Adaptive Fuzzy Funnel Control for Pure-Feedback Nonlinear System with Input Constraint

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***Abstract*—For pure-feedback nonlinear system with input saturation constraint and system internal interference, the problem of prescribed performance control is considered. Firstly, a smooth nonlinear function is introduced to approach the saturation function of the input signal, which solves the design difficulty caused by nonlinearity of non-differential saturation. Then, the pure-feedback system is decoupled by using mean-value theorem, so that the system controller can be constructed by utilizing the backstepping technique directly. Next, a novel system controller is proposed by combining prescribed performance control and fuzzy logic control technology. In the framework of Lyapunov stability theory, the controller can ensure that all closed-loop signals of the system are uniformly and ultimately bounded, and the tracking error of the system converges to the allowable range under the funnel function, thus the transient and steady-state performance of the system was guaranteed. Finally, the effectiveness of the control scheme is verified by a simulation example.**

***Index Terms*—Nonlinear systems, funnel control, backstepping, nonsymmetric input saturation, pure-feedback systems.**

I. Introduction

In recent years, nonlinear control systems are widely used in industry, military, aerospace and other fields. The research of nonlinear control system has become an important frontier direction, and great achievements have been made in nonlinear control systems [1], [2], [3], [4], [5]. However, the structure of today's nonlinear system is becoming more complex, and the requirements for the control accuracy of the controller are getting more higher. Therefore, it is significant to study the control theory and method of nonlinear system.

Backstepping technique is an effective method for control design of nonlinear systems. Starting from the design of the Lyapunov function and the intermediate virtual parameters of the first-order subsystem, the actual control law of the system can be obtained when "back" to the last-order system. For example, Zhou *et al.* designed a system state observer using backstepping technology, which guaranteed the steady-state performance of the system by estimating and compensating the unknown state variables in [6]. Furthermore, Chen *et al.* used the similar design idea to compensate the actuator failure of the system in [7]. Alternatively, Zhang *et al.* designed the system controller by using RBF and adaptive backstepping technology in [8]. However, many dynamic systems have non-affine structures in practical applications, which we call pure-feedback control systems. The structure of the pure-feedback system is more general than the strict-feedback system. In [9], Wang *et al.* used the mean-value theorem to deal with non-affine system functions, and proposed a robust adaptive control scheme based on the backstepping technology. Subsequently, many scholars further studied input constraint systems and achieved many meaningful results in [10], [11], [12], [13], [14], [15]. However, the backstepping method is difficult to deal with the nonlinear system with unknown structure or parameters, so it usually needs to be combined with other control methods.

Adaptive control is another important method to deal with nonlinear systems. The adaptive control has excellent approximation ability to unknown parameters and uncertain model structure. For example, a kind of fuzzy logic control is introduced to approximate the intermediate variables which are difficult to calculate directly in backstepping in [16]. Alternatively, a low complexity fault-tolerant controller is designed for strict feedback systems with unknown nonlinear functions by using smooth orientation functions and error transformation functions in [17]. From [16], [17], it is not difficult to find that fuzzy logic control does not need accurate system mathematical model. By utilizing this characteristic, Zhang and Yang solved a class of strict-feedback nonlinear problems with uncertain structure, unknown control direction and fault tolerance in [18]. Subsequently, Li *et al.* introduced a novel fuzzy sliding surface to suppress the chattering of the system in [19]. Moreover, a class of robust multivariable approach based on adaptive control technology is proposed for a class of multivariable linear systems with time-varying parameters in [20]. Meanwhile, in [8] [13] and [21], the system controller designed by using neural network control strategy not only has strong adaptive ability, but also has excellent fault-tolerant ability. However, adaptive control usually takes the asymptotic stability or error stability of the system as the control goal, without considering the dynamic performance of the system.

In 2008, prescribed performance control (PPC) was first proposed and quickly attracted extensive attention of scholars. In [22], [23], [24], [25], the designed controllers based on PPC method can ensure the system tracking error converges to a preset range, and improve the transient and steady-state performance of the system. Unfortunately, due to the large initial error in [24], [25], the overall performance of the system cannot be guaranteed. In [26], Han and Lee

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combined PPC technology with neural network control to realize accurate tracking of system output error.However, not all system functions can be derived. Liu *et al.* put forward further to propose a novel performance constraint function in [14] and combined with adaptive funnel technology to design the system controller, it avoids the repeated differentiation in backstepping method. while avoiding the repeated differentiation of virtual control variable in the backstepping method.

Inspired by the above discussion, different from the existing methods, a novel system controller is proposed for the pure-feedback nonlinear system with input saturation by using backstepping method, fuzzy logic control and prescribed performance control. The designed controller not only guarantees the performance of the system, but also ensures that all signals of the closed-loop system are uniformly bounded.

The reminder of this paper is arranged as follows: Section II is system description and the introduction of related basic knowledge. Section III is adaptive fuzzy controller design. Section IV is simulation and analysis. Section V is conclusion.

II. System descriptions and basic knowledge

A. System descriptions

Consider a pure feedback nonlinear system as follows:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i, x_{i+1}) + \lambda_i(t) \\ \dot{x}_n = f_i(\bar{x}_i, u) + \lambda_n(t) \\ y = x_1 \end{cases} \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ with $\bar{x}_i = [x_1, x_2 \cdots, x_i]^T \in R^i$ and $y \in R$ are the state vectors and system output. Both $f_i(\cdot) \in R^{i+1} \to R$ and λ_i are unknown smooth nonlinear functions. $\lambda_i(t)$ represents for system internal interference while meeting $|\lambda_i| < \bar{\lambda}_i, i = 1, 2, \ldots n-1$, where $\bar{\lambda}_i$ is unknown positive constant. For follow-up convenience, $\lambda_i(t)$ is denoted as λ_i in the following next. u denotes the system nonsymmetric saturation input. That is:

$$u = sat(v) = \begin{cases} u_{\max}, v \geq u_{\max} \\ v, u_{\min} < v < u_{\max} \\ u_{\min}, v \leq u_{\min} \end{cases} \tag{2}$$

where $u_{\max} > 0$ and $u_{\min} < 0$ are unknown constants, and v represents the input of the saturation nonlinearity.

