A Hybrid Metaheuristic Algorithm for the Heterogeneous School Bus Routing Problem and a Real Case Study

Yan-e Hou，Lanxue Dang，Yunfeng Kong，Zheye Wang and Qingjie Zhao

***Abstract*—In practice of school bus route planning, the bus fleet usually consists of a number of buses with different capacities, purchase costs and operation costs. However, the heterogeneous fleet school bus routing problem (HSBRP), especially for the problem with a limited fleet, has not been sufficiently investigated so far. This study deals with the HSBRP with unlimited fleet as well as limited fleet. The objective is to design a set of routes in such a way that the sum of fixed and variable costs is minimized. A metaheuristic algorithm for the HSBRP is proposed by hybridizing iterated local search (ILS) heuristic with a set partitioning (SP) procedure. The historical routes identified by ILS local search are globally selected by solving a set partitioning problem. The experimental results show that the proposed algorithm is quite effective and efficient. Furthermore, when the algorithm is applied to the routes plan of several real schools in china, it can obtain better school bus route planning solution and reduce the transportation costs.**

***Index Terms*—heterogeneous fleet, iterated local search, set partitioning, real case, school bus routing problem**

I. INTRODUCTION

THE school bus routing problem (SBRP), aims to plan an efficient schedule for a fleet of school buses where each bus picks up students at several bus stops and delivers them to their designated schools while meeting various constraints[1]. SBRP is a real-world transportation system problem that is important and challenging work for the school and education authorities. As a class of NP-hard problems in combinatorial optimization, SBRP has been constantly studied. Detailed descriptions on SBRP and a comprehensive survey of the related literature can be found in [2, 3].

In most SBRP literature, a fleet of homogeneous buses is considered. However, in practice of school bus route planning, the bus fleet always consists of a number of buses with different capacities, purchase costs and operation costs. More specifically, the number of each type of bus may be limited and fleet composition is known in advance. Hence, planning the routes for a heterogeneous fleet of buses is challenging but essential for bus operation companies. These practical problems make the researchers pay more attention on the extension of SBRP i.e., heterogeneous school bus routing problem (HSBRP).

The HSBRP is also a variant of heterogeneous vehicle routing problem (HVRP). In fact, the HSBRP tends to be a variant of HVRP with open routes [4, 5], because each bus does not necessarily return to the depot after servicing the students. Compared with the classical HVRP, the HSBRP has an additional constraint of the maximum student riding time.

In order to provide safety and cost efficient transportation, the riding time for any student shall not exceed a predefined maximum riding time. In addition, the bus service time at each bus stop depends on the number of students to be serviced at the stop [6]. Like the classification of HVRP [7, 8], the heterogeneous buses of HSBRP may be unlimited or limited. However, to the best of our knowledge, the HSBRP has received very limited attention, especially for the limited heterogeneous buses.

The aim of this paper is to deal with the HSBRP with unlimited fleet as well as limited fleet. The objective is to minimize the sum of fixed and variable costs. A hybrid metaheuristic algorithm (ILS-SP) is proposed based on an iterated local search (ILS) heuristic and a set partitioning procedure (SP). The intermediate routes identified by ILS are recorded in a route pool. The historical routes are recombined and selected by solving a set partitioning problem. This hybrid algorithm is to solve both the fleet size and mix school bus routing problem (FSMSBRP) and the heterogeneous fixed fleet school bus routing problem (HFSBRP). The performance of the proposed algorithm is tested on 20 benchmark instances. Finally, we also apply the proposed algorithm into a real case.

The remainder of this paper is organized as following. Section II reviews the related works in the literature. Section III defines the HSBRP. Section IV describes the proposed hybrid algorithm for the HSBRP in details. Computational results and a real case study are presented in Section V. Section VI offers some concluding remarks.

II. RELATED WORKS

The classification of SBRP based on fleet mix has been

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described in many existing studies [9-14]. In these researches, the solving methods including exact methods, heuristic methods, and metaheuristics are used.

For the school bus routing and scheduling problem with heterogeneous bus fleet, the mathematical formulations and exact algorithms are developed in [9]. The bus flow, single commodity flow, two-commodity flow, and multi-commodity flow mathematical models are solved by a branch-and-cut exact algorithm. However, only very small instances with heterogeneous fleet could be solved with the exact method.

There are only a few heuristic algorithms were proposed to solve the heterogeneous fleet problem. For a rural problem with heterogeneous buses, a simple taboo search algorithm combined with 1-exchange operator is presented in [10]. For a complex rural SBRP with multiple attributes such as mixed-fleet, multi-depot, site-dependent and split-delivery, a heuristic algorithm including six heuristic methods such as Student-interchange, Sharing, Reduction, Combine and Swap buses are used iteratively to reduce the travel cost[11]. There is an adaptive location based heuristic (ALBH) algorithm to solve the school transportation problems for 399 cities in Brazil [12]. In the algorithm in [12], the buses with biggest capacity are assigned to the routes at first. After building a route solution, the smaller buses could be adjusted more appropriately. All of these algorithms in [10-12] can manage the heterogeneous buses, but do not guarantee high-quality solutions.

In order to get better solutions, the metaheuristic algorithms are used in [13, 14]. Four metaheuristc algorithms are proposed in [13] to solve the mixed load capacitated rural SBRP with heterogeneous fleets but without considering the maximum riding time constraint. Then, the same problem with the multi-objectives is studied in [14]. Though there are some metaheuristic algorithms applied to solving the HSBRP, it is also necessary for the researcher to explore more and better metaheuristic algorithms to solve the HSBRP.

Since the fleets are usually heterogeneous in most practical distribution and transportation problems, the HVRP and its variants have received much attention in last 30 years [15]. Two major classes of HVRP problems are the fleet size and mix vehicle routing problem (FSMVRP) [7] and the heterogeneous fixed fleet vehicle routing problem (HFVRP) [8]. Like VRP, the HVRP and its variant problems are also NP-hard problems. Due to the intrinsic difficulty of the family of HVRP problems, most solution approaches in literature are the heuristic and metaheuristic algorithms such as tabu search [16,17], memetic algorithm[18], multi-start adaptive memory programming and path relinking heuristic[19], hybrid population heuristic[20], iterated local search[21,22], and unified hybrid genetic search metaheuristic [23]. In summary, significant progress has been made on the HVRP and its variants. A comprehensive comparison of recent metaheuristics shows that the standard versions of HVRP have been solved to near optimality by heuristics, and the algorithmic research on the standard problems has reached maturity [15]. However, these metaheuristic algorithms are successfully applied to solve VRPs, but they have not attracted much attention to solve the school bus routing problems [2].

