An On-line Calibration Method of Star Sensor/ Inertial Navigation System for Marine

Xiuwei Xia, Qian Sun

Abstract—**With the continuous application and development of marine navigation technology, the accuracy requirements of navigation information for ships are becoming higher and higher. The Inertial Navigation System (INS) has been widely used due to their advantages in accuracy and stability. S-tar sensor, which can provide high-precision carrier attitude quaternion information, has gradually gained more and more applications. With the continuous application and development of various integrated navigation systems, based on the above two sensors, the Star sensor/INS combined attitude positioning system has the advantages in high-precision, anti-interference and autonomy. However, sensor errors and system initial errors will affect the performance of this integrated navigation system significantly. To compensate the above errors is a common way to improved the accuracy of the integrated navigation system, and the error calibration is the premise of error compensation. Different from the traditional Star sensor/INS integrated navigation system which carries out data fusion on the computer, this study is based on the actual installation in which the Star sensor is installed directly and fixedly on the INS. In this case, not only the error sources within the INS, but also the installation error between the INS and the Star sensor should be considered. Thus, on the basis of considering the traditional error terms, introduced the new installation error term and established the extended INS error model, the errors can be estimated and compensated in real time by utilizing the state estimation method, improving the accuracy of the Star sensor/INS integrated navigation system greatly. Simulations are carried out and the results verify the effectiveness and availability of the improved on-line calibration method proposed in this manuscript.**

Index Terms—**Star sensor, Inertial Navigation System (INS), on-line calibration, quaternion.**

I. Introduction

CALIBRATION of the Inertial Navigation System (INS) is to determine the error of the produced instrument, and use a certain means to compensate the difference [1], [2]. The purpose is to determine the inertial measurement component error term to compensate in the device output to improve navigation accuracy [3], [4]. Although the error of the instrument can be determined in the laboratory, these parameters will change in the actual use afterwards, and there is no guarantee that the predetermined accuracy will always be maintained in the actual application [5], [6]. On-line calibration of fiber optic gyro inertial devices not only reduces the frequency of calibration on time, but also eliminates previous calibration methods, greatly reducing the complexity of use [7], [8].

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X. Xia is with College of Automation, Harbin Engineering University, Harbin, Heilongjiang, 150001 China e-mail:

Q. Sun is with College of Information and Communication Engineering, Harbin Engineering University, Harbin, Heilongjiang, 150001 China e-mail: qsun@hrbeu.edu.cn

The Global Position System (GPS) is utilized as an ancillary equipment in the traditional on-line calibration technology of the inertial navigation system and achieves good performance [9], [10]. However, due to the GPS signal being easily blocked and subject to environmental interference, this method is limited by the application area [11], [12]. In addition, the vehicle-based on-line calibration methods of the INS/Dead Reckoning (DR) integrated navigation system are not suitable for ships because the design of the calibration path is quite different from the motion of ships. As a result, the on-line calibration scheme should be redesigned for ships.

Since the Star sensor has the advantages in providing high-precision attitude for long-term, which makes it possible for the Star sensor/INS integrated navigation system to fully utilize the advantages of the Star sensor and the INS to achieve the goal of high-precision navigation [13], [14]. Therefore, this paper will focus on the Star sensor/INS integrated navigation system [15], [16], [17].

Theoretically, the output of Star sensor is considered as the same coordinate frame as the output of INS. But actually, there is installation error angle between the Star sensor and INS due to the influence of the external environment [18], [19], [20]. And the installation error angle may change with the influence of factors, such as swaying of ships, winds, and aging of the sensors[21], [22]. The traditional periodic off-line calibration technology has great limitations, and the error parameters cannot be estimated and compensated in real time, which has a great influence on the navigation accuracy. Therefore, the on-line calibration of the Star sensor/INS integrated navigation system has a profound significance.

The content of this work is organized in the following way. Section II presents the principle of the on-line calibration method based on multiplicative error quaternion. The on-line calibration model of the Star sensor/INS integrated navigation system is established in Section III. Section IV is the on-line calibration filter of the Star sensor/INS integrated navigation system. Section V gives the simulation results and verify the effectiveness of the novel on-line calibration method of the Star sensor/INS integrated navigation system. And Section VI concludes this manuscript.

