Optimal Multivariable Control Design based on a Fuzzy Model for an Unmanned Aerial Vehicle

M. López-Rivera, A.C. Cortés-Villada, E. Giraldo

Abstract—**This article presents an optimal tuning method for coupled Fuzzy multivariable controllers. The procedure is based on optimization using genetic algorithms of a square error cost function. The controllers tuning method can be applied to the design of systems required to meet design criteria, taking into account the system's inherent dynamics. The proposed method is evaluated on a two-degree-of-freedom scale helicopter that models a scale unmanned aerial vehicle system using the quadratic error as a cost function. The proposed optimal Fuzzy controller's performance is compared with a MIMO PID controller under two distinct structures: coupled and decoupled, considering the same optimal tuning method with genetic algorithms. Also, the system is validated by using a feedback state-space controller with a coupled structure.**

Index Terms—**MIMO, Fuzzy control, Genetic algorithms**

I. Introduction

THE control of multiple inputs multiple outputs (MIMO) systems is a complex task that require usually nonlinear [1], intelligent [2] or adaptive approaches [3]. Control of unmanned aerial vehicles by using PID multivariable structures has been proved as an efficient control strategy [4] but where the tuning is a high complex task. Systems based on fuzzy logic comprise a set of models and methods that allow us to approximate the actual behavior of a new system through the use of Fuzzy Logic as a modeling tool, using expert knowledge to converge on a series of fuzzy rules that allow obtaining an approximate linguistic model of a process [5], these rules are represented by membership functions, which are commonly Gaussian functions, according to the above, when the system has an input, it will calculate considering which membership function will have a high membership to perform an action. Fuzzy logic helps the academic community find solutions to industrial control problems in predicting time series, such as file methodologies and database search, operational research, strategies predictive maintenance, and other fields[6].

The most common way to use fuzzy logic is through 3 simple stages: The first stage is to convert the signal x into a set of fuzzy variables; this step is known as fuzzification, and it consists of assigning values to a group of membership functions. Each membership function's values are identified and labeled depending on the given signal. These, in turn, are assigned a level in the rank of membership, which can vary between (positive-medium-negative). Once this stage is completed, the fuzzifier element, given an input signal, provides the values of the fuzzy variables that correspond to the membership functions; thus, it determines the membership level connected to a system input signal [7].

In the second stage, the rule base is established. For this, a set of logical operators or fuzzy decision blocks must be taken into account, such that (AND-OR-NOT) allows to build logical rules according to the system's behavior according to the expert. This step shows one of the most significant advantages of fuzzy controllers over other types of controllers. And that is that they can be designed even if there is no exact mathematical model of the plant to be controlled, thanks to the fact that they are based on rules [8].

In the last stage, there is the defuzzification process, which consists of converting the fuzzy variables generated by the rule base into values with accurate interpretation [7]. It allows the transformation of the signals from the previous process to bring it to a real state required to generate the control action.

It is then that the objective of fuzzy logic is understood as a tool that allows many systems to be able to refine the degrees of the veracity of the output statements as the input ones are refined. These systems appear to be a learning task because of these properties, and they are excellent process control mechanisms. However, they are not perfect, and in the last decade, they have made significant progress using various optimization techniques such as genetic algorithms.

Genetic algorithms are based on biological evolution, so they continuously seek to optimize the individual with better characteristics. Where the worst features are discarded, and therefore, the cost function is obtaining better values in each iteration. They are widely used as techniques optimization design development of many controllers.

In this work is proposed an optimal fuzzy controller's tuned with genetic algorithms. The controller performance is compared with a MIMO PID controller under coupled and decoupled structures It considers the same optimal tuning method with genetic algorithms and uses a feedback state-space controller. The article structure is as follows: in section II, the error functions, the control signal, and the representation of the membership function are presented. Respectively, in section III, the simulation and implementation results on the plant are shown. The optimal Fuzzy coupled system's performance is evidenced in comparison with the optimal Fuzzy decoupled system to reduce oscillations. The proposed optimal Fuzzy coupled controller system is also compared to a multivariable state feedback controller. It is worth noting that the proposed

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approach obtains better results in terms of fluctuations around the reference for tracking performance.