In some literatures, the metaheuristic algorithm combined with set partitioning procedure methods are used to improve the VRP solutions, such as tabu with SP [24], genetic algorithm with SP [25], ILS with SP [22] and so on. The set partitioning procedure is introduced a post-optimization technique for solving vehicle routing problems [24]. This technique consists of saving all partial solutions identified during the tabu search algorithm, and improving the heuristic solution by solving a set partitioning problem. The genetic algorithm with set portioning procedure is proposed in [25] to solve the VRP with time windows. For both FSMVRP and HFVRP problems, an ILS-based heuristic algorithm is provided to generate columns in a set partitioning formulation, and the competitive results and new best-known solutions are obtained on benchmark instances [22]. It shows that the metaheuristic hybrid with set partitioning procedure is effective to solve VRP problems. For SBRP, a hybrid algorithm combined ILS with SP is used to solve the bi-objective single school SBRP with homogeneous fleets [26]. The experiment results show that the set partitioning procedure can effectively improve the solutions. It indicates that this hybrid metaheuristic algorithm has application potentials in solving SBRP and its variants.

III. Problem Description and Formulation

The HSBRP can be defined on a graph with a set of nodes and a set of edges. Let $G=(V,E)$ be a complete weighted graph, where $V=\{0,1,2,3...n,n+1\}$ is the node set, and $A=\{(i,j),i,j\in V \mid i\neq j\}$ is the edge set. Node 0 refers to the bus depot; node $n+1$ refers to the school; and a node set $C=\{1,2,3,...,n\}$ denotes the student stops. Each stop i has a known number of students q_i to be served and a service time t_i .For convenience, depot and school node have no demand and service time, that is $q_i=0,t_i=0(i\in\{0,n+1\})$. Each arc (i,j) is associated with a traveling distance d_{ij} and a traveling time t_{ij} . A fleet of school buses is located at the depot, and the set of bus types is denoted as $M=\{1,2,3...K\}$. Each school bus of type k has a carrying capacity Q_k , a fixed cost f_k and a variable cost per unit distance v_k . The number of school buses of type k is denoted as h_k .

The objective of HSBRP is to determine a set of bus routes with minimum fixed cost and variable cost while satisfying the following constraints. (1) Each bus leaves from the depot, visits to several student stops and travels to the school. (2) Every stop must be visited exactly once. (3) The number of students served by a bus cannot exceed its capacity. (4) The riding time for any student in the bus shall not exceed the predefined maximum riding time T . (5) The number of buses used of type k cannot exceed the limit of h_k . If the number of buses in the last constraints is unlimited ($h_k=+\infty,k\in M$), the HSBRP is denoted as FSMSBRP; otherwise, it is denoted as HFSBRP.

A mathematical formulation for the HSBRP is based on three types of decision variables. The variable x_{ijk} indicates that if a bus of type k travels from stop i to j , then $x_{ijk}=1$, otherwise, $x_{ijk}=0$. The variable y_{ik} donates the cumulative number of students in a bus of type k when the bus leaves from stop i ($y_{0k}=0$). The variable z_{ik} denotes the cumulative travelling time of a bus of type k when it arrives to stop i which is calculated from the school to the depot in reverse order ($z_{(n+1)k}=0$). A mixed integer programming (MIP) formulation for the HSBRP is shown as follows:

$$\min \quad \sum_{j\in C}\sum_{k\in M} f_k x_{ojk} + \sum_{i\in V\setminus\{n+1\}}\sum_{j\in V\setminus\{0,i\}}\sum_{k\in M} v_k d_{ij} x_{ijk} \tag{1}$$

$$\text{s.t.} \quad \sum_{j\in V\setminus\{i,n+1\}}\sum_{k\in M} x_{ijk}=1, \forall i\in C, k\in M \tag{2}$$

$$\sum_{i\in V/\{n+1\}} x_{ipk} - \sum_{j\in V/\{0\}} x_{pjk} = 0, \forall p\in C, k\in M \tag{3}$$

$$y_{ik}\le Q_k, \forall i\in V/\{n+1\}, k\in M \tag{4}$$

$$y_{ik}+q_j-y_{jk}\le M_1(1-x_{ijk}), \forall i\in V\setminus\{n+1\}, \quad j\in V\setminus\{0,i\},\ k\in M \tag{5}$$

$$z_{ik}\le T, \forall i\in V/\{0\}, k\in M \tag{6}$$

$$z_{jk}+t_{ij}+t_i-z_{ik}\le M_2\left(1-x_{ijk}\right), \forall i\in V\setminus\{n+1\}, \quad j\in V\setminus\{i\},\ k\in M \tag{7}$$

$$\sum_{i\in V\setminus\{0\}} x_{0ik}\le h_k, \forall k\in M \tag{8}$$

$$x_{ijk}\in\{0,1\}, \forall i\in V\setminus\{n+1\}, j\in V\setminus\{0,i\},\ k\in M \tag{9}$$

$$y_{ik}\in\{0,1,2...\}, \forall i\in V,\ k\in M \tag{10}$$

$$z_{ik}\in\{0,1,2...\}, \forall i\in V,\ k\in M \tag{11}$$

The objective function (1) is to minimize the sum of fixed and variable costs. Constraints (2) ensure that each bus stop must be visited exactly once. Constraints (3) make sure that if a bus of type k visits the stop p, it must leave from the stop. Constraints (4) guarantee that the total number of students in the bus of type k must not exceed its capacity. Constraints (5) express the accumulation of students in a bus. If $x_{ijk}=1$, then $(y_{ik}+q_j)x_{ijk}=y_{jk}$; otherwise $(y_{ik}+q_j)x_{ijk}<y_{jk}$. The non-linear constraints can be transformed to linear inequalities (5) by using a big number $M_1(M_1>2Q_k)$ .Constraints (6) guarantee that the riding time of each student in the bus is never exceeded the maximum riding time T . Constraints (7) express the accumulation of bus travelling time: $(z_{jk}+t_{ij}+t_i)x_{ijk}\le z_{ik}$. It can also be transformed to linear inequalities by introducing a big number $M_2(M_2>2T)$. Constraints (8) impose that each type of buses used cannot exceed its maximum number. For the FSMSBRP, constraints (8) are redundant. Constraints (9), (10) and (11) are the constraints on the decision variables.

IV. Hybrid Metaheuristic Algorithm

A. Algorithm Framework

The proposed hybrid algorithm (ILS-SP) for HSBRP integrates an iterated local search (ILS) heuristic with a set partitioning (SP) procedure. ILS has been successfully applied to various VRP problems. Its performance depends mainly on the choice of the local search, the perturbations and the acceptance criterion. The ILS in our algorithm consists of two methods for generating of an initial solution, eight inter and intra-route local search operators, three perturbation operators and a sequence of acceptance rules. In addition, the intermediate routes in the locally optimal solution identified by ILS are recorded in a route pool. After iterations of local search, a SP model will be build based on the routes in the route pool. A MIP solver then solves the model. The algorithm framework of ILS-SP is described in Algorithm 1.

