An Optimal Resource Allocation Scheme for Elastic Applications in Multipath Networks via Particle Swarm Optimization

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***Abstract*—In multipath networks each pair of source and destination has multiple paths between them so as to transmit data packets parallelly, and will obtain a certain level of satisfaction which is described as utility when obtaining a certain amount of bandwidth. In this paper we consider resource allocation for elastic applications in multipath networks with the objective of utility maximization, which is an intrinsically non-strictly convex optimization problem. We analyze the resource allocation model for elastic applications and discuss its implication from an economic point of view. In order to obtain the optimal resource allocation, we present a heuristic resource allocation algorithm via Particle Swarm Optimization (PSO), which is verified through some numerical examples.**

***Index Terms*—multipath networks, elastic applications, resource allocation, Particle Swarm Optimization.**

I. Introduction

In the last several years, there has been much interest in multipath resource allocation algorithms and applications ([1][2] [3][4]), where each source-destination pair can have several different routes for it to transmit data packets so as to improve throughput as well as transmission reliability. In multipath transmission schemes, maximizing the aggregated user utility over the network with multipath routing under the constraint of links' capacity is the objective of the *multipath network utility maximization* problems. They can be viewed as an example of cross-layer optimization framework [5] in Internet, where additional benefits are achieved by jointly optimizing at the routing (network layer) and transmission control (transport layer). However, the utility maximization problems to solve in multipath networks are usually concave but not strictly concave, resulting in non-unique optimums of the primal resource allocation problems and discontinuous dual price problems. Thus, most of researchers have to relax the multipath network utility maximization problems so as to make them strict concave ([6][7][8]), which means that the optimums of the network utility maximization problems are unique.

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In order to solve the mutipath utility maximization problems for resource allocation and their relaxation versions, roughly speaking, multipath resource allocation schemes can be generally classed into three categories: primal algorithms ([9][10][11]), dual algorithms ([12][7][13]), and primal-dual algorithms ([14][6][15]). The primal algorithms have a dynamical rule for adjusting user transmission rates and a static rule for generating link prices, and conversely, the dual algorithms have a dynamical law for adjusting link prices and a static law for generating user transmission rates. Then, the primal-dual algorithms have dynamical laws for adjusting both user transmission rates and link prices. The primal algorithms are usually based on a penalty function approach, i.e., they replace the capacity constraints by a penalty function in the optimization objective. They always tend to produce biased approximates of the optimal operating points, due to the fact that penalties are only incurred when the capacity constraints are violated. In contrast, the optimal operating point is defined to be one that satisfies the capacity constraints. As for the dual algorithms, the advantage is that they are designed to compute the exact optimal operating point including resource allocation and link price when the step-sizes are driven to near zero in an appropriate fashion.

It is very significant to study the resource allocation for elastic applications in multipath networks and investigate optimal resource allocation for these applications, so as to satisfy the QoS requirements of them while achieving the objective of utility maximization. In this paper we investigate the optimal resource allocation for elastic applications based on the idea of *network utility maximization* in multipath networks, which applies utility-based method from economics into the area of bandwidth allocation in multipath networks. We give an interpretation for the utility maximization problem and its sub-problems from an economic point of view. This paper assumes that users have access to two or more different routes in the multipath networks. We investigate the resource allocation for multipath network utility maximization problem and present an optimal resource allocation scheme for multipath networks by using Particle Swarm Optimization (PSO).

The rest of this paper is summarized as follows: We formulate the utility maximization model for resource allocation of elastic applications in multipath networks in Section II. Then we introduce the optimal resource allocation schemes for elastic applications in multipath networks in Section III and give some numerical examples to illustrate the performance of the proposed scheme in Section IV. Finally we conclude this paper in Section V.

II. Resource Allocation Model

A. Applications and Utility Functions

In the Internet there are mainly two classes of applications based on the shapes of their utility functions when users obtain these applications. That is, the utility of an application is regarded as the satisfaction or QoS of this application when one user obtain this application. One type of applications corresponds to the traditional data applications, such as file transfer and web application. They are tolerant of delay and can take advantage of even the minimal amounts of bandwidth resource. They are elastic and utility functions for them are usually considered to be concave. The other one corresponds to delay and rate sensitive real-time applications, such as real-time streaming video and audio applications. They are delay-sensitive and always require a fixed minimal amount of bandwidth resource so as to guarantee adequate level of QoS. They are regarded to be inelastic in their demand for bandwidth resource and utility functions for them are usually modelled to be nonconcave (e.g., sigmoidal or discontinuous).