II. Theoretical framework

A Sugeno-type Fuzzy MIMO coupled-based control signal for a system of two inputs. The controller is build-up from four Sugeno-type fuzzy controllers. Each controller is defined as follows:

$$e_i[k] = t_s e[k] + e_i[k-1] \tag{1}$$

$$e_d[k] = \frac{e[k] - e[k-1]}{t_s} \tag{2}$$

$$u[k] = \frac{\sum_{i=1}^{p}\sum_{j=1}^{q} w_{ij}\mu_i(e[k])\mu_j(e_d[k])}{\sum_{i=1}^{p}\sum_{j=1}^{q} \mu_i(e[k])\mu_j(e_d[k])} + w_i e_i[k] \tag{3}$$

being $e[k]$ the error, $e_d[k]$ the error derivative, $e_i[k]$ the error integral, t_s the sample time, p the number of membership functions of the error input, and q the number of membership functions of the derivative error input and being the membership function structure defined by a Gaussian function as follows:

$$\mu_i(x_n) = \exp^{-(\frac{c_i - x_n}{\sigma_i})^2} \tag{4}$$

with c_i the center and σ_i the width of the membership functions of the inputs, and being w_{ij} the output parameters.

The Fuzzy MIMO controller parameters w_{ij} are optimized by using a genetic algorithm [9] and taking into account the cost function (5)

$$J = \sum_{k=0}^{N} e^T[k]e[k] \tag{5}$$

The Fuzzy MIMO coupled controller structure is shown in Fig. 1.

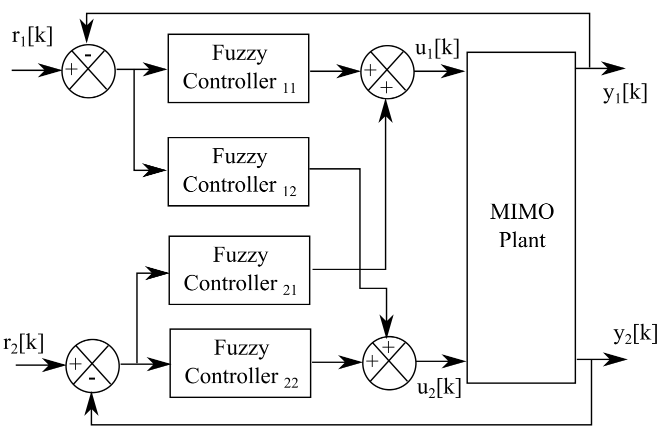

Fig. 1. Block diagram of the Fuzzy MIMO coupled controller structure

A decoupled version of the Fuzzy MIMO controller can also be proposed, as shown in Fig. 2.

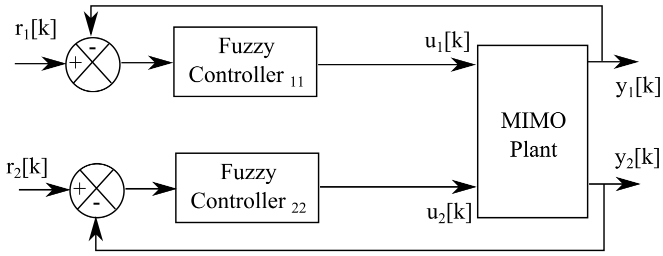

Fig. 2. Block diagram of the Fuzzy MIMO decoupled controller structure

III. Results and discussion

The proposed optimal MIMO Fuzzy approach is evaluated over a multivariable unmanned aerial vehicle system described by the following state space and output equations:

$$\begin{aligned} \dot{x} &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{6}$$

being matrices A, B, C defined by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -4.7059 & -0.088 & 0 & 0 & 0 & 1.359 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & -50 & 0 & 4.5 \\ 0 & 0 & 0 & 0 & -0.5 & 0.22 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.099 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

and

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -0.35 & 0 \\ 1 & 0 \\ 0 & 0.8 \end{bmatrix} \tag{8}$$

and

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

The model of (6) can be discretized by using the backward operator as

$$\begin{aligned} x[k+1] &= Fx[k] + Gu[k] \\ y[k] &= Cx[k] \end{aligned} \tag{10}$$

being $F = A + It_s$, $G = Bt_s$ the discrete matrices of the state-space differences equation.