Algorithm 1: ILS-SP (*Maxiter, Piter*)

(1) Generate an initial feasible solution S_0;
(2) *RoutePool*=*Null*, S_{best}=S=S_0,
    Update_route_pool(*RoutePool*, *S*);
(3) For (*i*=0; *i*<*Maxiter*; *i*++)
(4) For each local search operator *op* randomly selected
(5)   *S*=Localsearch(*op*,*S*,S_{best});
(6)   Update_route_pool(*RoutePool*, *S*);
(7) If (S_{best} has not been updated in *Piter* consecutive iterations)
(8)   S=Perturbation(*S*, S_{best});
(9)   Update_route_pool(*RoutePool*, *S*);
(10) *sp*=Build_sp_model(*RoutePool*);
(11) S*=MIPSolver(*sp*);
(12) Output S_{best} and *S**.

The procedure Localsearch(*op*, *S*, S_{best}) uses operator *op* to explore the neighborhood space of the current solution *S*. A neighborhood solution is accepted or rejected according to the acceptance rules. Once a neighborhood solution is better than the best solution S_{best}, the best solution will be updated. The procedure Update_route_pool(*RoutePool*,*S*) records all the routes of solution *S* into *RoutePool*. The procedure Build_sp_model (*RoutePool*) builds a SP model based on the routes in *RoutePool*. The procedure MIPSolver(*sp*) solves the set partitioning model *sp*.

B. Initial Solution Construction

ILS starts with an initial feasible solution. For the proposed algorithm, a giant tour method is used to construct the initial solution for the FSMSBRP. Since the available buses are limited and each route must satisfies the constraint of maximum riding time, finding a feasible solution for the HFSBRP would be quite difficult. Therefore, we adopt an improved cheapest insertion method to find an initial solution for the HSBRP. The algorithm is described in the following.

(1) Initialize the list of stops *U* and an empty solution S_0.

(2) For each bus, create a route that starts from the depot and ends at the school. The type of bus is assigned to its route. The number of routes equals to the total number of available buses v.

(3) Randomly select v stops from *U* insert them to the current routes, and then remove them from *U*. Note that the stop insertion must not violate the bus capacity. If a stop

cannot be inserted into any route, go to Step (1).

(4) For each unvisited stop in *U*, find the stop $u \in U$ with the cheapest insertion cost to S_0, and then insert u to S_0 as well as removing it from *U*. In case of an unvisited stop cannot be inserted into any route, go to Step (1). When all the stops are inserted into existing routes, an initial solution is constructed.

To decrease the number of trails in this algorithm, we also adopt a node sorting and selecting strategy in Step (3). The stops are first sorted by their demands in descending order. The routes are also sorted by their bus capacities in descending order. Then, for each route in the first half of the route list, a stop randomly selected from the top three unvisited stops in the stop list is inserted into it. This will insert most large stops into the routes with large capacities, and thus increases the opportunity of finding a feasible solution.

C. Neighborhood Structures

In the local search process of the proposed algorithm, multiple neighborhood operators with random neighborhood selection are performed. Both inter-route and intra-route neighborhood operators are used to improve the solution. In each iteration of the local search, the inter-route operators are randomly applied to the current solution. When a new neighborhood solution is accepted by an inter-route operator, the intra-route operators will be then performed in sequence.

The inter-route neighborhoods used in the proposed algorithm includes Shift(1,0), Shift(2,0), Swap(1,1), Swap(2,2) and 2-opt. These neighborhoods are described as follows.

(1)Shift(1,0).A student stop is removed from a route and then inserted to another route. In Fig 1 (a), the student stop 2 is removed from the top route and then inserted to the bottom route.

(2)Shift(2,0).Two adjacent student stops are moved from the same route to another route. In Fig 1 (b), two student stops 2 and 3 are moved from the top route to the bottom route.

(3)Swap(1,1).Two student stops on the two different routes are exchanged to obtain two new routes. In Fig 1(c), the student stop 2 and 7 on the two different routes are exchanged, so the two new routes are obtained.

(4)Swap(2,2).Two continuous student stops on the two different routes are exchanged. In Fig 1 (d), a pair of student stops 2 and 3 is exchanged with another pair of student stops 6 and 7.

(5)2-opt. 2-opt deletes two non-adjacent edges and then links the remaining segments. After 2-opt, two new routes are obtained. In Fig 1 (e), the edges e_1, e_2, e_3, e_4 are deleted from two different routes respectively. Two new edges e_5 and e_7 are added to get the top route, and two another edges e_6 and e_8 are added to get the bottom route.

The intra-route neighborhoods are used in the same route, which include Relocate, Route Reverse and 2-opt*. These intra-route neighborhoods are described in the following.

(1)Relocate. Relocate removes a student stop and then inserts it into another position of the same route. In Fig 2 (a), student stop 1 is removed and then inserted after student stop 3.

(2)Route Reverse. This operator reverses a route segment and reinserts it into the same position (Fig 2). In Fig 2 (b), the student stops from 1 to 4 are reversed and then reinserted into the same position. A new route is obtained by the operator.

(3)2-opt*. When 2-opt is performed in the same route (donated as 2-opt*), two non-adjacent edges are deleted and the student stops between these edges are all reversed. In Fig 2(c), the edges e_1 and e_2 are firstly deleted, and then the student stops from 2 to 5 between these two edges are reversed. Finally, two new edges e_3 and e_4 are inserted to obtain a new route.

D. Fleet Type Adjustment Strategy

In local search, adjusting the bus type for a route is necessary for two reasons. For a possible node move, if the number of the students in a route exceeds its capacity, the route should use a bus with enough capacity. Otherwise, if a low-cost bus could serve a route, assigning a low-cost bus to the route will reduce the route cost. Consequently, in each inter-route operator, the bus type for each related route could be changed either to keep the solution feasible or to reduce the routing cost.

Let the route r using bus type of k and the bus capacity is Q_k. The total cost of the route r is C_r^k, the number of the students in this route is Q. The set of bus types is denoted as $M = \{1, 2, 3 ... m\}$,which is ordered by capacity of bus type ascending. The fleet type adjustment strategy is defined as follow.

(1) If $Q = Q_k$, then leave it alone.

(2) If $Q < Q_k$, then it tries to find a low-cost bus to server this route. For each bus type in the set of bus types $\{1, 2, ..., k-1\}$, seek the bus type j that is satisfy with $Q <= Q_j$ and making $C_r^k - C_r^j$ with the minimum value. When it cannot find the bus type that meets the above constraints, or k is the smallest bus type in M, it will do nothing.

(3) If $Q > Q_k$, then it means the number of students in the route exceed the bus capacity constraints, it needs to find a new bus to server this route. For each bus type in the set of bus types $\{k+1, k+2, ..., m\}$, search the bus type j that is satisfy with $Q <= Q_j$ and making $C_r^j - C_r^k$ with the minimum value. When it cannot find the bus type that meets the above constraints, or k is the biggest bus type in M, it will do nothing.

