Cascading Failures in a New Multi-coupling-links Three-dimensional Coupled Networks Based on Coupled Map Lattices Model

Wen-ju Du, Jian-gang Zhang, Shuang Qin and Jian-ning Yu

***Abstract*—The paper presented a new multi-coupling-links scale-free coupled networks model based on the existing scale-free network model. More precisely, we investigated the cascading failures of multi-coupling-links three-dimensional coupled networks based on coupled map lattices model. And through Matlab simulation, this paper detailedly studied the cascading failure under different attack strategies, different coupling strength and different network's scale. The results indicate that the deliberate attack has a faster transmission speed than the random attack under the same perturbation amplitude; the greater coupling strength of the network, the more likely to occur global failure; the coupled networks performs more robust to resist cascading failures if each sub-network possess a larger average scale; and under the deliberate attack, the network in a isolated state is more vulnerable than in a coupled state.**

***Index Terms*—Three-dimensional coupled networks, Coupled map lattices, Cascading failures, Coupling strength**

I. INTRODUCTION

Since the end of the 20th century, the research of complex network has permeated to many different areas such as mathematical disciplines, life science and engineering disciplines and so on. The scientific understanding of quantitative and qualitative characteristics of complex networks has become an extremely important challenging task in the network era of scientific research, even referred to as "the new science of the networks" [1, 2]. Networking is the mainstream direction of the development of human society today, and the study of complex networks also become more and more important. In recent years, the study of complex networks and its applications has made significant achievements [3-6]. Cascading failures is an important research direction in the study of complex networks, sometimes also called "avalanche". In many real world networks, failure occurs in a single node or multiple nodes or edge will cause to the other node failure which leads to the whole network appears the cascading failures [7]. And in the real world the cascading failures frequent occurrence. For example, the three ultra-high voltage transmission lines overloads in Ohio which caused the America widespread power outages in August 14, 2003. In January 2008, the snow disaster in southern China has destroyed the southern city's power system, railway and highway transportation system. And on March 30, 2009, many places of Sydney downtown happens a massive power outage at the peak of the rush hour, which lead to more than 100 traffic lights not available and caused a serious traffic jam. These events are the result of the cascading failures of network, and caused serious influence on the city's social economy. Disasters like these have prompted many researchers to pay more attention to the robustness of the network, and call it dynamic robustness.

In order to meet the requirements of the safety and reliability of the complex network of people's livelihood, the researchers have also made a lot of effort. However, the large-scale cascading failures on the network still occur. Theoretical modeling is an important means to study cascading failures. Therefore, it is necessary to make an in-depth study on the generation mechanism of successive failures and the prevention and control of cascading failures from the aspect of cascading failure model. The temporal, spatial, and state variables of most spatiotemporal systems are continuous and are suitable for the partial differential equations to describe. And both theoretical analysis and numerical calculation are more complicated, and the amount of calculation is also large. Coupled map lattices is an effective method to discretize the time and space variables, and the state variables are maintained continuously, which can not only serve the above-mentioned shortcomings, but also show the complex spatial and temporal characteristics of the system. It is a nonlinear dynamic system with discrete time and space, and the state is continuous, and it has become a powerful tool to study spatiotemporal chaos. So the establishment and research of cascading failure of complex network model has a significant theory meaning and application value.

In recent decades, the research and application of cascading failures has become a hot topic [8-12]. Wang et al. [13] expounded the cascading failures in global coupling network, scale-free network and small world network based on CML. Cui et al. [14, 15] investigated the node's cascading

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failures of the small-world networks with community structure and scale-free networks. Chen et al. [16] constructed an urban traffic network model based on CML, and studied the cascading failures problem of the model. Fan and Yeung [17] proposed that by protecting the nodes which adjacent to initial fault nodes can effectively control the failure propagation of the scale-free network model based on CML. At present, the research on the cascading failures of network is mainly focused on the isolated network. But in real life, a lot of networks are not completely isolated, instead of coupled together on the basis of a certain logical relations or functions. Such as the road network, rail network, aviation network and shipping network constitutes the composite transportation network, and the electric power network and computer network constitute the coupled network through a certain logical relations. Therefore, the cascading failures of coupled networks have more realistic significance. And many scholars have been studied the two layers coupled network [18-22]. But the research on cascading failures of coupled networks is relatively few [23-28]. Therefore, we proposed a new multi-coupling-links three-dimensional coupled networks, and analyzed the coupled network's cascading failures under different attack strategy, different external perturbation and different coupling strength.

