Black-Scholes Equation Solution Using Laplace-Adomian Decomposition Method

Ira Sumiati, *Member, IAENG,* Endang Rusyaman, and Sukono, *Member, IAENG*

Abstract—**Black-Scholes partial differential equation is a very well-known model for pricing European option with the underlying financial assets being the stock price. The combination of the Adomian decomposition method and Laplace transform is called the Laplace-Adomian decomposition method. This method is effective and easy to solve ordinary or partial differential equations. Therefore, the purpose of this paper is to find the solution to the Black-Scholes equation using the Laplace-Adomian decomposition method (LADM). The results show that LADM is able and powerful to solve the Black-Scholes equation. Furthermore, the solution obtained is used to build a call and put option price model. The numerical simulation shows that the proposed model is very useful for pricing option properly and accurately.**

Index Terms—**Black-Scholes equation, Adomian decomposition method, Laplace transform, call and put option.**

I. Introduction

In recent years, investment has grown rapidly in the financial and economic fields. This is indicated by the increasing number of investors and funds involved in investment activities, as well as the increasingly diverse financial derivative products that are developed as alternative investments. Financial derivatives are investment instruments which are derivatives of a financial asset, so the value depends on the price of the financial asset, for example, an option contract [1]. An option is a right owned by the holder to call or put an underlying financial asset at a certain price for a certain period. Options can be used for hedging or speculation. Based on the implementation, the options consist of American and European type options. American options can be exercised at any time during the option period, while European options can only be exercised at the end of the option period. Therefore, the most traded option on the exchange is the American option, but the analysis and calculation of European options is easier than the American option. Keep in mind that options give the holder the right to call or put the underlying assets, the holder does not have to exercise this right [2].

One very well-known technique for pricing option is a binomial tree which assumes that time follows a simple discrete approach and its underlying assets are stock prices [3]. This tree illustrates that stock price movements during the option period have a probability to be going up or down. Other models or techniques that are also very well-known for pricing option is the Black-Scholes equations. The basic concept of this model is to price a European call option with the underlying asset being the stock price without paying dividends. The Black-Scholes equatios is a partial differential equations with a continuous-time approach [4]. Various methods are developed to solve partial differential equations, such as homotopy pertubation method [5], homotopy analysis method [6], variational iteration method [7] and static hand gesture recognition method based on Gaussian mixture model [8]. Several methods can also be used to solve the Black-Scholes partial differential equation.

The numerical solution of Black-Scholes partial differential equations can be obtained by the Merlin transformation approach [9] and semi discritization techniques [10]. Homotopy perturbation method [11], [12], homopoty analysis method, and variational iteration method [13], [14], [15] can be used to solve the Black-Scholes equation and the boundary conditions for European option pricing problems quickly and accurately. The finite difference method ensures that the scheme is stable for any volatility and interest rates, and shows accurate and effective method for solving the Black-Scholes equation [16], [17]. The projected differential transformation method is a modification of the classical differential transformation method applied to solve the Black-Scholes equation for pricing European and Asian option [18], [19], [20]. Another method used to find a solution to the Black-Scholes equation is the Adomian decomposition method. Analytical solutions from these equations are formed in infinite series that converge with components that are easily calculated and obtained by efficient recursive relationships, where nonlinear forms are decomposed into Adomian polynomials [21], [22], [23], [24].

Adomian decomposition method can be used to solve differential equations, including nonlinear partial differential equations. This method was first introduced by George Adomian to solve the system of stochastic equations [25]. This decomposition method can be an effective procedure for obtaining analytical solutions without linearization or weak nonlinear assumptions, perturbation theory, discretization, transformation or restrictive assumptions on stochastic cases [26]. This method can be used to solve algebraic, integral, differential and integrodifferential equations, even systems of equations. Differential equations that can be solved by this method can be an integer or fractional order, ordinary or partial, with initial or boundary value problems, with variable or constant coefficients, linear or nonlinear, homogeneous or nonhomogeneous [27], [28], [29]. Adomian decomposition method is a powerful and useful method for solving wave [30], Fokker-Planck [31], Riccati [32], heat [33], [34] and Chi-square quantile differential equations [35].

The numerical or algorithm scheme of the Laplace transform based on the Adomian decomposition method can be used to obtain an approximate solution of nonlinear

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differential equations. The main idea of this technique is to apply Laplace transforms to differential equations and assume the solution can be decomposed into an infinite series. The main advantage of this technique is that solutions can be expressed as infinite series that converge rapidly to exact solutions [36]. The Laplace-Adomian decomposition method is used to solve the Bratu problem [37], nonlinear Volterra integrodifferential equation [38], Burgers [39] and Kundu-Eckhaus differential equation [40].