According to the mean-value theorem, the functions $f_i(\cdot)$ can be described as follows:

$$\begin{cases} f_i(\bar{x}_i, x_{i+1}) - f_i(\bar{x}_i, 0) = g_i x_{i+1} \\ f_n(\bar{x}_n, u) - f_n(\bar{x}_n, u_0) = g_n(u - u_0) \end{cases} \tag{3}$$

where smooth function $f_i(\cdot,\cdot)$ is explicitly analyzed between $f_i(\bar{x}_i, x_{i+1})$ and $f_i(\bar{x}_i, x_0)$, $g_i = \partial f_i(\bar{x}_i, x_{i+1})/\partial x_{i+1}|x_{i+1}$ and $x_{n+1} = u$. Next, substituting (3) to (1) and choosing $u_0 = 0$, we can obtain

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i, 0) + g_i x_{i+1} + \lambda_i \\ \dot{x}_n = f_n(\bar{x}_n, 0) + g_n u + \lambda_n \\ y = x_1 \end{cases} \tag{4}$$

From (2), there are two sharp corners that $v = u_{\max}$ and $v = u_{\min}$ in saturation function. To solve this problem, Let's use two smooth functions to approach the saturation function $sat(v)$ results in

$$G(v) = \begin{cases} u_{\max} * \tanh(\frac{v}{u_{\max}}), v \geq 0 \\ u_{\min} * \tanh(\frac{v}{u_{\min}}), v < 0 \end{cases} = \begin{cases} u_{\max} * \frac{e^{\frac{v}{u_{\max}}} - e^{\frac{-v}{u_{\max}}}}{e^{\frac{v}{u_{\max}}} + e^{\frac{-v}{u_{\max}}}}, v \geq 0 \\ u_{\min} * \frac{e^{\frac{v}{u_{\min}}} - e^{\frac{-v}{u_{\min}}}}{e^{\frac{v}{u_{\min}}} + e^{\frac{-v}{u_{\min}}}}, v < 0 \end{cases} \tag{5}$$

Then, the nonsymmetric saturation system input can be expressed as

$$u = sat(v) = G(v) + d(v) \tag{6}$$

where $d(v)$ is a bounded function defined as

$$\begin{aligned} |d(v)| &= |sat(v) - G(v)| \\ &\leq \max\{u_{\max}(1 - \tanh(1)), u_{\min}(\tanh(1) - 1)\} \\ &= D \end{aligned} \tag{7}$$

In addition, by using the mean-value theorem, we can obtain a constant $G_\mu (0 < \mu < 1)$ such that

$$G(v) = G(v_0) + G_\mu(v - v_0) \tag{8}$$

where $G_\mu = (\partial G(v)/\partial v)|_{v=\mu}$, by choosing $v_0 = 0$, (8) can be redescribed as

$$G(v) = G_\mu v \tag{9}$$

Substituting (9) and (6) into (4) results in

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i, 0) + g_i x_{i+1} + \lambda_i \\ \dot{x}_n = f_i(\bar{x}_n, 0) + g_n(G_\mu v + d(v)) + \lambda_n \\ y = x_1 \end{cases} \tag{10}$$

Assumption 1. *The signs of g_i, $i = 1, 2, \ldots n$ are known, g_m and g_M are unknown constants, it has*

$$0 < g_m \leq |g_i| \leq g_M < \infty \tag{11}$$

Obviously, g_i is either positive or negative strictly. Without affecting the conclusion, we can further assume $0 < g_m \leq g_i \leq g_M$.

Assumption 2. *In (8), there exists an unknown positive constant G_m such that*

$$0 < G_m \leq G_\mu \leq 1 \tag{12}$$

According to 1-2, it has

$$0 < b \leq g_i, \quad 0 < b \leq g_n G_\mu \tag{13}$$

where $b = \min\{g_m, g_m G_m\}$ is an unknown constant.

Lemma 1. *[16] Young's Inequality: For $\forall(x, y) \in R^2$, the following inequality holds*

$$xy \leq \frac{\ell^p}{p}|x^p| + \frac{1}{q\ell^q}|y^q| \tag{14}$$

where $\ell > 0$, $p > 1$, $q > 1$, $(p-1)(q-1) = 1$ and ℓ, p, q are both constants.

Lemma 2. *[9] $F(Z)$ is defined as a continuous function on a compact set Ω_Z. Therefore, the fuzzy logic system is introduced. The fuzzy logic control system is mainly aimed at* *the approximation of any nonlinear continuous and uncertain link in the system.The mathematical expression as follows:*

$$F(Z) = W^T \varphi(Z) \tag{15}$$

where the input vector $z \in \Omega_Z$, $W = [w_1, w_2, \ldots, w_n]^T$ *and* $\varphi(Z)$ *are constant weight vector and Gaussian function vector respectively, that is*

$$\varphi_i(Z) = \exp\left[\frac{-(Z-\varsigma_i)^T(Z-\varsigma_i)}{\iota^2}\right],\ i = 1, \ldots, n \tag{16}$$

where $\varsigma_i = [\varsigma_{i1}, \varsigma_{i2}, \ldots, \varsigma_{in}]^T$ *and* ι *are center vector and the width of Gaussian function.*

Remark 1. *There must be a fuzzy logic system desired* $\sup_{x\in\Omega_z}|F(Z) - W^T\varphi(Z)| \leq \varepsilon$, *where* ε *is an unknown constant greater than 0.*

B. funnel control and adaptive controller design

By choosing a possible choice of boundary function $\omega(t)$ defined as

$$\omega(t) = (\rho_0 - \rho_\infty)e^{-\beta t} + \rho_\infty \tag{17}$$

where ρ_0 is the initial value of $\omega(t)$, β is the convergence speed, and $\lim_{t\to\infty}\omega(t) = \rho_\infty$. ρ_0, ρ_∞ , β are appropriately positive constants. In addition, z_i is constrained in the funnel function for the initial condition $|z_i(0)| < |\rho_0|$.

The error variable is defined as

$$\zeta_1 = \ln\frac{\omega + z_1}{\omega - z_1} \tag{18}$$

where z_1 is the tracking error and z_i satisfies

$$z_i = x_i - \alpha_{i-1}, \quad 1 < i < n-1 \tag{19}$$

where $\alpha_0 = y_r$ and α_i is a virtual control signal. The time-derivative ζ_1 will be used later and given by

$$\dot{\zeta}_1 = 2\tau_1(\dot{z}_1 - \frac{\dot{\omega}z_1}{\omega}) \tag{20}$$

where $\tau_1 = \omega/\omega^2 - z_1^2$. The construction forms of virtual control signal and actual control input are as follows:

$$\alpha_1 = -\frac{\zeta_1}{\tau_1}(a_1 + \frac{1}{2} + \frac{1}{2c_1^2}\hat{\theta}\varphi_1^T\varphi_1) \tag{21}$$

$$\alpha_i = -(a_i + \frac{1}{2})z_i - \frac{1}{2c_i^2}z_i\hat{\theta}\varphi_i^T\varphi_i \tag{22}$$

$$v_n = -(a_n + \frac{1}{2\eta^2})z_n - \frac{z_n}{2c_n^2}\hat{\theta}\varphi_n^T\varphi_n \tag{23}$$

where a_i, c_i $(i = 1, 2, \ldots n)$ and η are positive design parameters, $\hat{\theta}$ is the estimations of θ, θ is unknown constant and satisfies

$$\theta = \max\nolimits_{1\leq i\leq n}\{\frac{1}{b}||W_i^2||\} \tag{24}$$

where b is defined in (13) and W_i will be given later. The adaptive law will be updated by

$$\dot{\hat{\theta}} = \frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 + \sum_{i=2}^{n}\frac{\gamma}{2c_i^2}z_i^2\varphi_i^T\varphi_i - \sigma\hat{\theta} \tag{25}$$

where γ , σ are positive parameters to be designed.

