An Accurate Parameter Estimator for LFM Signals Based on Zoom Modified Discrete Chirp Fourier Transform

Jun Song, Yihan Xu, Guanghao Li

ABSTRACT **- An accurate parameter estimation of chirp rate and initial frequency of the linear frequency modulation (LFM) signals based on zoom modified discrete chirp Fourier transform (MDCFT) is investigated. The first step of the proposed algorithm returns a coarse estimate of the parameter by addressing the maximum MDCFT coefficient of a LFM signal. The coarse estimate is refined by fine search named zoom MDCFT (Zoom-MDCFT) in the second step. Compared to traditional brute fine search approaches, Zoom-MDCFT method has merit in high efficiency because it utilizes more prior information about the MDCFT results, thus requiring fewer extra computations. Finally, computer simulations are conducted to evaluate the performance by comparison with the Cramer–Rao lower bound. The proposed estimator shows accurate and robust performance with the addition in the additive white Gaussian noise.**

Index Terms **- Parameter estimation; Modified discrete chirp Fourier transform; Linear frequency modulation; Zoom-MDCFT**

I. INTRODUCTION

Linear frequency modulation (LFM) signals, also called chirp signals, have widely used in the field of radar [1–3], ultrasound [4], and communication [5]. Accurate estimation of the initial frequency and the chirp rate of an LFM signal without any prior knowledge is always a classical topic in these applications. There have been a variety of estimators based on several extensively studied methods.

The maximum likelihood (ML) estimation algorithms [2] have numerous local optima and high computational complexity, making their application in engineering impossible. The Fast Fourier transform (FFT)-based searching approach [6] is very convenient, but the necessary condition for the best performance is the high signal-to-noise ratio (SNR). The de-chirp methodology [7] does not require heavy numerical computation but cannot achieve expected performance in a noisy environment. The estimators based on the ambiguity function (AF) [8], and the Wigner–Ville distribution (WVD) [9], either have an insufficient resolution or suffer from interference from cross terms. The disturbance induced by cross terms would be suppressed through the selection of a matching kernel; however, the energy accumulation will be inevitably degraded. The short-time Fourier transform (STFT) [10] does not have the problem of cross terms but suffers from poor resolution due to its fixed window length. The fractional Fourier transform (FrFT) [11–15], as a generalized style of ordinary Fourier transform with a fractional order, attracts more and more attention in the field of parameter estimation because it has a remarkable energy concentration on the LFM signal. However, the FrFT-based approaches must search the fractional order in the whole fractional order space and thus have a heavy computational burden as their drawback. The linear canonical transform (LCT)-based estimator [16] can be implemented by FFT. But both the LCT method suffers from the "picket fence" effect. The phase-unwrapping and time-domain ML methods are combined in [17] to estimate chirp signal parameters effectively, and the estimation accuracy is really perfect together with the root mean square errors getting close to the Cramer–Rao lower bound (CRB). However, the drawback of this combination estimator is that it is highly time-consuming.

Each estimator has its merits and some drawbacks. However, further studies on LFM signal parameter

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estimation are still essential with respect to their efficiency and accuracy, which are decisive factors in engineering practice. In recent years, the modified discrete chirp Fourier transform (MDCFT) has been proposed as an efficient method for chirp signal processing [18–21]. The MDCFT of an LFM signal is optimal in the sense of maximum energy concentration since a matching chirp kernel is employed. However, the performance of parameter estimation cannot achieve an expected accuracy due to the discrete calculation.

This work aims to derive an accurate parameter estimator for LFM signals based on MDCFT. The proposed algorithm consists of two banks, namely, coarse search and zoom-based fine search. The coarse search returns a coarse estimate of the parameter by addressing the maximum MDCFT coefficient of a signal. The coarse estimate is refined by spectrum zooms method. Compared to conventional fine search approaches, spectrum zooms method is always more efficient because it utilizes more prior information about the MDCFT results, thus requiring fewer extra computations. Finally, computer simulations are conducted to demonstrate the performance of the proposed algorithms.