According to the background of the problem and previous studies that have been presented, we are motivated to solve the Black-Scholes equation using the Laplace-Adomian decomposition method (LADM). Then, the solution obtained is used to build a model for valuing the call and put options, which did not exist in previous studies. Numerical simulations are presented to show the accuracy of the proposed Black-Scholes model, and to compare it with existing classical Black-Scholes models [41].

II. Black-Scholes Equation

This section discusses the Black-Scholes option pricing equation. The option price is denoted by $V(S,t)$ is a function that depends on the current value of the underlying asset S and time t, where $C(S,t)$ and $P(S,t)$ respectively are call and put options. Option price also depends on the volatility of the underlying asset σ, exercise price E, expiry or maturity T and free-risk interest rate r. Black-Scholes partial differential equation for pricing option can be written [11], [41]

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=0. \quad (1)$$

The Black-Scholes equation for pricing call options based on Eq. (1) can be rewritten as follows

$$\frac{\partial C}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2}+rS\frac{\partial C}{\partial S}-rC=0 \quad (2)$$

with

$$C(0,t)=0, C(S,t)\sim S \text{ as } S\to\infty$$

and

$$C(S,T)=\max\{S-E,0\}.$$

Eq. (2) looks like the diffusion equation, but it has more terms, and each time C is differentiated concerning S it is multiplied by S, giving non-constant coefficients. Also, the equation is clearly in backward form, with final data given at $t=T$. The first thing to do is to get rid of the awkward S and S^2 terms multiplying $\partial C/\partial S$ and $\partial^2 C/\partial S^2$. At the same time take the opportunity of making the equation dimensionless, as defined in the technical point below, and turn it into a forward equation. Suppose

$$S=Ee^x, t=T-\frac{2\tau}{\sigma^2}, C(S,t)=Ev(x,\tau). \quad (3)$$

Use Eq. (3) and the partial derivatives of C is

$$\begin{aligned}\frac{\partial C}{\partial t} &= \frac{\partial Ev}{\partial \tau}\frac{\partial \tau}{\partial t}=-\frac{E\sigma^2}{2}\frac{\partial v}{\partial \tau}\\ \frac{\partial C}{\partial S} &= \frac{\partial Ev}{\partial x}\frac{\partial x}{\partial S}=\frac{E}{S}\frac{\partial v}{\partial x}\\ \frac{\partial^2 C}{\partial S^2} &= \frac{\partial}{\partial S}\left(\frac{E}{S}\frac{\partial v}{\partial x}\right)=-\frac{E}{S^2}\frac{\partial v}{\partial x}+\frac{E}{S^2}\frac{\partial^2 v}{\partial x^2}.\end{aligned}$$

Substitute Eq. (3) dan partial derivatives C to Black-Scholes Eq. (2), thus obtained

$$\frac{\partial v}{\partial \tau}-\frac{\partial^2 v}{\partial x^2}-\left(\frac{2r}{\sigma^2}-1\right)\frac{\partial v}{\partial x}+\frac{2r}{\sigma^2}v=0.$$

Suppose $k=\frac{2r}{\sigma^2}$, then the equation above can be written

$$\frac{\partial v}{\partial \tau}=\frac{\partial^2 v}{\partial x^2}+(k-1)\frac{\partial v}{\partial x}-kv \quad (4)$$

with the initial condition becomes $v(x,0)=\max\{e^x-1,0\}$.

This system of equations contains just two dimensionless parameters $k=\frac{2r}{\sigma^2}$ which k represents the balance between the rate of interest and the variability of stock returns and the dimensionless time to expiry $\frac{\sigma^2 T}{2}$, even though there are four-dimensional parameters, E, T, σ^2 and r, in the original statement of the problem.

III. Laplace-Adomian Decomposition Method

In this section, the Laplace Adomian Decomposition Method (LADM) discussed to solve the differential equation. Given a partial differential equation as follows

$$M_t u(x,t)+Nu(x,t)+Ru(x,t)=g(x,t) \quad (5)$$

with initial condition

$$u(x,0)=f(x)$$

where u is the two variables function, $M_t=\frac{\partial}{\partial t}$ is a partial derivative operator, N is a nonlinear operator, R is a linear operator and g is a given function. Solving for $M_t u(x,t)$, Eq. (5) can be written

$$M_t u(x,t)=g-Nu-Ru. \quad (6)$$

The Laplace transform is the transformation of the integral function of a real variable t to the function of a complex variable s. Laplace transform can be used to find solutions to differential equations by turning them into algebraic equations [42], [43]. Before using the Adomian decomposition method combined with Laplace transform, first explain some basic definitions and properties as follows.