III. ADAPTIVE FUZZY CONTROLLER DESIGN

In the following, we will design an adaptive fuzzy controller based on backstepping, and establish a fuzzy logic system $W_i^T\varphi_i(Z_i)$ at Step i to approximate the encapsulated unknown function $F_i(Z_i)$. The system controller is updated through the backstepping technology step by step, until the n step, the final controller of the system will be obtained. The specific design procedures are as follows:

Step 1: For the first subsystem in (10), a Lyapunov function candidate is selected as follows

$$V_1 = \frac{1}{4}\zeta_1^2 + \frac{b}{2\gamma}\tilde{\theta}^2 \tag{26}$$

The time derivatives of the two sides of the above formula are obtained as follows

$$\dot{V}_1 \leq \zeta_1\tau_1(f_1 + g_1x_2 + \lambda_1 - \dot{y}_r - \frac{\dot{\omega}z_1}{\omega}) - \frac{b}{\gamma}\tilde{\theta}\dot{\hat{\theta}} \tag{27}$$

By using Lemma 1. for $\zeta_1\tau_1\lambda_1$, one has

$$\zeta_1\tau_1\lambda_1 \leq \frac{\zeta_1^2\tau_1^2}{2} + \frac{\bar{\lambda}_1^2}{2} \tag{28}$$

A new function F_i is introduced to represent the logic fuzzy control and (27) can be rewritten as

$$\begin{aligned} F_1 &= \tau_1(f_1 + \frac{\zeta_1\tau_1}{2} - \dot{y}_r - \frac{\dot{\omega}z_1}{\omega}) + \frac{\zeta_1}{2} \\ \dot{V}_1 &\leq \zeta_1\tau_1g_1x_2 + \zeta_1F_1 - \frac{\zeta_1^2}{2} + \frac{\bar{\lambda}_1^2}{2} - \frac{b}{\gamma}\tilde{\theta}\dot{\hat{\theta}} \end{aligned} \tag{29}$$

According to Lemma 2, by employing a fuzzy logic system $W_1^T\varphi_1$ to approach F_1, there has

$$F_1 = W_1^T\varphi_1(Z_1) + \delta_1(Z_1),\ |\delta_1(Z_1)| \leq \varepsilon_1 \tag{30}$$

where $\delta_1(Z_1)$ is the approximation error and ε_1 is an unknown positive constant. By using Lemma 1 we get

$$\begin{aligned} \zeta_1F_1 &\leq \frac{b}{2c_1^2}\zeta_1^2\frac{||W_1||^2}{b}\varphi_1^T\varphi_1 + \frac{c_1^2}{2} + \frac{\zeta_1^2}{2} + \frac{\bar{\varepsilon}_1^2}{2} \\ &\leq \frac{b}{2c_1^2}\zeta_1^2\theta\varphi_1^T\varphi_1 + \frac{c_1^2}{2} + \frac{\zeta_1^2}{2} + \frac{\bar{\varepsilon}_1^2}{2} \end{aligned} \tag{31}$$

Substituting (30) into (29) and combining (31) results in

$$\dot{V}_1 \leq \zeta_1\tau_1g_1x_2 + \frac{b}{2c_1^2}\zeta_1^2\theta\varphi_1^T\varphi_1 + \frac{c_1^2}{2} + \frac{\bar{\varepsilon}_1^2}{2} + \frac{\bar{\lambda}_1^2}{2} - \frac{b}{\gamma}\tilde{\theta}\dot{\hat{\theta}} \tag{32}$$

According to formula (19), (32) can be rewritten as

$$\begin{aligned} \dot{V}_1 \leq& \zeta_1\tau_1g_1z_2 + \zeta_1\tau_1g_1\alpha_1 + \frac{b}{2c_1^2}\zeta_1^2\theta\varphi_1^T\varphi_1 \\ &+ \frac{c_1^2}{2} + \frac{\bar{\varepsilon}_1^2}{2} + \frac{\bar{\lambda}_1^2}{2} - \frac{b}{\gamma}\tilde{\theta}\dot{\hat{\theta}} \end{aligned} \tag{33}$$

Bring definition of α_1 (31) into (33), that is

$$\zeta_1\tau_1g_1\alpha_1 \leq -a_1g_1\zeta_1^2 - \frac{1}{2}g_1\zeta_1^2 - \frac{b}{2c_1^2}\zeta_1^2\hat{\theta}\varphi_1^T\varphi_1 \tag{34}$$

By using Lemma 1 to the term $\zeta_1\tau_1g_1z_2$ produce, (33) can be redescribed as

$$\begin{aligned} \dot{V}_1 &\leq -a_1g_1\zeta_1^2 - \frac{1}{2}g_1\zeta_1^2 + \zeta_1\tau_1g_1z_2 + \frac{c_1^2}{2} + \frac{\bar{\lambda}_1^2}{2} \\ &\quad + \frac{\bar{\varepsilon}_1^2}{2} + \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 - \dot{\hat{\theta}}) \\ &\leq -\Gamma_1\zeta_1^2 + \frac{g_M\tau_1^2z_2^2}{2} + \Delta_1 + \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 - \dot{\hat{\theta}}) \end{aligned} \tag{35}$$

where $\Gamma_1 = a_1 g_1 > 0$ and $\Delta_1 = (1/2)c_1^2 + (1/2)\bar{\lambda}_1^2 + (1/2)\bar{\varepsilon}_1^2$, The term $(1/2)g_M\tau_1^2 z_2^2$ will be dealt with in the next step.

Step 2: For the second subsystem in (10), a suitable Lyapunov function is selected as follows

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{36}$$

Combining (35) and (19) into (36), then, by taking the time-derivative of V_2, there has

$$\begin{aligned}\dot{V}_2 \le &-\Gamma_1\zeta_1^2 + \frac{g_M\tau_1^2 z_2^2}{2} + \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 - \dot{\hat{\theta}}) \\ &+ \Delta_1 + z_2[f_2 + g_2x_3 + \lambda_2 - \dot{\alpha}_1]\end{aligned} \tag{37}$$