Utility functions for elastic applications have been proposed in [16] [17] for single-path network resource allocation, and then further discussed in [18] [19]. Some interesting resource allocation algorithms are presented accordingly to achieve network utility maximization. Recently, migrating the elastic applications into the cloud has become an interesting research topic, and some novel resource allocation schemes are also proposed to achieve the objective of maximizing applications migration utility functions [20] [21]. Generally, the concave utility functions for elastic applications are given by $U_s(y_s) = c_s(\log(a_s y_s + b_s) + d_s)$ with resource y_s of user s, where a_s, b_s, c_s and d_s are parameters of elastic applications for user s. Here all utility functions are increasing and no less than zero in their arguments, i.e., $U_s(y_s) \geq U_s(0) = 0$. Meanwhile, each user s incurs a cost for application providers when it obtains an application. The cost can be described by a non-decreasing, differentiable convex function $V_s(y_s)$ of the obtained resource y_s for its application providers, which satisfies $V_s(y_s) \geq V_s(0) = 0$. Here we choose the following cost function $V_s(y_s) = \eta_s y_s^2$.

B. Model Description

In a multipath network there are a set S of users where each user $s \in S$ identifies a source-destination pair and uses an application, a set L of unidirectional links with capacities $C_l, l \in L$, and a set P of paths where each path $p \in P$ is a collection of links. Each source can send packets to its destination over multiple paths. In following analysis, let $P(s)$ be the set of paths that user s uses, $P(l)$ be the set of paths transmitting along link l for user s, respectively, and $L(p)$ be the set of links on path p. Hence if a user s uses path p for transmission, then $p \in P(s)$; if a path p transmits along link l, then $p \in P(l)$; if a link l is on the path p, then $l \in L(p)$.

For user s, assume the flow rate on path $p \in P(s)$ is x_{sp}, then the total flow rate of user s is $y_s = \sum_{p \in P(s)} x_{sp}$, thus user s attains a utility $U_s(y_s)$ when obtaining an elastic application. Meanwhile the aggregated rate on link l is $z_l = \sum_{p \in P(l)} x_{sp}$, which should not exceed the link capacity C_l.

Every user wants to maximize its own utility when obtaining an elastic application, but the goal of network is to maximize the performance of all applications. Therefore, the resource allocation model for elastic applications in multipath networks can be described as the following optimization problem, which we consider as the **primal problem**.

$$\begin{aligned} \max \quad & \sum_{s \in S} U_s(y_s) - V_s(y_s) \\ \text{subject to} \quad & \sum_{p \in P(s)} x_{sp} = y_s \\ & \sum_{p \in P(l)} x_{sp} \leq C_l, \\ \text{over} \quad & x_{sp} \geq 0, s \in S, p \in P. \end{aligned} \tag{1}$$

Here, the objective is to maximize the aggregated utility of user flow rate y_s over all users with constraints of link capacity in the network where each user uses multiple paths for data transmission.

C. Model Analysis

Now we analyze the resource allocation model for elastic applications using multipath communications, i.e., the nonlinear programming problem (1). The Lagrangian of (1) is

$$\begin{aligned} L(\mathbf{x}, \mathbf{y}; \lambda, \mu) = \; & \sum_{s \in S} (U_s(y_s) - V_s(y_s)) \\ & + \sum_{s \in S} \lambda_s \left(\sum_{p \in P(s)} x_{sp} - y_s \right) \\ & + \sum_{l : l \in L} \mu_l \left(C_l - \sum_{p \in P(l)} x_{sp} \right), \end{aligned} \tag{2}$$

where λ is the price vector with elements λ_s, which can be considered as the price per unit bandwidth paid by user s when using an elastic application; μ is the price vector with elements μ_l, which can be considered as the price per unit bandwidth charged by link l when transmitting data packets of an application; $\mathbf{x}$ is the flow rate matrix with elements x_{sp}; $\mathbf{y}$ is the flow rate vector with elements y_s.

