Portfolio Optimization based on Risk Measures and Ensemble Empirical Mode Decomposition

Chengli Zheng, Yinhong Yao

Abstract—**This study proposes a novel way to improve investors' total return rate of portfolio optimization by de-noising the data using Ensemble Empirical Mode Decomposition (EEMD). Firstly, the authors briefly introduce risk measure theory and EEMD methodology. Then, empirically demonstrating that the de-noising technique using EEMD surely has some efficient impact on the portfolio, and the cumulative return rate of the portfolio when the objective function is CVaR with the data de-noised 3 Intrinsic Mode Functions (IMFs) is the highest one. It indicates that the impact of de-noising the data using EEMD is much more significant on the portfolio when the objective functions have less powerful risk discrimination, and vice versa.**

Index Terms—**portfolio optimization, risk measures, Ensemble Empirical Mode Decomposition (EEMD), hypothesis test**

I. Introduction

PORTFOLIO optimization of the asset is a necessary way to maximize investors' total return rate. As the most efficient method to quantify risk, the risk measure theory receives widespread attention. In order to improve the cumulative return rate of portfolio optimization, the traditional way is to improve the property of risk measures, while in this paper, we propose a novel methodology from the perspective of de-noising securities' data series using Ensemble Empirical Mode Decomposition (EEMD).

From the perspective of traditional way, the quantitative analysis of modern financial portfolio theory date from the Portfolio Theory of Markowitz [1], which means that the portfolio would minimize the risk under certain expected return, or realize the optimization under certain risk. This theory introduced quantitative analysis to the financial field, laid the foundation of modern finance. However, high standard deviation doesn't truly mean a high level of risk, non-monotonicity is one of its defects.

VaR (Value at risk) was proposed by Philippe Jorion at the end of 1980s [2] which contains both the uncertainty and loss. While VaR only considers the quantile of the distribution without caring about what is happening to the left and to the right of the quantile, and it is concerned only with the probability of the loss, while does not care about the size of the loss [3].

In the late of 20th, Artzner et al. proposed the concept of coherent risk measure based on the axiomatic foundation, the coherent risk measure must satisfy monotonicity, translation invariance, positive homogeneous, and sub-additivity [4]. Then, Föllmer and Schied extended the coherent risk measure to convex risk measure [5-7], Frittelli and Gianin [8] defined the convex risk measure based on axioms, i.e. monotonicity, translation invariance and convexity.

CVaR (Conditional VaR) is one of the best choices of coherent risk measure [9], Kusuoka proved that CVaR is the smallest law invariant coherent risk measure that dominates VaR [10]. Wang and Ma proved that VaR is consistent with the first-order stochastic dominance, CVaR is consistent with the second-order stochastic dominances [11]. However, CVaR takes into consideration only the tail of the distribution.

Then, risk measures that pay more attention to the left tail of distribution were proposed. Krokhmal proposed HMCR (Higher Moment Coherent Risk measure) based on CVaR [12], Chen and Wang gave some proofs and derivations of the proprieties of P-norm (i.e. HMCR) [13]. Zheng and Yao proved that HMCR(p=n) is consistent with (n+1) th order stochastic dominance from the perspective of Kusuoka representation [14]. Zheng and Chen proposed iso-entropic risk measure based on relative entropy, which is obtained under the theoretical framework of the coherent risk measure, and proved that it is consistent with stochastic dominance of almost all the orders and it has the highest power of risk discrimination compared with VaR and CVaR [15-16].

However, noise can affect real information, which affects the efficiency of portfolio optimization. From the perspective way of the de-noising method, Huang et al. introduced EMD (Empirical Mode Decomposition) method, it is an empirical, intuitive, direct and self-adaptive data processing method which is proposed especially for nonlinear and non-stationary data [17]. The core of EMD is decomposing the target data into a small number of independent and nearly periodic Intrinsic Modes Functions (IMFs) and one residue. EEMD (Empirical EMD) was an improved version by Wu and Huang, which add a series of finite, not infinitesimal, amplitude white noise to overcome the mode mixing problem of EMD, and it is a truly noise-assisted data analysis method [18].

