A Novel Fuzzy Time Series Model Based on Fuzzy Logical Relationships Tree

Xiongbiao Li, Yong Liu, Xuerong Gou, Yingzhe Li

Abstract—**Fuzzy time series have been widely used to deal with forecasting problems. In this paper, a novel fuzzy time series model is proposed, in which a fuzzy logical relationships tree containing multi-orders fuzzy relationships is constructed. Specifically, the robustness of lower orders fuzzy logical relationships and the precision of higher orders fuzzy logical relationships are exploited simultaneously to improve the forecasting accuracy. The trading data of Taiwan Capitalization Weighted Stock Index (TAIEX) and the enrollments of Alabama University are used as benchmark data for training and testing, and the forecasting results show that the proposed model gets higher forecasting accuracy.**

Index Terms—**fuzzy time series, fuzzy logical relationships, data forecasting, fuzzy systems.**

I. Introduction

TIME-SERIES forecasting is used for forecasting the future based on historical observations in various domains, such as air pollution, stock forecasting and etc. In the actual applications, noises will give rise to uncertain data. However, traditional time series analysis cannot handle the perturbation in data. Therefore, based on the fuzzy set theory [1], Song and Chissom proposed the concepts of fuzzy time series, which are popular in data forecasting recently due to the fact that they could provide resistance to the perturbation of input data without requiring complex certification and assumption of large samples [2], [3], [4].

The forecasting process in fuzzy time series composes with the following four steps: (1) partition of the universe of discourse, (2) definition of fuzzy sets and fuzzification of time series with the use of these fuzzy sets, (3) establishment of fuzzy logical relationships from the fuzzy time series, and (4) forecasting and defuzzification of the output of fuzzy time series. Based on fuzzy time series, some methods and models have been proposed to improve performance of forecasting. In [5], Chen proposes a model using simplified arithmetic operations instead of the complicated max-min composition operations used in [2], [3], [4] when establishing fuzzy logical relationships. In [6], Huarng presents a heuristic model by integrating problem-specific heuristic knowledge with Chen's model when defuzzifying the output. In [7], the author has a discussion on the effect of the forecasting accuracy from the partition to the universe of discourse. In [8], Wang and Chen propose a new model based on high-order and fuzzy-trend of logical relationships for forecasting. In [9], the particle swarm optimization technique is exploited in the forecasting model to improve the forecasting accuracy. In [10], the fuzzy logical relationships are replaced with the artificial neural networks. In [11], a forecasting model based on similarity measures of fuzzy logical relationships is proposed. In [12], a method of partitioning the fuzzy logical relationships based on support vector machine is proposed. In [13], [14], [15], multiple variables time series are considered simultaneously to improve the forecasting accuracy, where multiple variables include a main factor and at least a secondary factor. In [16], an adaptive selection of analysis windows and heuristic rules is proposed to improve forecasting accuracy. In [17], a forecasting model of fuzzy time series which exploits respectively particle swarm optimization algorithm and fuzzy K-means clustering algorithm to obtain the optimum partition of the universe of discourse is proposed. The aforementioned works, in the testing phase, mainly exploit a certain order of the fuzzy relationship to forecast values, regardless of fixed order (first-order, second-order or higher-order) or adaptive order of fuzzy relationships is used for forecasting. However, different orders of the fuzzy relationships have different information, i.e., the lower orders fuzzy logical relationships have robustness information, while the higher orders fuzzy logical relationships have precision information. Only considering a single order of the fuzzy relationship in the testing phase will not make full use of effective information of different orders.

In this paper, we propose a novel forecasting model considering multi-orders (first-order, second-order and higher-order) of fuzzy logical relationships simultaneously. Specifically, a multi-orders relationship tree is constructed, and thus the robustness of lower orders fuzzy logical relationships and the precision of higher orders fuzzy logical relationships are introduced simultaneously into the model to improve the forecasting accuracy.