II. Methodology

A noisy signal under consideration is modeled as follows:

$$\begin{aligned} x(t) &= s(t) + \mu(t) \\ &= b_0 \cdot \exp\left(j2\pi\left(\alpha_0 \cdot t + \beta_0 \cdot t^2\right)\right) + \mu(t) \quad t \in [0, T] \end{aligned} \tag{1}$$

where b_0, α_0 and β_0 refer to signal amplitude, initial frequency, and chirp rate, respectively. $\mu(t)$ is the zero-mean additive complex white Gaussian noise, and the real part and imaginary part of the noise are mutually independent and irrelevant. The variance of $\mu(t)$ is $2\sigma^2$. Moreover, T represents the total observation time. The total number of samples during the observation time is assumed to be N with sampling intervals $\tau = n{\cdot}T/N$, $n = 0,1,\ldots,(N-1)$. As a result, the discrete sequence of the LFM signal can be expressed as

$$\begin{aligned} x(n) &= s(n) + \mu(n) \\ &= b_0 \cdot \exp\left(j\,2\pi(f_0 \cdot n + k_0/N \cdot n^2)/N\right) + \mu(n), n = 0,1,\ldots,(N-1) \end{aligned} \tag{2}$$

where $f_0 = \alpha_0 T, k_0 = \beta_0 T^2$.

The MDCFT for the sequence $x(n)$ is defined as

$$\begin{aligned} X(f,k) &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) \cdot W_N^{fn+(k/N)n^2}, \\ f &= 0,1,\ldots,N-1, k = 0,1,\ldots,N-1 \end{aligned} \tag{3}$$

with $W_N = \exp(-j\,2\pi/N)$.

From (3), one can see that for each fixed k, MDCFT is the ordinary discrete Fourier transform (DFT) of the signal. When $k = 0$, the DCFT is the same as the DFT. Consequently, the MDCFT can be implemented by the fast Fourier transform algorithm, and the computational complexity is thus $O\left(N \log_2\left(N\right)\right)$ [18][19].

In addition, the definition of MDCFT indicates that the modulus of $X(f,k)$ will reach its peak of $\sqrt{N}$ when f and k precisely match $f = f_0$ and $k = k_0$, respectively. Therefore, by the computation of the MDCFT on the signal, it is shown that the signal will be kept compact and concentrated along a peak coefficient; then, the corresponding peak coordinate, denoted as (f_p, k_p), yields the estimate of the LFM signal parameters, which is known as the basic principle that MDCFT can be used as an estimator of the LFM signal parameters.

However, the parameters f_0 and k_0 are always arbitrary (in other words, they may be not integers in most cases), whereas the variables f and k in MDCFT are all integers due to discretization. The question of interest here is that there are inevitably some deviations in the parameter estimates if we regard f_p and k_p directly as the approaches of parameters f_0 and k_0 ,, respectively. What we are going to discuss in the present study is how to eliminate or minimize the deviations, thus improving the accuracy of parameter estimation.

For the sake of simplicity in expression, the sought coordinate (f_p, k_p), which is addressed by searching in the two-dimensional plane (f,k) after the calculation of MDCFT, is named quasi-peak. Meanwhile, the coordinate corresponding to the true peak can be denoted as (f_0, k_0) and its estimator is expressed as $(\hat{f}_0, \hat{k}_0)$.

III. Further Improvement

It should be noted that the cross interference of parameters f_0 and k_0 is surprisingly obvious when both of them or either of them is not an integer in MDCFT. In addition, evidence-based results for this cross interference are simulation outcomes presented in Tables I–IV. First, it can be observed in Table I that the peak coefficient of MDCFT reaches its maximum amplitude when the parameters of LFM signal are all integers, e.g., $\{f_0 = 70.0, k_0 = 90.0\}$. Moreover, the coordinate of the quasi-peak coincides with that of the true peak, resulting in $\left(f_p = f_0 = 70, k_p = k_0 = 90\right)$. By contrast, the parameters of the LFM signal vary from $\{f_0 = 70.0, k_0 = 90.0\}$ to

$\{f_0 = 70.4, k_0 = 90.0\}$ $\{f_0 = 70.2, k_0 = 90.2\}$ and $\{f_0 = 70.0, k_0 = 90.4\}$ in Tables I–IV, but the three peak coefficients of MDCFT have almost the same amplitude (marked in **bold** in Tables II–IV separately). Furthermore, the coefficients around the quasi-peak can hardly be distinguished from each other since they possess nearly equal amplitudes. For example, the coefficients at position $\left(f_p + 1 = 71, k_p = 90\right)$ in Tables II, III, and IV hold three amplitudes of 0.5046, 0.5043 and 0.5045, respectively. The difference between these coefficients is so small that we cannot differentiate them in a noisy environment; this indicates that the cross interference of parameters is so severe that it cannot be ignored. With the presence of cross interference and noise, there would be erroneous or reverse compensation if the MDCFT coefficients are utilized directly to estimate parameters, since the MDCFT coefficients with signal parameter $\{f_0 = 70.0, k_0 = 90.4\}$ are nearly indistinguishable from those with signal parameter $\{f_0 = 70.4, k_0 = 90.0\}$, and it is difficult to determine the parameter that should be compensated, and vice versa.