In the above fleet type adjustment strategies, strategy (2)

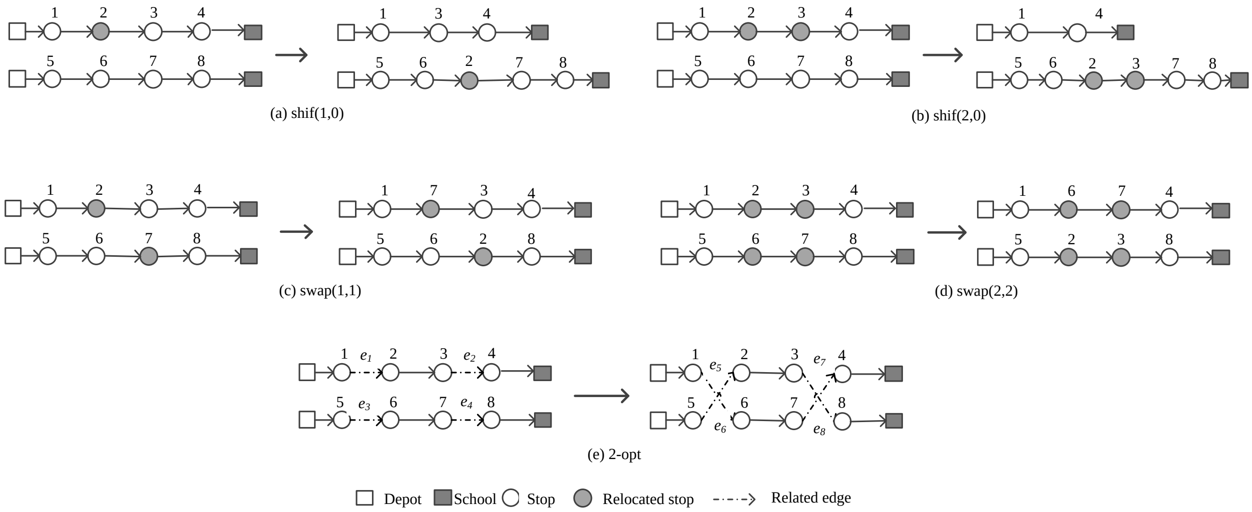

Fig 1. Examples of inter-route neighborhoods

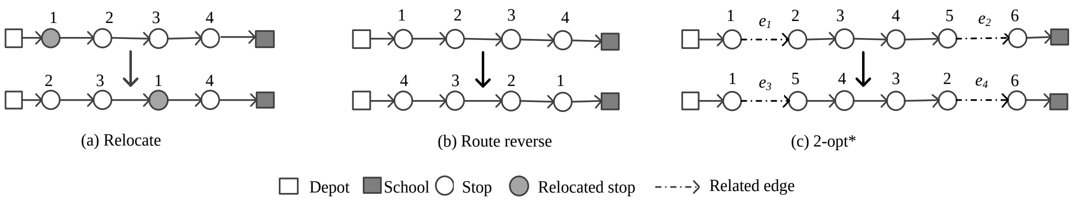

Fig 2. Examples of intra-route neighborhoods

tries to find the low-cost bus to reduce the total cost. While for strategy (3), it seeks the feasible solution through fleet type adjustment. Although it may cause the higher cost of the route, it brings the diversity of search and improves the probability of finding a better solution. It is possible to reduce the total cost of all the routes by adjusting fleet type again in the subsequent local search.

After the adjusting bus type for a route, it is necessary to determine whether the cost change caused by the fleet bus adjustment meets the acceptance rules of the solution. For HFSBRP, it is also essential to determine whether the limit on the number of vehicles of each type is met.

E. Acceptance Rules

The optimization objective defined in this paper is to minimize the sum of fixed and variable costs. The solution cost is related to the number of buses required and the combination of the bus types. Since the fixed cost per bus is often significantly higher than the travel cost per distance unit, the total cost for HSBRP solution could be saved by reducing the number of bus routes. However, minimizing the traveling distance may lead the search to solutions with a small traveling distance but it is difficult to remove routes effectively. In order to improve the performance of our algorithm for deleting as many unnecessary routes as possible, we introduce a supplementary function, described in (12), to evaluate the possible node moves in each inter-route operator.

$$Eval(i,j) = \min\{Q_m - Load_i, Q_p - Load_j\} - \min\{Q_k - Load_i^*, Q_l - Load_j^*\} \tag{12}$$

For routes i and j evaluated by an inter-route operator, $Load_i$ and $Load_j$ denote the total loads of the two routes respectively. A bus of type m and a bus of type p serve the two routes. For a possible node move, the total loads of the two routes are changed to $Load_i$* and $Load_j$*, and the bus types are adjusted to k and l respectively. If $Eval(i, j) > 0$, the node move will increase the bus utilization for either route i or route j .This function could encourage the search to delete some routes gradually.

Based on the discussion above, the acceptance rules for neighborhood solutions in our algorithm are designed as the following. For operators such as Shift(1,0) and Shift(2,0), if one route can be deleted, the neighborhood solution will be accepted. For all inter-route operators, if $Eval(i, j) > 0$, the neighborhood solution will also be accepted; otherwise, the cost saving in traveling distance will be considered. In summary, for a feasible neighborhood solution, we evaluate the number of routes, the supplementary function $Eval(i, j)$ and the total cost in sequence. In order to escape the local optimum, some worsening neighborhood solutions could be accepted according to the record-to-record travel (RRT) acceptance criterion [27].

F. Perturbation Mechanism

The success of ILS also depends on the perturbation operators and the perturbation strength. The simplest possibility to improve the performance of local search is to repeat the search from another starting point. To address this issue, we utilize three perturbation operators in ILS heuristic: multi-point shift, multi-point swap and ruin-and-recreate.

The first operator selects δ stops randomly and shifts them to other positions in the same or different routes. The second operator also randomly selects δ stops at first; then for each selected point, tries to swap it with one of its neighborhood points. These two perturbation operators are implemented by performing Shift(1,0) and Swap(1,0) neighborhood operators multiple times.

The ruin-and-recreate procedure destroys the current solution by removing δ stops and then uses a repair method to recreate a new solution [27]. The degree of ruin is evaluated by the number of stops to be removed. In the ruin method, a seed node is randomly selected at first. A list with $2*\delta$ neighborhood nodes of the seed is then constructed. Then, the seed node and $\delta-1$ nodes randomly selected from the list are removed from the solution. The lowest insertion algorithm is used to recreate the solution.

G.Set Partitioning Procedure

In the proposed algorithm, the final step is to improve the solution by a set partitioning procedure. ILS heuristics have been successfully applied to solving various routing problems. Various local search operators, perturbation methods and acceptance rules can be applied to ILS to design novel algorithms. However, all the algorithms based local search are short-sighted, since only a small neighborhood space can be explored by local search operators. In addition, most of routes explored by the local search operators are discarded when better routes are found. The set partitioning procedure in our ILS-SP algorithm aims to utilize the historical routes identified in local search and tries to optimize the HSBRP solution from a global point of view.

As described in the proposed algorithm, the route pool consists of many routes recorded in the local search. A SP model is build based on the routes in the route pool. Since the SP model is weakly NP-hard, the problem with thousands of routes can be solved efficiently by a MIP solver.