II. Principle of the on-line calibration method based on multiplicative error quaternion

The quaternion error model, which is a common used method to describe the pose, has the characteristics of linearity and convenience in filtering, so it is suitable for engineering applications. The implementation process is illustrated as followed. The quaternion error and the gyroscope error of the INS are set as state vectors, and the error quaternion are set as the measurement vector. And since Kalman filter is simple and has less calculation load, it is utilized to fuse the data on-line.

Since the Star sensor could provide the attitude information with high precision, the attitude quaternion information can be regarded as reference information. Thereby, it is possible to correct the error parameters by utilizing the Star sensor. The states equation and measurement equation are established based on the relationship between the attitude quaternion and angular rate. And Fig.1 is the on-line calibration schematic diagram of the Star sensor/INS integrated navigation system.

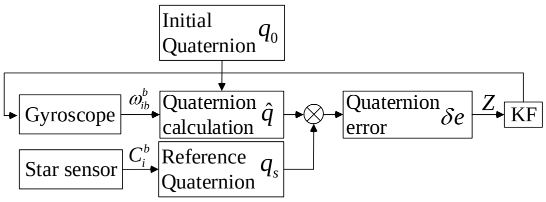

Fig. 1. On-line calibration schematic of the Star sensor/INS integrated navigation system

As shown in Fig.1, the output of the Star sensor is the attitude quaternion information of the body frame (b frame for short) relative to the inertial frame (i frame for short), and is considered as the reference for the INS. The output of the gyroscope ω_{ib}^b is the angular rate of the b frame relative to the i frame. The attitude quaternion obtained from ω_{ib}^b is denoted as calculated quaternion $\hat{q}$ while the output of the Star sensor is denoted as q_s in this paper. The error quaternion Δq, obtained by multiplication of $\hat{q}$ and q_s, is regarded as observation. The scale factor error of the gyroscope, constant bias of the gyroscope and other error terms will be estimated in real time by utilizing Kalman filter as the estimation method. And then the precision of navigation will be improved after the error compensation.

III. On-line calibration model of the Star sensor/INS integrated navigation system

Considering the actual application conditions, the coordinate frame of INS cannot coincide with that of Star sensor because these two different sensors are physically connected. As a result, the error model will be analyzed separately in the following.

A. Installation error model between the Star sensor and INS

Fig.2 shows the principle schematic of the INS. The accelerometer and the gyroscope measure the specific force and the angular rate of the vehicle relative to the i frame, respectively. After the integral operation, the attitude and position information of the carrier can be obtained. In this INS, the mathematical platform, implemented by the computer, replaces the actual physical platform and provides the attitude matrix C_b^n in real time, which transforms the acceleration information for subsequent update operations.

Fig.3 is the principle schematic diagram of the Star sensor. The attitude quaternion relative to the i frame, which is the output of the Star sensor, is utilized as the reference

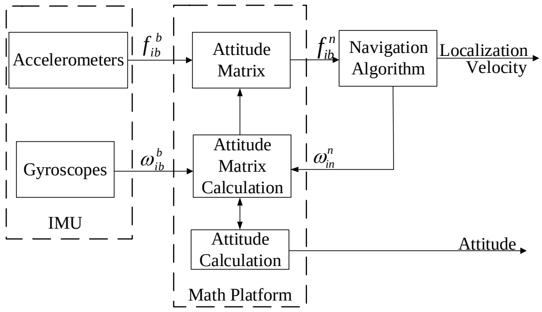

Fig. 2. Principle schematic of the Inertial Navigation System

information of the gyroscope. When the INS and Star sensor are physically connected together, there will be installation errors between them. When there is no rotation relationship between these two coordinate frame, the output of the Star sensor can be used as the gyroscopes' reference information because there is no error. However, when there is a rotation relationship between these two coordinate frame, the output of the Star sensor cannot be directly used as reference for INS if the rotation matrix has not been determined. Moreover, this installation error is much larger than the gyroscopes error. As a result, the installation error should be estimated and compensated in real time so that the quaternion information of the two sensors is in the same reference frame for further navigation.