Table I. Normalized amplitude of MDCFT coefficients with a signal parameter $\{f_0 = 70.0, k_0 = 90.0\}$

	Column				
Row	88	89	90	91	92
68	0.0501	0.0389	0.0000	0.2985	0.6283
69	0.0981	0.0918	0.0000	0.8946	0.4890
70	0.2225	0.2985	**1.0000**	0.2985	0.2225
71	0.4890	0.8946	0.0000	0.0918	0.0981
72	0.6283	0.2985	0.0000	0.0389	0.0501

Table II. Normalized amplitude of MDCFT coefficients with signal parameter $\{f_0 = 70.4, k_0 = 90.0\}$

	Column				
Row	88	89	90	91	92
68	0.0869	0.0979	0.1261	0.3032	0.5474
69	0.1294	0.1512	0.2162	0.6952	0.4999
70	0.2253	0.3039	**0.7568**	0.5103	0.3297
71	0.4171	0.6987	0.5046	0.2407	0.1857
72	0.5498	0.5066	0.1892	0.1336	0.1144

Table III. Normalized amplitude of MDCFT coefficients with signal parameter $\{f_0 = 70.2, k_0 = 90.2\}$

	Column				
Row	88	89	90	91	92
68	0.0914	0.1021	0.1271	0.2774	0.5804
69	0.1391	0.1626	0.2209	0.7164	0.5050
70	0.2435	0.3304	**0.7548**	0.5092	0.3129
71	0.4306	0.6741	0.5043	0.2242	0.3129
72	0.5159	0.5070	0.1915	0.1273	0.1729

Table IV. Normalized amplitude of MDCFT coefficients with signal parameter $\{f_0 = 70.0, k_0 = 90.4\}$

	Column				
Row	88	89	90	91	92
68	0.0966	0.1073	0.1300	0.2537	0.6123
69	0.1498	0.1760	0.2341	0.7334	0.5083
70	0.2619	0.3558	**0.7477**	0.5079	0.2951
71	0.4401	0.6463	0.5045	0.2100	0.1611
72	0.4827	0.5067	0.1984	0.1226	0.1027

In an attempt to avoid reverse compensation caused by incorrect or superfluous interpolation, we suggest an improved method called Zoom-MDCFT, which is a supplemental version of the high resolution MDCFT algorithm.

As demonstrated in Tables II–IV, the cross interference is most serious around the quasi-peak area when the signal parameters f_0 and k_0 are not integers. So, we can introduce the refining technology, namely, Zoom-MDCFT, to enrich the detailed spectrum character of MDCFT. More concretely, the non-standard coefficients located at $\left(f_p - n\alpha, k_p\right)$, $\left(f_p, k_p - n\alpha\right)$, $\left(f_p + n\alpha, k_p\right)$, and $\left(f_p, k_p + n\alpha\right)$ need to be calculated, where α is defined as the thickness of the Zoom-MDCFT; $n = 0, 1, \ldots, L-1$; L is the number of zooms; and $L \cdot \alpha = 1$. With the assistance of spectrum zooms, the detailed spectrum features of MDCFT near the quasi-peak would be presented completely. As shown in Fig. 1, nine coefficients near the quasi-peak location (70, 90) are plotted without spectrum zooms, and monotonous information is demonstrated.

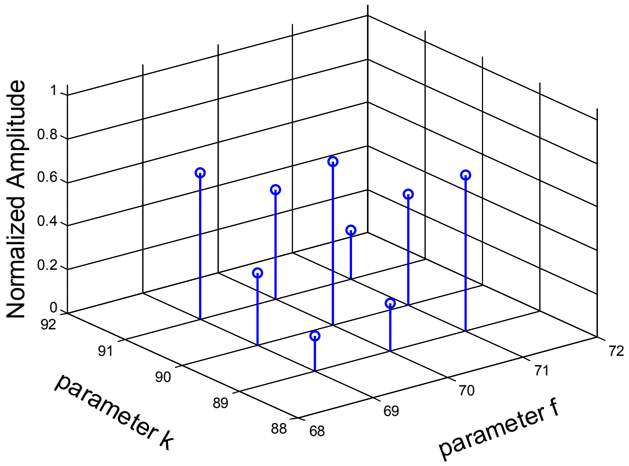

Fig. 1. Nine coefficients of MDCFT near the quasi-peak location $\left(f_p = 70, k_p = 90\right)$, with LFM signal parameters $f_0 = 70.2$ and $k_0 = 90.2$.