The rest of this paper is organized as follows. A brief review of the theory of fuzzy time series is described in Section 2. In Section 3, a novel forecasting model with multi-orders relationship tree is proposed. Experiments are presented in Section 4, and some concluding remarks are given in Section 5.

II. Review of fuzzy time series

In this section, we briefly review some basic concepts of fuzzy time series [2], [3], [4], where the values of fuzzy time series are represented by fuzzy sets [1].

Let U be the corresponding universe of discourse, where U= $\{u_1, u_2, \cdots, u_n\}$. A fuzzy set A_i in the U is defined as

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follows

$$A_i = f_{A_i}(u_1)/u_1 + \cdots + f_{A_j}(u_j)/u_j + \cdots + f_{A_i}(u_n)/u_n,$$

where f_{A_i} denotes the membership function of the fuzzy set A_i, $f_{A_i}(u_j)/u_j$ represents the degree of membership of u_j belonging to the fuzzy set A_i, $f_{A_i}(u_j) \in [0,1]$ and $1 \leqslant j \leqslant n$.

Definition 1: Let $Y(t)(t = \cdots, 0, 1, 2, \cdots)$, a subset of R, be the universe of discourse on which fuzzy sets $f_i(t)$ are defined. And let $F(t)$ is a collection of $f_1(t)$, $f_2(t)$, $\cdots$, then $F(t)$ is called a fuzzy time series defined on $Y(t)(t = \cdots, 0, 1, 2, \cdots)$.

Definition 2: When $F(t)$ is a fuzzy time series and $F(t)$=$F(t-1) \times R(t, t-1)$, where $R(t, t-1)$ is the fuzzy relationship between $F(t)$ and $F(t-1)$, and $\times$ is an operator, $F(t)$ is said to be caused only by $F(t-1)$. The relationship between $F(t)$ and $F(t-1)$ can be denoted by $F(t-1) \to F(t)$, which is called first-order fuzzy logic relationship. And $F(t)$=$F(t-1) \times R(t, t-1)$ is called first-order fuzzy time series model.

Definition 3: If $F(t)$ is caused by $F(t-1)$, $F(t-2)$, $\cdots, F(t-n)$, and the fuzzy relationship is represented by $F(t-n)$, $\cdots$, $F(t-2)$, $F(t-1) \to F(t)$, which is called nth-order fuzzy logic relationship. And $F(t)$ is called the nth-order fuzzy time series model.

Definition 4: Let $F(t-1)$=A_{i_1}, $F(t-2)$=A_{i_2}, $\cdots$, $F(t-n)$=A_{i_n} and $F(t)$=A_j, where A_{i_n}, $\cdots$, A_{i_2}, A_{i_1} and A_j are fuzzy sets. The fuzzy logical relationship among $n+1$ consecutive data can be denoted as $A_{i_n}, \cdots, A_{i_2}, A_{i_1} \to A_j$, where A_{i_n}, $\cdots$, A_{i_2}, A_{i_1} is the left-hand side (LHS), and A_j is the right-hand side (RHS).

We can group fuzzy logical relationships having the same LHS into a fuzzy logic relationship group (FLRG). For example, assume that the following fuzzy logic relationships exist

$$A_{i_n}, \cdots, A_{i_2}, A_{i_1} \to A_{j_1}$$

$$A_{i_n}, \cdots, A_{i_2}, A_{i_1} \to A_{j_2}$$

$$\cdots,$$

these fuzzy relationships can be group into a fuzzy relationship group

$$A_{i_n}, \cdots, A_{i_2}, A_{i_1} \to A_{j_1}, A_{j_2}, \cdots.$$

III. Proposed forecasting model based on fuzzy logical relationships tree

In this section, we present a novel fuzzy time series model considering multi-orders of fuzzy logic relationships simultaneously. Specifically, a tree of multi-orders fuzzy logic relationships is constructed and corresponding heuristic rules are proposed to improve forecasting accuracy. The proposed model is now presented as follows.