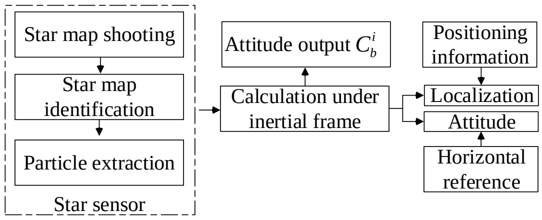

Fig. 3. Principle schematic diagram of the Star sensor

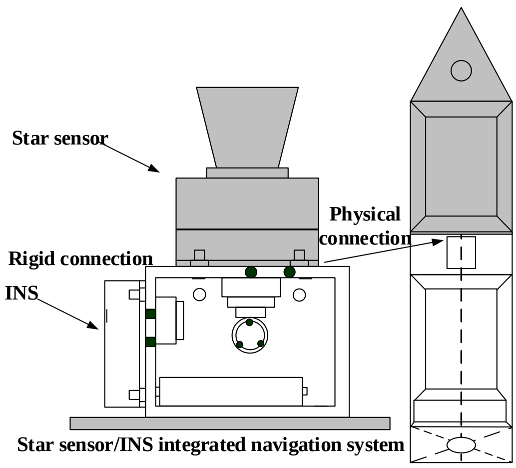

Fig. 4. Installation diagram of the Star sensor/INS integrated navigation system

The attitude information between the Star sensor frame and the navigation frame (n frame for short) θ_c, γ_c and ϕ_c and the attitude matrix C_{bc}^n can be further solved. It is supposed that the installation errror between the Star sensor frame and the INS frame are θ_{ci}, γ_{ci} and ϕ_{ci} and the attitude transformation matrix is C_{bi}^{bc}. The attitude matrix of INS can be obtained based on the relationship between C_{bc}^{n} and C_{bi}^{bc} shown as the following equation.

$$C_{bi}^{n}=C_{bc}^{n}C_{bi}^{bc} \tag{1}$$

When the installation error matrix δA exists between these two frame, we can obtain:

$$\widetilde{C}_{bi}^{bc}=C_{bc}^{bc}C_{bi}^{bc}=(I-[\delta A\times])C_{bi}^{bc} \tag{2}$$

We can know that the attitude matrix of INS obtained according to this method also has errors:

$$\widetilde{C}_{bi}^{n}=C_{bc}^{n}\widetilde{C}_{bi}^{bc}=C_{bi}^{n}-C_{bc}^{n}[\delta A\times]C_{bi}^{bc} \tag{3}$$

As a result, the error coefficient needs to be determined to ensure navigation accuracy for Star sensor/INS integrated navigation system.

B. Error model of gyroscope

The error model of the gyroscope is defined as follows:

$$\omega_g=\omega+\Delta\omega \tag{4}$$

where ω_g denotes the actual output of the gyroscope; ω is the theoretical output of the gyroscope; $\Delta\omega$ indicates the error of gyroscope and it is expressed as:

$$\Delta\omega=\omega_k+\omega_b+\omega_q+\eta_g \tag{5}$$

wherein ω_k denotes the error caused by the gyroscope's scale factor error and it can be expressed as:

$$\omega_k=\begin{bmatrix}\omega_{k1}\\ \omega_{k2}\\ \omega_{k3}\end{bmatrix}=\begin{bmatrix}k_1&0&0\\0&k_2&0\\0&0&k_3\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} \tag{6}$$

where $k_i(i=1,2,3)$ indicates the scale factor error of the three axis gyroscopes, respectively.

In Eq.(5), ω_b denotes the constant bias of the gyroscope and it can be shown as:

$$\omega_b=\begin{bmatrix}b_1&b_2&b_3\end{bmatrix} \tag{7}$$

In Eq.(5), ω_q denotes the installation error between the Star sensor and INS and it is expressed as

$$\omega_q=\begin{bmatrix}\omega_{q1}\\ \omega_{q2}\\ \omega_{q3}\end{bmatrix}=\begin{bmatrix}0&m_{12}&m_{13}\\m_{21}&0&m_{23}\\m_{31}&m_{32}&0\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} \tag{8}$$

In Eq.(5), η_g denotes the white noise. Since the Star sensor is utilized as reference, its error model is regarded as white noise.