According to $\alpha_i \in [\bar{x}_i, \hat{\theta}, \omega, \dot{\omega}, \dots \omega^{(i)}, y_r, \dot{y}_r, \dots y_r^{(i)}]$, there has

$$\begin{aligned}\dot{\alpha}_1 =& \frac{\partial\alpha_1}{\partial x_1}(f_1 + g_1x_2) + \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial\omega^{(i)}}\omega^{(i+1)} \\ &+ \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial y_r^{(i)}}y_r^{(i+1)} + \frac{\partial\alpha_1}{\partial\hat{\theta}}\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 \\ &+ \frac{\partial\alpha_1}{\partial\hat{\theta}}\frac{\gamma}{2c_2^2}z_2^2\varphi_2^T\varphi_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\sigma\hat{\theta} \\ &+ \frac{\partial\alpha_1}{\partial\hat{\theta}}\sum_{l=3}^{n}\frac{\gamma}{2c_l^2}z_l^2\varphi_l^T\varphi_l + \frac{\partial\alpha_1}{\partial x_1}\lambda_1\end{aligned} \tag{38}$$

By using Lemma 1 for (37) and (38), that is

$$\begin{aligned}z_2\lambda_2 \le& \frac{z_2^2}{2} + \frac{\bar{\lambda}_2^2}{2} \\ -z_2\frac{\partial\alpha_1}{\partial x_1}\lambda_1 \le& \left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 z_2^2 + \frac{\bar{\lambda}_1^2}{4}\end{aligned} \tag{39}$$

Then, taking (38) and (39) into (37) account, we get

$$\begin{aligned}\dot{V}_2 \le &-\Gamma_1\zeta_1^2 + \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 - \dot{\hat{\theta}}) + \Delta_1 \\ &+ z_2[\frac{g_M}{2}\tau_1^2z_2 + f_2 + g_2x_3 - \frac{\partial\alpha_1}{\partial x_1}(f_1 + g_1x_2) \\ &+ \left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 z_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 \\ &- \frac{\partial\alpha_1}{\partial\hat{\theta}}\frac{\gamma}{2c_2^2}z_2^2\varphi_2^T\varphi_2 - \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial\omega^{(i)}}\omega^{(i+1)} \\ &- \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial y_r^{(i)}}y_r^{(i+1)} + \frac{\partial\alpha_1}{\partial\hat{\theta}}\sigma\hat{\theta} + \frac{z_2}{2}] \\ &- \frac{\partial\alpha_1}{\partial\hat{\theta}}z_2\sum_{l=3}^{n}\frac{\gamma}{2c_l^2}z_l^2\varphi_l^T\varphi_l + \frac{\bar{\lambda}_2^2}{2} + \frac{\bar{\lambda}_1^2}{4}\end{aligned} \tag{40}$$

Similar to Step 1, by employing a fuzzy logic system $W_2^T\varphi_2(Z_2)$ to approach the uncertain contained function F_2, we get

$$\begin{aligned}F_2 =& \frac{g_M\tau_1^2z_2}{2} + f_2 + z_2 - \frac{\partial\alpha_1}{\partial x_1}(f_1 + g_1x_2) \\ &+ \left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 z_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 \\ &- \frac{\partial\alpha_1}{\partial\hat{\theta}}\frac{\gamma}{2c_2^2}z_2^2\varphi_2^T\varphi_2 + \frac{\partial\alpha_1}{\partial\hat{\theta}}\sigma\hat{\theta} \\ &- \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial\omega^{(i)}}\omega^{(i+1)} - \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial y_r^{(i)}}y_r^{(i+1)}\end{aligned} \tag{41}$$

Taking (41) into (40) and together with (19), results in

$$\begin{aligned}\dot{V}_2 \le &-\Gamma_1\zeta_1^2 + \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 - \dot{\hat{\theta}}) \\ &+ \Delta_1 + z_2[F_2 + g_2z_3 + g_2\alpha_2] - \frac{1}{2}z_2^2 \\ &- \frac{\partial\alpha_1}{\partial\hat{\theta}}z_2\sum_{l=3}^{n}\frac{\gamma}{2c_l^2}z_l^2\varphi_l^T\varphi_l + \frac{\bar{\lambda}_2^2}{2} + \frac{\bar{\lambda}_1^2}{4}\end{aligned} \tag{42}$$

By using Lemma 1 and combining (22), (30) into (42) similar to (39) , we can obtain

$$\begin{aligned}z_2g_2\alpha_2 \le& -a_2g_2z_2^2 - \frac{g_2}{2}z_2^2 - \frac{b}{2c_2^2}z_2^2\hat{\theta}\varphi_2^T\varphi_2 \\ z_2z_3g_2 \le& \frac{1}{2}g_2z_2^2 + \frac{1}{2}g_2z_3^2 \\ z_2F_2 \le& \frac{b}{2c_2^2}z_2^2\theta\varphi_2^T\varphi_2 + \frac{c_2^2}{2} + \frac{\bar{\varepsilon}_2^2}{2} + \frac{z_2^2}{2}\end{aligned} \tag{43}$$

Substituting (43) into (42). Then, simplifying the consequence like (35) results in

$$\begin{aligned}\dot{V}_2 \le &-\Gamma_1\zeta_1^2 - \Gamma_2z_2^2 + \frac{g_Mz_3^2}{2} + \Delta_1 + \Delta_2 \\ &+ \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 + \frac{\gamma}{2c_2^2}z_2^2\varphi_2^T\varphi_2 - \dot{\hat{\theta}}) \\ &- \frac{\partial\alpha_1}{\partial\hat{\theta}}z_2\sum_{l=3}^{n}\frac{\gamma}{2c_l^2}z_l^2\varphi_l^T\varphi_l\end{aligned} \tag{44}$$

where $\Delta_2 = (1/2)c_2^2 + (1/2)\bar{\varepsilon}_2^2 + (1/2)\bar{\lambda}_2^2 + (1/4)\bar{\lambda}_1^2$ and $\Gamma_2 = a_2g_2 > 0$.

Step i ($3 \le i \le n-1$): Let's select the following Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 \tag{45}$$

According to (35) and (44), it is not difficult to conclude that the time-derivative of V_{i-1} is

$$\begin{aligned}\dot{V}_{i-1} \le &-\Gamma_1\zeta_1^2 - \sum_{k=2}^{i-1}\Gamma_kz_k^2 + \sum_{k=1}^{i-1}\Delta_k + \frac{1}{2}g_Mz_i^2 \\ &+ \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 + \sum_{k=2}^{i-1}\frac{\gamma}{2c_k^2}z_k^2\varphi_k^T\varphi_k - \dot{\hat{\theta}}) \\ &- \sum_{k=1}^{i-2}\frac{\partial\alpha_k}{\partial\hat{\theta}}z_{k+1}\sum_{l=i}^{n}\frac{\gamma}{2c_l^2}z_l^2\varphi_l^T\varphi_l\end{aligned} \tag{46}$$