Step 1 Define the universe of discourse U, U=$[D_{min} - D_1, D_{max} + D_2]$, where D_{min} and D_{max} denote the minimum value and the maximum value of the historical training data respectively; D_1 and D_2 are two proper positive real values to partition the universe of discourse U into n equal intervals, denoted as $u_1, u_2, \cdots, u_n$. Note that the length of each interval is half of the standard deviation σ of historical data, and $D_1 + D_2 < \sigma/4$.

Step 2 Define the fuzzy linguistic terms A_1, A_2, $\cdots$, A_n represented by fuzzy sets, shown as follows

$$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + \cdots + 0/u_{n-1} + 0/u_n$$

$$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + \cdots + 0/u_{n-1} + 0/u_n$$

$$\cdots$$

$$A_n = 0/u_1 + 0/u_2 + 0/u_3 + \cdots + 0.5/u_{n-1} + 1/u_n,$$

where u_1, u_2, $\cdots$, u_n are the intervals defined in Step 1.

Step 3 Fuzzify each historical training datum into a fuzzy set defined in Step 2. If the historical training datum of time t belongs to the interval u_i defined in Step 1 and the maximum membership value of fuzzy set A_i happens at u_i, where $1 < i < n$, the fuzzified value of the historical training datum of time t is A_i.

Step 4 Construct the fuzzy logical relationships from the fuzzied historical training data obtained in Step 3. According to Definition 2 and Definition 3, first-order, second-order and third-order fuzzy logical relationships are constructed respectively. Moreover, according to Definition 4, FLRG for a certain order fuzzy logical relationships is constructed. For example, let us consider the following nth-order fuzzy logical relationships with the same LHS $A_{i_n}, \cdots, A_{i_2}, A_{i_1}$

$$A_{i_n}, \cdots, A_{i_2}, A_{i_1} \to A_{j_1}$$

$$A_{i_n}, \cdots, A_{i_2}, A_{i_1} \to A_{j_2}$$

$$\cdots,$$

these fuzzy relationships can be grouped into a fuzzy relationship group

$$A_{i_n}, \cdots, A_{i_2}, A_{i_1} \to A_{j_1}, A_{j_2}, \cdots.$$

Step 5 Construct fuzzy logical relationships tree (FLRT) based on the three orders FLRG obtained in Step 4.

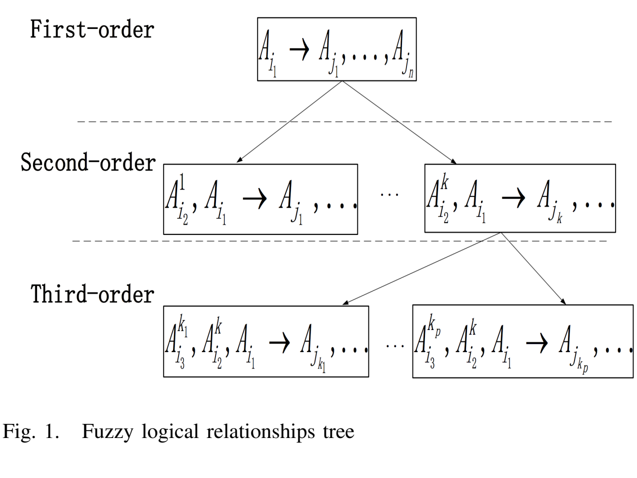

Fig. 1. Fuzzy logical relationships tree

In Fig.1, there are three layers from top to bottom. The top layer represents the first-order FLRG, where A_{i_1} represents the LHS of the first-order

FLRG, A_{j_1}, $\cdots$, A_{j_n} represents the RHS of the first-order FLRG, which means that there are n first-order fuzzy logic relationships in the group. The middle layer represents the second-order FLRG, where $A_{i_2}^k$, $A_{i_1} \rightarrow A_{j_k}$ represents the second-order FLRG containing LHS A_{i_1} of the first-order fuzzy relationships. In a similar way, the bottom layer represents the third-order FLRG, where $A_{i_3}^{k_p}$, $A_{i_2}^k$, $A_{i_1} \rightarrow A_{j_{k_p}} \cdots$ represents the third-order FLRG containing LHS $A_{i_2}^k, A_{i_1}$ of the second-order fuzzy relationships.

