Backorders Case in Inventory Model with Fuzzy Demand and Fuzzy Lead Time

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***Abstract*—In this paper, we present an inventory model with fuzzy parameters and back ordering is allowed. If the demands occurring when the system is out of stock, then there is shortage cost associated with incurring backorders to meet the demand. The annual demand and lead time are triangular fuzzy numbers. The cost parameters are crisps. Since the lead time is fuzzy, so holding cost and shortage cost are fuzzy. To determine holding cost and shortage cost, we need fuzzy integral. Hence, previously we define fuzzy integral through alpha-level. The total cost is summation of purchasing cost, ordering cost, holding cost and shortage cost. Furthermore, we get average annual cost with non linear membership function that depend on order quantity. We will determine the optimal order quantity and reorder point to minimize a fuzzy total cost by graded mean integration. We apply non linear optimization method by MATLAB to find order quantity. We give a numerical example.**

***Index Terms*—backorders, triangular fuzzy number, fuzzy integral.**

I. Introduction

THE classic economic order quantity (EOQ) inventory model is an inventory control system that demand and lead time are assumed to be constant and known [6]. Also the costs associated with maintaining inventory and meeting customer demand are deterministic. However, in some practical cases, demand, lead time and costs are imprecise. Many researchers have studied inventory model with fuzzy parameter. Buckley developed three EOQ models [2]. The first model is EOQ model with fuzzy cost and deterministic demand in the inventory system without backorders. The second model is EOQ model with fuzzy cost and fuzzy demand. The third model is EOQ model with fuzzy cost and fuzzy demand in the backorders case. The lead time in all models are zero. In the three models, the objective function are fuzzy numbers. The multiobjective optimization problem is used to transform fuzzy model to crisp model. Buckley et.al. also considered EOQ model with trapezoid fuzzy ordering cost. So that, the annual total cost is a fuzzy number. They minimized the annual cost by minimizing area under membership function of fuzzy annual cost. Buckley et.al. also investigated inventory model with fuzzy demand in the lost sales and backorders cases [3].

The other researchers considered EOQ model with different assumptions. Hsieh developed an inventory model with fuzzy demand per day and fuzzy lead time [7]. The optimal order quantity is obtained by using both function principle and graded mean integration representation method. Furthermore, he also introduce a fuzzy inventory model under safety stock based on fuzzy total annual safety stock cost combined by total annual holding cost of safety stock and fuzzy total annual stockout cost.

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Kazemi investigated backorders model with fuzzy ordering cost, fuzzy holding cost, fuzzy backorders cost, fuzzy demand and fuzzy decision variables [9]. The defuzzication of the cost function with the graded mean integration. Dinagar and Kannan developed the fuzzy inventory model with allowable shortage. The cost and demand are hexagonal fuzzy number. In this model, the order quantity is fuzzy [4].

For special inventory, Mandal and Islam [10] considered fuzzy inventory system for deterioration with time dependent demand with shortages under fully backlogged. There were two models, model with fixed cost and model with fuzzy cost. Meanwhile El-Wakeel and Al-Yazidi consider the continous review inventory model with shortage [5]. In the model, there is mixture shortage or partial backorders. The first model, the cost components are crisp value. In the second model, the cost components are trapezoidal fuzzy numbers.

Many applications of fuzzy sets have been developed. One of them is fuzzy optimization in inventory model which explains any imprecision in the parameters. Many approaches to solve fuzzy optimization. By fuzzy ranking, Jiang and Qiu propose the theorems of the alternative in the linear problem to solve fuzzy linear programming [8].

In this study, we propose an inventory model with backorders that demand and lead time are triangular fuzzy number. We define fuzzy integral to compute shortage cost and holding cost. Then we use graded mean integration ranking to solve the fuzzy optimization.

II. Preliminaries

We recall some basic concepts of fuzzy set theory from [1]. Our notation specifying a fuzzy set is to place a "bar" over a letter.