Definition 1 Suppose that f is a real or complex function of variables $t>0$ and s is a real or complex parameter. Laplace transform is defined

$$F(s)=\mathcal{L}[f(t)]=\int_0^\infty e^{-st}f(t)dt=\lim_{b\to\infty}\int_0^b e^{-st}f(t)dt$$

where the limit value exists and finite. If $\mathcal{L}[f(t)]=F(s)$, then the Laplace transform inverse is denoted as

$$\mathcal{L}^{-1}[F(s)]=f(t), t\geq 0.$$

Based on Definition 1, for $f(t)=t^n$ where $t\geq 0$, Laplace transform $f(t)$ is

$$\mathcal{L}[t^n]=\frac{n!}{s^{n+1}}, s>0$$

and Laplace transform for n-th derivative is

$$\mathcal{L}[f^{(n)}(t)]=s^nF(s)-\sum_{k=1}^{n}s^{n-k}f^{(n-1)}(0).$$

Apply the Laplace transform to Eq. (6), so that it is obtained

$$\mathcal{L}[M_t u(x,t)] = \mathcal{L}[g - Nu - Ru]$$

or equivalent with

$$su(x,s) - u(x,0) = \mathcal{L}[g - Nu - Ru]. \tag{7}$$

Substitute intial condition, Eq. (7) can be written

$$u(x,s) = \frac{f(x)}{s} + \frac{1}{s}\mathcal{L}[g] - \frac{1}{s}\mathcal{L}[Nu] - \frac{1}{s}\mathcal{L}[Ru] \tag{8}$$

furthermore, apply inverse Laplace transform to Eq. (8)

$$u(x,t) = f(x) + \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}[g] - \frac{1}{s}\mathcal{L}[Nu] - \frac{1}{s}\mathcal{L}[Ru]\right]. \tag{9}$$

Adomian decomposition method assumes that $u(x,t)$ can be decomposed into an infinite series [26], [28]

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{10}$$

and nonlinear term $Nu(x,t)$ is decomposed become

$$Nu(x,t) = \sum_{n=0}^{\infty} A_n \tag{11}$$

where $A_n = A_n(u_0, u_1, ..., u_n)$ are the Adomian polynomials defined by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{n}\lambda^k u_k\right)\right]_{\lambda=0} ; n = 0,1,2,...$$

with λ is a parameter, the A_n Adomian polynomial can be described as follows

$$\begin{aligned} A_0 &= N(u_0), \\ A_1 &= u_1 N'(u_0), \\ A_2 &= \frac{u_1^2}{2!}N'(u_0) + u_2 N(u_0), \\ &\vdots \end{aligned}$$

Substitute Eq. (10) and Eq. (11) to Eq. (9)

$$\begin{aligned} \sum_{n=0}^{\infty} u_n &= f(x) \\ &+ \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}[g] - \frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} A_n\right] - \frac{1}{s}\mathcal{L}\left[R\sum_{n=0}^{\infty} u_n\right]\right] \end{aligned} \tag{12}$$

therefore based on Eq. (12), a recursive relation of solution is obtained

$$\begin{aligned} u_0(x,t) &= f(x) + \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}[g(x,t)]\right], \\ u_{n+1}(x,t) &= -\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[A_n\right] + \frac{1}{s}\mathcal{L}\left[Ru_n\right]\right], \end{aligned}$$

where $n = 0, 1, 2, \cdots$.

Hence, an approximate solution of Eq. (5) is

$$u(x,t) \approx \sum_{n=0}^{k} u_n(x,t)$$

where

$$\lim_{k\to\infty}\sum_{n=0}^{k} u_n(x,t) = u(x,t).$$

The Adomian decomposition method that is combined with the Laplace transform needs less work in comparison with the standard Adomian decomposition method. The decomposition procedure of Adomian will be easy and efficient technique, without linearization or discretization of the problem. The approximate solution is found in the form of a convergent series with easily computed components and convergence quickly to the exact solution [36], [39], [40].

IV. Numerical Simulation

Based on the LADM algorithm, the following is a recursive solution of the Black-Scholes Eq. (4)

$$\begin{aligned} v_0 &= \max\{e^x - 1, 0\}, \\ v_{n+1} &= \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[\frac{\partial^2 v_n}{\partial x^2} + (k-1)\frac{\partial v_n}{\partial x} - kv_n\right]\right], \end{aligned}$$

where $n = 0, 1, 2, \cdots$.

If the recursive solution is described, then it is obtained

$$\begin{aligned} v_1 &= \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[\frac{\partial^2 v_0}{\partial x^2} + (k-1)\frac{\partial v_0}{\partial x} - kv_0\right]\right] \\ &= \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[k\max\{e^x, 0\} - k\max\{e^x - 1, 0\}\right]\right] \\ &= \mathcal{L}^{-1}\left[\frac{k\max\{e^x, 0\} - k\max\{e^x - 1, 0\}}{s^2}\right] \\ &= k\tau\max\{e^x, 0\} - k\tau\max\{e^x - 1, 0\} \end{aligned}$$

because $\frac{\partial v_1}{\partial x} = k\tau\max\{e^x, 0\} - k\tau\max\{e^x, 0\} = 0$, so

