A Novel Bandwidth Allocation Scheme for Elastic and Inelastic Services in Peer-to-peer Networks

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Abstract—This paper considers reasonable bandwidth allocation for both elastic and inelastic services in peer-to-peer (P2P) networks. We measure the satisfaction of each peer as customer when acquiring one service by a utility function, and develop a bandwidth allocation model with the objective of utility maximization, which is an intractable and difficult non-convex optimization problem. In order to overcome it, we approximate the non-convex problem to a serial of equivalent convex optimization problems by applying the successive approximation method. We then present a gradient-based bandwidth allocation scheme to achieve the optimum of each approximation problem. After a serial of approximations, the bandwidth allocation scheme finally converges to an optimum of the bandwidth allocation problem. Some numerical examples are finally given to illustrate that the scheme can efficiently converge to the optimal bandwidth allocation.

Index Terms—P2P networks, bandwidth allocation, elastic and inelastic services, utility function

I. INTRODUCTION

Peer-to-peer (P2P) networks are different from the traditional client-server architecture. They have many advantages such as high scalability and strong robustness. The main idea behind P2Ps is that each peer not only receives resources from the networks but also can provide resources to the networks. Thus they can support various network services and have caused a lot of Internet traffic by different P2P protocols [1][2], such as BitTorrent for file-sharing and VoIP for video conferencing. Indeed, these services can be mainly divided into two categories according to the satisfaction of peer when acquiring services [3], [4], [5], [6], [7]. One corresponds to the traditional data services, such as file download and upload. They are almost tolerant of transmission delay and can make use of even the minimal amounts of bandwidth. These services are known as elastic services and the satisfaction of a peer when requesting an elastic service can be described by a concave utility. The other one is related to delay or rate sensitive multimedia services, such as real-time streaming video service. They always have high requirements on time delay and service rate for receiving a certain of quality of service (QoS). These services are regarded as to be inelastic in their requirement for bandwidth. The satisfaction of each peer for obtaining an inelastic service can be modelled as nonconcave utility (e.g., sigmoidal).

Resource allocation for elastic and inelastic services in P2P networks has been an emerging area of research. Resource pricing is regarded as an interesting means to realize resource allocation. Eger and Killat [8] proposed a pricing mechanism to achieve a fair bandwidth allocation of service providers between service requesters. They [9] further studied the weighted fairness among service requesters and presented an extended bandwidth allocation mechanism where service requesters adjust their offered prices and service providers adjust their service rates. Then Kumar et al. [10] developed a scheme for pricing and resource allocation in P2P networks, which permits users in a firm to share computing resources effectively. Koutsopoulos and Iosifidis [11] investigated bandwidth allocation in a star topology P2P network where the access links to the backbone networks become the capacity bottleneck. The authors formulated the bandwidth allocation problem with the objective of maximizing total network utility through reasonably allocating the bandwidth of each peer to downloaders and uploaders. Recently, Li et al. [12] and [13] applied the first order Lagrangian idea and low-pass filtering method, and proposed a novel price-based bandwidth allocation mechanism. Thus a service customer receive its bandwidth allocation according to its offered price, which realizes the goal of fair resource allocation. Antal and Vinkó [14] considered max-min fairness bandwidth allocation in multi-swarm peer-to-peer content sharing community.

The research results above are mainly concerning resource allocation for elastic services, such as file-transfer in P2P file-sharing networks. However, how to achieve efficient resource allocation for inelastic services is also very important. Chen et al. [15] considered resource allocation for P2P multiparty conferencing applications where it is a crucial challenge to provide certain level of QoS. The authors described the quality of experience of the conferencing peer through a utility function and formulated the utility maximization model for P2P multiparty conferencing applications, which are constrained by peers’ uplink capacities. Li et al. [16] also developed the utility maximization problem for P2P inelastic services, and derived that the problem is difficult to resolve by using traditional gradient-based scheme due to the nonconvexity of the optimization. In order to overcome the difficulty and obtain the optimum, they applied PSO to resolve the problem.