EEMD have been applied in many areas, such as biomedical engineering, structured health monitoring, earthquake engineering, etc. In social science area, Zhang et

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al. extended EEMD to crude oil price analysis, and proved that it is a vital technique [19]. Li et al. demonstrated that EEMD can be statistically proved to be much stronger and more robust than other popular prediction models when forecasting country risk [20]. Xu et al. applied EMD and EEMD to the cross-correlation analysis of stock markets, and proved that the EEMD method performs better on the orthogonality of IMFs than EMD for the stock data [21]. Tang et al. demonstrated that the EEMD-based multi-scale fuzzy entropy approach can provide a new analysis tool to understand the complexity of clean energy markets [22].

The objective of this study is to analyze the impact of de-noising data using EEMD on portfolio optimization based on six risk measures in the Chinese stock market. Firstly, the closing price series of stocks with different time ranges and frequencies are decomposed into several IMFs, from high to low frequencies. Then, after the IMFs were de-noised from the original data gradually, we calculate the daily logarithmic return rate of these de-noising data series. Finally, build the portfolio optimization based on these return rate matrixes when the objective functions are standard deviation, VaR, CVaR, convex entropy risk measure, iso-entropic risk measure, and HMCR, and analyze the difference of de-noising effect from one to four IMFs of different risk measures.

The rest of the paper is organized as follows: Section II gives a brief introduction of risk measure theory and compares the difference of five risk measures. Section III introduces the basic theory and algorithm of EMD and EEMD. Section IV is the portfolio optimization based on six risk measures and EEMD. Section V concludes.

II. RISK MEASURE THEORY

A. Acceptance set and coherent risk measure

Here we introduce some concepts related to coherent risk measures. More details see Artzner [4], Föllmer and Shied [7], Föllmer and Knispel [23].

In financial theory, the uncertainty of value for a position (a set or a portfolio) in the future is usually described by a random variable $X:\Omega\rightarrow R$ on a probability space (Ω,F,P). The goal of risk measure is to determine a number $\rho(X)$ that quantifies the risk and can serve as a capital requirement, i.e. as the minimal amount of capital which, if added to the position and invested in a risk-free manner, makes the position acceptable. For an unacceptable risk, one remedy may be to alter the position, another remedy is to look for some commonly accepted instruments, which added to the current position, make its future value become acceptable to the investor/supervisor. The current cost of getting enough of this or these instruments is a good candidate for a measure of risk of the initially unacceptable position. Based on this, a series of definitions are given as follows. In the sequel, G denotes a given linear space of function $X:\Omega\rightarrow R$ containing the constants. Let G be the set of all risks, that is the set of all real valued functions on Ω.

Definition 2.1. A *measure of risk* ρ is a mapping from G into R.

Definition 2.2. An *acceptance set*: We call A a set of final values, expressed in currency, are accepted by one investor/supervisor.

It must be pointed out that there are different acceptance sets for different investors/superiors because they are heterogeneous when faced with risk assets. There is a correspondence between acceptance sets and measures of risk.

Definition 2.3. *Risk measure associated to an acceptance set*: the risk measure associated to the acceptance set A is the mapping from G to R denoted by ρ_{A} and defined by:

$$\rho_{\mathrm{A}}(X)=\inf\{m\in R|m+X\in \mathrm{A}\} \tag{1}$$

The risk measure is the smallest amount of units of date 0 money which invested in the admissible asset, must be added at date 0 to the planned future net worth X to make it acceptable. Note that we work with discounted quantities.

Definition 2.4. *Acceptance set associated to a risk measure:* the acceptance set associated to a risk measure ρ is the set denoted by A_ρ and defined by

$$\mathrm{A}_\rho=\inf\{X\in \mathrm{G}|\rho(X)\le 0\} \tag{2}$$

Definition 2.5. A measure of risk ρ is called a *monetary risk measure* if $\rho(0)$ is finite and if ρ satisfies the following conditions for all $X,Y\in \mathrm{G}$,

Monotonicity: If $X\le Y$, then $\rho(X)\ge\rho(Y)$.

Translation invariance: If $c\in\mathrm{R}$, then $\rho(X+c)=\rho(X)-c$.

The financial meaning of monotonicity is clear: the downside risk of a position is reduced if the payoff profile is increased. Translation invariance is also called cash invariance. This is motivated by the interpretation of $\rho(X)$ as a capital requirement, i.e, $\rho(X)$ is the amount which should be raised in order to make X acceptable from the point of view of an investor/supervisor, as Definition 2.3. Thus, if the risk-free amount c is appropriately added to the position or to the economic capital, then the capital requirement is reduced by the same amount.