C. State function and measurement function

1) State function: The output of the Star sensor, that is the quaternion, is indicated by $q=\begin{bmatrix}e&q_s\end{bmatrix}$. And wherein, q_s denotes its scalar part while $e=\begin{bmatrix}e_1&e_2&e_3\end{bmatrix}$ denotes its vector part, respectively. The quaternion differential equation is obtained:

$$\dot{q}=\frac{1}{2}\Omega(\omega)=\frac{1}{2}q\otimes\omega \tag{9}$$

wherein, $\omega=\begin{bmatrix}\omega_1&\omega_2&\omega_3\end{bmatrix}$ denotes the rotational angular velocity and $[\omega\times]$ denotes its anti-symmetric matrix. And we can get this:

$$[\omega\times]=\begin{bmatrix}0&-\omega_3&\omega_2\\ \omega_3&0&-\omega_1\\ -\omega_2&\omega_1&0\end{bmatrix} \tag{10}$$

It is supposed that $q(t)$ and $q(t+\Delta t)$ denote the quaternion of timestamp t and timestamp $t+\Delta t$, respectively. And based on the matrix theory, we know:

$$q(t+\Delta t)=q(t)\otimes\Delta q \tag{11}$$

where Δq is the quaternion under the small angular rotation. So we can obtain:

$$\begin{aligned}\Delta q&=\begin{bmatrix}e^{\top}\sin(\frac{\Delta\alpha}{2})&\cos(\frac{\Delta\alpha}{2})\end{bmatrix}^{\top}\\&\approx\begin{bmatrix}e^{\top}\sin(\frac{\Delta\alpha}{2})&1\end{bmatrix}^{\top}\end{aligned} \tag{12}$$

$$\dot{q}=\frac{1}{2}q\otimes\omega_{ob} \tag{13}$$

where q and ω_{ob} denote the attitude quaternion and angular rate of the b frame relative to the n frame. The attitude quaternion will be obtained by integrating the above equation. Since the actual attitude and angular velocity contain errors, the estimated value $\hat{q}$ and $\hat{\omega}_{ob}$ will be obtained as follows:

$$\dot{\hat{q}}=\frac{1}{2}\hat{q}\otimes\hat{\omega}_{ob} \tag{14}$$

Taken the inversion of Eq.11, we can get:

$$q^{-1}(t+\Delta t)=\Delta q^{-1}\otimes q^{-1}(t) \tag{15}$$

And its actual differential form is shown as:

$$\dot{\hat{q}}^{-1}=-\frac{1}{2}\hat{\omega}_{ob}\otimes\hat{q}^{-1} \tag{16}$$

wherein Δq means the difference between the theoretical quaternion and the actual quaternion. Based on its definition, Δq can be expressed as:

$$\Delta q=\begin{bmatrix}\Delta q_1&\Delta q_2&\Delta q_3&\Delta q_4\end{bmatrix} \tag{17}$$

Since $q=\hat{q}\otimes\Delta q$ and Δq is small enough, so $\Delta q_4\approx 1$. Therefore, the error quaternion is redefined as follows in this situation.

$$\Delta q=\hat{q}^{-1}\otimes q=\begin{bmatrix}\delta e&\delta q\end{bmatrix}^{\top} \tag{18}$$

where δe and δq denote the scalar component and vector component of Δq, respectively.

Based on the quaternion differential equation, we can get:

$$\dot{q}=\frac{1}{2}q\otimes\omega \tag{19}$$

$$\dot{\hat{q}}=\frac{1}{2}\hat{q}\otimes\hat{\omega} \tag{20}$$

where q denotes the reference quaternion and $\hat{q}$ denotes the calculated quaternion.