$$\begin{aligned} v_2 &= \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[\frac{\partial^2 v_1}{\partial x^2} + (k-1)\frac{\partial v_1}{\partial x} - kv_1\right]\right] \\ &= \mathcal{L}^{-1}\left[\frac{-k^2\max\{e^x, 0\} + k^2\max\{e^x - 1, 0\}}{s^3}\right] \\ &= -\frac{1}{2}(k\tau)^2\max\{e^x, 0\} + \frac{1}{2}(k\tau)^2\max\{e^x - 1, 0\} \\ v_3 &= \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[\frac{\partial^2 v_2}{\partial x^2} + (k-1)\frac{\partial v_2}{\partial x} - kv_2\right]\right] \\ &= \mathcal{L}^{-1}\left[\frac{k^3\max\{e^x, 0\} - k^3\max\{e^x - 1, 0\}}{s^4}\right] \\ &= \frac{1}{6}(k\tau)^3\max\{e^x, 0\} - \frac{1}{6}(k\tau)^2\max\{e^x - 1, 0\} \\ &\vdots \end{aligned}$$

Hence the solution of the Black-Scholes Eq. (4) can be formed into an infinite series that is convergent as follows

$$\begin{aligned} v(x,\tau) &= \lim_{k\to\infty}\sum_{n=0}^{k} v_n(x,\tau) \\ &= \max\{e^x - 1, 0\}e^{-k\tau} + \max\{e^x, 0\}(1 - e^{-k\tau}) \end{aligned}$$

with

$$e^x = \frac{S}{E}, \tau = \frac{\sigma^2}{2}(T-t), k = \frac{2r}{\sigma^2}$$

where S is asset price, t is time or date, E is excercise price, T is maturity date, σ is volatility of asset price and r is interest rate.

Based on the solution above, the call and put option price formula is obtained by the Black-Scholes equation using LADM and substitution of Eq. (3) are

$$C(S,t) = E \max\left\{\frac{S}{E} - 1, 0\right\} e^{-r(T-t)} + E \max\left\{\frac{S}{E}, 0\right\}(1 - e^{-r(T-t)}) \tag{13}$$

and

$$P(S,t) = Ee^{-r(T-t)} - S + E \max\left\{\frac{S}{E} - 1, 0\right\} e^{-r(T-t)} + E \max\left\{\frac{S}{E}, 0\right\}(1 - e^{-r(T-t)}). \tag{14}$$

The exact solution or we called the classical Black-Scholes model for pricing call and put option is given in [4], [41]

$$C(S,t) = SN(d_1) - Ee^{-r(T-t)}N(d_2)$$

and

$$P(S,t) = Ee^{-r(T-t)}N(-d_2) - SN(-d_1)$$

with

$$d_1 = \frac{\ln \frac{S}{E} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \text{ and } d_2 = d_1 - \sigma\sqrt{T-t}$$

where $N(d)$ is the cumulative normal density function.

Fig. 1 shows the call option price C of a stock variable S from the solution of Black-Scholes equation using LADM compared with exact solution, where an exercise price $E = 5$ and a risk-free interest rate $r = 0.05$ during a three-month option contract, even Fig. 2 for six months, Fig. 3 for one year, Fig. 4 for one and a half year.

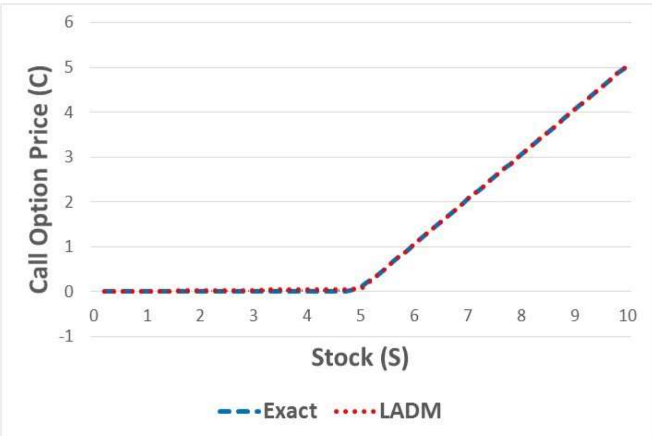

Fig. 1. The LADM Solution vs. Exact for Three-months Call Option Contract

Based on the Mean Absolute Error, each error for the different option contract periods (3 months, 6 months, 1 year and 1.5 years), respectively is 2%, 3%, 5%, and 8%. Based on the four case examples, all errors between the proposed Black-Scholes model and the classical Black-Scholes model for call option price have a percentage less than 10%. However, the longer the period of options contract used, then the error is getting greater. For all options contract periods, the call option price with the proposed Black-Scholes model is greater than the call option price with the classical Black-Scholes model when the stock price is less than the exercise price (more precisely when $S < 4.9$).