Theorem 2.1:
Let $\bar{A}$ be a fuzzy set in $\Re$. Then $\bar{A}$ is a fuzzy number if and only if there exists a closed interval $[a,b] \neq \emptyset$, such that

$$\mu_{\bar{A}}(x) = \begin{cases} \ell(x) & , x \in (-\infty, a) \\ 1 & , x \in [a,b] \\ r(x) & , x \in (b, \infty) \end{cases}$$

where $\mu_{\bar{A}}$ is the membership function for fuzzy set $\bar{A}$ with

i. $\ell : (-\infty, a) \to [0,1]$ is monotonely increasing, continuous from the right and $\ell(x) = 0$ for $x \in (-\infty, w)$, for any $w \leq a$ and
ii. $r : (b, \infty) \to [0,1]$ is monotonely decreasing, continuous from the left and $r(x) = 0$ for $x \in (v, \infty)$, for any $v \geq b$.

For requisite writing, we rewrite notation ℓ and r by $\mu^L_{\bar{A}}$ and $\mu^R_{\bar{A}}$ for fuzzy set $\bar{A}$, respectively.

For positive real number p and a fuzzy number $\bar{A}$ with membership function $\mu_{\bar{A}}(x)$, it is defined $p+\bar{A}=\bar{S}$ with membership function

$$\mu_{\bar{S}}(x)=\begin{cases}\ell(x-p) & , x\in(-\infty,a+p)\\ 1 & , x\in[a+p,b+p]\\ r(x-p) & , x\in(b+p,\infty).\end{cases}$$

In this paper, we use triangular fuzzy number that the definition is given below.

Definition 2.2: A fuzzy number $\bar{A}$ is said a triangular fuzzy number if its membership function $\mu_{\bar{A}}$ is given by

$$\mu_{\bar{A}}(x)=\begin{cases}\mu^L_{\bar{A}}(x)=\frac{x-a_l}{a-a_l} & , a_l\le x<a\\ 1 & , x=a\\ \mu^R_{\bar{A}}(x)=\frac{a_u-x}{a_u-a} & , a<x\le a_u\\ 0 & , \text{otherwise.}\end{cases}$$

The triangular fuzzy number (TFN) $\bar{A}$ is denoted by the triplet $\bar{A}=(a_l,a,a_u)$, with a_l and a_u are the lower and upper limits of the support of $\bar{A}$ respectively.
Further, we give a definition of α-cut of fuzzy set to study the fuzzy arithmetic.

Definition 2.3: Let $\bar{A}$ be a fuzzy set in X and $\alpha\in(0,1]$. The α-cut of fuzzy set $\bar{A}$ is the crisp set $\bar{A}_\alpha$ given by

$$\bar{A}_\alpha=\{x\in X:\mu_{\bar{A}}(x)\ge\alpha\}.$$

The α-cut of fuzzy number is a closed interval.

To understand the fundamentals of fuzzy arithmetic, one need to learn about interval arithmetic. We use approach based on α-cut to develop the arithmetic of fuzzy numbers. Let $\bar{A}$ and $\bar{B}$ be two fuzzy numbers and
$\bar{A}_\alpha=[a^L_\alpha,a^R_\alpha]$, $\bar{B}_\alpha=[b^L_\alpha,b^R_\alpha]$ be α-cut, $\alpha\in(0,1]$, of $\bar{A}$ and $\bar{B}$ respectively. The summation, division and multiplication in fuzzy number are defined through α-cut in the following:

(i.) $\bar{A}_\alpha+\bar{B}_\alpha=[a^L_\alpha+b^L_\alpha,a^R_\alpha+b^R_\alpha]$.
(ii.) $\bar{A}_\alpha-\bar{B}_\alpha=[a^L_\alpha-b^R_\alpha,a^R_\alpha-b^L_\alpha]$.
(iii.) $\bar{A}_\alpha:\bar{B}_\alpha=[\frac{a^L_\alpha}{b^R_\alpha},\frac{a^R_\alpha}{b^L_\alpha}],0\notin\bar{B}_\alpha$ for $\bar{A}$ and $\bar{B}$ in $\Re_+$.
(iv.) $\bar{A}_\alpha\cdot\bar{B}_\alpha=[\min\{a^L_\alpha\cdot b^L_\alpha,\ a^L_\alpha\cdot b^R_\alpha,\ a^R_\alpha\cdot b^L_\alpha,\ a^R_\alpha\cdot b^R_\alpha\},\ \max\{a^L_\alpha\cdot b^L_\alpha,\ a^L_\alpha\cdot b^R_\alpha,\ a^R_\alpha\cdot b^L_\alpha,\ a^R_\alpha\cdot b^R_\alpha\}]$
(v.) $(k\bar{A})_\alpha=k\bar{A}_\alpha=[ka^L_\alpha,ka^R_\alpha]$, for $k>0$.
(vi.) $(\frac{k}{\bar{A}})_\alpha=[\frac{k}{a^R_\alpha},\frac{k}{a^L_\alpha}]$, for $k>0$.

For real number r, we denoted (r,r,r) as fuzzy number and its $\alpha-$cut is $[r,r]$. For simplify, in this paper we let $\bar{A}<\bar{B}$ if $a^R_\alpha<b^L_\alpha$ for $\alpha\in(0,1]$.

III. Fuzzy Integral

Below we have a definition of the fuzzy integral as my preliminary result. Next we will use it to compute the fuzzy costs in the fuzzy inventory model.

Definition 3.1: Given a fuzzy function $\bar{f}:\bar{\mathcal{F}}\to\bar{\mathcal{F}}$, where $\bar{\mathcal{F}}$ is the set of all fuzzy numbers. For two fuzzy numbers $\bar{A}$, $\bar{B}$ and $\bar{A}<\bar{B}$, the fuzzy integral $\int_{\bar{A}}^{\bar{B}}\bar{f}tdt=\bar{H}$ is defined through α-level,

$$(\mu^\ell_{\bar{H}})^{-1}(\alpha)=\int_{(\mu^\ell_{\bar{A}})^{-1}(\alpha)}^{(\mu^\ell_{\bar{B}})^{-1}(\alpha)}(\mu^\ell_{\bar{f}})^{-1}(\alpha)tdt$$

and

$$(\mu^r_{\bar{H}})^{-1}(\alpha)=\int_{(\mu^r_{\bar{A}})^{-1}(\alpha)}^{(\mu^r_{\bar{B}})^{-1}(\alpha)}(\mu^r_{\bar{f}})^{-1}(\alpha)tdt\ .$$

The integral $\int_{\bar{A}}^{\bar{B}}\bar{f}tdt$ is exist, if $\bar{H}$ is a fuzzy number.

Example 3.2: Let constant fuzzy function $\bar{f}=\bar{G}=(8,10,12)$. We will compute $\int_{\bar{A}}^{\bar{B}}\bar{G}tdt$ through α-level, where $\bar{A}=(1,3,4)$ and $\bar{B}=(5,6,8)$.
We have $\bar{G}_\alpha=[2\alpha+8,12-2\alpha]$, $\bar{A}_\alpha=[2\alpha+1,4-\alpha]$ and $\bar{B}_\alpha=[\alpha+5,8-2\alpha]$.
Denote the $\int_{\bar{A}}^{\bar{B}}\bar{G}tdt$ by $\bar{H}$.
Then, we have

$$\begin{aligned}(\mu^\ell_{\bar{H}})^{-1}(\alpha)&=\int_{(\mu^\ell_{\bar{A}})^{-1}(\alpha)}^{(\mu^\ell_{\bar{B}})^{-1}(\alpha)}(\mu^\ell_{\bar{G}})^{-1}(\alpha)dt\\ &=\int_{2\alpha+1}^{\alpha+5}(2\alpha+8)tdt\\ &=-3\alpha^3-6\alpha^2+48\alpha+96\end{aligned}$$