The Lagrangian (2) can be rewritten as

$$\begin{aligned} L(\mathbf{x}, \mathbf{y}; \lambda, \mu) = \sum_{s \in S} (U_s(y_s) - V_s(y_s) - \lambda_s y_s) \\ + \sum_{s \in S} \sum_{p \in P(s)} x_{sp} \left(\lambda_s - \sum_{l \in L(p)} \mu_l \right) + \sum_{l \in L} \mu_l C_l. \end{aligned} \tag{3}$$

Notice that the first part in (3) is separable in y_s, and the second part in (3) is separable in x_{sp}. Thus the objective function of the dual problem is

$$\begin{aligned} D(\lambda, \mu) &= \max_{\mathbf{x}, \mathbf{y}} L(\mathbf{x}, \mathbf{y}; \lambda, \mu) \\ &= \sum_{s \in S} \mathcal{P}_s(\lambda_s) + \sum_{s \in S} \sum_{p \in P(s)} \mathcal{R}_{sp}(\lambda_s, \gamma_{sp}) + \sum_{l \in L} \mu_l C_l. \end{aligned} \tag{4}$$

Here

$$\mathcal{P}_s(\lambda_s) = \max_{y_s} U_s(y_s) - V_s(y_s) - \lambda_s y_s, \tag{5}$$

$$\mathcal{R}_{sp}(\lambda_s, \gamma_{sp}) = \max_{x_{sp}} x_{sp}(\lambda_s - \gamma_{sp}), \tag{6}$$

where $\gamma_{sp} = \sum_{l \in L(p)} \mu_l, p \in P(s)$.

The sub-problem (5) is regarded as USER problem. In this sub-problem, user s wants to maximize its own utility minus its own cost which depends on the total rate y_s when he obtains an elastic application. Meanwhile, the user has to pay a price for its using bandwidth resource when obtaining an elastic application. Since λ_s is the price per unit bandwidth paid by user s, then $\lambda_s y_s$ is the total cost that user s is willing to pay. Thus, USER problem (5) is an optimization problem that every user is to maximize its own *profit*.

The sub-problem (6) is regarded as PATH problem. In this sub-problem, μ_l is the price per unit bandwidth charged by link l, then γ_{sp} is the total price associated with the path p of user s. Then $x_{sp}\lambda_s$ is the cost paid by user s for path p, and $x_{sp}\gamma_{sp}$ is the total cost charged by path p of user s. Hence, PATH problem (6) is an optimization problem that every path is to maximize its own *revenue*.

Hence, the **dual problem** of resource allocation (1) for elastic applications is described as

$$\begin{array}{ll} \min & D(\lambda, \mu) \\ \text{over} & \lambda_s \geq 0, \mu_l \geq 0, s \in S, l \in L. \end{array} \tag{7}$$

The dual problem (7) can be considered as the NETWORK problem. The objective is to minimize the total price charged by all links under the constraints that users are guaranteed with certain levels of satisfaction. The traditional gradient-based price algorithms (e.g., [12], [1]) are not necessarily efficient to achieve the optimum since the optimization problem is not strictly convex. They may produce infeasible or suboptimal resource allocation of the primal model.

III. Resource Allocation Algorithm

A. PSO scheme

1) PSO basic methodology: In a PSO [22] system, particles fly around in multidimensional search space. During flight process, each particle adjusts its own position according to its experience, and the experience of its neighboring particles, making full use of the best position obtained by itself and its neighbors. The swarm direction of a particle is defined by the set of particles neighboring the particle and its history experience. PSO has been found very useful to deal with very complicated optimization problems, such as power optimization [23], function optimization problems[24][25], and communication networks [19][26].

Each particle keeps track of its coordinates in the space of interest, which are associated with the best solution (fitness) it has achieved so far. This value is called *Pbest*. Another best value that is tracked by the global version of the particle swarm optimizer is the overall best value, and its location, obtained so far by any particle in the population. This location is called *Gbest*. At each iteration step, the particle swarm optimization concept consists of velocity changes of each particle toward *Pbest* and *Gbest* locations. Let x and v denote a particle coordinates (position) and its corresponding flight speed (velocity) in a search space, respectively.