In (46), the last term also can be rewritten as

$$\begin{aligned}\sum_{k=1}^{i-2}\frac{\partial\alpha_k}{\partial\hat{\theta}}z_{k+1}\sum_{l=i}^{n}\frac{\gamma}{2c_l^2}z_l^2\varphi_l^T\varphi_l =& \sum_{k=1}^{i-2}\frac{\partial\alpha_k}{\partial\hat{\theta}}z_{k+1} \\ * \frac{\gamma}{2c_i^2}z_i^2\varphi_i^T\varphi_i + \sum_{k=1}^{i-2}\frac{\partial\alpha_k}{\partial\hat{\theta}}z_{k+1}&\sum_{l=i+1}^{n}\frac{\gamma}{2c_l^2}z_l^2\varphi_l^T\varphi_l\end{aligned} \tag{47}$$

Combining (10) and (47) into (46) results in

$$\begin{aligned}\dot{V}_i \leq & -\Gamma_1\zeta_1^2 - \sum_{k=2}^{i-1}\Gamma_k z_k^2 + \sum_{k=1}^{i-1}\Delta_k - \sum_{k=1}^{i-2}\frac{\partial\alpha_k}{\partial\hat{\theta}}z_{k+1} \\ & * \sum_{l=i+1}^{n}\frac{\gamma}{2c_l^2}z_l^2\varphi_l^T\varphi_l + z_i[\frac{1}{2}g_M z_i - \sum_{k=1}^{i-2}\frac{\partial\alpha_k}{\partial\hat{\theta}}z_{k+1} \\ & * \frac{\gamma}{2c_i^2}z_i\varphi_i^T\varphi_i + f_i + g_i x_{i+1} + \lambda_i - \dot{\alpha}_{i-1}] \\ & + \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 + \sum_{k=2}^{i-1}\frac{\gamma}{2c_k^2}z_k^2\varphi_k^T\varphi_k - \dot{\hat{\theta}})\end{aligned} \tag{48}$$

where $\dot{\alpha}_{i-1}$ can be obtained as following form by repeating the same method as (38) in Step 2. That is

$$\begin{aligned}\dot{\alpha}_{i-1} \leq & \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}(f_k + g_k x_{k+1} + \lambda_k) \\ & + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\omega^{(k)}}\omega^{(k+1)} + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(k)}}y_r^{(k+1)} \\ & + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\sigma\hat{\theta} \\ & + \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\omega^{(k)}}\omega^{(k+1)} + \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(k)}}y_r^{(k+1)} \\ & + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\sum_{l=2}^{n}\frac{\gamma}{2c_l^2}z_l^2\varphi_l^T\varphi_l\end{aligned} \tag{49}$$

Substituting (49) into (48) and using Lemma 1 similar to procedure (39) results in

$$\begin{aligned}\dot{V}_i \leq & -\Gamma_1\zeta_1^2 - \sum_{k=2}^{i-1}\Gamma_k z_k^2 + \sum_{k=1}^{i-1}\Delta_k - \sum_{k=1}^{i-2}\frac{\partial\alpha_k}{\partial\hat{\theta}}z_{k+1} \\ & * \sum_{l=i+1}^{n}\frac{\gamma}{2c_l^2}z_l^2\varphi_l^T\varphi_l - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}z_i\sum_{l=i+1}^{n}\frac{\gamma}{2c_l^2}z_l^2\varphi_l^T\varphi_l \\ & + z_i[F_i + g_i x_{i+1} + g_i\alpha_i] - \frac{z_i^2}{2} + \frac{\bar{\lambda}_i^2}{2} + \sum_{k=1}^{i-1}\frac{\bar{\lambda}_k^2}{4} \\ & + \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 + \sum_{k=2}^{i-1}\frac{\gamma}{2c_k^2}z_k^2\varphi_k^T\varphi_k - \dot{\hat{\theta}})\end{aligned} \tag{50}$$

where

$$\begin{aligned}F_i = & \frac{1}{2}g_M z_i - \frac{\gamma}{2c_i^2}z_i\varphi_i^T\varphi_i\sum_{k=1}^{i-2}\frac{\partial\alpha_k}{\partial\hat{\theta}}z_{k+1} \\ & + f_i + z_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}(f_k + g_k x_{k+1}) \\ & + \sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}\right)^2 z_i - \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\omega^{(k)}}\omega^{(k+1)} \\ & - \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(k)}}y_r{}^{(k+1)} - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 \\ & - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\sum_{k=2}^{i}\frac{\gamma}{2c_k^2}z_k^2\varphi_k^T\varphi_k + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\sigma\hat{\theta}\end{aligned} \tag{51}$$

Similar to (43), by utilizing Young's Inequality and combining with (22) results in

$$\begin{aligned} z_i g_i\alpha_i &\leq -a_i g_i z_i^2 - \frac{g_i}{2}z_i^2 - \frac{b}{2c_i^2}z_i^2\hat{\theta}\varphi_i^T\varphi_i \\ z_i z_{i+1}g_i &\leq \frac{1}{2}g_i z_i^2 + \frac{1}{2}g_i z_{i+1}^2 \\ z_i F_i &\leq \frac{b}{2c_i^2}z_i^2\theta\varphi_i^T\varphi_i + \frac{c_i^2}{2} + \frac{\bar{\varepsilon}_i^2}{2} + \frac{z_i^2}{2}\end{aligned} \tag{52}$$

Substituting (52) into (50), there has

$$\begin{aligned}\dot{V}_i \leq & -\Gamma_1\zeta_1^2 - \sum_{k=2}^{i}\Gamma_k z_k^2 + \sum_{k=1}^{i}\Delta_k + \frac{g_M}{2}z_{i+1}^2 \\ & - \sum_{k=1}^{i-1}\frac{\partial\alpha_k}{\partial\hat{\theta}}z_{k+1}\sum_{l=i+1}^{n}\frac{\gamma}{2c_l^2}z_l^2\varphi_l^T\varphi_l \\ & + \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 + \sum_{k=2}^{i}\frac{\gamma}{2c_k^2}z_k^2\varphi_k^T\varphi_k - \dot{\hat{\theta}})\end{aligned} \tag{53}$$

where $\Delta_i = (1/2)c_i^2 + (1/2)\bar{\varepsilon}_i^2 + (1/2)\bar{\lambda}_i^2 + \sum_{k=1}^{i-1}\sum_{l=1}^{k}(1/4)\bar{\lambda}_l^2$ and $\Gamma_i = a_i g_i > 0$.