A sample time of $t_s = 50$ miliseconds is used for the simulation.

In addition the proposed approach is evaluated with a multivariable proportional-integral-derivative (PID) controller defined by:

$$e_i[k] = t_s e[k] + e_i[k-1] \tag{11}$$

$$e_d[k] = \frac{e[k] - e[k-1]}{t_s} \tag{12}$$

$$u[k] = K_p e[k] + K_i e_i[k] + K_d e_d[k] \tag{13}$$

being K_p, K_d and K_i defined as:

$$K_p = \begin{bmatrix} K_{p11} & K_{p12} \\ K_{p21} & K_{p22} \end{bmatrix}, \tag{14}$$

$$K_i = \begin{bmatrix} K_{i11} & K_{i12} \\ K_{i21} & K_{i22} \end{bmatrix}, \tag{15}$$

$$K_d = \begin{bmatrix} K_{d11} & K_{d12} \\ K_{d21} & K_{d22} \end{bmatrix} \tag{16}$$

In Fig. 3 is presented the structure of the PID MIMO coupled controller.

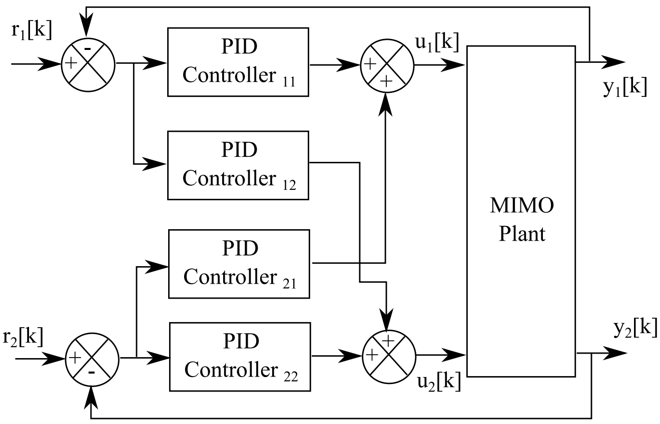

Fig. 3. Block diagram of the PID MIMO coupled controller structure

The tuning of the PID parameters is performed by using a simple genetic algorithm with the cost function of (5) used for the optimal MIMO Fuzzy approach.

The coupled PID parameters obtained by using a genetic algorithm are presented in (17), (18) and (19).

$$K_p = \begin{bmatrix} 2.6842 & -1.4075 \\ 3.0733 & 16.2590 \end{bmatrix}, \tag{17}$$

$$K_i = \begin{bmatrix} 7.0249 & -9.4807 \\ 8.2199 & 14.2184 \end{bmatrix}, \tag{18}$$

$$K_d = \begin{bmatrix} 4.7069 & 0.4789 \\ 0.7589 & 9.4771 \end{bmatrix} \tag{19}$$

It is worth noting, that the positions of the matrices in (17), (18) and (19) are directly related to the PID controller of Fig. 3.

The cost function evolution of the coupled PID is shown in Fig. 4 for 200 generations.

In Fig.4, the cost function needed this number of generations to see that it begins to get similar results over the generations; therefore, the PID controller will late more time while the process of each generation.

The Sugeno-type Fuzzy controller system parameters are obtained by using a genetic algorithm. The cost function of the coupled Fuzzy is shown in Fig. 5 for 20 generations.

In Fig.5, the cost function just needed 4 generations to get behavior similar a constant around one value, these means that cost function evolution for the fuzzy will late less time than the cost function evolution for the PID; therefore, this controller begin to present better response respect to PID controller.

Reference tracking of the closed-loop system's pitch for the coupled PID control and the Sugeno-type Fuzzy are shown in Fig.6.