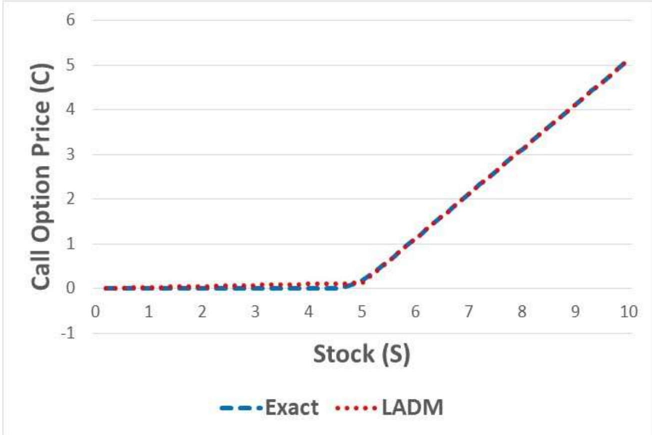

Fig. 2. The LADM Solution vs. Exact for Six-months Call Option Contract

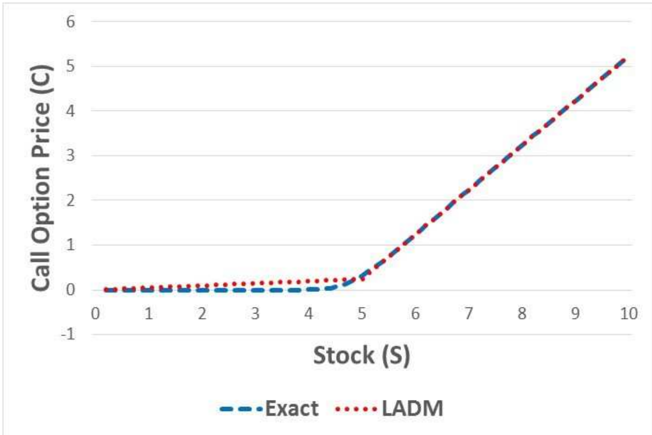

Fig. 3. The LADM Solution vs. Exact for One-year Call Option Contract

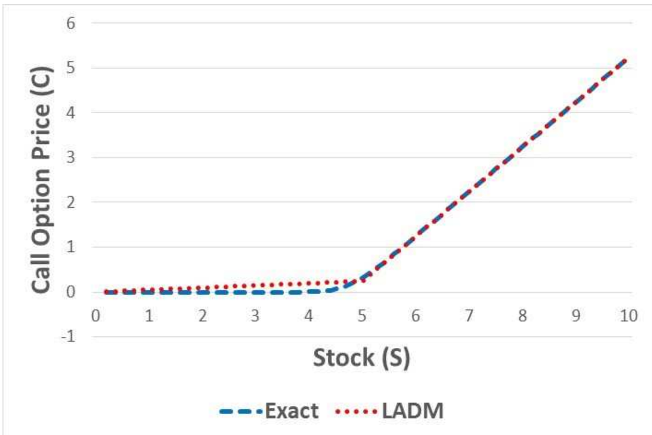

Fig. 4. The LADM Solution vs. Exact for One and a half year Call Option Contract

In Fig. 1 and 2, the call option prices for the two models tend to be the same. Whereas in Fig. 3 and 4, the call option price with the proposed Black-Scholes model has a smoother graph than the call option price with the classical Black-Scholes model. Fig. 5 shows the put option price P of a stock variable S from the solution of Black-Scholes equation using LADM compared with the exact solution, where an exercise price $E = 5$ and a risk-free interest rate $r = 0.05$ during a three-month option contract, even Fig. 6 for six months, Fig. 7 for one year, Fig. 8 for one and a half year.