In this paper we consider bandwidth allocation problem for P2Ps where both elastic and inelastic services are coexisting, and formulate the utility maximization (social welfare) model for resource allocation, i.e., the total satisfaction of all peers in the networks when they acquire these services. However, the utility maximization model is a non-convex
problem because of non-concavity of inelastic services. By applying the successive approximation method, we transform the non-convex problem into a series of equivalent convex optimization problems and develop a gradient-based resource allocation scheme to achieve the optimal solution of the approximations. After a serial of approximations, the proposed scheme can finally converge to a suboptimal solution of the primal utility maximization model which also satisfies the Karush-Kuhn-Tucker (KKT) conditions.

The rest of this paper is summarized as follows: We introduce the utility maximization model for resource allocation of both elastic and inelastic services and formulate a serial of approximations of the primal model in Section 2. In Section 3 we develop a gradient-based resource allocation scheme to converge to the optimal solution of the primal model, and give some numerical examples to illustrate the performance of the proposed scheme in Section 4. Finally we conclude this paper in Section 5.

II. BANDWIDTH ALLOCATION MODEL

A. Services and utility functions

P2P networks support a wide variety of network services. Each peer who acquires a service will receive a certain level of satisfaction if the service is provided with a certain amount of resource. The utility function derived from economics is found useful to describe the satisfaction of a peer when acquiring a service. According to the different shapes of the utility functions, they can be divided into two categories: elastic services and inelastic services [4] [5] [7]. The former one mainly refers to traditional data services such as file uploading and downloading services in P2P file-sharing networks. The utility functions for this type of services are generally concave. The latter one is mainly related to multimedia video and audio services such as P2P multiparty conferencing services and VoIP over P2P networks. This type of services are usually very sensitive to granted resources and the QoS will drop drastically if resources are below a certain threshold. They usually have non-concave utility functions, such as sigmoidal functions. We adopt the utility functions proposed for resource allocation in IP networks. The shapes of elastic and inelastic services are illustrated in [3] [4] [5].

If a peer \( c \) acquires an elastic service, it will have a concave utility as follows

\[
U_c(y_c) = \begin{cases} 
  w_c \log(y_c + 1), & \text{if } \alpha = 1, \\
  w_c \frac{(y_c + 1)^{1-\alpha} - 1}{1 - \alpha} & \text{if } \alpha > 0 \text{ and } \alpha \neq 1,
\end{cases}
\]

and if it acquires an inelastic service, then it will have a sigmoidal utility as follows

\[
U_c(y_c) = w_c \frac{(y_c)^\beta}{(y_c)^\beta + m}, \forall \beta > 1, m > 0,
\]

where \( y_c \) is the service rate, \( w_c \) is the willingness-to-pay of the peer.

B. Model description

Consider a P2P network which is composed of a set of peers, a set \( S \) of elastic services and a set \( R \) of inelastic services. Each peer in the network acquires at least one service, elastic or inelastic. For example, a peer who is downloading files from other peers is also interested in taking part in a P2P multiparty conferencing. On the other hand each peer can also provide one or several services for others. Therefore, each peer acts as both a service customer and a service provider. In P2P networks, each peer uses its access link not only to obtain services from other peers, such as downloading files, but also to provide services for other peers, such as uploading files. Therefore, the upload bandwidth of a peer becomes a rare resource in the network, and other peers will compete for the upload bandwidth so as to obtain services. Therefore, the P2P networks are faced with the problem of how to allocate the peers’ upload bandwidth reasonably and effectively among service requesters, which is the main aim of this work.

Let the set \( P \) be peers acting as service providers that offer upload bandwidth to requesters, and the set \( C \) of peers acting as service customers that request elastic services or inelastic services. Introduce \( x_{pc} \) as the service rates offered by service provider \( p \) for customer \( c \) who requests elastic service or inelastic service. Then, each peer \( c \in C \) receives a total bandwidth \( y_c \) offered by its providers \( P(c) \) when it requests a service. The obtained bandwidth \( y_c \) satisfies \( y_c^\text{min} \leq y_c \leq y_c^\text{max} \), where \( y_c^\text{min} \) and \( y_c^\text{max} \) are the minimal and maximal bandwidth requirements of peer \( c \), respectively. Finally, the total bandwidth allocation of service provider \( p \) does not exceed its access link capacity \( C_p \).