The paper is organized as follows. A new multi-coupling-links three-dimensional coupled network is established based on coupled map lattices model in section 2. In section 3, we investigated the cascading failures under different attack strategies, different coupling strength and different network's scale through Matlab simulation. In section 4, we conclude the paper.

II. MULTI-COUPLING-LINKS THREE-DIMENSIONAL COUPLED NETWORK BASED ON CML

A. Multi-coupling-links scale-free coupled networks model

We first constructed a new coupled network, as shown in Fig. 1. And three sub-networks A, B and C constitute the three-dimensional coupled network. The number N_A, N_B and N_C represents the size of sub-networks A, B, C, respectively. The edges of sub-networks A, B, C are defined as internal edges, and the edges which connected sub-network A, B and C are called coupling edges. We do not restrict that each node has at most one coupling edges, and each node can possess the multiple coupling edges. So, the edges of each node consist of two components: internal edges and coupling edges. Accordingly, the degree of each node also consists of two components: internal degree and external degree. The internal degree k_I is defined as the number of the internal edges of each node and the external degree k_O refers to the number of the coupling edges of each node has. The degree of each node is equal to the sum of internal degree and external degree.

The empirical study found that the node degree of majority network obey power-law distribution, and it can be described as $P(k) \propto k^{-\gamma}$, where k is the node degree, $\gamma > 0$ is the scale-free networks exponent. And the network which degree distribution obeys power-law distribution called a scale-free network. We adopt the classical BA scale-free network model to generate the sub-networks A, B, C, and using both growth and preferential attachment mechanism to generate BA scale-free model [29].

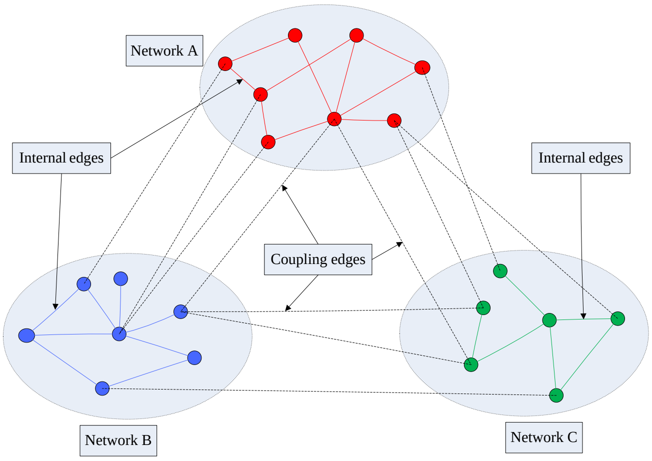


Fig. 1. Three-dimensional coupled networks model and schematic diagram for cascading failures propagation

B. Coupled networks cascading failures model based on the CML

Consider the following three CMLs of N_A, N_B, N_C nodes described as follows:

$$\begin{aligned} x_i(t+1) = \Bigg| &(1-\varepsilon_A-\varepsilon_{AB}-\varepsilon_{AC}) f(x_i(t)) + \varepsilon_A \sum_{j=1, j\neq i}^{N_A} \frac{a_{ij} f(x_j(t))}{k_I(i)} \\ &+ \varepsilon_{AB} \sum_{k=1}^{N_B} \frac{d_{ik} f(y_k(t))}{k_{OB}(i)} + \varepsilon_{AC} \sum_{l=1}^{N_C} \frac{d_{il} f(z_l(t))}{k_{OC}(i)} \Bigg|, \quad i = 1, 2, \cdots, N_A, \end{aligned} \tag{1}$$