Step 6 Analyse the fuzzy variations in FLRGs. In the first-order FLRG, ie., $A_{i_1} \rightarrow A_{j_1}$, $\cdots$, A_{j_n}, $i_1 < j_k (k = 1, \cdots, n)$ represents the uptrend of fuzzy relationships, $i_1 > j_k (k = 1, \cdots, n)$ represents the downtrend of fuzzy relationships, and i_1=$j_k (k = 1, \cdots, n)$ represents the invariant trend of fuzzy relationships. Let $p_{1,u}$ be the number of the uptrend in the first-order FLRG, $p_{1,d}$ be the number of the downtrend and $p_{1,e}$ be the number of the invariant trend. For the second-order FLRG, ie., $A_{i_2}^k$, $A_{i_1} \rightarrow A_{j_k}$, $\cdots$, and the third-order FLRG, ie., $A_{i_3}^{k_p}$, $A_{i_2}^k$, $A_{i_1} \rightarrow A_{j_{k_P}}$, the uptrend, downtrend and invariant trend of fuzzy relationships are subject to the first-order variations therein.

Step 7 To calculate the forecasting value, the following heuristic rules are defined

- Rule 1: If the multiple orders fuzzy logical relationships are found as follows

$$A_{i_1} \rightarrow A_{j_1}, \cdots$$

$$A_{i_2}^k, A_{i_1} \rightarrow A_{j_k}, \cdots$$

$$A_{i_3}^{k_p}, A_{i_2}^k, A_{i_1} \rightarrow A_{j_{k_p}}, \cdots ,$$

the forecasting values $F(t)_1, F(t)_2$ and $F(t)_3$ based on first-order, second-order and third-order FLRGs are respectively

$$\begin{aligned} F(t)_i = (Y(t-1) - \frac{k}{16}) \times \frac{p_{i,d}}{p_{i,d}+p_{i,e}+p_{i,u}} \\ +Y(t-1) \times \frac{p_{i,e}}{p_{i,d}+p_{i,e}+p_{i,u}} \\ +(Y(t-1) + \frac{k}{16}) \times \frac{p_{i,u}}{p_{i,d}+p_{i,e}+p_{i,u}} \end{aligned} \quad (1)$$

where $F(t)_i (i = 1, 2, 3)$ denotes the forecasting value derived from the ith-order FLRG, $Y(t-1)$ stands for the actual value at the time $t-1$, k denotes the length of intervals, and $p_{i,d}, p_{i,u}$ and $p_{i,e}$ $(i = 1, 2, 3)$ respectively denote the numbers of forecasting downtrend, uptrend and invariant trend of the ith-order fuzzy logical relationship group. Then, different weights are assigned to the different orders of fuzzy logical relationships, in which higher orders have larger weights and lower orders have smaller weights. Here we set the weight n for nth-order FLRG according to the empirical researches. Finally, the final forecasting values $F(t)$ could be achieved

$$F(t) = \frac{F(t)_1 \times 1 + F(t)_2 \times 2 + F(t)_3 \times 3}{1+2+3} \quad (2)$$

- Rule 2: If only the first-order fuzzy logical relationships are found, the final forecasting values $F(t)$ could be achieved

$$F(t) = F(t)_1 \quad (3)$$

- Rule 3: If only the first-order and the second-order fuzzy logical relationships are found, the final forecasting values $F(t)$ could be achieved

$$F(t) = \frac{F(t)_1 \times 1 + F(t)_2 \times 2}{1+2} \quad (4)$$

- Rule 4: If there is no fuzzy logical relationship is found, the final forecasting values $F(t)$ could be achieved

$$F(t) = \frac{Y(t-1) \times 3 + Y(t-2) \times 2 + Y(t-3) \times 1}{1+2+3} \quad (5)$$

where $Y(t-1)$, $Y(t-2)$ and $Y(t-3)$ represent respectively the actual values of time $t-1$, time $t-2$ and time $t-3$.