Definition 2.6. A monetary risk measure ρ is called a *convex risk measure* if ρ satisfies the following conditions,

Convexity: $\rho(\lambda X+(1-\lambda)Y)\le\lambda\rho(X)+(1-\lambda)\rho(Y)$, for $0\le\rho\le 1$.

The axiom of convexity give a precise meaning to the idea that diversification should not increase the risk, convex duality shows that a convex risk measure typically takes the following form,

$$\rho(X)=\sup_{Q\in\mathrm{Q}_\rho}\{E_Q[-X]-\alpha(Q)\} \tag{3}$$

Where, Q_ρ is some class of plausible probability measures on the underlying set of possible scenarios, and $\alpha(Q)$ is some penalty function on Q_ρ. The capital requirement is thus determined as follows: the expected loss of a position is calculated for the probability measures in Q_ρ, but these models are taken seriously to a different degree as prescribed by the penalty function.

Definition 2.7. A convex risk measure ρ is called a *coherent risk measure* if ρ satisfies the following conditions

Positive homogeneity: if $\lambda \ge 0$, then $\rho(\lambda X)=\lambda\rho(X)$.

Under the assumption of positive homogeneity, the convexity of a monetary risk measure is equivalent to *Subadditivity*: $\rho(X+Y)\le\rho(X)+\rho(Y)$.

Based on the convex risk measure, if investors take the worst penalized expected loss over the class Q_ρ, the penalty will vanish, i.e. $\alpha(Q)=0$, thus we get the special coherent case

$$\rho(X)=\sup_{Q\in Q_\rho} E_Q[-X] \tag{4}$$

So, a monetary risk measure must satisfy two axioms: monotonicity, translation invariance; a convex risk measure must add the axioms of convexity, a coherent risk measure must add the axioms of positive homogeneity and convexity or subadditivity.

B. *Definition of risk measure*

Here we introduce VaR, CVaR, convex entropic risk measure, iso-entropic risk measure and HMCR.

The first one is VaR at level $\alpha\in(0,1]$, VaR defined for X on a probability space (Ω, F, P) is

$$VaR_\alpha(X)=\inf\{m\in R \mid P[X+m<0]\le\alpha\} \tag{5}$$

From the Eq. 5, it is seen that the VaR is quantile-based, because $VaR_\alpha(X)=-q_\alpha(X)$, where $q_\alpha(X)$ is the $\alpha-$quantile of X. VaR satisfies monotonicity, translation invariance and positive homogeneity but not subadditivity, it is not convex.

The second one is CVaR, at level $\alpha\in(0,1]$, CVaR is defined as,

$$CVaR_\alpha(X)=\frac{1}{\alpha}\int_0^\alpha VaR_u(X)du=\frac{1}{\alpha}\int_0^\alpha -q_u(X)du \tag{6}$$

CVaR is also called Expected Shortfall (ES), Average Value at Risk (AVaR). CVaR satisfies monotonicity, translation invariance, positive homogeneity and subadditivity. It is a coherent risk measure.

The third one is convex entropic risk measure based on relative entropy. Firstly, the relative entropy is defined by

$$H(Q\mid P):=\begin{cases} E_Q\left[\log\dfrac{dQ}{dP}\right] & if\ Q=\ P \\ +\infty & otherwise\end{cases} \tag{7}$$

Relative entropy is also called Kullback-Leibler distance or information divergence. The penalty function of convex risk measure is noted by $\alpha(Q):=\frac{1}{m}H(Q\mid P), m>0$, m is a parameter, it can be calculated by relative entropy, then,

$$e_m(X):=\sup_{Q\in M_1}\left\{E_Q[-X]-\frac{1}{m}H(Q\mid P)\right\},\quad m>0 \tag{8}$$

Using the well-known variational principle,

$$H(Q\mid P)=\sup_{X\in L^\infty(\Omega,F,P)}\left\{E_Q[-X]-\log E_P\left[e^{-X}\right]\right\} \tag{9}$$

for the relative entropy, it follows that e_m takes the explicit form,

$$e_m(X)=\frac{1}{m}\log E_P\left[e^{-mX}\right] \tag{10}$$

Convex entropic risk measure satisfies monotonicity, translation invariance, convexity, it is a convex risk measure.