Then we formulate the resource allocation for multiclass services in P2P networks as the following primal optimization problem

\[
\text{POP:} \quad \max \sum_{c \in C} U_c(y_c) \\
\text{subject to} \quad \sum_{p \in P(c)} x_{pc} = y_c, \quad \sum_{c \in C(p)} x_{pc} \leq C_p, \quad y_c^\text{min} \leq y_c \leq y_c^\text{max}, \quad x_{pc} \geq 0.
\]

Here, in the resource allocation problem POP, the objective is to maximize the aggregated utility of obtained bandwidth \( y_c \) over all service customers under the constraints that each service provider offers no more than its own access link capacity. As described by the equality of the resource allocation model, the aggregated bandwidth provision \( y_c \) for a service of service customer \( c \) is the sum of the rates \( x_{pc} \) that its service providers grant. \( y_c^\text{min} \) and \( y_c^\text{max} \) are the minimal and maximal bandwidth allocation that customer \( c \) requires. Also, as described by the inequality in the optimization problem above, the bandwidth provision of provider \( p \) is constrained by its own upload capacity of access link, i.e. \( C_p \).

C. Model analysis

In this part we give an analysis on the resource allocation problem POP for multiclass services in peer-to-peer networks. Firstly we obtain the Lagrangian
where $\phi$ is the price vector with elements $\phi_c$, which can be interpreted as the prices per unit bandwidth paid by customer $c$ when acquiring a service; $\varphi$ is the price vector with element $\varphi_p$, which can be thought as the price per unit bandwidth charged by provider $p$ when offering bandwidth allocation for customers; $x$ is the service rate matrix with elements $x_{pc}$ for the elastic or inelastic service of customer $c$; $y$ is the rate vector with elements $y_c$ for the elastic or inelastic service of customer $c$.

We rewrite the Lagrangian (4) as following

$$L_{POP}(x, y; \phi, \varphi) = \sum_{c \in C} U_c(y_c) + \sum_{c \in C} \sum_{p \in P(c)} x_{pc} (\phi_c - \varphi_p) + \sum_{p \in P} \varphi_p C_p,$$

(4)

We find the first part in (5) is separable in variables $y_c$, and the second part is separable in variables $x_{pc}$. Thus the objective function of the dual problem is described as

$$D(\phi, \varphi) = \max_{x, y} L_{POP}(x, y; \phi, \varphi) = \sum_{c \in C} \mathcal{P}(\phi_c) + \sum_{c \in C} \sum_{p \in P(c)} \mathcal{R}_{pc}(\varphi_c, \varphi_p) + \sum_{p \in P} \varphi_p C_p,$$

(5)

where

$$\mathcal{P}(\phi_c) = \min_{y_c} \max_{y_c \in \mathcal{C}_c} U_c(y_c) - \phi_c y_c,$$

(6)

$$\mathcal{R}_{pc}(\varphi_c, \varphi_p) = \max_{x_{pc}} x_{pc}(\phi_c - \varphi_p).$$

(7)

We give an economic interpretation for equations (7)-(8) as follows. In (7), service customer $c$ acquires an elastic or inelastic service with total bandwidth provision $y_c$, and wants to maximize its own utility $U_c(y_c)$. Meanwhile, it needs to pay a fee for its obtained bandwidth to support the service. Recall that $\phi_c$ is the price per unit bandwidth paid by customer $c$ for the service, then $\phi_c y_c$ is regarded as the total fee paid by customer $c$. Therefore, the economic meaning of (7) is that each peer as service customer aims at achieving the objective of maximizing its own profit. As for (8), $\varphi_p$ is considered as the price per unit bandwidth charged by service provider $p$. The product $x_{pc} \varphi_p$ is the fee paid by customer $c$ to provider $p$ for the elastic or inelastic service, and $x_{pc} \varphi_p$ is the expense demanded by provider $p$ for its granting bandwidth $x_{pc}$. Then, the economic meaning of (8) is that each peer as service provider wants to achieve the objective of maximizing its own revenue.