In the following, we use an example to illustrate the forecasting process of the proposed model based on the TAIEX. The data from January 2004 to October 2004 is used for training, and the data from November 2004 to December 2004 is used for testing.

[Step 1] Based on the aforementioned training data from TAIEX, the minimum value D_{min} and the maximum value D_{max} can be known, i.e., D_{min}=5316.87 and D_{max}=7034.1 respectively, and the calculated standard deviation σ=456. With the length of each interval be half of the standard deviation σ, the positive real values D_1 and D_2 could be set 16.87 and 89.9 respectively. Then, the universe of discourse becomes U=$[D_{min} - D_1, D_{max} + D_2]$=$[5300, 7124]$, which could be divided into 8 intervals u_1, u_2, $\cdots$, u_8 of equal length, where u_1=$[5300, 5528)$, u_2=$[5528, 5756)$, $\cdots$, u_8=$[6896, 7124]$.

[Step 2] Based on the generated 8 intervals $u_1, u_2, \cdots, u_8$, the fuzzy sets A_1, A_2, $\cdots$, A_8 could be defined as follows

$$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + \cdots + 0/u_7 + 0/u_8$$

$$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + \cdots + 0/u_7 + 0/u_8$$

$$\cdots$$

$$A_8 = 0/u_1 + 0/u_2 + 0/u_3 + \cdots + 0.5/u_7 + 1/u_8$$

[Step 3] Fuzzify each training datum into the corresponding fuzzy set based on the fuzzy sets A_1, A_2, $\cdots$, A_8 defined in *[step 2]*. For example, the historical training data of TAIEX on January 2^{nd} is 6041.56, which belongs to the interval u_4=$[5985, 6212]$, and the maximum membership value of fuzzy set A_4 occurs at u_4, so the historical training data 6041.56 is fuzzified into A_4.

[Step 4] Construct first-order, second-order and third-order fuzzy logical relationships respectively based on the fuzzied historical training data from TAIEX. Moreover, FLRGs of first-order, second-order and third-order are constructed respectively. For example, the first-order FLRG with the LHS be A_4 of day $t-1$ is $A_4 \rightarrow A_4 \times 14, A_3 \times 6, A_6, A_5 \times 3$, where ”×” represents the number of the same first-order fuzzy logical relationship based on the historical training data from TAIEX. Next, the second-order FLRGs constructed based on the first-order FLRG are as follows

$$A_4, A_4 \rightarrow A_4 \times 9, A_3 \times 3, A_5, A_6$$

$$A_5, A_4 \rightarrow A_5 \times 2, A_4 \times 2$$

$$A_3, A_4 \rightarrow A_3 \times 3, A_4 \times 2.$$

One of the constructed second-order FLRGs is A_4, $A_4 \rightarrow A_4 \times 9$, $A_3 \times 3$, A_5, A_6. Further, the third-order FLRGs are constructed based on the second-order FLRGs in a similar way. One of the constructed third-order FLRGs is $A_4, A_4, A_4 \rightarrow A_4 \times 5, A_5$.

[Step 5] Construct fuzzy logical relationships trees with three layers based on the FLRGs of three orders obtained in Step 4 as shown in Fig .2.