$$\begin{aligned} y_n(t+1) = \Bigg| &(1-\varepsilon_B-\varepsilon_{BA}-\varepsilon_{BC}) f(y_n(t)) + \varepsilon_B \sum_{k=1, k\neq n}^{N_B} \frac{b_{nk} f(y_k(t))}{k_I(n)} \\ &+ \varepsilon_{BA} \sum_{j=1}^{N_A} \frac{d_{nj} f(x_j(t))}{k_{OA}(n)} + \varepsilon_{BC} \sum_{l=1}^{N_C} \frac{d_{nl} f(z_l(t))}{k_{OC}(n)} \Bigg|, \quad n = 1, 2, \cdots, N_B, \end{aligned} \tag{2}$$

$$\begin{aligned} z_m(t+1) = \Bigg| &(1-\varepsilon_C-\varepsilon_{CA}-\varepsilon_{CB}) f(z_m(t)) + \varepsilon_C \sum_{l=1, l\neq m}^{N_C} \frac{c_{ml} f(z_l(t))}{k_I(m)} \\ &+ \varepsilon_{CA} \sum_{j=1}^{N_A} \frac{d_{mj} f(x_j(t))}{k_{OA}(m)} + \varepsilon_{CB} \sum_{k=1}^{N_B} \frac{d_{mk} f(y_k(t))}{k_{OB}(m)} \Bigg|, \quad m = 1, 2, \cdots, N_C, \end{aligned} \tag{3}$$

where $x_i(t), y_n(t), z_m(t)$ are the state variables of the i th node of sub-network A, the n th node of sub-network B and the m th node of sub-network C at the t th time step, respectively. The adjacency matrix $(a_{ij})_{N_A \times N_A}, (b_{nk})_{N_B \times N_B}$ and $(c_{ml})_{N_C \times N_C}$ refers to the topology of sub-networks A, B, C, respectively. If there exist an edge between node i and j in sub-network A, then $a_{ij} = a_{ji} = 1$; otherwise, $a_{ij} = a_{ji} = 0$. If there exist an edge between node n and k in sub-network B, then $b_{nk} = b_{kn} = 1$; otherwise, $b_{nk} = b_{kn} = 0$. And if there exist an edge between node m and l in sub-network C, then $c_{ml} = c_{lm} = 1$; otherwise, $c_{ml} = c_{lm} = 0$. d_{ik} is the coupling matrix between sub-networks A and B, d_{il} is the coupling matrix between sub-networks A and C, d_{nj} is the coupling matrix between sub-networks B and A, d_{nl} is the coupling matrix between sub-networks B and C, d_{mj} is the coupling matrix between sub-networks C and A, d_{mk} is the coupling matrix between sub-networks C and B, and $d_{ik}=(d_{nj})^{\mathrm{T}}$, $d_{il}=(d_{mj})^{\mathrm{T}}$, $d_{nl}=(d_{mk})^{\mathrm{T}}$. $k_I(i)$ and $k_{OB}(i)+k_{OC}(i)$ represents the internal degree and external degree of node i, $k_I(n)$ and $k_{OA}(n)+k_{OC}(n)$ represents the internal degree and external degree of node n, $k_I(m)$ and $k_{OA}(m)+k_{OB}(m)$ represents the internal degree and external degree of node m. $\varepsilon_A,\varepsilon_B$ and ε_C are the coupling strength between the internal nodes of sub-networks A,B,C, respectively. $\varepsilon_{AB},\varepsilon_{BA}$ represents the coupling strength between the coupling nodes of sub-networks A and B, $\varepsilon_{AC},\varepsilon_{CA}$ represents the coupling strength between the coupling nodes of sub-networks A and C, $\varepsilon_{BC},\varepsilon_{CB}$ represents the coupling strength between the coupling nodes of sub-networks B and C. The function f defines the local dynamics which is chosen as the chaotic Logistic map $f(x)=4x(1-x)$ in this work. And the absolute value in (1), (2) and (3) are to guarantee that the state of each node is always nonnegative.

The node i is said to be in a normal state at the s th time step if $0<x_i(t)<1, t\le s$. The node i is failed when $x_i(s)\ge 1$ at time s and in this case we assume that $x_i(t)\equiv 0, t>m$. If the initial state of all nodes in (1), (2) and (3) are random selected in interval $(0,1)$ and there is no any external perturbation, then the state of all nodes will be in normal states forever.