The fourth one is iso-entropic risk measure, it was proposed by Zheng and Chen [15] based on the representation theorem of coherent risk measure,

$$\rho_\alpha(X)=-E(Z^*X)=\int_0^1 -q_u(X)z_u(X,\alpha)du \tag{11}$$

where, $Z^*=z(X,\alpha)=\frac{e^{-mX}}{c}$, $z_u(X,\alpha)=\frac{e^{-mq_u(X)}}{c}$, $c=E\left[e^{-mX}\right]$.

And $m\ge 0$, it satisfies the following equation,

$$H_\alpha=E\left[\frac{e^{-mX}(-mX-\log c)}{c}\right] \tag{12}$$

Here $m=m(X,\alpha)$ is determined by X and α, H_α is the given relative entropy. Iso-entropic risk measure satisfies monotonicity, translation invariance, positive homogeneity and sub-additivity, is coherent risk measure.

The fifth risk measure is higher moment coherent risk measure base on higher moment norm, which is defined by

$$\begin{aligned}\rho_{c,p}(X)&=\inf_{\eta\in R}\left\{c\|(\eta-X)_+\|_p-\eta\right\}\\ &=\inf_{\eta\in R}\left\{\frac{1}{\alpha}\left\{E\left[((\eta-X)_+)^p\right]\right\}^{1/p}-\eta\right\}\end{aligned} \tag{13}$$

where, $p>1, c>1, \alpha\in(0,1]$.

When $p=1$, CVaR a special case of HMCR

$$CVaR_\alpha(X)=\inf_{\eta\in R}\left\{\frac{1}{\alpha}E\left[(\eta-X)_+\right]-\eta\right\} \tag{14}$$

When $p>1$

$$HMCR_\alpha(X)=\int_0^1 -q_t(X)\phi_t(X,\alpha)dt \tag{15}$$

Where, $\phi_t(X,\alpha)=\dfrac{\left|(\eta-q_t(X))_+\right|^{p-1}\cdot 1_{\eta>q_t(X)}}{E\left[\left|(\eta-q_t(X))_+\right|^{p-1}\cdot 1_{\eta>q_t(X)}\right]}$.

HMCR satisfies the coherent four axioms, it is also a coherent risk measure.

C. *Comparisons of these risk measures*

The properties of coherence are introduced in section B. Now, we compare these five risk measures in perspective of convexity, the volume of information which is used to measure risk and stochastic dominance.

Firstly, we consider the convexity of these risk measures. According to the four axioms, VaR is not convex on the whole space, while convex entropic risk measure is convex obviously, since convexity is a necessary condition of coherence, so CVaR, iso-entropy risk measure, and HMCR are coherent risk measures, also convex.

Secondly, we consider the volume of information which is used to measure the risk. For the quantile-based risk measures, a unified expression can be given as follows

$$R_\alpha(X)=-\int_0^1 q_t(X)\phi_t(X,\alpha)dt \tag{16}$$

According to the Section B, VaR only reflects information of one point of distribution, CVaR and HMCR reflect information of quantiles where $t\le\alpha$, convex entropic risk measure, iso-entropic risk measure reflects information of the whole distribution of risk asset. These differences of the volume of information used by these risk measures might be one of the reasons for the difference of risk discrimination.

Lastly, we consider the relationship between these risk measures and stochastic dominance in this section. According to Wong and Ma[11], Zheng and Chen[16], Zheng and Yao[14], under certain conditions, VaR is only consistent with the first-order stochastic dominance, it means that VaR can recognize two kinds of risk assets with first-order stochastic dominance; CVaR is consistent with the second-order stochastic dominances, it means that CVaR can recognize two kinds of risk assets with second or lower order stochastic dominances; iso-entropic risk measure is consistent with the nth-order stochastic dominances, and according to formula (16), convex entropic risk measure is the integral of iso-entropic risk measure, so we can deduce that entropy is consistent with the nth-order stochastic dominances, HMCR(p=n) is consistent with (n+1)th order stochastic dominances, these three kinds of risk measures can recognize two kinds of risk assets with high or lower order stochastic dominances. i.e. convex entropic risk measure, iso-entropic risk measure, and HMCR(p>3) have higher power risk discrimination.