For example, to forecast the value on December 30^{th}, the fuzzy values of 27^{th}, 28^{th} and 29^{th}, i.e., A_4, A_4 and A_4, are used to search the FLRGs from top to bottom according to the constructed tree in Fig.2. The resulting FLRGs are as follows

First-order	$A_4 \rightarrow A_4 \times 14, A_3 \times 6, A_6, A_5 \times 3$
Second-order	$A_4, A_4 \rightarrow A_4 \times 9, A_3 \times 3, A_5, A_6$
Third-order	$A_4, A_4, A_4 \rightarrow A_4 \times 5, A_5$

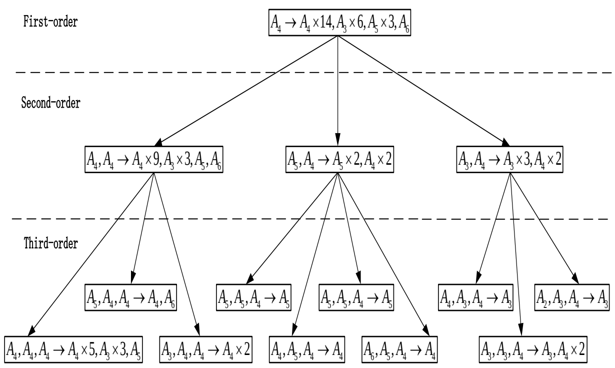

Fig. 2. A fuzzy logical relationships tree of TAIEX

[Step 6] Analyse the fuzzy variations in FLRGs. According to Fig.2, for the each order FLRG, $A_4 \rightarrow A_3$ represents the downtrend, $A_4 \rightarrow A_4$ represents the invariant trend, and $A_4 \rightarrow A_5$, A_6 represents the uptend. The numbers of the uptrend, downtrend and invariant trend of each order FLRG are illustrated in Table I.

TABLE I
PARAMETERS OF FORECASTING TREND

	$p_{i,d}$	$p_{i,e}$	$p_{i,u}$
$i=1$	6	14	4
$i=2$	3	9	2
$i=3$	3	5	1

In Table I, $p_{i,d}$, $p_{i,e}$ and $p_{i,u}$ represent respectively the downtrend, invariant trend and uptrend for the ith order FLRG.

[Step 7] From *[step 6]*, we can see that three orders of fuzzy logical relationship groups exist for the forecasting of the value on December 30^{th}, 2004, so Rule 1 is adopted. We know the actual value of December 29^{th}, 2004 is 6088.49, the forecasting value is calculated by Rule 1 with the parameter $Y(t)$=6088.49, k=328. According to Eq.(1), $F(t)_1$, $F(t)_2$ and $F(t)_3$ can be calculated

$$F(t)_1 = \frac{6074.24\times6+6088.49\times14+6102.74\times4}{6+14+4},$$

$$F(t)_2 = \frac{6074.24\times3+6088.49\times9+6102.74\times2}{3+9+2},$$

$$F(t)_3 = \frac{6074.24\times3+6088.49\times5+6102.74\times1}{3+5+1}.$$

Then the final forecasting value $F(t)$ on December 30^{th}, 2004 could be achieved according to Eq.(2)

$$F(t) = \frac{F(t)_1 \times 1 + F(t)_2 \times 2 + F(t)_3 \times 3}{1+2+3} = 6086.37.$$

IV. EXPERIMENTAL RESULTS

In this section, the forecasting performance of the proposed model will be examined based on two datasets. One is the TAIEX from 2001 to 2004, where the data from January to October of each year is used as training data, and the data in November and December of each year is used as testing data. The other dataset is the enrollments of Alabama University from 1971 to 1992, where the enrollments data from 1971 to 1991 is used as training data, and the enrollments data from 1994 to 1992 is used as testing data.