In order to show how the initial disturbance of a single node triggers cascading failures, an external perturbation $R\ge 1$ is added to the node c of sub-network A at the s th time step as following:

$$x_c(s)=\left|(1-\varepsilon_A-\varepsilon_{AB}-\varepsilon_{AC})f(x_c(s-1))+\varepsilon_A\sum_{j=1,j\neq i}^{N_A}\frac{a_{cj}f(x_j(s-1))}{k_I(c)}+\varepsilon_{AB}\sum_{k=1}^{N_B}\frac{d_{ck}f(y_k(s-1))}{k_{OB}(c)}+\varepsilon_{AC}\sum_{l=1}^{N_C}\frac{d_{cl}f(z_l(s-1))}{k_{OC}(c)}\right|+R, \quad (4)$$

In this case, the node c will be failed at s th time step and we have $x_c(t)\equiv 0$ for all $t>s$. At next time step, the state of the nodes which directly connected with node c will be affected by the state of $x_c(t)\ge 1$. The state of these nodes may be greater than 1 and this may lead to a new round of nodes failures. The number of failed nodes is defined as $S=\lim_{t\to\infty}S(t)$ when the number of fault nodes no longer increases, where $S(t)$ is the total number of failed nodes in coupled network before $(t+1)$ th time step.

III. Simulations and Results

In this paper, we use the Matlab software to simulate and the data in the figure is the average value of 100 trials. The initial states of all nodes are randomly generated from interval $(0,1)$. The fault scale $I=S/(N_A+N_B+N_C)$ is defined as the ratio of network failure nodes, which denotes the ratio of the number of failure nodes and the total number of network nodes.

In order to trigger the cascade events in the coupled networks, we just attack a node in sub-network A. Attacking a node is equivalent to adding a perturbation R to this node. We adopt two different strategies to trigger the cascade events of a CML model: random attack and deliberate attack. The former to attack randomly selected nodes, the latter to attack the key node of the network. In this paper, we consider two kinds of deliberate attack strategies: attack the node with largest internal degrees or attack the node with largest total degrees.

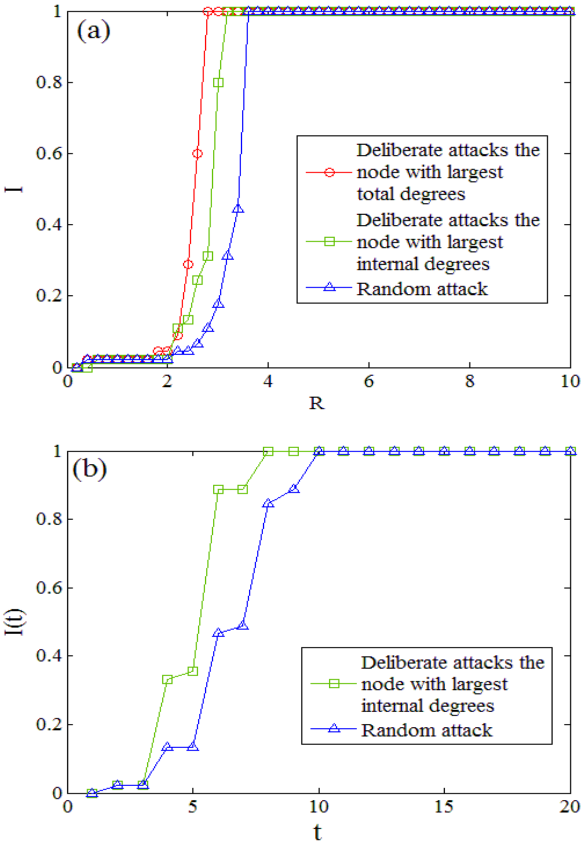
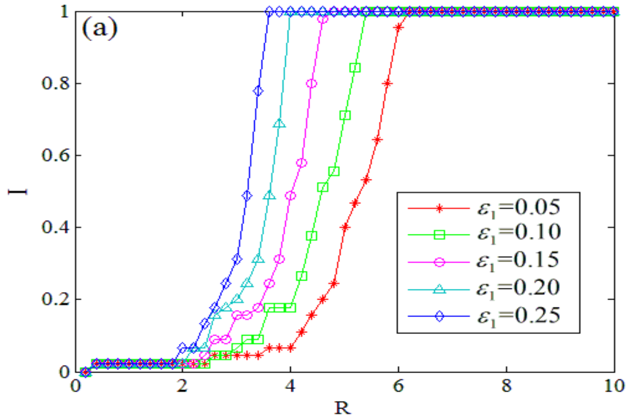