On the contrary, complex characterization of the MDCFT spectra near the same area is depicted by Zoom-MDCFT (as shown in Fig. 2), which is the basis of accurate parameter estimation. In addition, numerical simulation indicates that the cross interference is significantly suppressed. However, the computational effort would be increased sharply if the number of zooms is fairly large, even though the non-standard coefficients can be calculated by the FFT algorithm. Then, several simulations are conducted to evaluate the range of number L.

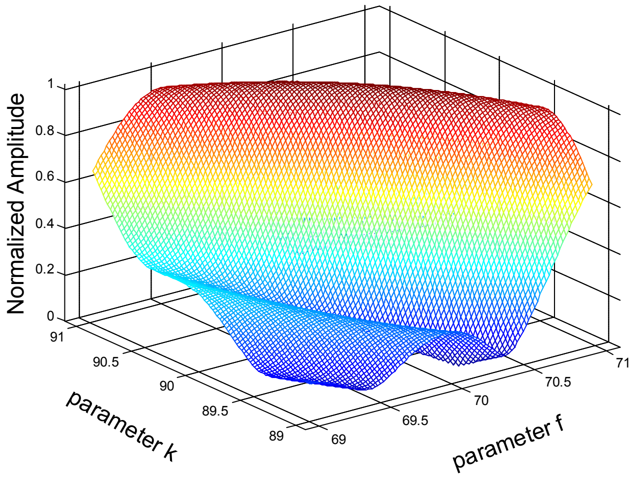

Fig. 2. Spectrogram of MDCFT near the quasi-peak location $\left(f_p = 70, k_p = 90\right)$ by Zoom-MDCFT, with LFM signal parameters $f_0 = 70.2$ and $k_0 = 90.2$.

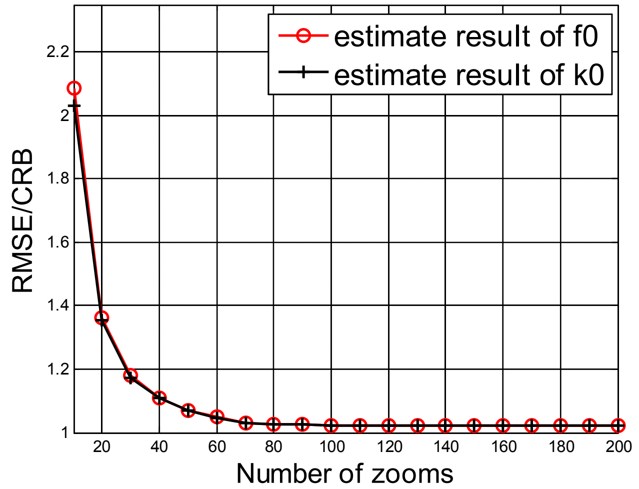

Fig. 3. The relationship of the number of zooms and RMSE, with SNR = 3 dB.

In the experiments, five LFM signals were constructed with five different parameters in the range of $[70, 71]$, and parameter k_0 is treated similarly within $[90, 91]$. Number L varies from 10 to 200 with step size 10, and the signal-to-noise ratio (SNR) level is 3 dB. Then, Monte Carlo simulations are performed 1000 times to compute the root mean square errors (RMSEs) of parameters f_0 and k_0. After the five LFM signals are all simulated, the relationships between the mean RMSE and number L are illustrated in Fig. 3, and the CRB [6] is also measured for comparison. As shown in Fig. 3, the RMSE/CRB of estimation will not decrease when the number of zooms is more than approximately 70, even though the number L increases further, let alone the extra computation with the increase of L. As a result, L is generally taken as a value within 50–80 from the perspective of engineering implementation.

Finally, the flow of the Zoom-MDCFT algorithm is summarized in Table V.

Table V. Flow of the Zoom-MDCFT algorithm.