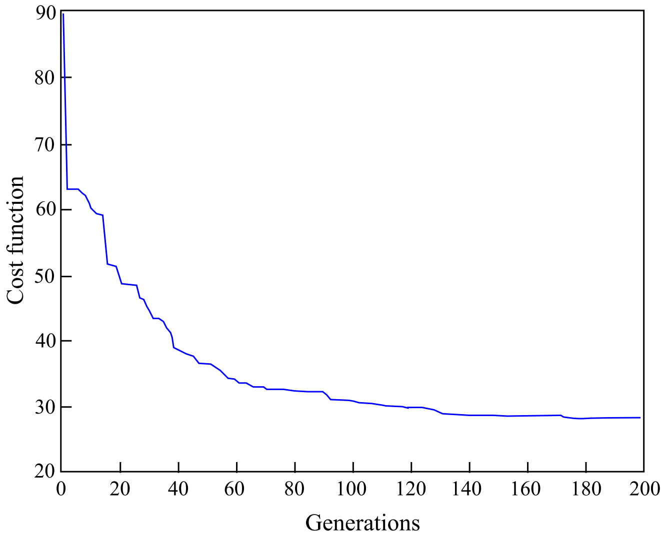

Fig. 4. Cost function evolution for the PID

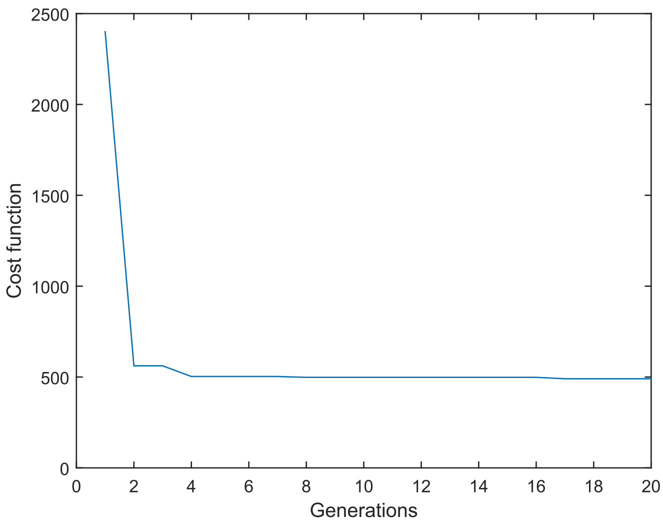

Fig. 5. Cost function evolution for the Fuzzy

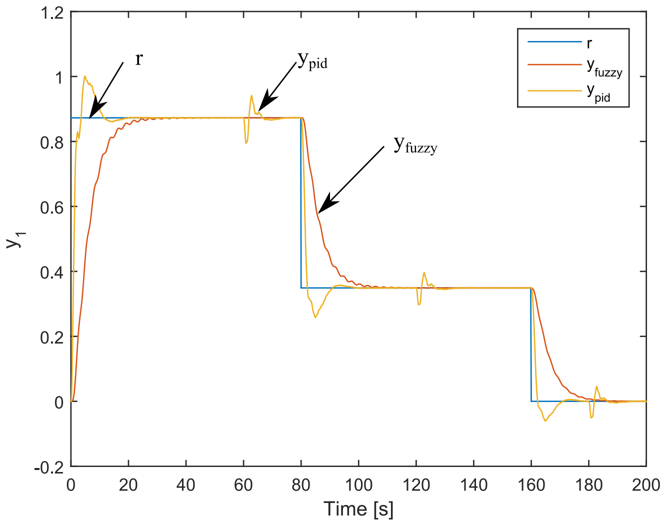

Fig. 6. Reference tracking of pitch for the closed loop system: PID and Fuzzy comparison

In Fig.6, we can compare the tracking to the reference signal, the pitch output signal of the fuzzy controller, and the pitch output signal of the PID controller, in the coupled