Then, we can obtain the dual problem of resource allocation model POP

$$\min_{\phi_c, \varphi_p} D(\phi, \varphi) \quad \text{over} \quad \phi_c \geq 0, \varphi_p \geq 0.$$

(9)

The objective of the dual problem (9) is to minimize the total price charged by all service providers under the constraints that service customers are guaranteed with certain levels of satisfaction. In order to obtain the optimal price and bandwidth allocation, distributed algorithm should be developed to resolve the resource allocation model (3) and its dual problem (9). Traditional gradient-based schemes can converge to the global optimum when only considering elastic services since their utility functions are all concave. However, these schemes do not work well when considering both elastic and inelastic services since the resource allocation problem becomes an intractable and difficult non-convex problem. They may produce suboptimal or even infeasible bandwidth allocation for each peer.

**D. Approximation problem**

The resource allocation problem POP equals to the following approximate optimization problem

**AOP**: \[
\begin{align*}
\max & \log \left( \sum_{c \in C} U_c(y_c) \right) \\
\text{subject to} & \sum_{p \in P(c)} x_{pc} = y_c, \\
& \sum_{c \in C(p)} x_{pc} \leq C_p, \\
& y_c^{min} \leq y_c \leq y_c^{max}, \\
& x_{pc} \geq 0.
\end{align*}
\]

(10)

Now the problem AOP is still a nonconvex optimization problem, since The objective is still nonconvex. In order to obtain a convex approximate problem, we derive an inequality to replace the nonconvex constraint with a convex one. Following Jensens inequality, we introduce an important result as follows.

**Lemma 1** For any vector $\xi = (\xi_c, c \in C)$ where $\xi_c > 0$, and $\sum_{c \in C} \xi_c = 1$, the following inequality holds

$$\log \left( \sum_{c \in C} U_c(y_c) \right) \geq \sum_{c \in C} \xi_c \log \left( \frac{U_c(y_c)}{\xi_c} \right).$$

(11)

The equality (11) holds if and only if

$$\xi_c = \frac{U_c(y_c)}{\sum_{c \in C} U_c(y_c)}.$$

(12)

Then, we consider the canonical form for optimization problem, and deduce the following equivalent approximation problem based on **Lemma 1**

**APP**: \[
\begin{align*}
\max & \sum_{c \in C} U_c(y_c, \xi_c) \\
\text{subject to} & \sum_{p \in P(c)} x_{pc} = y_c, \\
& \sum_{c \in C(p)} x_{pc} \leq C_p, \\
& y_c^{min} \leq y_c \leq y_c^{max}, \\
& x_{pc} \geq 0,
\end{align*}
\]

(13)

where $U_c(y_c, \xi_c) = \xi_c \log \left( \frac{U_c(y_c)}{\xi_c} \right)$. Now the approximation problem APP indeed includes a series of approximations when we choose different values of $\xi$. Given an initial value
of $\xi$, the solution to APP is a suboptimal solution to POP. After substituting this suboptimal solution, a new value $\xi$ is deduced by the update rule (12). With this new value $\xi$, the corresponding new APP is solved. After a sequence of approximations, the solution to APP will finally converge to the global solution to POP. We will provide a resource allocation scheme for convergence to the stationary point of APP, which satisfies the KKT conditions for an optimization problem. At the stationary point the APP is equivalent to POP, thus the point is exactly the optimal resource allocation of POP.

It is not difficult to find that the extended utility $U_c(y_c, \xi)$ for elastic service is still a concave function since $U_c(y_c)$ is concave. For inelastic service, we analyze the extended utility function $U_c(y_c, \xi)$, and obtain the following result.

**Lemma 2** The extended utility functions $U_c(y_c, \xi)$ are continuously differentiable and strictly concave for inelastic services.

**Proof** We prove the result by verifying the second derivative of $U_c(y_c, \xi)$ with respect to variable $y_c$.

$$
\frac{d^2U_c(y_c, \xi)}{dy_c^2} = \frac{\xi^2}{U_c^2(y_c)} \left( U_c(y_c) \frac{d^2U_c(y_c)}{dy_c^2} - \left( \frac{dU_c(y_c)}{dy_c} \right)^2 \right).
$$

(14)

Thus, utility functions $U(y_c, \xi)$ are strictly concave since the second derivatives are negative, that is, $U_c(y_c)\frac{d^2U_c(y_c)}{dy_c^2} - \left( \frac{dU_c(y_c)}{dy_c} \right)^2 < 0$. Thus the result is obtained.