The formulation for the HSBRP based on the set partitioning problem is defined as following. Let R be the set of all possible routes for HSBRP and R^* be a subset of $R(R^* \in R)$. Let R_k be the subset of routes using bus type of k and $R^* = \cup_{k \in M} R_k$. Each route $r(r \in R_k)$ has an associated cost c_r and a binary variable x_r. Let R^i be the subset of the routes covering stop $i(i \in C, R^i \subseteq R^*)$. A set partitioning formulation for the HSBRP is given as follows:

$$\min \quad \sum_{r \in R^*} c_r x_r \tag{13}$$

$$\text{s.t.} \quad \sum_{r \in R^i} x_r = 1, \forall i \in C \tag{14}$$

$$\sum_{r \in R_k} x_r \le n_k, \forall k \in M \tag{15}$$

$$x_r \in \{0,1\}, \forall r \in R^* \tag{16}$$

This SP model tries to select an optimal HSBRP solution from the possible routes. The objective function (13) minimizes the sum of the route costs. Constraints (14) guarantee that each stop must be covered exactly once. Constraints (15) impose the upper bound on the number of buses for each bus type. In the case of FSMSBRP, the constraints (15) can be eliminated. Constraints (16) define the binary decision variables.

V. Computational Results and Real Case

A. Benchmark Instances

The benchmark instances for HSBRP, as shown in TABLE I, are created based on benchmark instances for multi-school SBRP, which were proposed by [29]. The original instances are classified into two groups: random spatial distribution of schools and bus stops (RSRB) and clustered distribution (CSCB). A set of small instances (S01-S08), each with at most 13 bus stops, are selected from instances RSRB02 and CSCB02. The second set of instances (C01-C06) with 17~75 bus stops is derived from instance CSCB01. The third set of instances (R01-R06) with 38~51 bus stops is derived from the instance RSRB01. Instead of the homogeneous fleet defined in the original instances, a heterogeneous fleet with two or three types of buses is carefully designed for each new instance.

In TABLE I, the columns, N and TD, represent the number of stops and the total number of students to be served respectively. For each bus type k, its capacity, fixed cost, variable cost per mile and available number are listed in columns Q_k, f_k, v_k and h_k ($k = A, B$ and C).

B. Comparison of Exact Algorithm and ILS-SP

The newly designed instances for HSBRP were solved by ILS-SP algorithm and CPLEX respectively. In each instance, the distance between any two nodes was calculated by Manhattan distance. The average speed of school bus was assumed to be 29.333333 feet per second (20 miles per hour). The bus service time t_i at student stop was an integer number estimated by the formula 19+2.6*q_i, where q_i is the number of students at the stop i. The maximum riding time (T) of a student in a bus was set to 2700 seconds.

The ILS-SP algorithm described in Section IV was implemented by C# programming on a personal computer with an Intel(R) Core 2 3.06GHz CPU and 4GB RAM running 32-bit Windows 7 operating system. The parameters for ILS-SP algorithm were estimated based on several experiments. The parameters *Maxiter* and *piter* were set to 500 and 10 respectively. The deviation used in the acceptance criteria was set to 0.00001. For the perturbation procedure, the number of perturbed stops was defined as 20% of the student stops. The SP model was solved by IBM ILOG CPLEX 12.6. The parameters for CPLEX solver were set to their default values, except that the maximum computation time was set to 60 seconds and the MIPGap was set to 10^{-10}. The ILS-SP algorithm was executed ten times over each instance.

In order to evaluate the performance of our algorithm, we also tried to solve the benchmark instances by exact method. The optimal or sub-optimal obtained from some instances were used as the baseline data for comparisons

TABLE I
THE BENCHMARK INSTANCES FOR HSBRP

Instance	N	TD	Bus type A				Bus type B				Bus type C			
			Q_A	h_A	h_A	h_A	Q_B	f_B	h_B	h_B	Q_C	f_C	v_C	h_C
S01	5	50	12	1000	0.6	1	20	2000	0.9	2	-	-	-	-
S02	10	83	20	1200	1.2	1	30	2000	1.5	2	40	3000	1.8	1
S03	9	49	10	800	0.6	1	20	1000	1.2	2	30	1500	1.5	1
S04	12	212	40	1200	1.2	2	50	1500	1.5	1	66	1800	1.8	2
S05	7	29	10	1000	0.6	2	20	1500	1.2	2	-	-	-	-
S06	13	136	10	500	0.6	1	30	1200	1.2	3	40	1600	1.5	2
S07	7	80	20	1200	1.2	1	40	2400	1.5	2	-	-	-	-
S08	6	131	40	1500	1.2	2	66	1800	1.5	1	-	-	-	-
C01	70	887	27	1000	1.0	5	54	2500	1.5	12	72	3000	1.7	9
C02	35	674	40	2200	1.2	4	60	2700	1.3	6	72	3000	1.6	4
C03	30	492	30	1200	1.0	3	60	2500	1.3	3	70	3000	1.4	4
C04	23	402	30	1000	1.1	2	50	2200	1.3	5	60	2500	1.4	2
C05	75	1116	40	2500	1.2	4	60	3000	1.4	16	70	3500	1.5	6
C06	17	336	30	2000	1.1	2	40	2500	1.3	3	60	3000	1.5	4
R01	38	569	40	2000	1.1	1	50	2200	1.3	5	70	2500	1.4	6
R02	40	557	30	2400	1.0	2	50	3000	1.2	5	70	3500	1.5	6
R03	51	794	40	2500	1.2	2	50	3000	1.5	6	70	3500	1.7	9
R04	35	427	30	1800	1.0	5	45	2500	1.2	5	60	3200	1.5	3
R05	42	550	40	2600	1.0	5	60	3200	1.3	6	70	3500	1.6	2
R06	44	512	30	1800	1.0	4	40	2500	1.2	3	60	3000	1.4	6