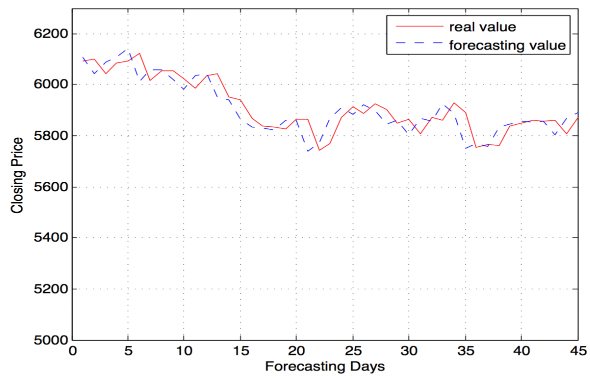

Fig. 3. Forecasting value of 2003

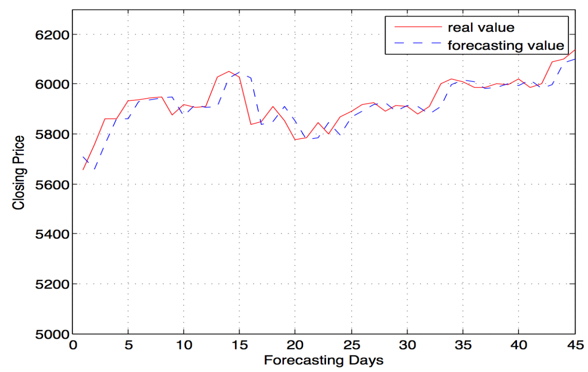

Fig. 4. Forecasting value of 2004

TABLE II
COMPARISON OF THE RMSEs AND THE AVERAGE RMSEs FOR DIFFERENT MODELS

Models			2001	2002	2003	2004	Average RMSE
Uni-variate	Conventional Regression Model[21], [22]		1070	116	329	146	415.25
	Neural Network Model[19], [20]		259	78	57	60	113.5
	Neural Network-Based Fuzzy Time Series Model[23], [19], [20]		130	84	56	116	96.5
	Neural Network-Based Fuzzy Time Series Model with Substitutes[23], [19], [20]		130	84	56	116	96.5
Multi-variate	Huarng et al.'s Model[18]	Use NASDAQ	136.49	95.15	65.51	73.57	92.68
		Use DOW Jones	138.25	93.73	72.95	73.49	94.605
		Use M1B	133.26	97.1	75.23	82.01	96.9
		Use DOW Jones, M1B,NASDAQ	124.02	95.73	70.76	72.35	90.715
	Chen's Fuzzy Time Series Model[5], [19], [20]		148	101	74	84	101.75
	Bivariate Conventional Regression Model[19], [20]		120	77	54	85	84
	Bivariate Neural Network Model[19], [20]		130	80	58	67	83.75
	Chen and Chang's Model[14]	Use NASDAQ	115.08	73.06	66.36	60.48	78.745
		Use Dow Jones	113.7	79.81	64.02	82.32	84.994
		Use Dow Jones, NASDAQ	113.33	72.33	60.29	68.07	78.505
		Use NASDAQ,M1B	116.59	76.48	53.51	69.29	78.9675
		Use Dow Jones, NASDAQ,M1B	113.67	66.82	56.1	64.76	75.3375
	Chen and Chen's Model[24]	Use Dow Jones	121.98	74.65	66.02	58.89	80.385
		Use NASDAQ	123.12	71.01	65.14	61.94	80.3025
		Use Dow Jones, NASDAQ	123.85	71.98	58.06	57.73	77.905
		Use M1B,Dow Jones	115.33	77.96	60.32	65.86	79.8675
		Use M1B,NASDAQ	123.15	74.05	67.83	65.09	82.53
	Chen and Chu's Model[25]	Use TAIFEX	120.3	72.23	56.89	55.4	76.205
		Use Dow Jones	117.18	68.45	53.96	52.55	73.035
		Use NASDAQ	114.81	69.07	53.16	53.57	72.6525
		Use M1B	117.75	70.63	54.92	55.29	74.6475
The Proposed Model			113.10	66.71	52.24	54.89	71.73

The performance of the proposed model is evaluated using the root-mean square error (RMSE), which is defined as follows

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n} (F(t) - Y(t))^2}{n}}$$

where n denotes the number of days needed to be forecasted, $F(t)$ is the final forecasting value of day t, $Y(t)$ is the actual value of day t.