Fig. 2 (a) Relationship of the scale of failure and the perturbation amplitude, (b) Cascading failures process in a multi-coupling-links three-dimensional coupled networks based on coupled map lattices model when attack a single node of sub-network A

A. The cascading failures under different strategies

We attack a single node in sub-network A by using different attack strategies, and investigate the cascading failures of the coupled networks based on coupled map lattices model. We let $\varepsilon_A=\varepsilon_B=\varepsilon_C=\varepsilon_{AB}=\varepsilon_{BA}=\varepsilon_{AC}=\varepsilon_{CA}=\varepsilon_{BC}$ $=\varepsilon_{CB}=0.1, N_A=2000, N_B=1500, N_C=1000, m_0=3, m=3$ and plot the relation curve of the scale of failure I and the perturbation amplitude R, as shown in Fig. 2(a). From the simulation results we can find that the scale of failure is relatively small when the perturbation amplitude is small, and with the increase of perturbation amplitude the cascading failure of network is more likely to occur. And there is a threshold R_c for multi-coupling-links three-dimensional coupled networks. The failure scale is very small when $R<R_c$, namely at most a few nodes are failed. And when $R>R_c$, the failure scale increases rapidly to near or equal to 1. Obviously, the I-R curve of random attack is rise slowly than deliberate attacks when $R>R_c$. When we fixed disturbance amplitude R, and found that the most serious damage to the network when we attack the node with largest total degree, attack the node with largest internal degree damaged smaller and random attack damaged smallest.

Fig. 2(b) shows the cascading failures process of the multi-coupling-links three-dimensional coupled networks, which triggered by an initial perturbation R=3.6 . When we added the perturbation and before all nodes are failed $(2 < t < 10)$, in each moment the number of failed node under deliberate attack strategy is always greater than the random attack strategy's. The networks collapse within seven evolution steps under deliberate attack, and nine evolution steps under random attack. This shows that when the disturbance R is a certain value, the diffusion speed of destruction caused by deliberate attack is faster than the random attack case.



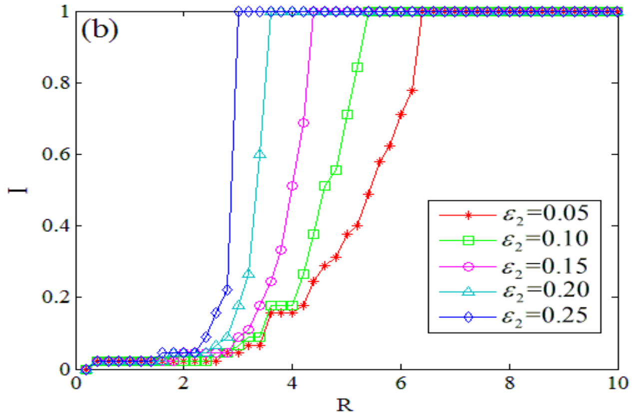
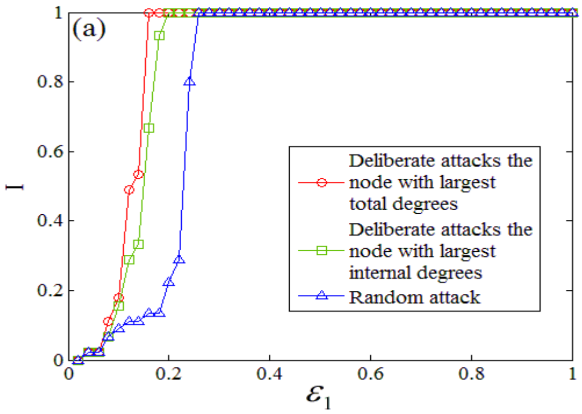