Step 1. $X(f_i,k_j)=MDCFT\left[x(n)\right]$ and $Y(f_i,k_j)=\left|X(f_i,k_j)\right|$,

where $i,j=0,1,...,N-1$; .

Step 2. $\left\{f_p,k_p\right\}=\arg\underset{f,k}{Max}\left\{Y(f_i,k_i)\right\}$.

Step 3. Let $L=50\sim100$, by spectrum zooms to perform

$\left\{f_M,k_M\right\}=\arg\underset{f,k}{Max}\left(Y(f_m,k_n)=\left|X(f_p\pm m/L,k_p\pm n/L)\right|\right)$,

where $m,n=0,1,...,L-1$;

Step 4. End: Obtain the estimation results $\left\{\hat{f}_0=f_M,\hat{k}_0=k_M\right\}$.

The proposed Zoom-MDCFT algorithm can be easily popularized and applied to the case of multi-component signals. In engineering applications, the intensity of each component always varies from that of other components, and the presence of an intensive component signal may affect the parameter estimation of a weak component. As a result, certain measures must be taken to suppress the influence of an intensive component on weak ones. In light of the good concentration of MDCFT, we can use the chirp Fourier domain signal separation technology to avoid the interference of the intensive component on weak ones. The flow is arranged as follows. First, the peak masking process can be introduced to eliminate the most intensive component [11] to repeatedly detect the most intensive component and estimate the parameters by the Zoom-MDCFT algorithm and the second intensive component and process.

IV. Performance Analysis and Simulation Results

A. Computation Issue

The computational complexity of the proposed Zoom-MDCFT algorithm consists of two banks, namely, coarse search and fine search. For the coarse search, MDCFT can be implemented by the fast Fourier transform algorithm, and the computational complexity is thus $O\left(N\log_2\left(N\right)\right)$ [18][19]. During the fine search procedure, spectrum zooms instead of a brute-force search is introduced. For each step of spectrum zoom calculation, the computational complexity is about 4N complex multiplication; and the total computational complexity of spectrum zooms is about 4N*L complex multiplication, where L is the number of spectrum zooms. As a result, the total computational complexity during our fine search procedure is about 4N*L complex multiplication. However, the computation load of a fine search by increasing the number of zooms is about 4N*N complex multiplication, which is far greater than 4N*L, as the number of samples N is usually far greater than the number of zooms L, e.g. the N is 512,and L is 50 in our simulations.

As for the FrFT-based method [15], discrete FrFT should be performed once for each fractional order in the range [0,2π]. As a result, the primary computation complexity during the procedure of coarse search in the FrFT-based method is about $O\left(2\pi/M\cdot N\log_2 N\right)$, where M is the searching step of fractional order, not mention the fine search.

Then the computational complexity of algorithm named time-domain maximum likelihood (TDML) estimator [17] is taken for comparison too. The TDML method always needs two 1D searches, thus requiring quantity of computation $O\left(N^2\right)$, which is much larger than $O\left(N\log_2 N\right)$ when N is large enough.

B. Simulation Results

In an effort to validate the effectiveness of the proposed Zoom-MDCFT algorithms, several experiments were conducted to evaluate the estimation performance in additive Gaussian noise environments, and CRB was also employed for comparison.

First, an LFM signal with parameters $\left\{f_0=70.0,k_0=90.4\right\}$ was taken into consideration. In the experiments, the number of spectrum slices is fixed at $L=50$, and the Zoom-MDCFT algorithm is conducted. The length of signal was assumed to be N = 1000. The value of SNR varies from −9 dB to 6 dB, with increments of 1 dB. At each SNR level, 1000 Monte Carlo simulations were conducted to obtain the RMSE of the estimation result. The CRBs were also examined for comparison. The estimation results of f_0 and k_0 are plotted in Figs. 3 and 4, respectively. As comparison, the CRBs[6] and algorithms reported in [11],[15] and [17], denoted as algorithm [Qi], [Song] and [Deng] respectively, are also employed, and the ratio of RMSE / CRB is set as the *y*-axis.

As plotted in Fig.3 and Fig.4, the estimation precision improves gradually with the increase of SNR level. When the Zoom-MDCFT algorithm is applied, the RMSE of $\hat{f}_0$ and $\hat{k}_0$ is greater than 1.05 times CRB when the SNR level is lower than -3 dB. In addition, the RMSE and CRB agree closely in both figures when SNR is higher than 0 dB. It can also be confirmed that, the traditional MDCFT algorithm is inferior to the Zoom-MDCFT. The performance of the proposed algorithm then slightly outperforms the other three algorithms introduced in [11] (denoted as [Qi]), [15] (denoted as [Song]) and [17] (denoted as [Deng]) when the SNR is lower than -1dB.