TABLE II
COMPUTATIONAL RESULTS ON FSMSBRP PROBLEM

Instance	CPLEX		ILS-SP					
	C_{obj}	T_c/s	C_{best}	C_{avg}	C_{dev}	Fleet	CUR	T_I/s
S01	5062.57*	0.17	**5062.57***	5062.57*	0.00%	1A2B0C	96.15%	0.04
S02	6537.06*	4.23	**6537.06***	6537.06*	0.00%	2A2B0C	83.00%	0.17
S03	3090.16*	0.91	**3090.16***	3090.47	0.01%	0A3B0C	81.67%	0.11
S04	6469.90*	31.36	**6469.90***	6469.90*	0.00%	1A1B2C	95.50%	0.15
S05	3567.46*	0.28	**3567.46***	3567.53	0.01%	2A1B0C	72.50%	0.06
S06	6261.82*	144.6	**6261.82***	6262.37	0.01%	1A2B2C	90.67%	0.17
S07	6122.95*	0.28	**6122.95***	6122.95*	0.00%	1A2B0C	80.00%	0.05
S08	4894.26*	0.17	**4894.26***	4894.26*	0.00%	2A1B0C	89.73%	0.04
C01	50956.36	7201	**39702.44**	39705.32	0.01%	4A2B10C	94.76%	54.11
C02	33758.32	7252	**31123.66**	31123.82	0.01%	1A5B5C	96.29%	1.91
C03	21983.99	7204	**21268.22**	21268.53	0.00%	2A5B2C	98.40%	0.91
C04	18298.82	7248	**17582.47**	17582.71	0.00%	1A4B3C	98.05%	0.73
C05	73431.26	7510	**57192.93**	57197.77	0.01%	0A13B5C	98.76%	62.57
C06	18776.70*	7157	**18776.70***	18776.90	0.01%	2A1B4C	98.82%	0.27
R01	25327.03	7299	**21998.65**	22040.57	0.19%	0A3B6C	99.82%	4.43
R02	34004.71	7204	**30202.44**	30203.87	0.00%	1A1B7C	96.03%	2.01
R03	48725.23	7205	**41618.62**	41625.13	0.01%	1A0B11C	98.02%	66.1
R04	26432.46	7205	**26028.24**	26031.15	0.01%	3A3B4C	91.83%	1.75
R05	35846.40	7363	**29806.23**	29806.89	0.00%	0A7B2C	98.21%	2.47
R06	32555.19	7318	**27374.87**	27423.03	0.17%	0A0B9C	94.81%	15.27
Avg	23105.13	4367	**20234.08**	20239.64	0.02%	-	92.65%	10.67

with the performance of the proposed algorithm. According to the MIP formulation (1)-(11) for the HSBRP described in Section III, we built the MIP models for all HSBRP instances. In constraints (5) and (8), let M_1=200 and M_2=20000 respectively. The models were solved by CPLEX 12.6.The parameters for CPLEX solver were set to its default values, except that the maximum computation time was set to 7200 seconds and the MIPGap was set to 10^{-10}.

TABLE II and TABLE III show the solution results obtained by ILS-SP and CPLEX for HSBRP instances with unlimited and limited fleet respectively. Columns, CPLEX and ILS-SP, denote the solution costs from CPLEX and ILS-SP algorithm respectively. The columns, C_{obj} and T_C represent the best objective value and the computation times of CPLEX. The number with asterisk (*) indicates that it is an optimal solution verified by CPLEX. The columns, C_{best}, C_{avg} and C_{dev}, represent the best, average and percentage deviation of costs among the 10 solutions, respectively. The numbers in bold indicated that it is equal or better than CPLEX. The column, Fleet and CUR, denote the best fleet composition and its capacity utilization rate. The column T_I gives the average computation time in seconds.

There are several findings from TABLE II and TABLE III. (1) The small instances (S01~S08) can be solved by CPLEX with optimal solutions in 0.14~144.6 seconds. The instance C06 with 17 stops is also solved with optimal solution in 2 hours. (2) Compared with ILS-SP, for instances C01~C05 and R01~R06, feasible solutions for the

FSMSBRP can be obtained by CPLEX with MIPGap between 1.53%~22.21% in 2 hours. Meanwhile, for instances C01~C06 and R01~R06, just only 3 feasible solutions for the HFSBRP can be found by CPLEX with MIPGap between 0.00%~7.52%. (3) Compared with the optimal solutions on small instances (S01~S08 and C06), the ILS-SP can find the optimal solutions. This confirms the excellent performance of the ILS-SP algorithm for solving HSBRP. Compared with the feasible solutions on other instances, the ILS-SP can find equivalent or much better solutions. (4) The variances among 10 solutions of ILS-SP algorithm for each instance are quite small. All the percentage deviations, except the FSMSBRP instance R01 and R06 and the HFSBRP instance R01, are less than 0.10%. The bus capacity utilization rates ranges between 91.83% and 99.82% for instances C01~06 and R01~R06, which are high enough for most of vehicle routing problems. (5) The ILS-SP algorithm solved most the problem instances efficiently. For FSMSBRP and HSBRP, the average computation times of ILS-SP are just only 10.67 seconds and 11.79 seconds respectively.

C. Performance of Set Partitioning Procedure

This section evaluates the performance improvement by set partitioning. The set partitioning procedure introduced in the proposed algorithm is expected to improve solution quality by using the historic routes explored by ILS. TABLE IV compares the SP results with the ILS results on problem instances. Column ILS denotes the best solution cost obtained by ILS. Column ILS-SP denotes the best solution cost obtained by ILS with additional SP procedure. The average computation times for ILS and ILS-SP are shown in columns T_1 and T_2, respectively. The column Gap indicates the cost savings from the set partitioning compared with ILS.

TABLE IV shows that a better or equivalent solution can be generated by solving a set partitioning model for every instance. For the FMSSBRP instances, an average of 0.81% solution cost can be reduced, and the costs on instances C01, C03, C04, C05, R01, and R03 are all reduced significantly. For the HFSBRP instances, 1.33% of solution cost can be reduced, and the costs on instances C01, C05, R01, R02 and R06 are reduced significantly. At the same time, additional computation time is needed for updating the route pool, building the SP model and solving the model. The large part of the increasing time is consumed by the MIP solver, especially for instances C01, C05, R03 and R06.

D. Influence of the local search strategy on ILS-SP

In order to verify the effectiveness of ILS-SP algorithm, We also developed four versions of local search strategies for ILS: random sequence of local search (RLS) as described in Section IV, fixed sequence of local search (FLS), basic variable neighborhood decent (BVND) and random variable neighborhood decent (RVND). They all have the same parameters values and other settings expect with the different local search strategies. Each new algorithm was executed ten times over each instance. For every local search strategies for ILS, we computed the average of best solution costs on all the instances obtained from each algorithm. The results of FSMSBRP and HFSBRP are shown in Fig 3 and Fig 4 respectively.

As shown from Fig 3 and Fig 4, the set partitioning procedure coupled with ILS is capable of improving the solutions quality significantly. For FSMSBRP, the four ILS algorithms with set partitioning procedure can be improved between 0.64% and 1.19%. The ILS algorithm with FLS and BVND local search strategies are improved by 1.19% and 1.18% respectively. While for HFSBRP, the four ILS algorithms with set partitioning procedure are all improved bigger than 1.11%. This could be explained by the fact that the locally optimal routes explored by the ILS local search are globally recombined and selected by set partitioning. While the performance of ILS depends much on its heuristic components, the set partitioning procedure is able to generate robust solutions by using the routes identified by the commonly used LS heuristic.