and

$$\begin{aligned}(\mu^r_{\bar{H}})^{-1}(\alpha)&=\int_{(\mu^r_{\bar{A}})^{-1}(\alpha)}^{(\mu^r_{\bar{B}})^{-1}(\alpha)}(\mu^r_{\bar{G}})^{-1}(\alpha)dt\\ &=\int_{4-\alpha}^{8-2\alpha}(12-2\alpha)tdt\\ &=-3\alpha^3+42\alpha^2-192\alpha+288.\end{aligned}$$

Here, we have $(\mu^\ell_{\bar{H}})^{-1}(\alpha)$ is an increasing function for $\alpha\in[0,1]$ and the $(\mu^r_{\bar{H}})^{-1}(\alpha)$ is a decreasing function for $\alpha\in[0,1]$ and $(\mu^\ell_{\bar{H}})^{-1}(1)=(\mu^r_{\bar{H}})^{-1}(1)$. So $\bar{H}$ is a fuzzy number.

IV. Model

In the following research we investigate the EOQ inventory model with backorder under deterministic costs, fuzzy demand and fuzzy lead time. We have some assumptions:

a. The rate of demand per year is a triangular fuzzy number $\bar{\lambda}$.
b. If an order of any size is placed, then an ordering and setup cost K is incurred.
c. The lead time for each order is a triangular fuzzy number $\bar{\tau}$.
d. The cost per unit per year of holding inventory is h.
e. Back ordering is allowed. Shortage cost is s per unit per year.
f. Unit purchasing cost C is deterministic.
g. The orders arrive instantaneously.

We will determine the quantity of each ordering (q) and maximum of inventory level (M) that minimize total cost per year. We define $q-M$ is a maximum shortage that occurs under an ordering policy. In this model, if the order is arrival, $q-M$ units are used to met old demand (shortage).

In the first model, we assume that the lead time for each order is zero. Because the demand is fuzzy, so we have the length of time per cycle is fuzzy. Furthermore, the holding cost of inventory per cycle is a fuzzy number. We define $\bar{T}_b$ the length of time which shortage is incurred,

$$\bar{T}_b=(q-M)/\bar{\lambda}.$$

If q units are ordered, then the length of one cycle is

$$\bar{T}=(M/\bar{\lambda})+\bar{T}_b$$

or

$$\bar{T}=q/\bar{\lambda}.$$

The holding cost per cycle is given by

$$\bar{I} = h\int_0^{M/\bar{\lambda}} (M - \bar{\lambda}t)dt. \quad (1)$$

We will compute shortage cost per cycle as follow

$$\bar{S} = s\int_0^{\bar{T}_b} ((q - M) - \bar{\lambda}t)dt. \quad (2)$$

Although, the $\bar{\lambda}$ is a TNF, if we compute $\bar{I}$ and $\bar{S}$ using Definition 3.1 then the fuzzy numbers $\bar{I}$ in (1) and $\bar{S}$ in (2) are not TNF.
Since purchasing cost is not a variable cost, we do not involve the cost in the total cost per cycle. So the total cost per cycle is a fuzzy number:

$\overline{TC}$ =ordering cost + holding cost + shortage cost

$= K + \bar{I} + \bar{S}.$

Since there are $1/\bar{T}$ cycles in one year, so we obtain average annual cost per year as

$$\overline{TY} = \overline{TC}/\bar{T}. \quad (3)$$

The annual cost $\overline{TY}$ is a fuzzy number.

We will minimize the cost by graded mean integration to convert a fuzzy model to a crisp model. The definition graded mean integration is given in the following.

