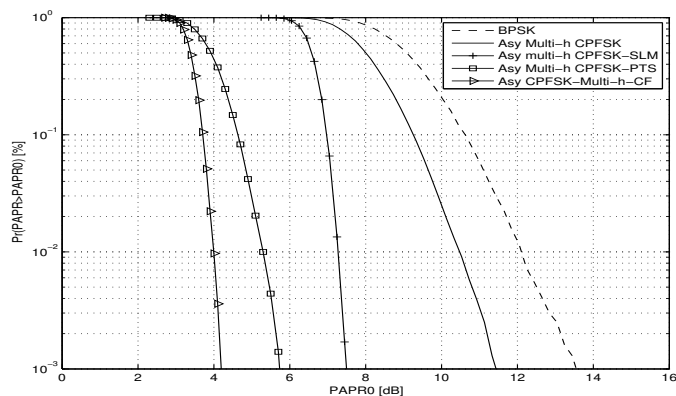


Fig. 18: PAPR Performance of 128 subcarrier for SLM, PTS and CF of asymmetric multi- $h$  CPFSK mapper  $H_+ = \{\frac{2}{3}, \frac{1}{4}\}$  and  $H_- = \{\frac{1}{4}, \frac{2}{3}\}$

## IX. CONCLUSIONS

The CPM mappers in OFDM system were simulated and three of the existing PAPR reduction techniques, Selected Mapping (SLM), Partial Transmit Sequence (PTS) and Clipping and Filtering (CF) methods have been applied into the system and the results have been studied. The results show better PAPR reduction in PTS method and less compared with SLM and CF methods. In an extensive study needs to be carried out to determine the optimum CPM mappers with least values of PAPR. Also, one needs to determine best CPM mappers in OFDM systems that achieve not only least probability of bit error but also least PAPR. Also, it would be interesting to obtain analytical bounds on PAPR when CPM mappers are used.

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