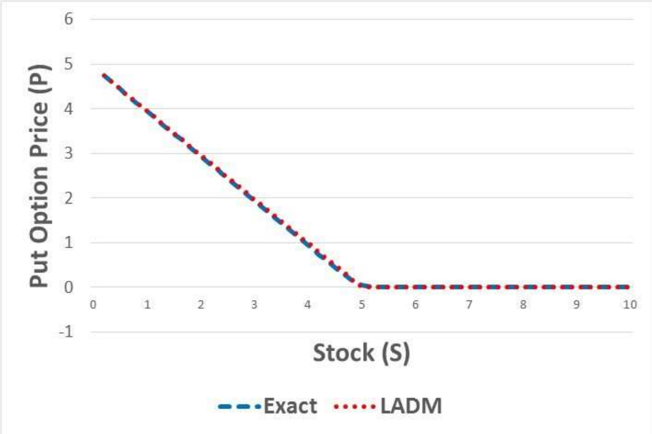

Fig. 5. The LADM Solution vs. Exact for Three-months Put Option Contract

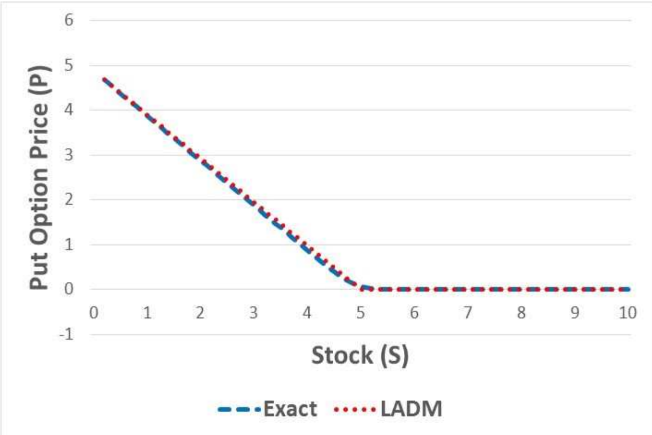

Fig. 6. The LADM Solution vs. Exact for Six-months Put Option Contract

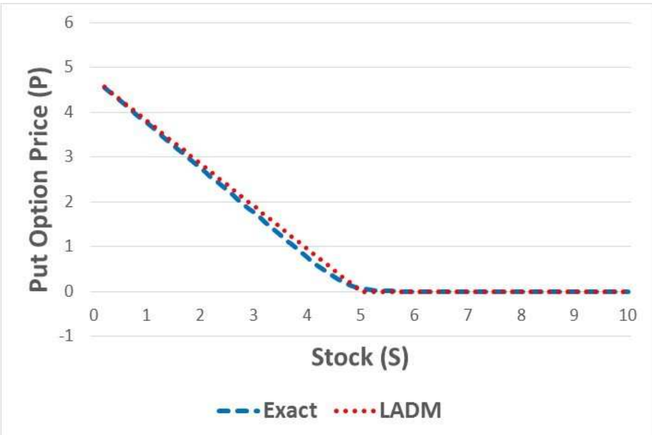

Fig. 7. The LADM Solution vs. Exact for One-year Put Option Contract

The same analysis also applies to value the price of put options. Based on the Mean Absolute Error, each error for the different option contract periods (3 months, 6 months, 1 year and 1.5 years), respectively is 2%, 3%, 5%, and 8%. Based on the four case examples, all errors between the proposed Black-Scholes model and the classical Black-Scholes model for put option price have a percentage less than 10%. However, the longer the period of options contract used, then the error is getting greater. For all options contract periods, the put option price with the proposed Black-Scholes

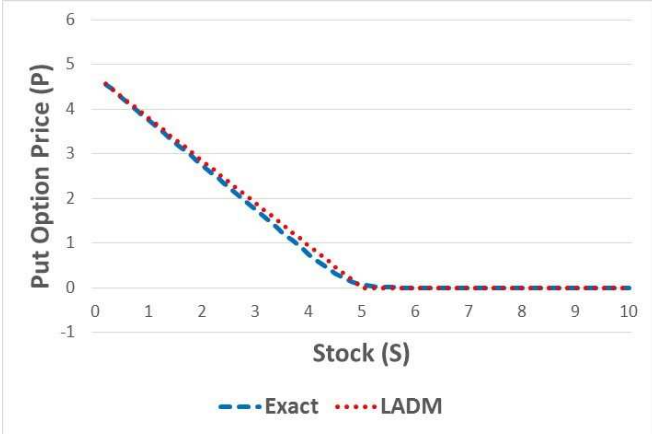

Fig. 8. The LADM Solution vs. Exact for One and a half year Put Option Contract

model is greater than the put option price with the classical Black-Scholes model when the stock price is less than the exercise price (more precisely when $S < 4.9$). In Fig. 5 and 6, the put option prices for the two models tend to be the same. Whereas in Fig. 7 and 8, the put option price with the proposed Black-Scholes model has a smoother graph than the put option price with the classical Black-Scholes model.

V. Conclusion

Laplace-Adomian decomposition method (LADM) is an effective and easy algorithm for solving differential equations. Especially the Black-Scholes partial differential equation presented in this paper. The solution obtained is used to build a call and put option price model. Numerical simulations show that the proposed model is accurate and powerful for pricing option. Because for all the case studies presented with various option periods, the model has an error of less than 10%. However, the longer the period of options contract used, then the error is getting greater. For all options contract periods, the call or put option price with the proposed Black-Scholes model is greater than the call or put option price with the classical Black-Scholes model when the stock price is less than the exercise price. Besides that, it can be concluded that the model obtained using LADM is better than the classical Black-Scholes model, because it has smoother graphics, especially for 1 and 1.5-year option contracts.

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References

[1] M. A. M. Ghandehari and M. Ranjbar, "European Option Pricing of Fractional Version of the Black-Scholes Model: Approach Via Expansion in Series," *Int. J. Nonlinear Sci.*, vol. 17, no. 2, pp. 105-110, 2014.