Now the objective of approximation problem APP is strictly concave with respect to variables $y_c$, but is not strictly concave with respect to variables $x_{pc}$. Meanwhile, the constraint conditions are linear, thus the constraint set of this optimization problem is convex. Thus, based on the convex optimization theory [17], we can obtain the following result.

**Theorem 1** For the approximation problem APP of resource allocation for multiclass services in P2P network, there exists unique optimal resource allocation $y^*_c$ for customer $c$ when requesting elastic or inelastic service. However, the optimal resource provision $x^*_{pc}$ from provider $p$ to customer $c$ is not necessarily unique.

### III. Resource Allocation Scheme

A. Algorithm description

In order to obtain the optimum of approximation problem APP, we firstly introduce the the Lagrangian of problem APP

$$
L_{APP}(x, y; \lambda, \mu; \xi) = \sum_{c \in C} U_c(y_c, \xi) + \sum_{c \in C} \lambda_c \left( \sum_{p \in \mathcal{P}(c)} x_{pc} - y_c \right) + \sum_{p \in \mathcal{P}} \mu_p \left( C_p - \sum_{c \in \mathcal{C}(p)} x_{pc} \right).
$$

(15)

We apply the gradient projection method to solve approximate problem APP and present the following resource allocation algorithm to achieve the optimum. The distributed algorithm only depends on locally available information of each service provider.

Each service provider $p$ updates its resource allocation for customer $c$ who requests a service with the following rule

$$
x_{pc}(t+1) = (x_{pc}(t) + \kappa x_{pc}(t)(\lambda_c(t) - \mu_p(t)))^+.
$$

(16)

$$
\lambda_c(t) = \frac{\partial U_c(y_c(t), \xi)}{\partial y_c(t)},
$$

(17)

$$
y_c(t) = \left( \sum_{p \in \mathcal{P}(c)} x_{pc}(t) \right)^{\max},
$$

(18)

where $\kappa > 0$ is the step size; $a = (b)_d^{\max}$ means $a = \min\{d, \max\{b, c\}\}$.

Each service provider $p$ updates its price $\mu_p(t)$ with the following rule

$$
\mu_p(t+1) = \left( \mu_p(t) + \eta_p(t) - C_p \right)^+ \mu_p(t),
$$

(19)

$$
z_p(t) = \sum_{c \in \mathcal{C}(p)} x_{pc}(t),
$$

(20)

where $\eta > 0$ is the step size; $a = (b)_d^{\max}$ means $a = \min\{d, \max\{b, c\}\}$ if $c = 0$.

Note that the approximation problem APP indeed includes a series of approximations, each one is identified by a value $\xi$. If we select an appropriate value $\xi$, we can achieve the optimum of the corresponding approximation problem by applying the resource allocation scheme above. In order to guarantee that the approximations of APP finally become exact where the equality (11) always holds, each customer $c$ updates its parameter $\xi_c(t)$ with the following rule

$$
\xi_c(t) = \frac{U_c(y_c(t))}{\sum_{c \in C} U_c(y_c(t)).
$$

(21)

In the proposed resource allocation scheme above, if customer $c$ requests an elastic or inelastic service from provider $p$, it computes the price $\lambda_c(t)$ paid for provider $p$ according to (17). And provider $p$ calculates its charged price $\mu_p(t)$ according to (19), and updates its resource allocation $x_{pc}(t)$ for customer $c$ with the rule of (16). We observe that resource allocation scheme (16) is a gradient-based fluid model which depends on the difference between the price $\lambda_c(t)$ paid by customer $c$ and the price $\mu_p(t)$ charged by provider $p$. On the other hand, provider $p$ observes the aggregated load $z_p(t)$ from (20), and updates its charged price $\mu_p(t)$ according to (19). Thus the update rules for resource allocation and price are both a scaled gradient-based algorithm, which has been proven to be efficiently convergent to the optimum when choosing appropriate step sizes. However, each customer needs to learn the total utility values of all customers so as to update $\xi_c(t)$ with the law of (21). Therefore, after each iteration each customer $c$ communicates its utility value to all other customers in the network. In a new iteration the initial value is the stationary value of the previous iteration.
IV. SIMULATION RESULTS