Therefore, the ath particle is represented as $X^a = (x^a_{11}, \ldots, x^a_{1p}; \ldots; x^a_{s1}, \ldots, x^a_{sp})$, and $V^a = (v^a_{11}, \ldots, v^a_{1p}; \ldots; v^a_{s1}, \ldots, v^a_{sp})$ in PSO-based resource allocation scheme. In the following analysis, let $X^a = (x^a_{jp})$ and $V^a = (v^a_{jp})$ for simplicity. And let $Pbest^a = (x^{aPbest}_{jp})$ and $Gbest = (x^{Gbest}_{jp})$ be the best position of individual a and its neighbors' best position so far, respectively. Using the information above, the velocity and position of individual a are updated by the following law

$$\begin{aligned} V^a(k+1) &= \omega V^a(k) + c_1 * \Re_1 * (Pbest^a(k) - X^a(k)) \\ &\quad + c_2 * \Re_2 * (Gbest(k) - X^a(k)), \\ X^a(k+1) &= X^a(k) + V^a(k+1), \end{aligned}$$

where ω is the inertia weight factor; c_1, c_2 are acceleration constants; $\Re_1, \Re_2$ are uniform random values between 0 and 1; $X^a(k)$ is the current position of individual a at the iteration step k; $V^a(k)$ is the velocity of individual a at the iteration step k, $V^a_{\min} < V^a(k) < V^a_{\max}$; $Pbest^a(k)$ is the best position of the individual a at the iteration step k; $Gbest(k)$ is the best position of the group.

In the update law above, the parameters $V^a_{\min}$ and $V^a_{\max}$ determine the resolution, or fitness, with which regions between the present position and target position are searched. The constants c_1 and c_2 represent the weighting of the stochastic acceleration terms that pull each particle toward *Pbest* and *Gbest* positions.

2) The fitness function: Since the utility maximization problem is subjected to inequality constraints, we use the PSO with penalty function in the algorithm. In the penalty method, the fitness function is described as follows:

$$F_f = \begin{cases} f(X), & \text{if the solution is feasible} \\ f(X) + h(k)H(X), & \text{otherwise} \end{cases} \tag{8}$$

where $f(X)$ is the original objective function to be optimized, $h(k)$ is a penalty value, and $H(X)$ is a penalty factor.

3) The adaptive inertia weight factor (AIWF): In PSO scheme, global exploration and local exploitation should be coordinated so as to efficiently find the optimum [27]. A larger inertia weight pressures toward global exploration, while a smaller inertia weight pressures toward fine-tuning of the current search area. In [28] an adaptively varying inertia weight was presented to achieve trade-off between exploration and exploitation. The adaptive inertia weight factor (AIWF) is generalized as the following expression

$$\omega = \begin{cases} \omega_{min} + \dfrac{(\omega_{max} - \omega_{min})(f - f_{min})}{f_{avg} - f_{min}}, \\ \qquad \text{if } \ f \leq f_{avg}, \\ \omega_{max}, \quad \text{otherwise}, \end{cases} \tag{9}$$

where ω_{max} and ω_{min} are the maximum and minimum of ω respectively, f is the current objective value of the particle, f_{avg} and f_{min} are the average and minimum objective values of all particles, respectively. The improved inertia weight factor (9) varies depending on the objective values of the particles. Thus particles with low objective values can be protected while particles with objective values over the average will be dropped. That is, good particles tend to perform exploitation to improve results by local search, while bad particles tend to perform large modification to explore space with large steps.

B. Algorithm Description

In this subsection, we present a heuristic algorithm using PSO to resolve the utility maximization model (1) for resource allocation of elastic applications in multipath networks. The important notations in the proposed resource allocation algorithm using PSO are listed as following:

Particle: Position of each particle within the population in PSO represents a candidate solution for solving the resource allocation problem (1). For example, each particle X^a represents one possible bandwidth resource allocation for model (1), and the jpth dimension of the particle corresponds to the flow rate allocation of user j on path p.

Velocity: V^a, the velocity in PSO, is the auxiliary variable for the algorithm to find the final optimal solution.

Fitness function: The objective of model (1) is to maximize the aggregated utility with constraints of links' capacities in the network, so when there are elastic applications in the network the fitness function here given according to (8) has the following form (note that the first term of F_f is obtained by substituting the equality $y_s = \sum_{p\in P(s)} x_{sp}$ into the objective):

$$F_f = \begin{cases} \sum_{s\in S} U_s\left(\sum_{p\in P(s)} x_{sp}\right) - V_s\left(\sum_{p\in P(s)} x_{sp}\right), \\ \text{if} \sum_{p\in P(l)} x_{sp} \le C_l \\ \sum_{s\in S} U_s\left(\sum_{p\in P(s)} x_{sp}\right) - V_s\left(\sum_{p\in P(s)} x_{sp}\right) \\ +\sum_{l\in L} \mu_l \left(C_l - \sum_{p\in P(l)} x_{sp}\right), \text{otherwise.} \end{cases} \tag{10}$$

So if one particle satisfies all constraints, it is a feasible particle. Otherwise, an extra charge should be paid which is proportional to the amount of violation with very large positive constant.