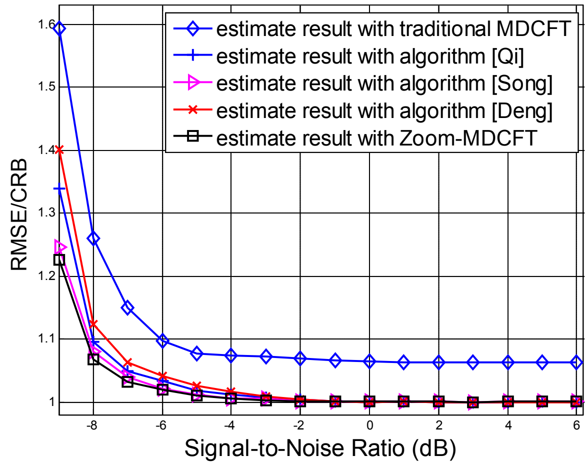

Fig. 3. The estimation results of parameter f_0 with several different iterative interpolation times.

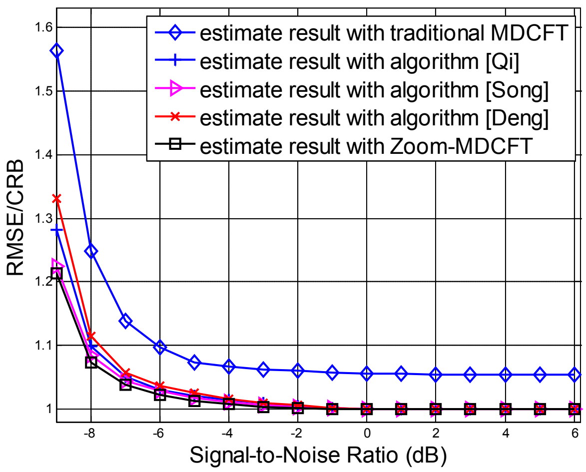

Fig. 4. The estimation results of parameter k_0 with several different iterative interpolation times.

To demonstrate the relationship between estimation performance and cross interference caused by decimal parameters, an extra simulation was conducted. In this experiment, SNR was fixed at 3 dB, and the parameter k_0 was fixed as an integer 90; at the same time, the other parameter f_0 varied from 69.5 to 70.5 with a step size of 0.1, and Monte Carlo simulations were performed 1000 times at each step. The relationship of RMSE and parameter f_0 is plotted in Fig. 5. The RMSEs of estimated parameters decrease with the parameter f_0 as it varies from 69.5 to 70.5, whereas the curves rise with the increase of f_0 from 70.0 to 70.5. As expected, the RMSEs reach their bottom when f_0 equals exactly 70.0. This means that the further the parameter deviates from the integer, the more serious the cross interference and, at the same time, the worse the estimation performance.

Fig. 5 indicates that as long as the parameters are not all integers, the cross interference between parameters will be tangible, which results in a decline in the estimation performance.

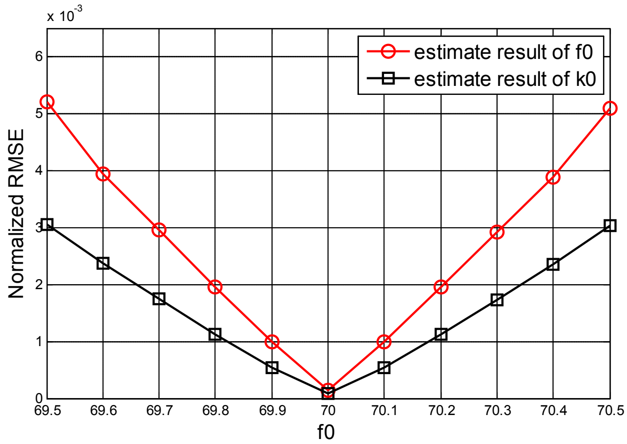

Fig. 5. The estimation results when f_0 varies from 69.5 to 70.5, with a fixed $k_0 = 90$ at the 3-dB SNR level.