In this part we analyze the performance of the proposed bandwidth allocation scheme in a P2P network through some numerical examples. Consider a simple P2P network which consists of two service providers and four service customers. The access link capacity of service providers is $C = (C_1, C_2) = (10, 20)$Mbps. The willingness-to-pay of peers as customer is $w = (w_1, w_2, w_3, w_4) = (40, 30, 20, 10)$. In the algorithm we choose step sizes $\kappa = \nu = 0.2$.

A. Elastic services

We firstly consider the customers are only requesting elastic services and analyze the proposed bandwidth allocation scheme. Consider the concave utility function (1) for elastic services, and choose parameter $\alpha = 2$.

The simulation results of the bandwidth allocation scheme are illustrated in Fig. 1, which show the service rates of each customer granted by providers and the performance of the proposed scheme. We observe from the simulation results that the bandwidth allocation scheme gradually tends to a steady state where the access link of each provider is approximately 100% utilized. This can be understood from the selfish feature of each peer as customer, acquiring as much resource as possible. The scheme can achieve the optimal bandwidth allocation within reasonable iterations, i.e., $x^*(3.3554, 6.7083, 2.8602, 5.7212, 2.2735, 4.5497, 1.5109, 3.0209)$Mbps, providing an efficient bandwidth allocation for elastic services. Also, we deduce that the utility of each customer is also driven to a steady state where the optimal bandwidth allocation is achieved.

B. Inelastic services

Now we consider the customers are only requesting inelastic services and analyze the proposed bandwidth allocation scheme. Consider the utility function (2) for inelastic services, and choose parameters $\beta = 3$ and $m = 5$.

We obtain the simulation results of the proposed bandwidth allocation scheme for inelastic services and illustrate them in Fig. 2. Similar to the case for only elastic services, the optimal bandwidth allocation for customers can also be achieved within reasonable iterations, i.e., $x^*(2.8847, 5.7598, 2.6798, 5.3572, 2.4155, 4.8343, 2.0200, 4.0487)$Mbps, despite of the fact that the bandwidth allocation for inelastic services is a difficult nonconvex problem. We find the proposed bandwidth allocation works well in this case and can be efficient to resolve the nonconvex problem.

C. Heterogeneous services

In this part, we consider the bandwidth allocation for heterogeneous services, that is, the services requested by customer are not only elastic but also inelastic, and investigate the performance of proposed bandwidth allocation scheme. The first two customers are requesting elastic services, and the utility functions are given in form of (1) with parameter $\alpha = 2$. The other customers are requesting inelastic services. They have the utility functions (2) with parameters $\beta = 3$ and $m = 5$.

We obtain the simulation results of the proposed bandwidth allocation scheme for heterogeneous services and illustrate them in Fig. 3. Obviously, we observe that the scheme can converge to the optimal bandwidth allocation for heterogeneous services within reasonable iterations, i.e., $x^*(2.5109, 4.9574, 2.3316, 4.6083, 3.0779, 6.2273, 2.0797, 4.2071)$Mbps.

V. CONCLUSIONS

In this paper we consider optimal bandwidth allocation for both elastic and inelastic services in P2P networks, and formulate the utility maximization model for peers who request these services. We find that the utility maximization
model is an intractable and difficult non-convex optimization problem, since the inelastic services have non-concave utility functions. In order to obtain the optimal bandwidth allocation, we approximate the utility maximization problem to an equivalent convex optimization problem by applying the successive approximation method, and design a gradient-based resource allocation scheme to achieve the optimal solution of the approximations. We finally give some numerical examples to verify the performance of bandwidth allocation scheme for both elastic and inelastic services.

Fig. 2. Optimal bandwidth allocation for inelastic services

Fig. 3. Optimal bandwidth allocation for heterogeneous services

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