Step n: The actual controller v of the system will be designed in this step. Using (10) and (19), there has

$$\dot{z}_n = f_n + g_n(G_\mu v + d(v)) + \lambda_n - \dot{\alpha}_{n-1} \tag{54}$$

By choosing an appropriate Lyapunov function, that is

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 \tag{55}$$

According to (53) with $i = n-1$, we can obtain

$$\begin{aligned}\dot{V}_n \leq & -\Gamma_1\zeta_1^2 - \sum_{k=2}^{n-1}\Gamma_k z_k^2 + \sum_{k=1}^{n-1}\Delta_k - \frac{z_n^2}{2} + \frac{\bar{\lambda}_n^2}{2} \\ & + \sum_{k=1}^{n-1}\frac{\bar{\lambda}_k^2}{4} + z_n[F_n + g_n(G_\mu v + d(v))] \\ & + \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 + \sum_{k=2}^{n-1}\frac{\gamma}{2c_k^2}z_k^2\varphi_k^T\varphi_k - \dot{\hat{\theta}})\end{aligned} \tag{56}$$

where

$$\begin{aligned}F_n = & \frac{g_M z_n}{2} - \frac{\gamma}{2c_n^2}z_n\varphi_n^T\varphi_n\sum_{k=1}^{n-2}\frac{\partial\alpha_k}{\partial\hat{\theta}}z_{k+1} + z_n \\ & - \sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\omega^{(k)}}\omega^{(k+1)} - \sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(k)}}y_r{}^{(k+1)} \\ & + \sum_{k=1}^{n-1}\left(\frac{\partial\alpha_{n-1}}{\partial x_k}\right)^2 z_n - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} + f_n \\ & - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}(f_k + g_k x_{k+1})\end{aligned} \tag{57}$$

By using Lemma 1 for (56) similar to (31) results in

$$z_n F_n \leq \frac{b}{2c_n^2}z_n^2\theta\varphi_n^T\varphi_n + \frac{c_n^2}{2} + \frac{z_n^2}{2} + \frac{\bar{\varepsilon}_n^2}{2} \tag{58}$$

Furthermore, substituting (58) into (56), we get

$$\begin{aligned}\dot{V}_n \leq & -\Gamma_1\zeta_1^2 - \sum_{k=2}^{n-1}\Gamma_k z_k^2 + \sum_{k=1}^{n-1}\Delta_k + \frac{\bar{\lambda}_n^2}{2} + \frac{c_n^2}{2} \\ & + \frac{\bar{\varepsilon}_n^2}{2} + \sum_{k=1}^{n-1}\frac{\bar{\lambda}_k^2}{4} + z_n g_n(G_\mu v + d(v)) \\ & + \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 + \sum_{k=2}^{n-1}\frac{\gamma}{2c_k^2}z_k^2\varphi_k^T\varphi_k - \dot{\hat{\theta}}) \\ & + \frac{b}{2c_n^2}z_n^2\theta\varphi_n^T\varphi_n\end{aligned} \tag{59}$$

Substituting (23) and (7) into (59) results in

$$\begin{aligned} z_n g_n G_\mu v &\leq -a_n g_n G_\mu z_n^2 - \frac{g_n G_m}{2\eta^2}z_n^2 - \frac{b}{2c_n^2}z_n^2\hat{\theta}\varphi_n^T\varphi_n \\ z_n g_n d &\leq \frac{1}{2\eta^2}g_n G_m z_n^2 + \frac{1}{2G_m}g_n\eta^2 D^2 \end{aligned} \tag{60}$$

Combining (60), (59) can be formulated as

$$\begin{aligned}\dot{V}_n \leq & -\Gamma_1\zeta_1^2 - \sum_{k=2}^{n-1}\Gamma_k z_k^2 - a_n g_n G_\mu z_n^2 + \sum_{k=1}^{n-1}\Delta_k \\ & + \frac{\bar{\lambda}_n^2}{2} + \frac{c_n^2}{2} + \frac{\bar{\varepsilon}_n^2}{2} + \sum_{k=1}^{n-1}\frac{\bar{\lambda}_k^2}{4} + \frac{1}{2G_m}g_n\eta^2 D^2 \\ & + \frac{b}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1 + \sum_{k=2}^{n}\frac{\gamma}{2c_k^2}z_k^2\varphi_k^T\varphi_k - \dot{\hat{\theta}})\end{aligned} \tag{61}$$

Then, taking the law of adaption $\dot{\hat{\theta}}$ into (25), we have

$$\begin{aligned}\dot{V}_n \leq & -\Gamma_1\zeta_1^2 - \sum_{k=2}^{n-1}\Gamma_k z_k^2 - a_n g_n G_\mu z_n^2 + \sum_{k=1}^{n-1}\Delta_k \\ & + \frac{\bar{\lambda}_n^2}{2} + \frac{c_n^2}{2} + \frac{\bar{\varepsilon}_n^2}{2} + \sum_{k=1}^{n-1}\frac{\bar{\lambda}_k^2}{4} + \frac{1}{2G_m}g_n\eta^2 D^2 \\ & + \frac{b\sigma}{\gamma}\tilde{\theta}\hat{\theta}\end{aligned} \tag{62}$$

According to the define of θ, the last term in (62) can be written as $(b\sigma/\gamma)\tilde{\theta}\hat{\theta} \leq -(b\sigma/2\gamma)\tilde{\theta}^2 + (b\sigma/2\gamma)\theta^2$, there has

$$\dot{V}_n \leq -\Gamma_1\zeta_1^2 - \sum_{k=2}^{n}\Gamma_k z_k^2 - \frac{b\sigma}{2\gamma}\tilde{\theta}^2 + \sum_{k=1}^{n}\Delta_k \tag{63}$$

where $\Gamma_k = a_k g_k > 0$, $\Delta_k = \frac{c_k^2}{2} + \frac{\bar{\lambda}_k^2}{2} + \frac{\bar{\varepsilon}_k^2}{2}$,$k = 1, 2, \ldots, n-1$, $\Gamma_n = a_n g_n G_\mu > 0$, and $\Delta_n = \frac{c_n^2}{2} + \frac{\bar{\lambda}_n^2}{2} + \frac{\bar{\varepsilon}_n^2}{2} + \sum_{k=1}^{n-1}\frac{\bar{\lambda}_k^2}{4} + \frac{b\sigma}{2\gamma}\theta^2 + \frac{1}{2G_m}g_n\eta^2 D^2$.

According to the above deduction, the following theorem is given.

Theorem 1. *Consider the closed-loop pure-feedback nonlinear system (1) with unknown system internal interference Λ_i and unknown input saturation signals (5), and the system controller (23) and the adaptive law (25). Under the conditions of Assumptions 1 and 2, the following results are true:*

(1)All signals of the closed-loop system are consistent and ultimately bounded.

(2)The system tracking error z_1 converges asymptotically.

The stability of the closed-loop system is proved as follows.