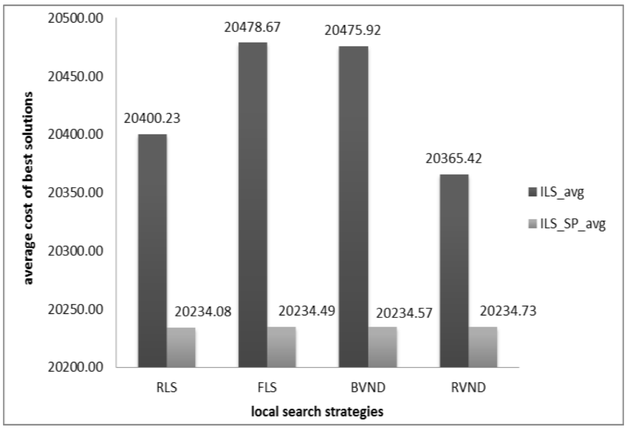

Fig 3. Solution costs from four ILS heuristics with and without set partitioning for FSMSBRP

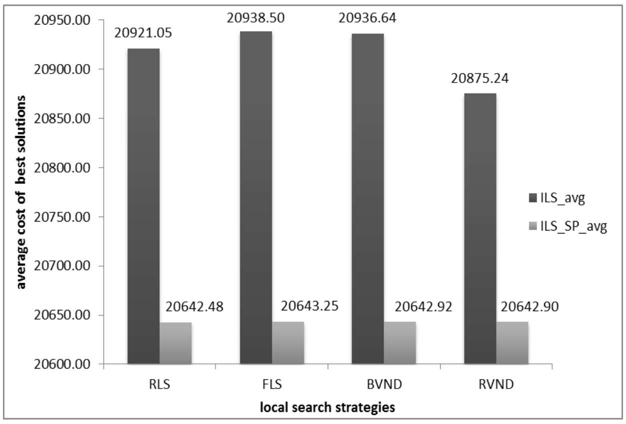

Fig 4. Solution costs from four ILS heuristics with and without set partitioning for HFSBRP

In additional, we also find that the four different local search strategies for ILS affect the quality of the ILS algorithm, especially for the HFSBRP. It is shown that the RLS and RVND search strategies could find the better solutions than FLS and BVND search strategies for the ILS algorithm. But for the ILS-SP, it has an advantage in that its performance does not highly depend on the local search strategy in ILS.

Further, we also calculate the number of best solutions of these two types of HFSBRP on different instances. For small instances S01~S08, four ILS algorithms with and without set partitioning can obtain all the best solutions quickly. For all the instances, the four algorithms find the number of best solutions is shown in Fig 5.

There are some findings from the Fig 5. Four ILS algorithms combined with set partitioning procedure can all find more optional or best solutions than those without set partitioning procedure. For R01~R06 and C01~C06 instances, the ILS heuristics with FLS and BVND local search strategies cannot find any best solutions on FSMSBRP and HFSBRP. But for four ILS with set partitioning procedure, ILS algorithms coupled with different local search strategies have little or no effect on FSMSBRP and HFSBRP. These results again prove the advantage of the set partitioning procedure.

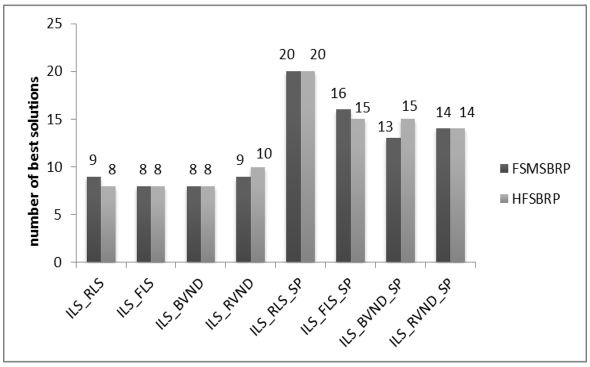

Fig 5. Number of best solutions obtained by four ILS heuristics with and without set partitioning for FSMSBRP and HFSBRP

E. A Case Study

We use the proposed algorithm to solve the routing planning of several schools in Wuxi City, Jiangsu province of China. Wuxi City is located in the south of Jiangsu province, which is the main traffic center in the Yangtze River Delta. The Huishan District of Wuxi was set up in 2001, with an area of 325 square kilometers and 580 thousand permanent residents. There are a provincial Economic Development Zone, five streets (Bridge Street, Chang'an street, Qian Qiao street, Qian Zhou street, Yuqi Street) and two built towns (Luoshe Town, Yangshan town) in the Huishan District of Wuxi. A Jinjiang primary and secondary school delivery service center was specially built to provide the school bus services for primary schools, secondary schools and kindergartens. The service center are equipped with three types of standardized school buses, which it has 27 seats, 54 seats and 57 seats respectively.

We prepared 4 primary schools(donated as PS01~PS04) of Huishan District to carry out case study, including QianQiao center primary school, Yangshi primary school, Yangshan center primary school and Luoshe zhangzhen primary school. The steps of the case study are described in the following.

Firstly, we build the road network data set and obtain the time and distance cost unit between stations in ArcGIS 10.2. The average speed of each road section is simulated according to the speed limit of the road. The speed of the high-speed road and expressway is set to 60 kilometers per hour. The nation road and main road are set to 40 kilometers per hour. The other road speeds are set to 30 kilometers per hour. The network analysis tool in ArcGIS 10.2 is used to calculate the time cost and distance cost matrix between the depot station, the student stops and the school. The time and distance cost unit between these stations are set to minutes and meter respectively.

Secondly, it is assumed that the earliest time of school bus departure from the school is 6:30, the lasted time of school bus arriving the school is 7:50, and the maximum driving time of each school bus is limited to 40 minutes (2400 seconds). The service time of each student stop is also estimated by the number of students, which is same with the service time defined in section V.

Thirdly, we defined the types of school buses and the costs of them. There are three types of school buses donated as type A, type B and type C respectively. In order to facilitate planning, the fixed cost of these school buses just take the purchase cost into account, and the unit distance cost per day is the same, the unit of distance is kilometer (km). The fixed cost of 27 seats, 54 seats and 57 seats are set to 300,000 RMB, 480,000 RMB and 500,000 RMB, and the fuel consumption cost per kilometer are 1.2 RMB, 2.2 RMB and 2.5 RMB respectively. There are also a driver and a teacher in the school bus, so the capacity of three types of school buses is 25 seats, 52 seats and 55 seats. The number of three types of school buses is unlimited.

Finally, we used the proposed algorithm and the VRP tool in ArcGIS 10.2 to plan the school buses routing solution of four schools respectively.

As shown from TABLE V, the ILS-SP algorithm finds the better solutions as well as the average capacity utilization rate than ArcGIS 10.2 VRP tool. The ILS-SP algorithm reduces the total cost by 3.07% in average. For PS03, the ILS-SP algorithm can get the best fleet composition, reducing the total cost by 6.08%; but for PS04, it use less the school buses, improving by 6.18%. The routing planning results shows that our proposed algorithm is effective.