III. ENSEMBLE EMPIRICAL MODE DECOMPOSITION (EEMD)

Ensemble Empirical Mode Decomposition (EEMD) proposed by Wu and Huang [18] is an improved version of Empirical Mode Decomposition (EMD) proposed by Huang et al.[17]. Therefore, this subsection firstly introduces the EMD technique, and then presents the EEMD technique in brief.

A. Empirical Mode Decomposition (EMD)

Empirical Mode Decomposition (EMD) technique is an adaptive time series decomposition technique for nonlinear and non-stationary data [17, 25]. It assumes that the data, depending on its complexity, may have many different modes of oscillations coexisting at the same time. The main purpose of EMD is to decompose the original data into a series of Intrinsic Mode Functions (IMFs), which must satisfy the following two conditions. ① In the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one. ② At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

With these two conditions, meaningful IMFs can be well defined. Usually, an IMF represents a single oscillatory mode, in contrast to simple harmonic function. Using the definition, any complicated data series $x_t\left(t=1,2,\cdots,T\right)$ can be decomposed, according to the following sifting process

1) Identify the local extrema of x_t, including both maxima and minima;

2) Generate its upper and lower envelopes, $x_{up,t}$ and $x_{low,t}$, with cubic spline interpolation;

3) Calculate the point-by-point mean m_i from upper and lower envelopes: $m_t=\left(x_{up,t}+x_{low,t}\right)/2$;

4) Extract the mean from the time series and define the difference between x_t and m_t as c_t, $c_t=x_t-m_t$;

5) Check the properties of c_t:

i) If c_t meets the above two conditions, and IMF is extracted and replace x_t with the residue $r_t=x_t-c_t$;

ii) If c_t is not an IMF, replace x_t with c_t;

6) Repeat steps 1-5 until the stop criterion is satisfied, i.e., when the residue r_t becomes a monotonic function from which no more IMFs can be extracted.

The total number of IMFs of a data set is close to $\log_2 N$ with N, the length of total data points. Using this sifting procedure, the original data series x_t can finally be expressed as a sum of IMFs and a residue,

$$x_t=\sum_{j=1}^{n}c_{j,t}+r_{n,t} \tag{17}$$

Where n is the number of IMFs, $r_{n,t}$ is the final residue, and $c_{j,t}\left(j=1,2,\cdots,n\right)$ is the j th IMF. All IMFs are nearly orthogonal to each other, and all have nearly zero means. Thus, the data series can be decomposed into n IMFs and one residue. The IMF components contained in each frequency band are different and they change with variation of time series x_t, while $r_{n,t}$ represents the central tendency of data series x_t.

B. Ensemble EMD(EEMD)

An obvious drawback of the original EMD is the frequent appearance of mode mixing, which is defined as either a single IMF consisting of signals of widely disparate scales, or a signal of a similar scale residing in different IMF components. To overcome this drawback, Wu and Huang [18] proposed Ensemble EMD (EEMD) technique. The basic principle of EEMD is that the added white noise would populate the whole time-frequency space uniformly with the constituting components of different scales separated by the filter bank. Substantially, an additional step of adding white noise is taken to help extract the true signal in the data. The process of EEMD is developed as follows,

1) Add a white noise series to the original data;

2) Decompose the data with added white noise into IMFs using the EMD procedure;

3) Repeat step1 and step 2 iteratively, but use different white noise series each time and obtain ensemble means of corresponding IMFs as the final results.

The added white noise series can help extract the true IMFs, and can offset themselves via ensemble averaging after serving their purpose. Therefore, this can substantially reduce the chance of mode mixing and represents a significant improvement over the original EMD. The effect of the added white noise can be controlled according to the well-established statistical rule proved by Wu and Huang [18].

$$\varepsilon_{ne}=\varepsilon/\sqrt{NE} \tag{18}$$

Where NE is the number of ensemble members, ε is the amplitude of the added noise, and ε_{ne} is the final standard deviation of error, defined as the difference between the input signal and the corresponding IMFs. The example was provided by Wu and Huang [18], and they demonstrated that this noise-assisted data analysis using EEMD significantly improved the capability of extracting signals from the data compared with the decomposition method using EMD. In practice, the number of ensemble members is often set to 100 and the standard deviation of white noise series is set to 0.1 or 0.2.