Definition 4.1: Given a fuzzy number $\bar{A}$ with membership function respect to Definition 2.1. Graded mean integration of $\bar{A}$, denoted by $G(\bar{A})$,

$G(\bar{A}) = \int_0^1 \alpha(\ell^{-1}(\alpha) + r^{-1}(\alpha))d\alpha.$

Our purpose is to minimize the mean integration of $G(\overline{TY})$. The $G(\overline{TY})$ is a function of two variables of q and M and is a convex function. So we get the optimal q^* and M^*.

In the second model, the lead time is not zero. The optimal order quantity q^* and optimal maximum of inventory level M^* are the same in the first model. Since the lead time is a triangular fuzzy number $\bar{\tau}$, we will determine inventory level (fuzzy reorder point) $\bar{r}$ when we replace an order. It's hard to describe the fuzzy inventory system with figure. To help the illustration, we use deterministic inventory system in Fig.1.

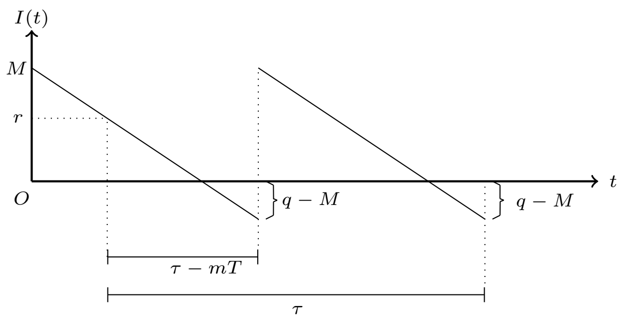

Fig. 1. Backorders with Lead Time

In the Fig 1, notation $I(t)$ represent an inventory level at time t. Before we determine reorder point, we give two definitions in the following.

Definition 4.2:
The net inventory is the amount on hand minus backorders. The hand inventory is the amount on hand plus order minus backorders.
The net inventory may be negative or positive.

In the backorder case, the reorder point is base on net inventory. Find t such that $\mu_{\bar{\tau}/\bar{T}}(t) = 1$. Let $m = \lfloor t \rfloor$. The optimal reorder point as

$$\bar{r^*} = \bar{d} - mq^* - b^*, \quad (4)$$

where $\bar{d} = \bar{\lambda}\bar{\tau}$ is the fuzzy demand during lead time, b^* is the amount optimal backorders and q^* is the amount optimal orders. It is possible for $\bar{r^*}$ to be negative fuzzy number. This means that an order is placed when the backorders reach a level $-\bar{r^*}$.

V. Numerical Example

Given an inventory system with backorders case where the parameters are:

$\bar{\lambda} = (150, 200, 250)$ unit per year
$K =$ IDR 50000
$h =$ IDR 500 per unit per year
$s =$ IDR 1000 per unit per year
$\bar{\tau} = (2/3, 3/4, 5/6)$ year

We will determine q, M and $\bar{r}$ to minimize average annual cost.
Base on $\bar{\lambda}$ and $\bar{\tau}$, we have α-cuts:

$$\bar{\lambda}_\alpha = [\lambda_\alpha^L, \lambda_\alpha^R] = [50\alpha + 150, 250 - 50\alpha]$$

and

$$\bar{\tau}_\alpha = [\tau_\alpha^L, \tau_\alpha^R] = [\frac{\alpha + 8}{12}, \frac{10 - \alpha}{12}].$$

We write $\frac{M}{\bar{\lambda}} = \bar{P}$, then we find $\bar{P}$ through α-cut in the below:

$$\bar{P}_\alpha = [M, M] : [50\alpha + 150, 250 - 50\alpha]$$
$$= [\tfrac{M}{250-50\alpha}, \tfrac{M}{50\alpha+150}].$$