Then, to validate the stability of the Zoom-MDCFT algorithm when the number of spectrum zooms is greater than a certain value, we tweaked the number from 50 to 100 in steps of 5 and conducted the following simulation. In the experiment, the LFM signal parameters were $\{f_0 = 70.0, k_0 = 90.4\}$, and the SNR level is fixed at 3 dB; then the RMSEs of estimation are calculated. As shown in Fig. 6, the Zoom-MDCFT algorithm had a nearly stable performance when the number L increases from 65 to 100, and the RMSEs of $\hat{f}_0$ and $\hat{k}_0$ fluctuate within a fairly narrow range, namely, 1.015 to 1.025.

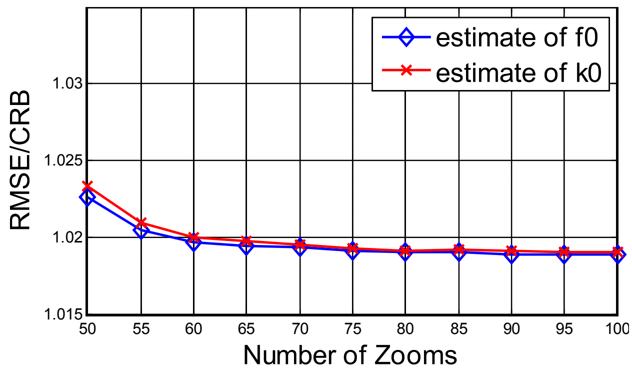

Fig. 6. The estimation results with a different number of zooms at an SNR level of 3 dB.

In the third part, experiments are performed to test and verify the robustness of the Zoom-MDCFT algorithm when the LFM signal parameter varies. In simulation experiments, the LFM signal parameters were tweaked in a range $\{f_0 = [69.5 \sim 70.5], k_0 = 90.0\}$ and $\{f_0 = [69.5 \sim 70.5], k_0 = 90.4\}$ separately, and the SNR level was set to 3 dB; then we applied the Zoom-MDCFT algorithm with 1000 Monte Carlo cycles. As illustrated in Fig. 7, the RMSEs of estimated parameters fluctuate slightly, and the maximum value of the RMSE / CRB ratio does not exceed 1.020 when the parameters vary in the range $\{f_0 = [69.5 \sim 70.5], k_0 = 90.0\}$ or $\{f_0 = [69.5 \sim 70.5], k_0 = 90.4\}$. Therefore, the proposed algorithm can ensure a steady accuracy of estimation with different LFM signal parameters.

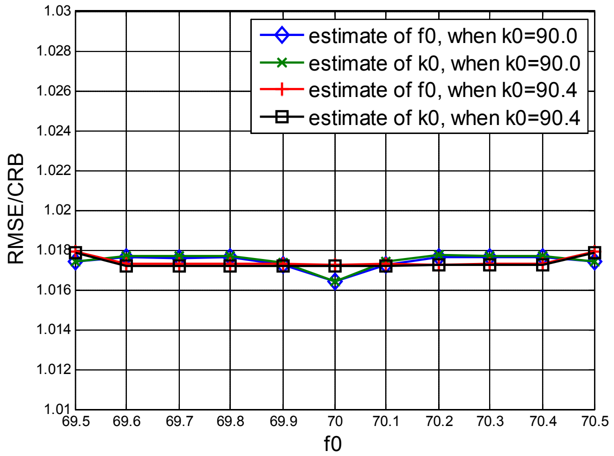

Fig. 7. The estimation results when LFM signal parameters change in the ranges $\{f_0 = [69.5 \sim 70.5], k_0 = 90.0\}$ and $\{f_0 = [69.5 \sim 70.5], k_0 = 90.4\}$ at the 3-dB SNR level.

V. CONCLUSIONS

An accurate parameter estimation method named Zoom-MDCFT has been investigated for LFM signals in the present study. First, the shortcoming of MDCFT in parameter estimation has been analyzed in detail; Second, the severe cross interference between LFM signal parameters when performing MDCFT is proposed, and the corresponding solution called the Zoom-MDCFT method is presented accordingly. Finally, numerical simulations were conducted to validate the proposed algorithm, and experimental results have shown that the RMSEs of the parameters estimated by Zoom-MDCFT converge asymptotically to the CRB under low SNR levels, and our algorithm exhibits a steady performance with different LFM signal parameters. For future work, the performance evaluation of Zoom-MDCFT with the presence of colored noise should be taken under consideration, and the golden cut method would be more time-friendly than brute searching in the procedure of peak location.

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