IV. EMPIRICAL ANALYSIS OF PORTFOLIO OPTIMIZATION

A. Portfolio optimization

Suppose there are N stocks, and each return rate is denoted by r_i, we structure a combination P, the weight of each stock is $\omega_i, i=1,2,\ldots,N$, so the return rate of the combination is $r_p=\sum\omega_i r_i$, under the given expected return rate $E\left[r_p\right]=\mu$, we don't consider short selling, the portfolio selection problems at a given level α is as follows:

$$\begin{aligned}&\min \rho_\alpha\left(r_p\right)\\ &s.t.\ E\left[r_p\right]=\mu,\ \sum\omega_i=1,\ \omega_i\geq 0\end{aligned} \tag{19}$$

Where the optimization objective functions are standard deviation, VaR, CVaR, convex entropic risk measure and iso-entropic risk measure based on modified relative entropy, and HMCR(p=1,2,3). We use the historical data to estimate these risk measures, calculated by the frequency of data. Then we get the in-sample coefficients of the portfolio, and calculate the cumulative return rate of the period out-of-sample.

B. Hypothesis test

In this paper, we use the hypothesis testing method to verify the result, and this method has ever been used in Chen et al. [26]. Let μ_0 and μ_1 denote two series of average cumulative return rate of the portfolio, in order to compare the value of these two series. we give the null hypotheses, i.e.,

$$H_0:\mu_0-\mu_1\geq 0,\ H_1:\mu_0-\mu_1<0$$

Then, we use paired z-test to compare the means of the two samples, the test statistic is calculated as

$$z=\frac{\bar{d}}{s_d/\sqrt{n}} \tag{20}$$

Where, $\bar{d}=\sum_{i=1}^{n}d_i/n$ is the mean difference between two paired samples, d_i is the value difference of i the paired samples, $i=1,2,\ldots,n$; $s_d^2=\sum_{i=1}^{n}\left(d_i-\bar{d}\right)/\left(n-1\right)$ is the sample variance, n is the sample size and z is a paired sample z-test with $n-1$ degrees of freedom.

C. Data Description

In order to reflect the impact of the data de-noising using EEMD to the portfolio in the stock market, we choose the constituent stocks of SSE 50 index from 2009.08.03 to 2014.03.21, which is much smoother, totally 1122 trading days, eliminating 15 stocks of missing data or halting more than 12 days, remaining 35 stocks. The data is separated into two periods, in-sample period (from 2009.08.03 to 2012.01.18, totally 800 trading days) as the historical sample to calculate optimization coefficient and out-of-sample period (from 2012.01.19 to 2014.03.21, totally 321 trading days) as the test data to investigate the performance.

Firstly, we use the daily logarithmic return rate of original data to build the portfolio, and investigate the performance of six risk measures. Then, de-noising the original data using EEMD technique from 1 to 4 IMFs, and using the logarithmic return rate of the data to build the portfolio based on risk measures, respectively, to investigate the impact of de-noising using EEMD to the performance in the portfolio of each risk measure during the out-of-sample period. We get the data form Wind, and use Matlab 8.2 to complete the calculation.

D. Portfolio optimization of original data

Now we get the results of portfolio optimization under different expected return rate. Firstly, taking equidistant 100 points between the minimum and maximum of average return rate of 35 stocks, so we get 101 points of $E\left[r_p\right]=\mu$, the level $\alpha=0.03$. Then we optimize the calculation of the portfolio whose objective functions are standard deviation, VaR, CVaR, convex entropic risk measure, iso-entropic risk measure, HMCR(p=3), and get the efficient frontier of these six risk measures like traditional Markowitz portfolio frontier, since HMCR(p=1) is equal to CVaR, HMCR(p=2) is quietly close to HMCR(p=3) theoretically, and the research on HMCR(p>3) is not included in this paper [14].

The efficient frontiers of MV, VaR, CVaR, entropy, iso-entropy and HMCR(p=3) was obtained by applying Formula (19). It indicates that VaR is not satisfied with convexity, and the size of five risk measure is mostly consistent with Formula (2.23) when the value of expected return rate covering $\mu=0.5e-3$ and $\mu=1.5e-3$, i.e. the value of HMCR(p=3) is larger than CVaR, iso-entropy, entropy, VaR, MV, which proved that HMCR(p=3), CVaR, iso-entropy, entropy are much more prudent that VaR and MV.