From (1), we compute holding cost per cycle $\bar{I}$,

$$\bar{I} = h\int_0^{\frac{M}{\bar{\lambda}}} (M - \bar{\lambda}t)dt.$$

We write $\int_0^{\frac{M}{\bar{\lambda}}} Mdt = \bar{A}$. By Definition 3.1 , we have

$$(\mu_{\bar{A}}^\ell)^{-1}(\alpha) = \int_0^{\frac{M}{250-50\alpha}} Mdt = \frac{M^2}{250 - 50\alpha}$$

and

$$(\mu_{\bar{A}}^r)^{-1}(\alpha) = \int_0^{\frac{M}{50\alpha+150}} Mdt = \frac{M^2}{50\alpha + 150}.$$

The α-cut for $\bar{A}$,

$$\bar{A}_\alpha = [\frac{M^2}{250 - 50\alpha}, \frac{M^2}{50\alpha + 150}].$$

We call $\int_0^{\frac{M}{\lambda}} \bar{\lambda} t dt = \bar{B}$. Compute,

$$(\mu_{\bar{B}}^{\ell})^{-1}(\alpha) = \int_0^{\frac{M}{250-50\alpha}} (50\alpha + 150) t dt$$

$$= \tfrac{1}{2}[(50\alpha + 150)t^2]_0^{\frac{M}{250-50\alpha}}$$

$$= (25\alpha + 75)(\tfrac{M}{250-50\alpha})^2.$$

and

$$(\mu_{\bar{B}}^{r})^{-1}(\alpha) = \int_0^{\frac{M}{50\alpha+150}} (250 - 50\alpha) t dt$$

$$= \tfrac{1}{2}[(250 - 50\alpha)t^2]_0^{\frac{M}{50\alpha+150}}$$

$$= (125 - 25\alpha)(\tfrac{M}{50\alpha+150})^2,$$

we have

$$\bar{B}_\alpha = [(25\alpha+75)(\frac{M}{250-50\alpha})^2, (125-25\alpha)(\frac{M}{50\alpha+150})^2].$$

So,

$$\tfrac{1}{h}\bar{I}_\alpha = \bar{A}_\alpha + \bar{B}_\alpha$$

$$= [\tfrac{M^2}{250-50\alpha}, \tfrac{M^2}{50\alpha+150}] +$$

$$[(25\alpha + 75)(\tfrac{M}{250-50\alpha})^2, (125 - 25\alpha)(\tfrac{M}{50\alpha+150})^2]$$

$$= [\tfrac{25M^2\alpha+76M^2}{(250-50\alpha)^2}, \tfrac{126M^2-25M^2\alpha}{(50\alpha+150)^2}],$$

or

$$\bar{I}_\alpha = h[\frac{25M^2\alpha + 76M^2}{(250-50\alpha)^2}, \frac{126M^2 - 25M^2\alpha}{(50\alpha+150)^2}]. \quad (5)$$

We compute $\bar{T}_b = \frac{b}{\bar{\lambda}}$ with $b = q - M$. Compute α-cut for $\bar{T}_b$,

$$\bar{T}_{b\alpha} = [b, b] : [50\alpha + 150, 250 - 50\alpha]$$

$$= [\frac{b}{250-50\alpha}, \frac{b}{50\alpha+150}].$$

We determine shortage cost per cycle $\bar{S}$,

$$\bar{S} = s\int_0^{\bar{T}_b} (b - \bar{\lambda} t) dt.$$

We call $\int_0^{\bar{T}_b} b dt = \bar{V}$, we calculate

$$(\mu_{\bar{V}}^{\ell})^{-1}(\alpha) = \int_0^{\frac{b}{250-50\alpha}} b dt = \frac{b^2}{250-50\alpha}$$

and

$$(\mu_{\bar{V}}^{r})^{-1}(\alpha) = \int_0^{\frac{b}{50\alpha+150}} b dt = \frac{b^2}{50\alpha+150},$$