C. Main Steps

Next we present the resource allocation scheme using PSO to resolve the utility maximization model (1) of resource allocation problem for elastic applications in multipath networks (i.e., position) according to the objective function (i.e., fitness function) through the auxiliary variable (velocity in PSO).

Step1: Initialize the variables and the parameters.

Let k be zero and set the maximum number of iterations as K. Initialize position of particle X^a, which corresponds to a set of bandwidth resource allocation, and initialize velocity of particle V^a, which is the auxiliary variable in the scheme. The particle must be one of the feasible candidate solutions satisfying the inequality constraints on link capacities. Initialize PSO parameters (ω, c_1, c_2).

Step2: Calculate fitness.

The current searching points are evaluated by using the objective functions of the target problem, i.e., F_f in (10). $Pbest^a(k)$ is set to be the searching point, and $Gbest(k)$ is the best evaluated value among all $Pbest^a(k)$ at the iteration k, respectively.

Step 3: Update the searching points.

If the evaluation of each point is better than the previous $Pbest^a(k)$, the value is set to $Pbest^a(k)$. If the best $Pbest^a(k)$ is better than $Gbest(k)$, the value is set to be $Gbest(k)$. All the $Gbest(k)$ are candidates for the final control strategy. Update velocities and positions according to the update law.

Step 4: Set stop criterion.

When the link capacity constraint is violated, an extra charge should be paid, which is proportional to the amount of violation with the penalty value. When the number of iterations reaches the maximum or the optimal objective is achieved (i.e., the objective of resource allocation model does not change any more), the searching procedure can be stopped. The last $Gbest(k)$ can be drawn as the optimal position, and the corresponding $x_{jp}^{Gbest}(k)$ is the optimal allocation allocation for user j on path p in the network.

IV. Numerical Examples

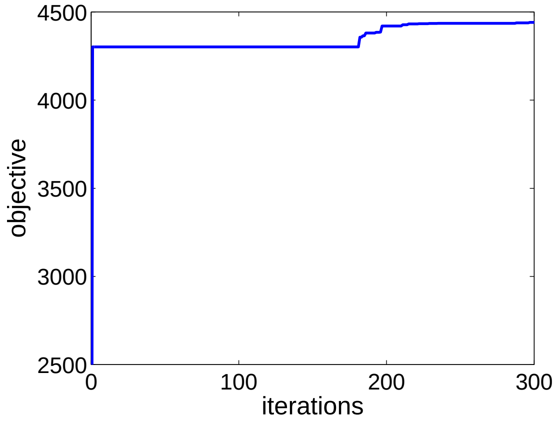
Consider the following concave utility function $U_s(y_s) = \xi_s \log(y_s + 1)$ when user s requests and obtains an elastic application, where ξ_s is a private parameter which captures the willingness-to-pay of user s for the obtained elastic application. Choose the convex cost function $V_s(y_s) = \eta_s y_s^2$, where η_s is a parameter which describes the cost valuation of user s when he is granted an amount y_s of bandwidth resource for the elastic application.

We first consider a simple multipath network consisting of 50 users who transmit data packet to each other. Each user has the same access link capacity C_l =20Mbps. In order to ensure the convergence of the proposed scheme, the acceleration coefficients $c1$, $c2$ are set to 2, and the inertia weight factor range is set to [0.4, 0.9]. The swarm size is chosen to be 20.