Fig. 3 Relationship of the scale of failure and the perturbation amplitude with different value of (a) ε_1 , (b) ε_2

B. The cascading failures under different coupling strength

Based on the previous description, we know $\varepsilon_A, \varepsilon_B, \varepsilon_C$ are represents the internal coupling strength of sub-networks A, B, C , and $\varepsilon_{AB}, \varepsilon_{BA}, \varepsilon_{AC}, \varepsilon_{CA}, \varepsilon_{BC}, \varepsilon_{CB}$ are refers to the external coupling strength between two sub-networks. In order to facilitate the analysis, we let $\varepsilon_A = \varepsilon_B = \varepsilon_C = \varepsilon_1, \varepsilon_{AB} = \varepsilon_{BA} = \varepsilon_{AC} = \varepsilon_{CA} = \varepsilon_{BC} = \varepsilon_{CB} = \varepsilon_2$. In this section, we take $N_A = 2000, N_B = 1500, N_C = 1000, m_0 = 3, m = 3, R = 3.6$ and investigate how the size of ε_1 and ε_2 affect the scale of cascading failures of the networks.

First, we attack the node with largest internal degree of sub-network A to trigger the cascade events, and the Fig. 3 shows the relationship of the scale of failure and the perturbation amplitude with different coupling strength value. Obviously, with the increase of ε_1 and ε_2 , the threshold R_c of cascading failures has gradually decreased. That's means a small perturbation can cause the networks occurs global failure when the coupled networks possess a larger internal coupling strength or external coupling strength.



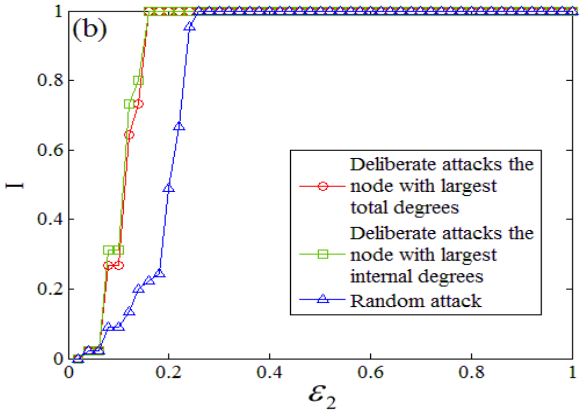


Fig. 4 (a) Relationship of the scale of failure and the internal coupling strength ε_1 , (b) Relationship of the scale of failure and the external coupling strength ε_2

Next, we attack a single node in the sub-network A , and investigate the cascading failures under the different attack strategies. We take $\varepsilon_1 = 0.1$ and then plot the relation curve of failure scale I and internal coupling strength ε_1 , as shown in Fig. 4(a). The Fig. 4(a) shows the failure scale is relatively small when the internal coupling strength ε_1 take a small value, and the larger internal coupling strength, the easier for cascading failure occurs. Fixed internal coupling strength ε_1 , and we find that compared with the random attack, the cascading failures are more likely to occur under deliberate attack. When $\varepsilon_1 = 0.1$, the Fig. 4(b) shows the relation curve of failure scale I and coupling strength ε_2 , and we can get a similar conclusion. Results indicate that the most serious damage to the network when we attack the node with largest total degree, attack the node with largest internal degree almost the same damaged, and above two deliberate strategies have more damage than random attack.

C. The effect of network's scale on the cascading failures

First, we let $\varepsilon_A = \varepsilon_B = \varepsilon_C = \varepsilon_{AB} = \varepsilon_{BA} = \varepsilon_{AC} = \varepsilon_{CA} = \varepsilon_{BC} = \varepsilon_{CB} = 0.1$, $R = 3.6, N_B = 1500, N_C = 1000, m_0 = 3, m = 3$ and investigate how the size of sub-network A affect the scale of cascading failures. As shown in Fig. 5, with the increase of N_A , the threshold R_c of cascading failures has gradually increased.