so we have

$$\bar{V}_\alpha = [\frac{b^2}{250-50\alpha}, \frac{b^2}{50\alpha+150}].$$

We call $\int_0^{\bar{T}_b} \bar{\lambda} t dt = \bar{W}$. Compute,

$$(\mu_{\bar{W}}^{\ell})^{-1}(\alpha) = \int_0^{\frac{b}{250-50\alpha}} (50\alpha + 150) t dt$$

$$= \tfrac{1}{2}[(50\alpha + 150)t^2]_0^{\frac{b}{250-50\alpha}}$$

$$= (25\alpha + 75)(\tfrac{b}{250-50\alpha})^2.$$

$$(\mu_{\bar{W}}^{r})^{-1}(\alpha) = \int_0^{\frac{b}{50\alpha+150}} (250 - 50\alpha) t dt$$

$$= \tfrac{1}{2}[(250 - 50\alpha)t^2]_0^{\frac{b}{50\alpha+150}}$$

$$= (125 - 25\alpha)(\tfrac{b}{50\alpha+150})^2,$$

so we have

$$\bar{W}_\alpha = [(25\alpha+75)(\frac{b}{250-50\alpha})^2, (125-25\alpha)(\frac{b}{50\alpha+150})^2].$$

Then,

$$\tfrac{1}{s}\bar{S}_\alpha = \bar{V}_\alpha + \bar{W}_\alpha$$

$$= [\tfrac{b^2}{250-50\alpha}, \tfrac{b^2}{50\alpha+150}] +$$

$$[(25\alpha + 75)(\tfrac{b}{250-50\alpha})^2, (125 - 25\alpha)(\tfrac{b}{50\alpha+150})^2]$$

$$= [\tfrac{25b^2\alpha+76b^2}{(250-50\alpha)^2}, \tfrac{126b^2-25b^2\alpha}{(50\alpha+150)^2}],$$

or

$$\bar{S}_\alpha = s[\frac{25b^2\alpha + 76b^2}{(250-50\alpha)^2}, \frac{126b^2 - 25b^2\alpha}{(50\alpha+150)^2}]. \quad (6)$$

The total cost per cycle is

$$\overline{TC} = K + \bar{I} + \bar{S}.$$

By equations (5) and (6) we have α-cut for $\overline{TC}$,

$$\overline{TC}_\alpha = [K, K] + h[\tfrac{25M^2\alpha+76M^2}{(250-50\alpha)^2}, \tfrac{126M^2-25M^2\alpha}{(50\alpha+150)^2}] + s[\tfrac{25b^2\alpha+76b^2}{(250-50\alpha)^2}, \tfrac{126b^2-25b^2\alpha}{(50\alpha+150)^2}]$$

$$= [K + \tfrac{h(25M^2\alpha+76M^2)+s(25b^2\alpha+76b^2)}{(250-50\alpha)^2}, K + \tfrac{h(126M^2-25M^2\alpha)+s(126b^2-25b^2\alpha)}{(50\alpha+150)^2}]$$

$$= [\tfrac{5000(5)(5-\alpha)^2+25M^2\alpha+76M^2+50b^2\alpha+152b^2}{5(5-\alpha)^2}, \tfrac{5000(5)(\alpha+3)^2+126M^2-25M^2\alpha+252b^2-50b^2\alpha}{5(\alpha+3)^2}].$$

Since we have $\frac{1}{T} = \frac{\bar{\lambda}}{q}$ cycles in one year, then we compute annual cost $\overline{TY} = \frac{1}{T}\overline{TC}$ through α-cut,

$$\overline{TY}_\alpha = \tfrac{1}{q}[\tfrac{5000(5)(5-\alpha)^2+25M^2\alpha+76M^2+50b^2\alpha+152b^2}{5(5-\alpha)^2}, \tfrac{5000(5)(\alpha+3)^2+126M^2-25M^2\alpha+252b^2-50b^2\alpha}{5(\alpha+3)^2}] \times [50\alpha + 150, 250 - 50\alpha]$$