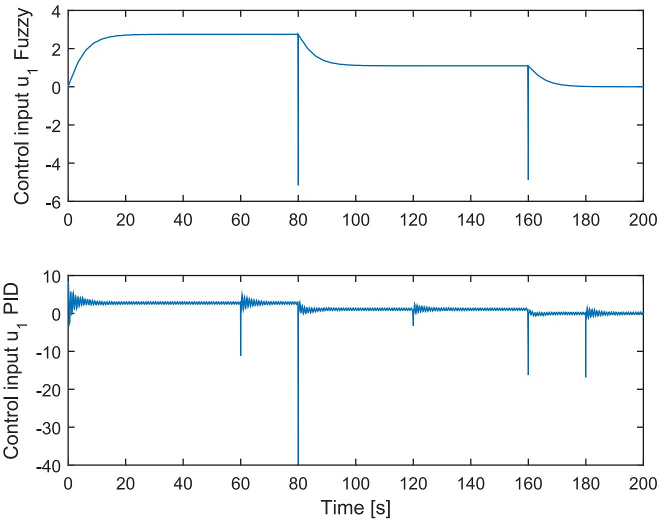

Fig. 8. Control signal u_1 of pitch: Fuzzy and PID

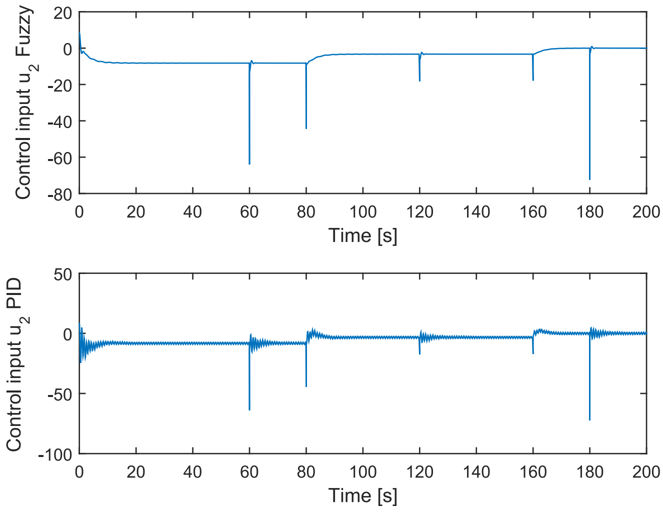

Fig. 9. Control signal u_2 of yaw: Fuzzy and PID

system. During this tracking can be noticed how the PID output signal reflects an overshoot of 13% before reaching the reference; additionally, for each change of reference, an overshoot is observed in the same output; compared to the fuzzy output where we have a reference tracking without overshoot and disturbances.

Reference tracking of yaw for the closed loop system for the coupled PID control and the Sugeno-type Fuzzy are shown in Fig.7.

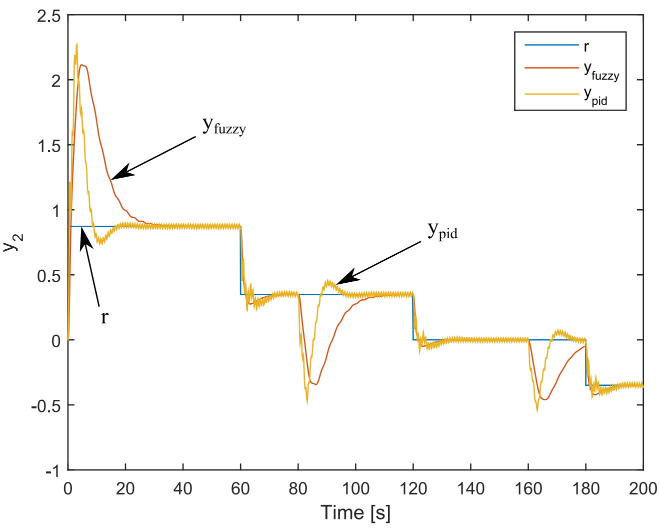

Fig. 7. Reference tracking of yaw for the closed loop system: PID and Fuzzy comparison

In Fig.7, we can see the yaw output results in the same coupled system, comparing the PID signal and the fuzzy signal again, in the tracking to the reference signal. During this tracking, it can be observed that both output signals, both the fuzzy controller and the PID controller have similar behavior and a very similar overshoot; however, the PID output signal has small oscillations at each reference change, which denotes some instability of the PID controller compared to the fuzzy controller signal.

Control signals for pitch and yaw of the Fuzzy and PIDs are shown in Fig. 8 and Fig. 9.

In Fig.8, it can be analyzed in u_1, the pitch control signal, for the fuzzy controller, and the PID controller, with a time of 200 seconds. The control signal from the fuzzy controller is more precise and less disturbed than the control signal from the PID controller, in which small oscillations are observed for each reference change.