For rational investors, they would utilize the portfolio coefficients during the efficient level in the efficient frontier to build their portfolio, which might maximize their profit and reduce risk. Therefore, we build our portfolio based on the portfolio coefficients under the expected return rate between $\mu=0.8e-3$ and $\mu=1.2e-3$, the average cumulative return rate of the 15 portfolios with the objective function of these risk measures is shown in Table 1.

TABLE 1
CUMULATIVE RETURN RATE OF PORTFOLIO WITH ORIGINAL DATA

risk measures	cum return	risk measures	cum return
MV	0.0392	VaR	-0.007
CVaR	-0.0468	Entropy	0.1604
Iso-entropy	0.1153	HMCR(p=3)	0.1671

NOTE: cum. return represents the cumulative return rate

From Table 1, We find that the cumulative return rates of portfolios with the objective functions of HMCR(p=3), entropy and iso-entropy are larger than that of standard deviation, VaR and CVaR, which represents that risk measure that equipped with powerful risk discrimination also have strong power of portfolio optimization. While, the result of the portfolio with the objective functions are standard deviation, VaR, CVaR is contracted to the theoretical analysis, it might be related to the properties of the Chinese stock market.

E. Portfolio optimization of de-noising data

In this subsection, we use the daily closing price of Shanghai Pudong Development Bank as an example to illustrate the result of the decomposition using EEMD. The number of ensemble members is set to 100, and the standard deviation of white noise series is set to 0.1. As shown in Figure 1, S1 to S8 represents the component of the original data in different average frequencies respectively, and the residue (S0) represents the trend of the original data (S) by filtering out the high-frequency components, 34 other stocks are also decomposed in this way. Then, we use the logarithmic return rate of de-noising data that remove 1 IMF to 4 IMFs to build portfolios.

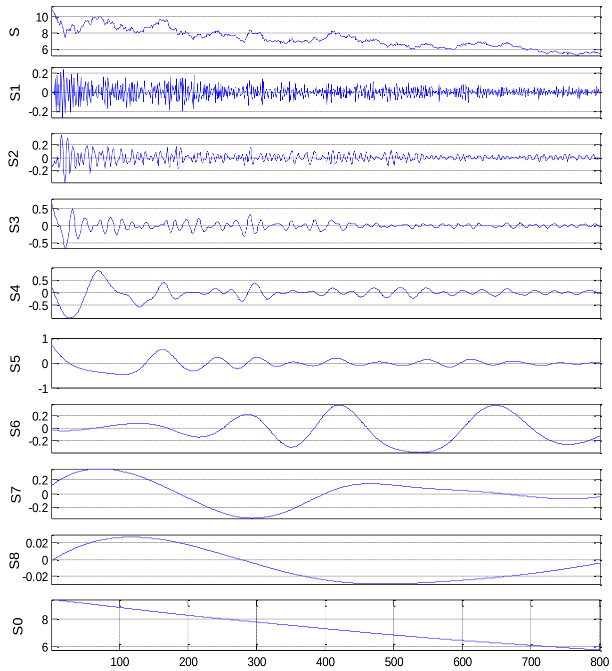

Fig. 1. Daily Pudong Development Bank closing price data decomposed into its IMFs and residue using EEMD

According to the same procedure as the portfolio using original data, we get the average cumulative return rate of portfolios with the objective of six risk measures using de-noising data. The hypothesis test results of the average cumulative return rate under six risk measures are shown in Table 2. where, *MV_3 f MV_4* represents that the null hypothesis is the cumulative return rate out-of-sample of the portfolio's objective function is MV with the data de-noised 3 IMFs is larger than that of MV with the data de-noised 4 IMFs, the result is "1" means that we would reject the null hypothesis under the 95% confidence level, the latter is the larger one, while the "0" means that we would not reject the null hypothesis, the latter is not significant the larger one.