Taken the derivative of Eq.(17) and substituted Eq.(19) and Eq.(20) into it, we can know that:

$$\delta\dot{q}=\frac{1}{2}[\delta q\otimes\omega-\hat{\omega}\otimes\delta q] \tag{21}$$

wherein

$$\hat{\omega}=\delta\omega+\omega \tag{22}$$

It is known from the error model that

$$\hat{\omega} = \omega_g - (\hat{\omega}_k + \hat{\omega}_b + \hat{\omega}_q) \tag{23}$$

$$\omega = \omega_g - (\omega_k + \omega_b + \omega_q + \eta_q) \tag{24}$$

$$\delta\omega = \begin{bmatrix} \omega_k - \hat{\omega}_k \\ \omega_b - \hat{\omega}_b \\ \omega_q - \hat{\omega}_q \\ \eta_q \end{bmatrix} = \begin{bmatrix} \delta\omega_k \\ \delta\omega_b \\ \delta\omega_q \\ \eta_q \end{bmatrix} \tag{25}$$

$$\delta\dot{q} = \frac{1}{2}[\delta q \otimes \hat{\omega} - \hat{\omega} \otimes \delta q] + \frac{1}{2}\delta q \otimes \omega \tag{26}$$

wherein

$$\delta q \otimes \hat{\omega} - \hat{\omega} \otimes \delta q = -2\hat{\omega} \otimes \delta e \tag{27}$$

After linearization and ignoring the second-order small value, Eq.26 can be simplified as follows:

$$\delta\dot{q} = -\hat{\omega} \otimes \delta e - \frac{1}{2}(\delta\omega_k + \delta\omega_b + \delta\omega_q + \eta_q) \tag{28}$$

And its corresponding matrix form is expressed as:

$$\begin{aligned} \begin{bmatrix} \delta\dot{q} \\ \delta\dot{e}_1 \\ \delta\dot{e}_2 \\ \delta\dot{e}_3 \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \hat{\omega}_3 & -\hat{\omega}_2 \\ 0 & -\hat{\omega}_3 & 0 & \hat{\omega}_1 \\ 0 & \hat{\omega}_2 & -\hat{\omega}_1 & 0 \end{bmatrix} \begin{bmatrix} \delta q \\ \delta e_1 \\ \delta e_2 \\ \delta e_3 \end{bmatrix} \\ &- \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \delta k_1 & 0 & 0 \\ 0 & 0 & \delta k_2 & 0 \\ 0 & 0 & 0 & \delta k_3 \end{bmatrix} \begin{bmatrix} 0 \\ \hat{\omega}_1 \\ \hat{\omega}_2 \\ \hat{\omega}_3 \end{bmatrix} + \\ & \begin{bmatrix} 0 \\ \frac{\delta b_1}{2} \\ \frac{\delta b_2}{2} \\ \frac{\delta b_3}{2} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & m_{12} & m_{13} \\ 0 & m_{21} & 0 & m_{23} \\ 0 & m_{31} & m_{32} & 0 \end{bmatrix} \\ & \cdot \begin{bmatrix} 0 \\ \hat{\omega}_1 \\ \hat{\omega}_2 \\ \hat{\omega}_3 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 \\ \eta_{g1} \\ \eta_{g2} \\ \eta_{g3} \end{bmatrix} \end{aligned} \tag{29}$$

It is obvious from Eq.29 that $\delta\dot{q} = 0$.

Then we selected the system state vector as:

$$X = \begin{bmatrix} \delta e & \delta k & \delta b & \delta\phi \end{bmatrix}^\top \tag{30}$$

wherein $\delta\phi = \begin{bmatrix} m_{12} & m_{13} & m_{21} & m_{23} & m_{31} & m_{32} \end{bmatrix}^\top$; $\delta e = \begin{bmatrix} \delta e_1 & \delta e_2 & \delta e_3 \end{bmatrix}^\top$; $\delta k = \begin{bmatrix} \delta k_1 & \delta k_2 & \delta k_3 \end{bmatrix}^\top$; $\delta b = \begin{bmatrix} \delta b_1 & \delta b_2 & \delta b_3 \end{bmatrix}^\top$.