We write $\overline{TY}_\alpha = \frac{10}{q}[a(\alpha), (b(\alpha)]$,

where

$$a(\alpha) = \tfrac{1875000-125000\alpha+151M^2\alpha+302b^2\alpha-175000\alpha^2}{(5-\alpha)^2} + \tfrac{25000\alpha^3+50b^2\alpha^2+228M^2+456b^2+25M^2\alpha^2}{(5-\alpha)^2}$$

and

$$b(\alpha) = \frac{1125000+525000\alpha-251M^2\alpha-502b^2\alpha-25000\alpha^2}{(\alpha+3)^2} + \frac{-25000\alpha^3+50b^2\alpha^2+630M^2+1260b^2+25M^2\alpha^2}{(\alpha+3)^2}.$$

Since the average annual cost is a fuzzy number and we want to minimize the cost, here we choose defuzzification with graded mean integration. We will find graded mean integration for $\overline{TY}$ without we determine the membership function for $\overline{TY}$. Since we have α-cut for $\overline{TY}$, so we obtain

$$G(\overline{TY}) = \int_0^1 \alpha(\ell^{-1}(\alpha) + r^{-1}(\alpha))d\alpha. \quad (7)$$

The first, we compute

$$\int_0^1 \alpha\ell^{-1}(\alpha)d\alpha$$
$$= \frac{10}{q}\int_0^1 \frac{1875000-125000\alpha+151M^2\alpha+302b^2\alpha-175000\alpha^2}{(5-\alpha)^2}\alpha d\alpha + \frac{10}{q}\int_0^1 \frac{25000\alpha^3+50b^2\alpha^2+228M^2+456b^2+25M^2\alpha^2}{(5-\alpha)^2}\alpha d\alpha$$

$$\approx \frac{459175}{q} + \frac{275.354^2}{q} + 185.936q - 371.872M. \quad (8)$$

The second we compute,

$$\int_0^1 \alpha r^{-1}(\alpha)d\alpha$$
$$= \frac{10}{q}\int_0^1 \frac{1125000+525000\alpha-251M^2\alpha-502b^2\alpha-25000\alpha^2}{(\alpha+3)^2}\alpha d\alpha + \frac{10}{q}\int_0^1 \frac{-25000\alpha^3+50b^2\alpha^2+630M^2+1260b^2+25M^2\alpha^2}{(\alpha+3)^2}\alpha d\alpha$$

$$\approx \frac{541825}{q} + \frac{536.859M^2}{q} + 355.042q - 710.084M. \quad (9)$$

We substitute equations (8) and (9) to equation (7), so we have

$$G(\overline{TY}) \simeq \frac{1001000}{q} + \frac{812.213M^2}{q} + 540.978q - 1081.956M.$$

The function $G(\overline{TY})$ is a function of two variables q and M. To minimize $G(\overline{TY})$, we solve it by MATLAB and we have $q^* \approx 74.435$ and $M^* \approx 49.578$. From equation (4), we have a fuzzy number $\bar{r^*}$ with $\bar{r^*}_\alpha = [\frac{25}{6}\alpha^2 + \frac{275}{6}\alpha + 0.708,\ \frac{25}{6}\alpha^2 - \frac{375}{6}\alpha + 109,042]$. It means we get the membership function for fuzzy reorder point,

$$\mu_{\bar{r}}(x) = \begin{cases} \sqrt{0.24x+30.42} - 5.5 & , x \in [0.708, 50.708] \\ 7.5 - \sqrt{0.24x+30.08} & , x \in (50.708, 109.042] \\ 0 & \text{, otherwise.} \end{cases}$$

VI. Conclusion

In this research, we have triangular fuzzy number that its membership function is linear. Although, the fuzzy holding cost has non linear membership function. Furthermore, the membership function of fuzzy annual cost is non linear and depend on order quantity variable. In this case, we apply non linear optimization method to determine optimal order quantity that minimize fuzzy annual cost.

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