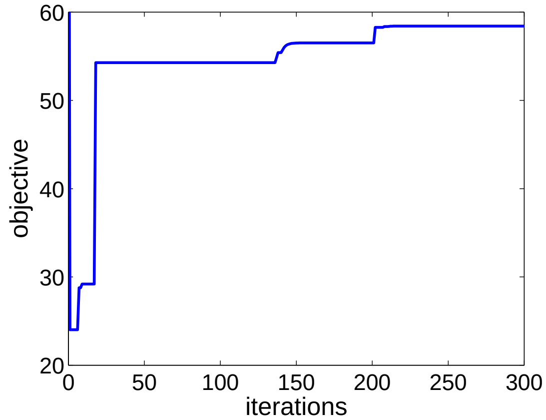
We consider the impacts of parameters ξ_s and η_s on the performance of the resource allocation scheme, and depict the evolution of aggregated utility for this network in Fig. 1. The scheme is gradually driven to a steady state where the utilization of each access link is approximately 100% as we observed in the simulation, since each user tries to best utilize the bandwidth resource of each access link so as to maximize its own utility. From the results, we also observe that in the case with larger ξ_s/η_s utility functions users obtain more bandwidth resource and finally higher perceived utility. This is expected since the original problem is to maximize the aggregated utility of all users.

Now we analyze the influence of the swarm size on the convergence performance of the proposed resource allocation scheme and depict the simulation results in Fig. 2. We observe that increasing the swarm size from 20 to 160 slightly improves the convergence performance of the resource allocation scheme. In fact, recall that the global exploration and local exploitation discussed in Section III, the convergence performance mainly depends on algorithm parameters such as swarm size other than the number of users.

Finally we investigate the impact of user numbers on the performance of our resource allocation scheme. The simulation setup is identical with the one above except that the users are not static, i.e., after a period time some users leave the network while other new users join. Assume at iteration $k = 300$, 20 users leave the network after completion of one elastic application (e.g., transmission) and at iteration $k = 600$ new 10 users join the network for requesting new elastic applications. We depict the evolution of aggregated utility for the dynamic network in Fig. 3. We find that the



(a) $\xi_s = 10, \eta_s = 1$



(b) $\xi_s = 1, \eta_s = 10$

Fig. 1. Performance of the resource allocation scheme for the multipath network

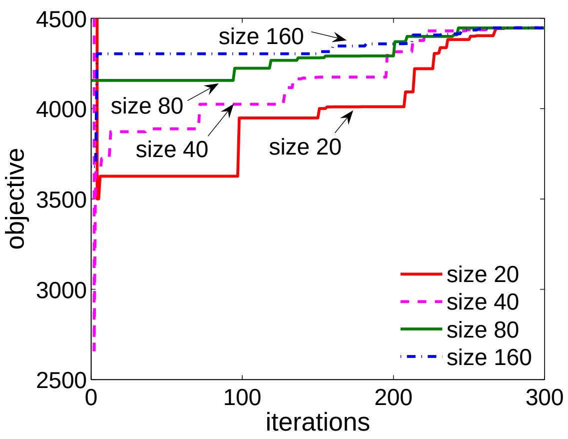


Fig. 2. Performance of the scheme with different swarm sizes

proposed resource allocation scheme behaves well after the transitional points of user arrivals and departures and also finally converges to the optimum within reasonable iteration times.

V. Conclusions

We considered resource allocation for elastic applications in multipath networks in this paper and investigated resource allocation for elastic applications with the objective of utility maximization, which can be summarized as how to maximize the aggregated utility of all users under the constraint of

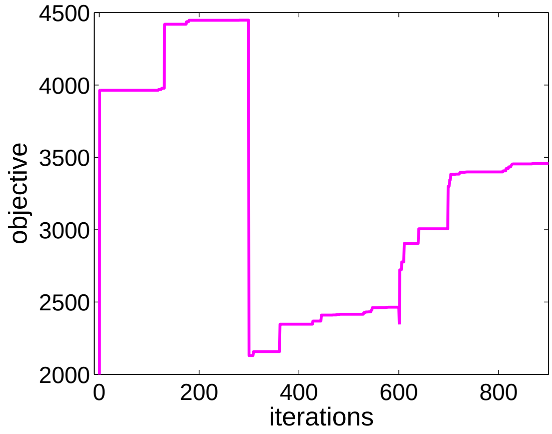


Fig. 3. Performance of the scheme with user arrivals and departures

links' capacities. Moreover, we presented a heuristic resource allocation scheme via PSO to achieve the optimal resource allocation. Finally, we gave some numerical examples to verify the research results obtained. For future research work, we will investigate the resource allocation for inelastic applications in multipath networks and try to develop the resource allocation scheme for inelastic applications which can be implemented into multipath networks.

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