So the state equation can be represented as:

$$\hat{X}(t) = F(t)X(t) + G(t)W(t) \tag{31}$$

wherein F denotes the state transition matrix and G is the noise driven matrix; and W indicates the system noise matrix. And they can be expressed in detail as:

$$F = \begin{bmatrix} -[\hat{\omega}\times]_{3\times3} & -\frac{1}{2}G_{k3\times3} & -\frac{1}{2}I_{3\times3} & -\frac{1}{2}G_{d3\times3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

wherein $[\omega\times] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$;

$$G_k = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \omega_2 & 0 \\ 0 & 0 & \omega_3 \end{bmatrix};$$

$$G_d = \begin{bmatrix} \omega_2 & \omega_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_1 & \omega_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_1 & \omega_2 \end{bmatrix}.$$

And

$$G = \begin{bmatrix} -\frac{1}{2}I_{3\times3} & 0_{3\times12} \\ 0_{12\times3} & I_{12\times12} \end{bmatrix};$$

$$W = \begin{bmatrix} \eta_g & \eta_k & \eta_b & \eta_m \end{bmatrix}^\top.$$

2) Measurement function: The vector part Δe of the error Δq between the Star sensor's output and the gyroscope's output is utilized as measurement vector, which is defined as follows:

$$Z = \begin{bmatrix} \delta e_1 & \delta e_2 & \delta e_3 \end{bmatrix}^\top \tag{32}$$

So the measurement equation can be expressed as:

$$Z(t) = HX(t) + V(t) \tag{33}$$

where H is the measurement matrix while $V(t)$ is the measurement noise vector. And we know that $H = \begin{bmatrix} I_{3\times3} & 0_{3\times12} \end{bmatrix}$.

IV. Filter establishment of the on-line calibration method for the Star sensor/INS integrated navigation system

Based on the establishment of the state equation and the measurement equation, the Kalman filter is used to estimate the system state. Because the sampling frequency of gyroscope is 100Hz, which is much higher than that of the Star sensor (no higher than 20Hz usually), the output of the Star sensor is not synchronous with the gyroscope. As a result, when the Star sensor has no output, that means there is no measurement, only the state update is performed. And when the measurement of the Star sensor is obtained, the measurement update is performed then. Specific steps of the filter are as follows.

A. When no measurement information is obtained, the prediction is performed.

When there is no measurement information, the quaternion value $(\hat{q}_b)_{k|k-1}$ of timestamp k will be predicted by utilizing the quaternion value of timestamp $k-1$:

$$(\dot{\hat{q}}_b)_{k-1} = \frac{1}{2}(\hat{q}_b)_{k-1} \otimes (\hat{\omega}_b)_{k-1} \tag{34}$$

where $(\hat{\omega}_b)_{k-1}$ denotes the gyroscope's output minus the sensor's error. The constant bias and scale factor error of the gyroscope have the following properties, respectively.

$$\hat{b}_{k|k-1} = \hat{b}_{k-1} \tag{35}$$

$$\hat{k}_{k|k-1} = \hat{k}_{k-1} \tag{36}$$

B. When measurement information is obtained, the filtering calculation is performed.

According to the definition, the error covariance matrix $P = E\left\{\Delta X \Delta X^T\right\}$. And its prediction is:

$$P_{k|k-1} = \Phi_{k|k-1}P_{k-1}\Phi_{k|k-1}^\top + Q_{k-1} \tag{37}$$

The filter gain K is:

$$K_k = P_{k|k-1}H_k^\top(H_kP_{k|k-1}H_k^\top + R_k)^{-1} \quad (38)$$

Then the measurement update is performed and we can obtain the updated state vector as follows:

$$\hat{X}_k = \hat{X}_{k|k-1} + K_k[Z_k - H_k\hat{X}_{k|k-1}] \quad (39)$$

The estimated state vector $\hat{X}$ is compensated and the corrected quaternion is expressed as:

$$(\hat{q}_b)_k = (\hat{q}_b)_{k|k-1} \otimes (\Delta\hat{q}_b)_k \quad (40)$$

The error covariance matrix P can be updated by

$$P_k = (I - K_kH_k)P_{k|k-1}(I - K_kH_k)^\top + K_kR_kK_k^\top \quad (41)$$

Then the error terms of the Star sensor/INS integrated navigation system can be estimated on-line avaliably, reducing the influence on navigation accuracy and improving the performance of integrated navigation system.

V. SIMULATION AND ANALYSIS

In order to verify the effectiveness of the on-line calibration method, relative simulations are carried out in this section.