[2] J. C. Hull, *OPTIONS, FUTURES, AND OTHER DERIVATIVES*, 8th ed. Totonto, USA: Prentice Hall, pp. 1-329, 2012.
[3] J. C. Cox, S. A. Ross and M. Rubinstein, "Option pricing: A simplified approach," *J. financ. econ.*, vol. 7, no. 3, pp. 229-263, 1979.
[4] F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities," *J. Polit. Econ.*, vol. 81, no. 3, pp. 637-654, 1973.
[5] S. T. Mohyud-Din and M. A. Noor, "Homotopy Perturbation Method for Solving Partial Differential Equations," *Z. Naturforsch.*, vol. 64a, pp. 157-170, 2009.
[6] Z. M. A. Abbas, "Homotopy Analysis Method for Solving Nonlinear Various Problem of Partial Differential Equations," *Mathematical Theory and Modeling*, vol. 4, no. 14, pp. 113-126, 2014.
[7] M. Datar and K. Takale, "On Variational Iteration Method For Solving Partial Differential Equations And Its Applications," *Int. Jr. of Engineering, Contemporary Mathematics and Sciences*, vol. 1, no. 2, pp. 13-20, 2015.
[8] Q. Zheng, X. Tian, S. Liu, M. Yang, H. Wang and J. Yang, "Static Hand Gesture Recognition Based on Gaussian Mixture Model and Partial Differential Equation," *IAENG International Journal of Computer Science*, vol. 45, no. 4, pp. 569-583, 2018.
[9] J. C. Cortés, L. Jódar, R. Sala and P. Sevilla-Perris, "Exact and numerical solution of Black-Scholes matrix equation," *Appl. Math. Comput.*, vol. 160, pp. 607-613, 2005.
[10] R. Company, E. Navaro, J. Ramón Pintos and E. Ponsoda, "Numerical solution of linear and nonlinear Black-Scholes option pricing equations," *Comput. Math. with Appl.*, vol. 56, pp. 813-821, 2008.
[11] V. Gülkaç, "The homotopy perturbation method for the Black – Scholes equation," *J. Stat. Comput. Simul.*, vol. 80, no. 12, pp. 1349-1354, 2010.
[12] S. Kumar, A. Yildirim, Y. Khan, H. Jafari, K. Sayevand and I. Wei, "Analytical Solution of Fractiona Black-Scholes European Option Pricing Equation by Using Laplace Transform," *Journal of Fractional Calculus and Applications*, vol. 2, no. 8, pp. 1-9, 2012.
[13] T. Allahviranloo and S. S. Behzadi, "The use of iterative methods for solving Black-Scholes equation," *Int. J. Ind. Math.*, vol. 5, no. 1, pp. 1-11, 2013.
[14] S. Kumar, D. Devendra and J. Singh, "Numerical computation of fractional Black-Scholes equation arising in financial market," *Egyptian Journal of Basic and Applied Sciences*, vol. 1, pp. 177-183, 2014.
[15] S. Eshaghi, A. Ansari, R. K. Ghaziani and M. A. Darani, "Fractional Black-Scholes Model with Regularized Prabhakar Derivatives," *PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE, Nouvelle série, tome*, vol. 102, no. 116, pp. 121-132, 2017.
[16] Z. Cen and A. Le, "A robust and accurate finite difference method for a generalized BlackScholes equation," *J. Comput. Appl. Math.*, vol. 235, no. 13, pp. 3728-3733, 2011.
[17] M. H. Akrami and G. H. Erjaee, "Numerical Solutions for Fractional Black-Scholes Option Pricing Equation," *Global Analysis and Discrete Mathematics*, vol. 1, no. 1, pp. 9-14, 2016.
[18] S. O. Edeki, O. O. Ugbebor and E. A. Owoloka, "Analytical Solutions of the Black–Scholes Pricing Model for European Option Valuation via a Projected Differential Transformation Method," *Entropy*, vol. 17, pp. 7510-7521, 2015.
[19] S. O. Edeki, O. O. Ugbebor and E. A. Owoloko, "Analytical Solution of the Time-fractional Order Black-Scholes Model for Stock Option Valuation on No Dividend Yield Basis," *IAENG International Journal of Applied Mathematics*, vol. 47, no. 42, pp. 407-416, 2017.
[20] S. O. Edeki, O. O. Ugbebor and O. O. Ogundile, "Analytical Solutions of a Continuous Arithmetic Asian Model for Option Pricing using Projected Differential Transform Method," *Engineering Letters*, vol. 27, no. 2, pp. 303-310, 2019.
[21] M. Bohner and Y. Zheng, "On analytical solutions of the Black-Scholes equation," *Appl. Math. Lett.*, vol. 22, pp. 309-313, 2009.