Proof 1. *Let the Lyapunov function $V = V_n$, and the control gains are selected as*

$$\begin{aligned}\Gamma_1 &= \frac{1}{4}\Lambda_1 \\ \Gamma_j &= \frac{1}{4}\Lambda_j, \quad j = 2, 3, \ldots, n \\ \sigma_j &= \Lambda_j, \quad j = 1, 2, \ldots, n\end{aligned} \tag{64}$$

Let $\Lambda = \min\{\Lambda_1, \Lambda_2, \ldots, \Lambda_n\}$ and $\vartheta = \sum_{k=1}^{n}\Delta_k$. Furthermore, we have:

$$\dot{V} \leq -\Lambda V + \vartheta \tag{65}$$

Furthermore, solving inequality (65) gives can be obtained directly

$$\begin{aligned} 0 \leq V(t) &\leq (V(0) - \frac{\vartheta}{\Lambda})e^{-\Lambda t} + \frac{\vartheta}{\Lambda} \\ &\leq V(0) - e^{-\Lambda t} + \frac{\vartheta}{\Lambda}, \quad \forall t > 0\end{aligned} \tag{66}$$

As $t \to \infty$, we have

$$0 \leq V(t) \leq \frac{\vartheta}{\Lambda} \tag{67}$$

Remark 2. *For $j = 1, 2, \ldots, n$, the error signals z_j and $\tilde{\theta}$ eventally probability of bounded. Under Assumption 1-2 and initial condition, we can deduce that θ is a constant, $\hat{\theta}$ is also bounded in probability. Then, for $||\varphi_i|| \leq \varepsilon$, $||\varphi_i||$ are also bounded in probability, by parity of reasoning, we can conclude that all the signals of the closed-loop system are uniformly ultimately bounded in probability, and the tracking error of the system will converge to $(-\omega,\ \omega)$.*

Remark 3. *Here θ is used as the estimated parameter. For n-order nonlinear system, we only need an adaptive law to realize online update.*

IV. Simulation Example

To show the applicability of the proposed control scheme, Brusselator model describes a simplified chemical reactions model, which is a typical nonlinear control model, consider the following model in dimensionless from [9]:

$$\begin{cases}\dot{x}_1 = C - (D+1)x_1 + x_1^2 x_2 + \lambda(t) \\ \dot{x}_2 = Dx_1 + (2 + \cos(x_1))u - x_1^2 x_2 \\ y = x_1\end{cases} \tag{68}$$

where x_1 and x_2 denote the concentrations of the reaction intermediates and $C, D > 0$ are parameters which describe the supply of "reservoir" chemicals. $\lambda(t)$ stands for unknown interference and choose $\lambda(t) = \frac{1}{3}\sin(x_2^3)$. Eleven fuzzy sets are defined over interval $[-5; 5]$ for all state variables by choosing the partitioning points as $-5; -4; -3; -2; -1;\ 0;\ 1;\ 2;\ 3;\ 4;\ 5$. The fuzzy membership functions are given as follows:

$$
\begin{aligned}
\varphi_1(Z_i) &= \exp(-0.5(Z_i+5)^2)\\
\varphi_2(Z_i) &= \exp(-0.5(Z_i+4)^2)\\
\varphi_3(Z_i) &= \exp(-0.5(Z_i+3)^2)\\
\varphi_4(Z_i) &= \exp(-0.5(Z_i+2)^2)\\
\varphi_5(Z_i) &= \exp(-0.5(Z_i+1)^2)\\
\varphi_6(Z_i) &= \exp(-0.5(Z_i+0)^2)\\
\varphi_7(Z_i) &= \exp(-0.5(Z_i-1)^2)\\
\varphi_8(Z_i) &= \exp(-0.5(Z_i-2)^2)\\
\varphi_9(Z_i) &= \exp(-0.5(Z_i-3)^2)\\
\varphi_{10}(Z_i) &= \exp(-0.5(Z_i-4)^2)\\
\varphi_{11}(Z_i) &= \exp(-0.5(Z_i-5)^2)
\end{aligned}
\tag{69}
$$

Here, select $\rho_0 = 4$, $\rho_\infty = 0.05$ and $\beta = 2$. Thus, (17) can be written as $(4-0.05)e^{-2t}+0.05$. From (17) with $\alpha_0 = y_r$ and funnel function variable $\zeta_1 = \ln\frac{\omega+z_1}{\omega-z_1}$, the virtual control α_1 and adaptive $\dot{\hat{\theta}}_1$ of the first subsystem can be described as follows:

$$
\begin{aligned}
\alpha_1 &= \frac{\zeta_1}{\tau_1}(a_1+\frac{1}{2}+\frac{1}{2c_1^2}\hat{\theta}\varphi_1^T\varphi_1)\\
\dot{\hat{\theta}} &= \frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1-\sigma\hat{\theta}
\end{aligned}
\tag{70}
$$

where a_1, c_1, γ and σ are properly selected positive parameters.

According to (19) have $z_2 = x_2-\alpha_1$, the input with saturation u and the adaptive law as follows:

$$
\begin{aligned}
u &= (a_2+\frac{1}{2})z_2-\frac{z_2}{2c_2^2}\hat{\theta}\varphi_2^T\varphi_2\\
\dot{\hat{\theta}} &= \frac{\gamma}{2c_1^2}\zeta_1^2\varphi_1^T\varphi_1+\frac{\gamma}{2c_2^2}z_2^2\varphi_2^T\varphi_2-\sigma\hat{\theta}
\end{aligned}
\tag{71}
$$

where a_2, c_2, γ and σ are appropriately selected positive parameters.

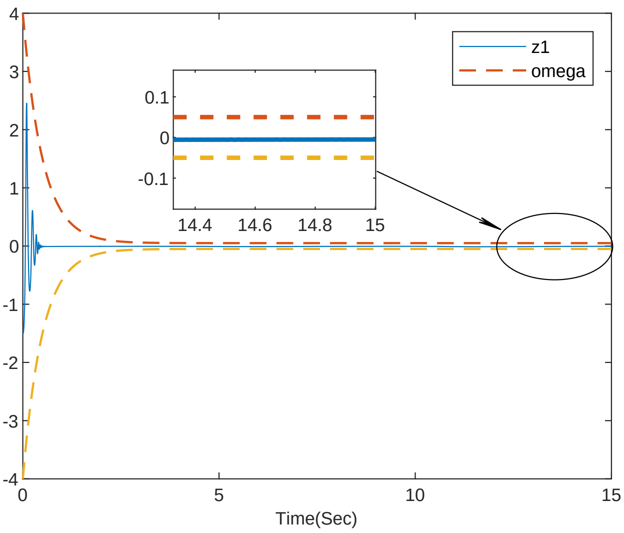

Fig. 1: The boundary function ω and the controlled function z_1.

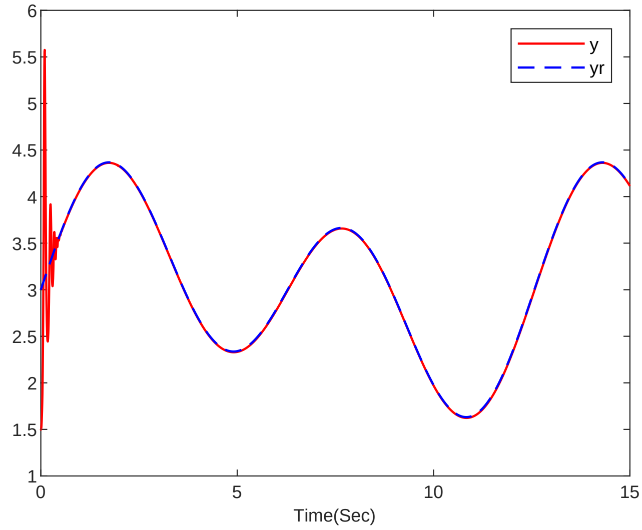

Fig. 2: System output $y(t)$ and reference signal $y_r(t)$.