A. Simulation parameters setting

In the simulation environment, it is assumed that the vehicle locates at $(126.6705°E, 45.7796°N)$ and moves with a constant speed whose value is $10m/s$. The initial heading and horizontal attitude of the vehicle are $(30°, 0°, 0°)$ adn the initial horizontal and heading misalignment angle errors are $5^{'}$ and $20^{'}$, respectively. It is also assumed that the vehicle's movement contains not only the linear motion, but also the swaying motion. And the swaying model is:

$$\begin{cases} \theta & = & \theta_m sin(\omega_\theta t + \theta_0) \\ \gamma & = & \gamma_m sin(\omega_\gamma t + \gamma_0) \\ H & = & H_m sin(\omega_H t + H_0) \end{cases} \quad (42)$$

wherein θ_m, γ_m and H_m are the swing amplitudes and thay are set as $\theta_m = 3°, \gamma_m = 2°, H_m = 1°$. $\omega_\theta, \omega_\gamma and \omega_H$ are the angular rates of the swing motion, and the swing periods are $T_\theta = \frac{2\pi}{\omega_\theta} = 6s$, $T_\gamma = \frac{2\pi}{\omega_\gamma} = 8s$ and $T_H = \frac{2\pi}{\omega_H} = 10s$, respectively. θ_0, γ_0 and H_0 are the initial phase angles of the swing motion and they are set as $\theta_0 = \gamma_0 = H_0 = 0°$. The system motion parameters are set as Table I.

TABLE I
MOTION PARAMETERS SETTING

Parameter	Value
Initial position	$126.6705°E, 45.7796°N$
Initial horizontal angle error	$5'$
Initial heading error	$20'$
Initial heading	$30°$
Initial horizontal angle	$0°$
Velocity	$10m/s$
Pitch swaying	$3°sin(\frac{\pi}{3}t)$
Roll swaying	$2°sin(\frac{\pi}{4}t)$
Yaw swaying	$1°sin(\frac{\pi}{5}t)$

The Kalman filter is established to estimate and calibrate the error state vector. The initial parameters of the Kalman filter including the covariance matrix, the observation noise, process noise are set as Table II.

TABLE II
KALMAN FILTER PARAMETERS SETTING

Parameter	Value
Covariance of gyroscopes' constant bias	$\sigma_{bg}^2 = (10^{-6\circ}/h)I_{3\times3}$
Covariance of the Star sensor error	$\sigma_q^2 = (10^{-5})^2 I_{3\times3}$
Covariance of scale factor error	$\sigma_k^2 = (3\times10^{-6})^2 I_{3\times3}$
Covariance of installation error	$\sigma_m^2 = (4\times10^{-6}rad)^2 I_{6\times6}$
Error covariance matrix	$P = 0.01 \cdot I_{15\times15}$
Covariance of the observation noise	$R = diag(\sigma_q^2)$
Covariance of the process noise	$Q = diag(\sigma_q^2, \sigma_k^2, \sigma_m^2, \sigma_{bq}^2)$
Initial quaternion	$q_0 = [1 \quad 0 \quad 0 \quad 0]$

The expected values of the state to be estimated including gyroscopes' constant bias, gyroscopes' scale factor error and installation error are set as Table III.

TABLE III
STATE PARAMETERS SETTING

Parameter	Value
Gyroscope constant bias	$[\delta b_1, \delta b_2, \delta b_3] = [0.01, 0.01, 0.01]°/h$
Gyroscope scale factor error	$[\delta k_1, \delta k_2, \delta k_3] = [20, 20, 20]ppm$
Installation error	$[m_{12}, m_{13}, m_{21}] = [3.6, 3.6, 3.6]''$ $[m_{23}, m_{31}, m_{32}] = [3.6, 3.6, 3.6]''$

B. Simulation results and analysis

Fig.5, Fig.6 and Fig.7 show the estimated curve of the gyroscopes' constant bias, scale factor error and installation error, respectively. It is known from Fig.5 that the estimated value of gyroscopes' constant bias converged to 0.01°/h in 2 hours; from Fig.6 that the estimated value of gyroscopes' scale factor converged to 20 ppm in 5 hours; from Fig.7 that the estimated value of installation error converged to 3.6″ in 2 hours. The conclusion is obtained from the simulation results that the estimated value can converge to the theoretical value in a short time. It proves the effectiveness of the novel on-line calibration methods. However the convergence speed of different variables is different. It is because that different variables have different observability values.