This means that a small perturbation can cause the global failure of the coupling networks when the size of sub-network A is relatively small. Namely, the larger the scale of sub-network A, the more difficult to occur global failure. The multi-coupling-links three-dimensional coupled networks perform more robust to resist cascading failures if each sub-network holds a larger average scale. Next, we take $N_A = 2000$, and attack the sub-network's node and the isolated network's node respectively, then the simulation results are shown in Fig. 6. Due to the mutual coupling effect of sub-networks A, B, C, the coupled network's complexity has increased. Obviously, compared with attack isolated network, the cascading failures are more likely to occur when attack the sub-network. That is to say, the isolated network is more vulnerable than coupled networks under deliberate attack.

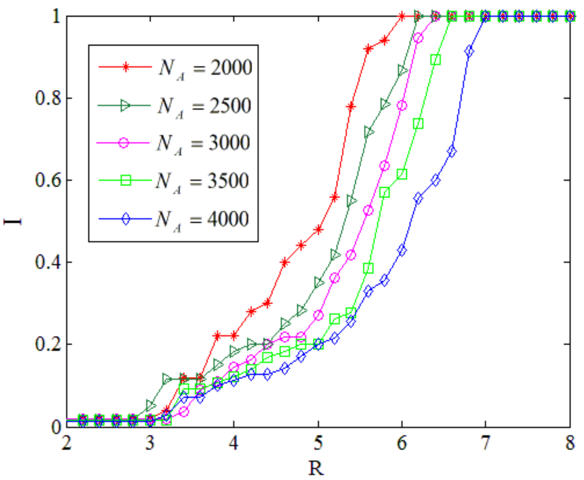


Fig. 5 Relationship of the scale of failure and the perturbation amplitude with different scale of sub-network A

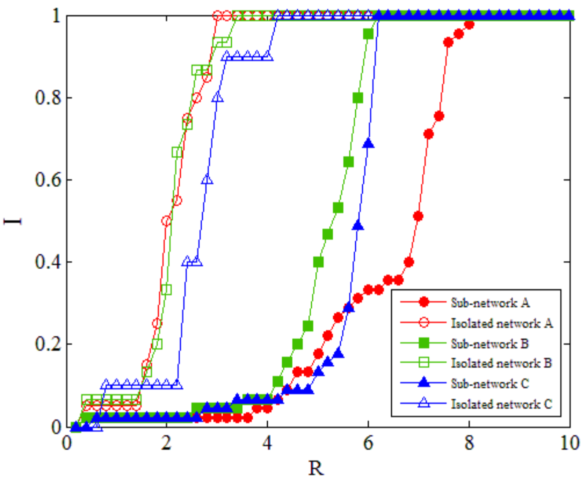


Fig. 6 Relationship of the scale of failure and the perturbation amplitude for the isolated network and the sub-network of coupled networks

IV. Conclusion

In real life, the possibility of occurrence of cascading failures is ubiquitous in many infrastructures, such as power network, traffic network and communication network. If there is a cascading failures in these networks, it will bring great losses to the development of economic and social. Therefore, in order to reduce and prevent the occurrence of cascading failures on these infrastructure networks. It is necessary for us to research the cascading failures, and take effective measures to prevent and control the occurrence of this phenomenon. In reality, most of the nodes of complex systems have many functions, and these functions are different in quality and cannot be superimposed, thus forming a multi-layer network. A variety of multi-layer networks around us has been everywhere, and just research on single-layer network is far from meeting the needs of reality. We must change the current relative deficient situation of lack of theory, methods and technical means in the multi-layer network. So we proposed a new multi-coupling-links scale-free coupled networks model and investigated the cascading failures of the networks based on coupled map lattices model. Through Matlab simulation technology, the cascading failures under different attack strategies, different coupling strength and different network's scale are studied respectively. In order to improve the anti-destroying ability of coupling networks, we should choose the appropriate coupling strength and network's scale to construct the coupled networks. To improve network robustness and optimize the network structure, these conclusions can provide decision-making basis for real network's planning, control, and management.

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