As shown in Fig.9, we have the yaw control signal in u_2, for the fuzzy controller and the PID controller, with a time of 200 seconds. Starting from the analysis of the control signal of the PID controller, which has small oscillations in each change of reference, certain instability in its behavior can be observed, compared to the control signal of the fuzzy controller, which has a signal more accurate, with fewer oscillations for the same tracking time.

The decoupled closed-loop reference tracking responses for PID and Fuzzy are shown in Fig. 10 and Fig. 11.

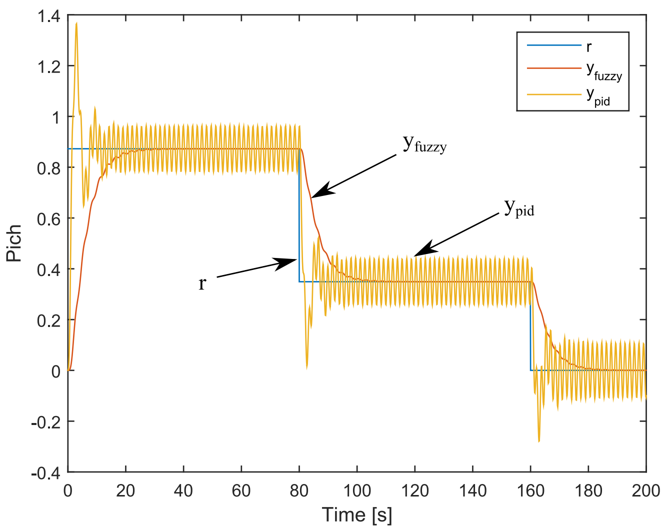

Fig. 10. Reference tracking of pitch for the closed loop decoupled system: PID and Fuzzy comparison

As shown in Fig.10, the PID controller's pitch output signal and the fuzzy controller are found, but this time in the decoupled system. Analyzing each of the output signals, it can be observed how the PID signal has the largest over-impulse present at $t = 0$ of around 57% of the reference signal; then the signal presents small dampened oscillations before reaching the Steady-state, this is compared to the output signal of the fuzzy controller, which presents a better behavior because it does not have oscillations in the output signal, quickly reaching the steady-state. Now, we track the Yaw output of both controllers, presented in Fig.11.

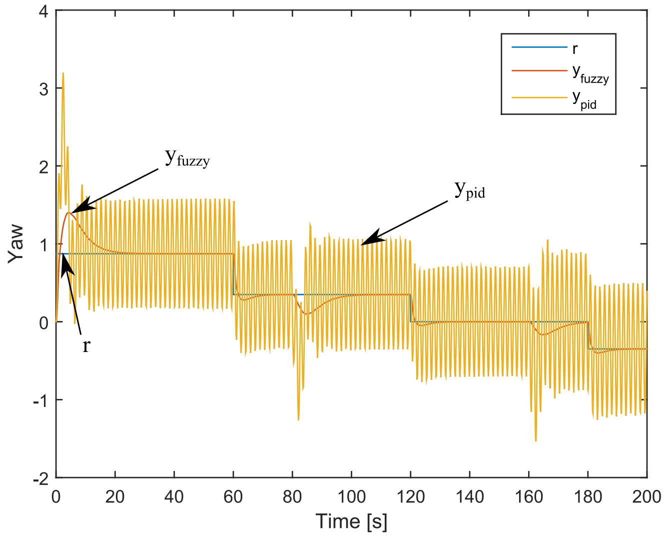

Fig. 11. Reference tracking of yaw for the closed loop decoupled system: PID and Fuzzy comparison

We can notice more significant damped oscillations in the control signal showing behavior similar to the previous one for this second output. But with more tremendous perturbations to the change of reference, noting less stability in the signal due to a more significant overshoot than for the evolution of reference at $t = 0$ it reaches 255% of the reference signal.

An additional validation is performed by using a feedback state space controller, where the tracking results are shown in Fig. 12 and Fig. 13.