[22] J. Biazar and F. Goldoust, "The Adomian Decomposition Method for the Black-Scholes Equation," *3rd Int. Conf. Appl. Math. Pharm. Scincences, Singapore*, pp. 321-323, 2013.
[23] O. González-Gaxiola, J. Ruíz de Chávez and J. A. Santiago, "A Nonlinear Option Pricing Model Through the Adomian Decomposition Method," *Int. J. Appl. Comput. Math.*, vol. 2, pp. 435-467, 2016.
[24] M. Yavuz and N. Ozdemir, "A Quantitative Approach to Fractional Option Pricing Problems with Decomposition Series," *Konuralp Journal of Mathematics*, vol. 6, no. 1, pp. 102-109, 2018.
[25] G. Adomian, "STOCHASTIC SYSTEMS ANALYSIS*," *J. Polit. Econ.*, pp. 1-17, 1980.
[26] G. Adomian, "A review of the decomposition method in applied mathematics," *J. Math. Anal. Appl.*, vol. 135, no. 2, pp. 501-544, 1988.
[27] J. Duan, R. Rach, D. Baleanu and A. Wazwaz, "A review of the Adomian decomposition method and its applications to fractional differential equations," *Commun. Frac. Calc.*, vol. 3, no. 2, pp. 73-99, 2012.
[28] E. Al awawdah, *Partial Differential Equations: New Methods for Their Treatment and Solution*. Palestine: M. Sc. Thesis, Department of Mathematics, Birzeit University, pp. 1-64, 2016.
[29] A. A. Opaguna, E. A. Owoloko, O. O. Agboola and H. I. Okagbue, "Application of Homotopy Perturbation and Modified Adomian Decomposition Methods for Higher Order Boundary Value Problems," *Proceedings of the World Congress on Engineering*, vol. 1, pp. 130-134, 2017.
[30] X. Luo, Q. Wu and B. Zhang, "Revisit on partial solutions in the Adomian decomposition method: Solving heat and wave equations," *J. Math. Anal. Appl.*, vol. 321, pp. 353-363, 2006.
[31] M. Tatari, M. Dehghan and M. Razzaghi, "Application of the Adomian decomposition method for the Fokker – Planck equation," *Math. Comput. Model.*, vol. 45, pp. 639-650, 2007.
[32] A. El-Ajou, Z. Odibat, S. Momani and A. Alawneh, "Construction of Analytical Solutions to Fractional Differential Equations Using Homotopy Analysis Method," *IAENG International Journal of Applied Mathematics*, vol. 40, no. 2, pp. 43-51, 2010.
[33] A. Cheniguel and A. Ayadi, "Solving Heat Equation by the Adomian Decomposition Method," *Proceedings of the World Congress on Engineering*, vol. 1, pp. 288-290, 2011.
[34] A. A. Opaguna, E. A. Owoloko and H. I. Okagbue, "Comparison Homotopy Perturbation and Adomian Decomposition Techniques for Parabolic Equations," *Proceedings of the World Congress on Engineering*, vol. 1, pp. 24-27, 2017.
[35] H. I. Okagbue, M. O. Adamu and T. A. Anake, "Solutions of Chi-square Quantile Differential Equation," *Proceedings of the World Congress on Engineering and Computer Science*, vol. 2, pp. 813-818, 2017.
[36] S. A. Khuri, "A Laplace Decomposition Algoritm to a Class of Nonlinear Differential Equations," *Journal of Applied Mathematics*, vol. 1, no. 4, pp. 141-155, 2001.
[37] S. A. Khuri, "A new approach to Bratu's problem," *Applied Mathematics and Computation*, vol. 147, pp. 131-136, 2004.
[38] A. Wazwaz, "The combined Laplace transform–Adomian decomposition method for handling nonlinear Volterra integro–differential equations," *Applied Mathematics and Computation*, vol. 216, pp. 1304-1309, 2010.
[39] A. Naghipour and J. Manafian, "Application of the Laplace Adomian Decomposition and Implicit Mehods for Solving Burgers' Equation," *TWMS J. Pure Appl. Math.*, vol. 6, no. 1, pp. 68-77, 2015.
[40] O. Gonzalez-Gaxiola, "The Laplace-Adomian Decomposition Method Applied to the Kundu-Eckhaus Equation," *International Journal of Mathematics And its Applications*, vol. 5, no. 1-A, pp. 1-12, 2017.
[41] P. Wilmot, S. Howison and J. Dewynne, *The Mathematics of Financial Derivatives: A Student Introduction*. Cambrige, England: Press Syndicate, pp. 18-83, 1996.
[42] J. L. Schiff, *The Laplace Transform: Theory and Applications*. New York, USA: Spinger-Verlag, pp. 1-35, 1999.
[43] M. C. Potter, *Engineering Analysis*. Cham, Switzerland: Springer, pp. 111-152, 2019.

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