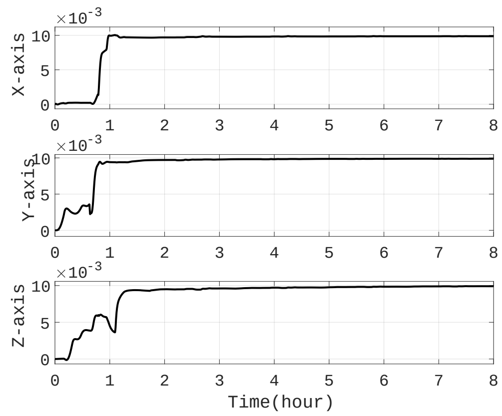

Fig. 5. Estimation results of the gyroscopes' constant bias

VI. CONCLUSION

According to the principle of the Star sensor/INS integrated navigation system, an on-line calibration algorithm

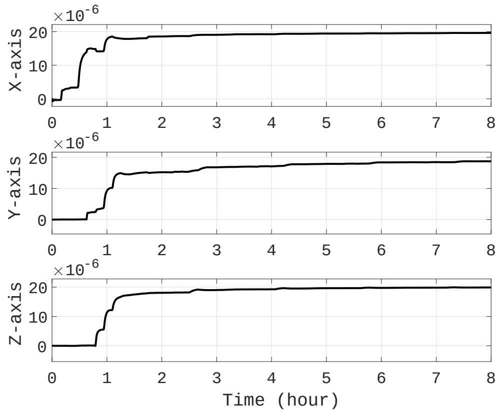

Fig. 6. Estimation results of the gyroscopes' scale factor error

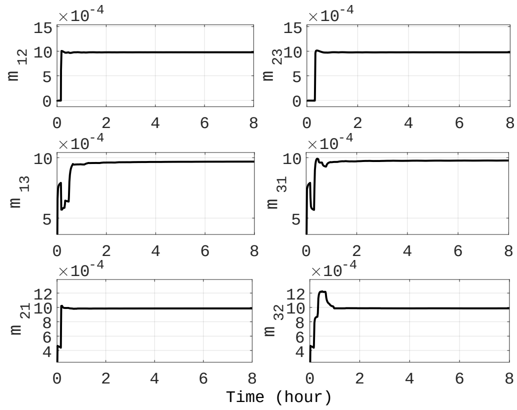

Fig. 7. Estimation results of the installation error

for gyroscope's error based on error quaternion is proposed in this paper. Based on the attitude calculation method of quaternion differential equation, the novel on-line calibration algorithm considers the installation error between the Star sensor and INS as well as the gyroscope errors. Expanding the dimension of the INS error model, the state vector is expended and re-established. Taken the multiplicative quaternion error of the INS and the Star sensor as the measurement, the vector part is selected as the measurement and the measurement equation is established. Considered the carrier actual motion, motion speed and swaying are both set in the simulation. The simulation results show that the error of gyroscopes and installation can be both estimated and the dynamic error is within the allowable range by utilizing the novel on-line calibration algorithm, verified the feasibility and effectiveness of the proposed on-line calibration method for the Star sensor/INS integrated navigation system.

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Xiuwei Xia was born in Dehui, Jilin in 1987. He received his M.Eng. degree from Harbin Engineering University, Harbin, Heilongjiang, China in 2014. He is pursuing the Ph.D. degree from Harbin Engineering University, Harbin, Heilongjiang, China. His research interest includes integrated navigation and inertial navigation system.

Qian Sun was born in Fuxin, Liaoning in 1988. He received her Ph.D. degree from Harbin Engineering University, Harbin, Heilongjiang, China in 2016. He is now an assistant professor in College of Information and Communication Engineering, Harbin Engineering University, Harbin, Heilongjiang, China. His research interest includes integrated navigation and inertial navigation system.