Firstly, the proposed model is verified based on the TAIEX data of 2003 and 2004. In each year, 250 samples are provided, in which, 200 samples are used for training, 45 samples are used for testing. The forecasting closing prices in 2003 and 2004 are shown in Fig.3 and Fig.4 respectively.

In Fig.3 and Fig.4, the solid line represents real values of 45 testing samples, while the dotted line represents their forecasting values. From the figures we can see that the two lines fit well. The RMSE of 2003 is 53.33 and RMSE of 2004 is 54.89 respectively.

Next, based on the same TAIEX data, the performance of the proposed model is verified by comparing forecasting results with those of the existing models proposed in [5], [14],[18],[19],[20],[21],[22],[23], [24], [25] based on data from 2001 to 2004.

In those models for comparison, some models adopt univariate models, while the others adopt multi-variate models to improve the forecasting accuracy. The univariate models for comparison include the conventional regression model [21], [22], the neural network model [19], [20] and the neural network-based fuzzy time series model [19], [20], [23]. The conventional regression model exploits linear regression without considering fuzziness. The neural network model exploits neural network to train the relationship of input and output. And the neural network-based fuzzy time series model employs the neural network model with the fuzzy value instead of numerical value being the input and the output. While the multi-variate models for comparison contain Huarng's model [18], bivariate conventional regression model [19], [20], bivariate neural network model [19], [20] and Chen's models [5], [14], [19], [20], [24], [25]. Huarng's model considers the fuzzy logical relationships of multiple time series. The bivariate conventional regression model exploits the bivariate linear regression method without considering the fuzziness. The bivariate neural network model extends the input of neural network from single time series to multiple time series. Chen's models exploit respectively multivariate fuzzy clustering, fuzzy variation groups and automatically generated weights of multiple factors to improve forecasting accuracy. The forecasting results in terms of RMSEs and the average RMSEs from 2001 to 2004 are all shown in Table II.

TABLE III
COMPARISON OF THE RMSES FOR FORECASTING THE ENROLLMENTS OF ALABAMA UNIVERSITY

Models	Song's Model[2]	Chen's Model [5]	Huarng's Model [6]	Singh's Model [26]	Aladag's Model [27]	Proposed Model
RMSE	642	638	353	295	279	216

It can be seen from Table II that the RMSE of each year and the average RMSE of all years of the proposed model are all smaller than the comparative models. Moreover, only univariate is adopted for simplification in our proposed model, the higher forecasting accuracy could be achieved, which is even higher than those of the existing multivariate fuzzy time series forecasting models.

Finally, the proposed fuzzy time series model is applied on the enrollments of Alabama University from 1971 to 1992. The performance of the proposed model is compared with those of existing models by RMSE. In those models, Singh's fuzzy time series model exploit robust method to improve the forecasting performance; Aladag's model exploit neural networks to replace the fuzzy logical relationships in training and testing stage. The experimental results are shown in Table III.

It can be seen from Table III that the forecasting RMSE of the proposed model is 216, the forecasting RMSEs of Song's, Chen's, Huarng's, Singh's and Aladag's model are 642, 638, 353, 295 and 279 respectively. The proposed model gets better forecasting performance.

V. CONCLUSION

In this paper, we propose a novel fuzzy time series model based on fuzzy logical relationships tree containing multi-orders relationships, in which the robustness of lower orders fuzzy logical relationships and the precision of higher orders fuzzy logical relationships are combined together to improve the forecasting accuracy. Moreover, univariate is adopted in our proposed model. We compare the proposed model with some existing forecasting models including univariate and multivariate models. Experiment results show that the proposed model has better forecasting performance, and is more simple and easy to be implemented. In the future, we will introduce fuzzy logical relationships tree into multi-variate models.

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