TABLE 2
SINGLE HYPOTHESIS TEST RESULT

H_0	result	H_0	result
MV		VaR	
MV_3 f MV_4	1	*VaR_4 f VaR_3*	1
MV_2 f MV_3	1	*VaR_1 f VaR_4*	1
MV_1 f MV_2	1	*VaR_2 f VaR_1*	1
MV f MV_1	1	*VaR f VaR_2*	1
CVaR		entropy	
CVaR_4 f CVaR_3	1	*e f e_3*	1
CVaR_2 f CVaR_4	1	*e_4 f e*	1
CVaR_1 f CVaR_2	1	*e_1 f e_4*	1
CVaR f CVaR_1	1	*e_2 f e_1*	1
Iso-entropy		HMCR(p=3)	
is_4 f is_3	1	*n f n_1*	1
is_2 f is_4	1	*n_3 f n*	1
is f is_2	1	*n_4 f n_3*	1
is_1 f is	1	*n_2 f n_4*	1

NOTE: MV_3 represents the average return rate of the portfolio when the objective function is MV with the data de-noised 3 IMFs.

As shown in Table 2, the cumulative return rate of the portfolio increases with the IMF de-noised from 1 to 4 from the data gradually when the objective function is the standard deviation. It is significantly improved when the data de-noised 1 IMF and 2 IMFs. The cumulative return rate of portfolio increases with the IMF de-noised from 1 to 2 when the objective function is VaR, while change to decrease with the data de-noised 4 IMFs. It is significantly improved when the data de-noised 1 IMF and 3 IMFs. In addition, the result is much more significant when the objective function of the portfolio is CVaR, which has the same change trend in VaR. However, when the objective function of the portfolio is convex entropic risk measure, iso-entropic risk measure and HMCR(p=3), which have a higher power of risk discrimination, is not very significant with the IMFs de-noised from the original data. Similarly, the cumulative return rates are also improved with the IMF removed, especially for iso-entropy risk measure.

In order to compare the value of the average cumulative return rate of the portfolio when the objective functions are these six risk measures with the de-noising data. We give a rank of total hypothesis test in Table 3.

TABLE 3
TOTAL HYPOTHESIS TEST RESULT

Rank	H_0	result	Rank	H_0	result
1	*is_3 f CVaR_3*	1	16	*is f MV_3*	0
2	*e_3 f is_3*	1	17	*is_1 f is*	1
3	*n_1 f e_3*	0	18	*e_2 f is_1*	0
4	*n f n_1*	1	19	*n_2 f e_2*	1
5	*e f n*	1	20	*CVaR_2 f n_2*	1
6	*n_3 f e*	1	21	*MV_2 f CVaR_2*	1
7	*is_4 f n_3*	1	22	*VaR_4 f MV_2*	0
8	*CVaR_4 f is_4*	0	23	*MV_1 f VaR_4*	1
9	*e_4 f CVaR_4*	0	24	*VaR_1 f MV_1*	1
10	*n_4 f e_4*	1	25	*MV f VaR_1*	1
11	*MV_4 f n_4*	1	26	*CVaR_1 f MV*	0
12	*VaR_3 f MV_4*	1	27	*VaR_2 f CVaR_1*	1
13	*is_2 f VaR_3*	0	28	*VaR f VaR_2*	1
14	*e_1 f is_2*	1	29	*CVaR f VaR*	1
15	*MV_3 f e_1*	0			

As shown in Table 3, the ability of the optimization when the objective functions are CVaR, iso-entropy and entropy with the data de-noising 3 IMF are capable to obtain higher cumulative return rate, while less capable when the objective functions are CVaR, VaR and CVaR with the data de-noising 2 IMFs.

V. Conclusion

Firstly, the de-noising technique using EEMD surely have some impact to the portfolio, and it is much more significant when the portfolio' objective function is MV, VaR and CVaR compared with that of entropy, iso-entropy and HMCR(p=3), which have much more powerful risk discrimination theoretically. Therefore, it reflects that the impact of de-noising using EEMD is much more significant to the portfolio when the objective functions have less powerful risk discrimination. Then, compared with the highest cumulative return rate out-of-sample of the portfolio with the original data when the objective function is HCMR (p=3), the optimization is much more efficient when the portfolios' objective functions are CVaR, iso-entropy, entropy with the data de-noised with 3 IMFs, i.e. de-noised the high frequency IMFs in general. And the cumulative return rate of the portfolio when the objective function is CVaR with the data de-noised 3 IMFs is the highest one.

Therefore, using the de-noising technique to the data must be an efficient way to improve the profit of the portfolio, especially when the portfolios' objective functions have less powerful risk discrimination.

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