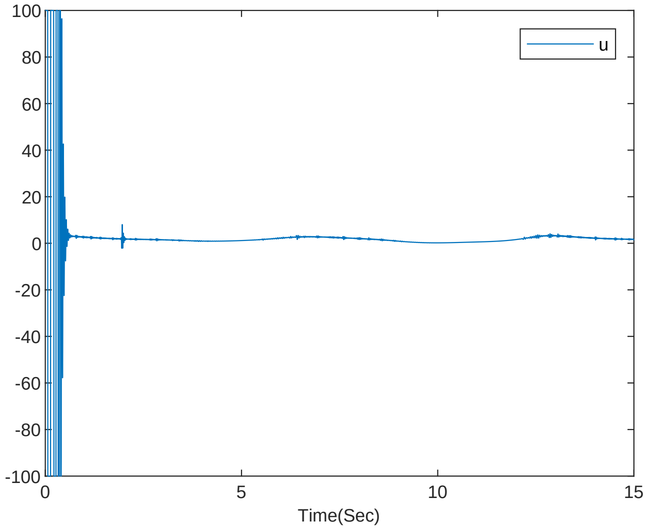

Fig. 3: Saturation function output signal u.

The model parameters are appropriately are chosen as $C = 1$ and $D = 0.5$. The control parameters are selected as $a_1 = 31$, $a_2 = 31$, $c_1 = 80$, $c_2 = 80$, $\eta = 1$, $\gamma = 1$ and $\sigma = 5.25$. The initial conditions are $[x_1(0), x_2(0)]^T = [1.5, 0]^T$ and $\theta^T = [0, 0]^T$. The input saturation limits are chosen as $u_{\max} = 100 \text{and} u_{\min} = -100$. Select reference signal as $y_r = 3+\sin(t)+0.5\sin(0.5t)$. The boundary constraint function and tracking error of the system are shown in $Fig.1$. The tracking performance of the system controller is shown in $Fig.2$. The actual control signals u and input signals v are shown in $Fig.3$ and $Fig.4$ respectively. The adaptive law $\dot{\hat{\theta}}$ is shown in $Fig.5$. The state variable x_2 is shown in $Fig.6$.

V. Conclusion

In this paper, for a pure-feedback nonlinear control system with input saturation constrains, by using mean-value theorem to decouple the pure-feedback system. Then, introducing a class of smooth function to approximate the input saturation signal. Next, a logarithmic type Lyapunov function is

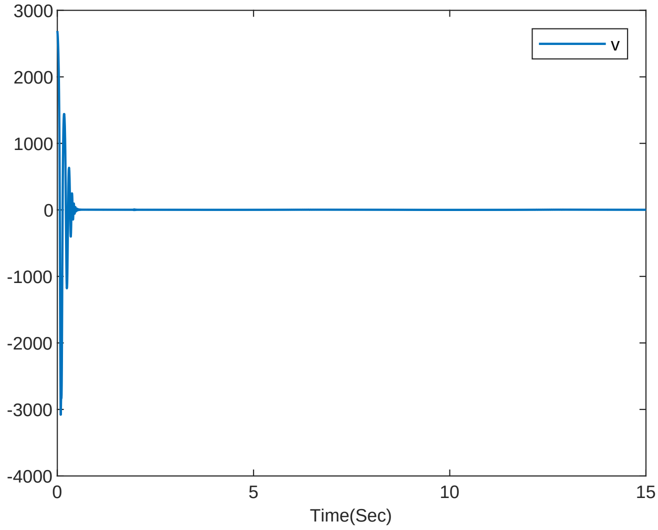

Fig. 4: Control input signal v.

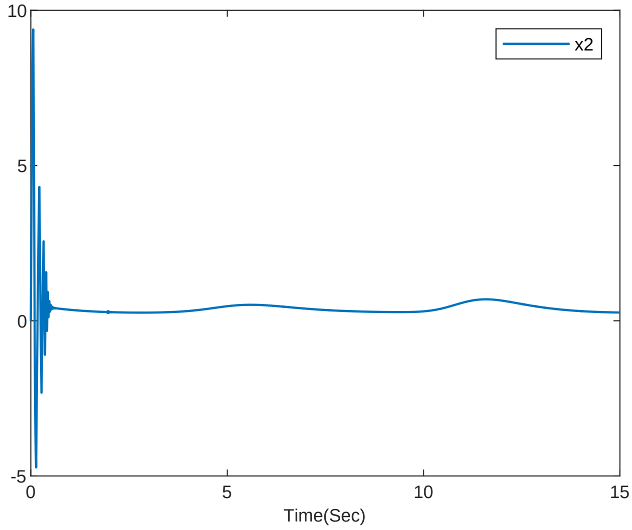

Fig. 6: State variables x_2.

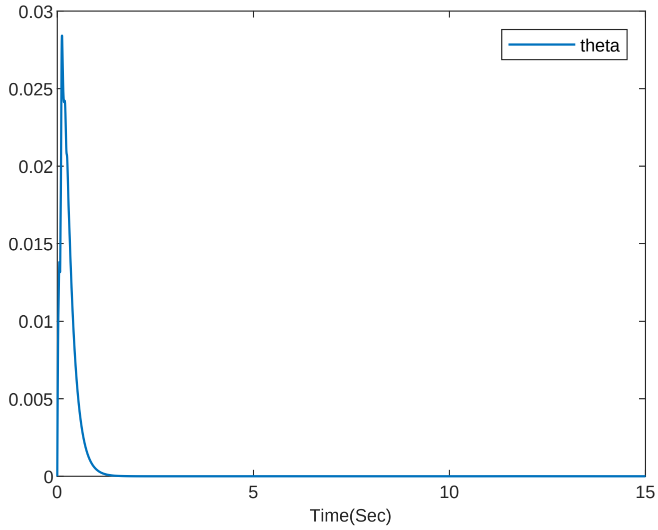

Fig. 5: Adaptive parameter $\dot{\hat{\theta}}(t)$.

proposed, which can make the system signal always in the time-varying boundary function with prescribed exponential decay. The transient and steady state performance of the system output also can be guaranteed. Furthermore, combining with Backstepping technology and logic fuzzy control used to design the prescribed performance adaptive controller, which make the tracking error more accurate and faster for the excepted function, also satisfied the output constrains of the system. Finally, Based on Lyapunov stability theorem proved that all signals of the closed-loop system are always finally bounded. The simulation results of Model Brusselator model show that the proposed method is effective.

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