VI. Conclusions

This paper proposes a hybrid metaheuristic algorithm (ILS-SP) for the heterogeneous fleet school bus routing problem (HSBRP). We implemented the algorithm by combining an iterated local search (ILS) heuristic and a set partitioning (SP) procedure. We evaluated the performance of the algorithm on a set of benchmark instances. The experimental results on the FSMSBRP and the HFSBRP show that the proposed algorithm is quite effective and efficient. Further, we also use the algorithm to solve the school bus routing planning of several schools in Wuxi City of China, and the result shows that the algorithm is effective. The success of hybriding ILS with set partitioning could be explained by the fact that the locally optimal routes identified by ILS local search are globally selected by set partitioning. We also found that the set partitioning approach to the HSBRP has an advantage in that its performance does not highly depend on the local search strategy. Compared with the competitive but sophisticated metaheuristics for solving the HVRP and its variants, the local search based set partitioning method is relatively easy to be implemented.

As for future work, we intent to improve our ILS-SP algorithm to solve large-sized instances, as well as other SBRP variants that include additional attributes such as multiple depots and multiple schools.

TABLE III
COMPUTATIONAL RESULTS ON HFSBRP PROBLEM

Instance	CPLEX		ILS-SP					
	C_{obj}	T_c/s	C_{best}	C_{avg}	C_{dev}	Fleet	CUR	T_I/s
S01	5062.57*	0.16	**5062.57***	5062.57*	0.00%	1A2B0C	96.15%	0.05
S02	7135.77*	4.16	**7135.77***	7135.77*	0.00%	0A2B1C	83.00%	0.10
S03	3386.92*	1.08	**3386.92***	3387.14	0.01%	1A1B1C	81.67%	0.08
S04	6469.90*	13.18	**6469.90***	6470.35	0.01%	1A1B2C	95.50%	0.15
S05	3567.46*	0.14	**3567.46***	3567.98	0.01%	2A1B0C	72.50%	0.07
S06	6261.82*	112.1	**6261.82***	6262.84	0.02%	1A2B2C	90.67%	0.19
S07	6122.95*	0.22	**6122.95***	6122.95*	0.00%	1A2B0C	80.00%	0.06
S08	4894.26*	0.17	**4894.26***	4894.26*	0.00%	2A1B0C	89.73%	0.05
C01	Infeasible	7203	**39727.47**	39730.58	0.01%	5A4B8C	96.72%	41.98
C02	35178.81	7279	**32533.43**	32533.97	0.00%	3A5B4C	95.20%	3.04
C03	Infeasible	7203	**22272.00**	22273.98	0.01%	2A3B4C	94.62%	4.90
C04	18297.01	7202	**18297.01**	18297.01	0.00%	2A5B2C	93.49%	1.05
C05	Infeasible	7228	**57201.12**	57201.91	0.00%	0A13B5C	98.76%	67.88
C06	18776.70*	2895	**18776.70***	18778.43	0.01%	2A1B4C	98.82%	0.34
R01	Infeasible	7204	**21998.65**	22140.11	1.91%	0A3B6C	99.82%	9.98
R02	Infeasible	7205	**30299.61**	30300.80	0.00%	0A3B6C	97.72%	2.64
R03	Infeasible	7204	**43129.10**	43144.07	0.03%	2A2B9C	98.02%	63.07
R04	Infeasible	7205	**26432.46**	26436.45	0.02%	5A3B3C	91.83%	2.26
R05	Infeasible	7204	**31198.45**	31199.81	0.00%	3A5B2C	98.21%	5.32
R06	Infeasible	7204	**28082.87**	28083.58	0.00%	4A1B6C	98.46%	32.67
Avg	-	4118	**20642.53**	20651.23	0.10%	-	92.54%	11.79

TABLE IV
ILS AND ILS-SP RESULTS ON HSBRP INSTANCES

Instance	FSMSBRP					HFSBRP				
	ILS	ILS-SP	T_1	T_2	Gap	ILS	ILS-SP	T_1	T_2	Gap
S01	5062.57	5062.57	0.04	0.04	0.00%	5062.57	5062.57	0.03	0.05	0.00%
S02	6537.06	6537.06	0.16	0.17	0.00%	7135.77	7135.77	0.08	0.10	0.00%
S03	3090.16	3090.16	0.10	0.11	0.00%	3386.93	3386.93	0.06	0.08	0.00%
S04	6469.90	6469.90	0.11	0.15	0.00%	6469.90	6469.90	0.12	0.15	0.00%
S05	3567.46	3567.46	0.04	0.06	0.00%	3567.46	3567.46	0.05	0.07	0.00%
S06	6261.82	6261.82	0.14	0.17	0.00%	6261.82	6261.82	0.16	0.19	0.00%
S07	6122.95	6122.95	0.05	0.05	0.00%	6122.95	6122.95	0.05	0.06	0.00%
S08	4896.26	4894.26	0.03	0.04	0.04%	4894.26	4894.26	0.04	0.05	0.00%
C01	40734.57	39702.44	9.84	54.11	2.53%	40732.60	39727.47	10.5	41.98	2.47%
C02	31242.31	31123.66	1.53	1.91	0.38%	32539.79	32533.43	2.99	3.04	0.02%
C03	21576.36	21268.22	0.73	0.91	1.43%	22276.43	22272.00	1.51	4.90	0.02%
C04	17887.65	17582.47	0.44	0.73	1.71%	18297.01	18297.01	0.98	1.05	0.00%
C05	57740.79	57192.93	11.4	62.57	0.95%	57724.26	57201.12	12.7	67.88	0.91%
C06	18780.91	18776.70	0.25	0.27	0.02%	18780.61	18776.70	0.55	0.34	0.02%
R01	22289.01	21998.65	2.45	4.43	1.30%	23435.75	21998.65	3.31	9.98	6.13%
R02	30208.88	30202.44	1.34	2.01	0.02%	32104.50	30299.61	2.33	2.64	5.62%
R03	42166.38	41618.62	5.07	66.10	1.30%	43156.24	43129.10	5.75	63.07	0.06%
R04	26038.12	26028.24	1.52	1.75	0.04%	26434.09	26432.46	2.09	2.26	0.01%
R05	29838.68	29806.23	2.21	2.47	0.11%	31228.18	31198.45	4.11	5.32	0.10%
R06	27492.74	27374.87	3.47	15.27	0.43%	28809.85	28082.87	3.7	32.67	2.52%
Avg	20400.23	20234.08	2.05	10.67	0.81%	20921.05	20642.53	2.56	11.79	1.33%

TABLE V
RESULTS OBTAINED BY ILS-SP AND ARCGIS 10.2 VRP TOOL

#schools	#stops	#studs	ILS-SP			ArcGIS 10.2 VRP Tool			Gap
			TC	Fleet	CRU	TC	Fleet	CRU	
PS01	5	90	960075.61	0A2B0C	86.53%	960075.61	0A2B0C	86.53%	0.00%
PS02	20	237	2260123.78	1A2B2C	99.15%	2260127.82	1A2B2C	99.15%	0.00%
PS03	27	296	2780172.02	1A1B4C	99.66%	2960178.99	0A2B4C	91.36%	6.08%
PS04	45	790	7280576.81	0A11B4C	99.75%	7760543.94	0A12B4C	93.60%	6.18%
Avg	24.25	353.25	3320237.06	-	96.27%	3485231.59	-	92.66%	3.07%

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