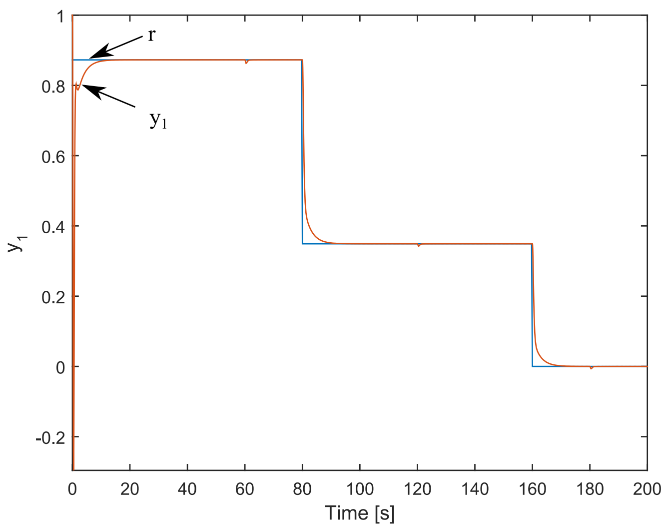

Fig. 12. Reference tracking of pitch for the closed loop system: feedback state space controller

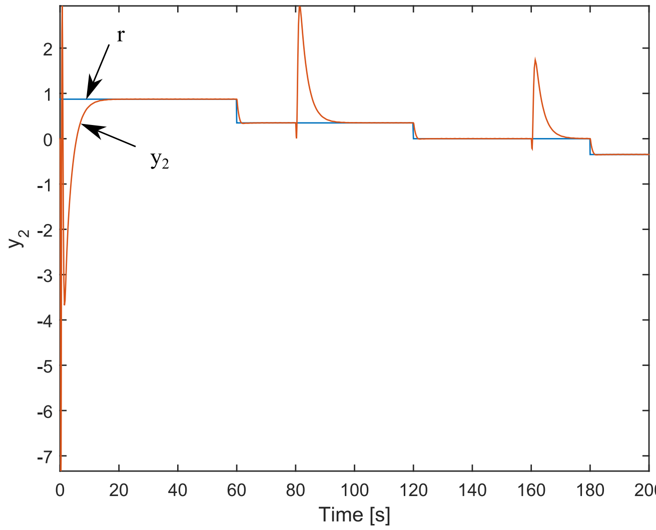

Fig. 13. Reference tracking of yaw for the closed loop system: feedback state space controller

In addition, the control inputs are shown in Fig. 14.

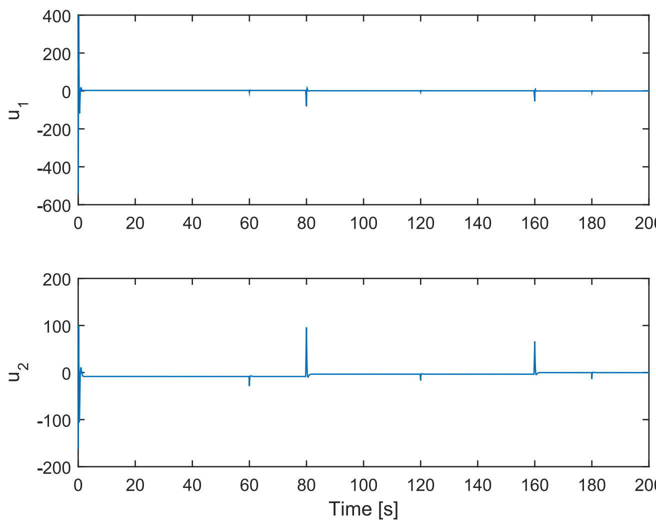

Fig. 14. Control inputs for pitch and yaw for the closed loop system: feedback state space controller

IV. Conclusions

An optimal fuzzy controller is proposed in this work and compared with a MIMO PID controller under a coupled and decoupled system. It is possible to conclude that the fuzzy controller has optimal behavior since fewer oscillations and less overshoot or zero overshoot are obtained. The PID showed good behavior but with more fluctuations and a more significant overshoot in comparison with the fuzzy controller.

The coupled system's behavior is compared to the decoupled system that uses the coupled system, and fewer oscillations are obtained in the PID controller. The method is also validated using a feedback state-space controller where a similar response is obtained but with a more significant amplitude requirement.

Since a similar behavior for both controllers' output signals is obtained but with a faster and less oscillatory response from the fuzzy controller, we can say that this controller is more optimal for this type of system.

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