Fixed-Time Stabilization for A Wheeled Mobile Robot With Actuator Dead-Zones

Zicheng Yang, Yulu Zhao and Fangzheng Gao

***Abstract*—In this paper, the problem of fixed-time stabilization is addressed for a unicycle-type wheeled mobile robot with actuator dead-zones. A novel switching control strategy is given to overcome the obstacle that the presence of actuator dead-zones renders the traditional feedback control technique inapplicable to such mobile robot. Then, by employing the adding a power integrator(API) technique, a state feedback controller is successfully developed to regulate all states of closed-loop system (CLS) to zero in a given fixed time. Finally, simulation results are given to confirm the efficacy of the proposed method.**

***Index Terms*—wheeled mobile robot, actuator dead-zones, adding a power integrator (API), fixed-time stabilization.**

I. Introduction

THE wheeled mobile robot (WMR) has attracted much attention in the past years because it wide applications in entertainment, security, war, rescue missions, spacial missions, assistant health-care, etc [1-3]. An important feature of WMR that the number of control inputs is fewer than the number of freedom degrees, leads to the control of WMR challenging. As pointed out by Brockett in [4], there is not any continuous time-invariant state feedback to stabilize such category of nonlinear systems. To address this difficulty, a number of control approaches have been proposed, mainly including time-varying feedback [5-7] and discontinuous time-invariant feedback [8,9]. Thanks to these valid approaches, many significant results have been made, e.g., [10-16] and the references therein.

Noted that majority existing results mainly centre around the asymptotic behavior of system trajectories as time verges to infinity. However, in practical applications, the CLS is desired to have the property that trajectories converge to the equilibrium in finite time. Moreover finite-time stable system possesses the superior properties of fast response, good robustness and disturbance rejection [17]. Motivated by these facts, the research on finite-time control has become popular recently [18-20]. Especially, as the preliminary research, the work [21] addressed the problem of finite-time stabilization by state feedback for a family of nonholonomic systems with some weak drifts. Whereafter, the problems of adaptive finite-time stabilization with linear/nonlinear parameterization were studied in [22] and [23]. By relaxing the limitation on system growth, the authors in [24] extended the work of [21] to a general category of nonholonomic systems. An output feedback controller was developed in [25] to finite-time stabilize a category of nonholonomic systems in feedforward-like form. Later, this result is further extended to the high order case in [26] and [27]. But a common drawback of the aforementioned studies is that the convergence time depends on initial system conditions, which renders that the desired performance in an exact preset time cannot be achieved. Recently, to remove the limitation of finite-time algorithm, a novel finite-time stability concept that requires the convergence time of a global finite-time stable system being bounded independent of initial conditions, was introduced in [28]. Such stability, usually called fixed-time stability, offers a new perspective to study the finite-time control problems and has stimulated some interesting results [29-32]. However, the effect of the actuator dead-zone is ignored in the aforementioned results.

In reality, owing to physical limitations of device, input dead-zone nonlinearity inevitably are suffered during operation in many real systems. Such unexpected property could seriously degrade the system's performance [33-35]. Therefore, the interesting question naturally arises: *For a WMR with actuator dead-zones, is it possible to devise a controller to achieve the fixed-time stabilization? If possible, how can one design it?*

Motivated by the above observations, this paper focuses on solving the problem of fixed-time stabilization of nonholonomic WMR with actuator dead-zones. The contributions are highlighted as follows. (i) The fixed-time stabilization problem of nonholonomic WMR with actuator dead-zones is studied. (ii) A novel switching control strategy is given to overcome the obstacle that the presence of actuator dead-zones renders the traditional discontinuous feedback control technique inapplicable to nonholonomic systems. (iii) By employing the API technique, a systematic state feedback control design procedure is proposed to ensure all states of the CLS to zero for any given fixed time.

Notations. In this paper, the notations used are fairly standard. Specifically, for any $c > 0$ and $\eta \in \mathbb{R}$, the function $[\eta]^c$ is defined as $[\eta]^c = \mathrm{sign}(\eta)|\eta|^c$ with the standard signum function $\mathrm{sign}(\cdot)$.

II. Problem Statement and Preliminaries

Consider a tricycle-type WMR shown in Fig.1. The kinematic equations of this robot are represented by

$$\begin{aligned} \dot{x}_c &= v\cos\theta, \\ \dot{y}_c &= v\sin\theta, \\ \dot{\theta} &= \omega, \end{aligned} \tag{1}$$

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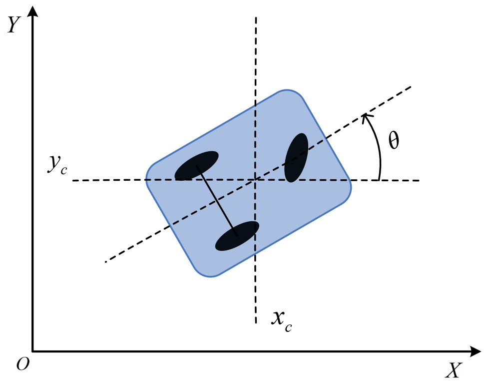

Fig. 1. The planar graph of a mobile robot.

where (x_c, y_c) denotes the position of the center of mass of the robot, θ is the heading angle of the robot, v is the forward velocity while ω is the angular velocity of the robot.

Introducing

$$\begin{cases} x_0 = \theta, \\ x_1 = x_c \sin\theta - y_c \cos\theta, \\ x_2 = x_c \cos\theta + y_c \sin\theta, \\ u_0 = w, \\ u_1 = v, \end{cases} \tag{2}$$

and taking into the inevitably presence of the actuator dead-zones in reality, the dynamics of (1) can be modelled as

$$\begin{cases} \dot{x}_0 = D_0(u_0), \\ \dot{x}_1 = x_2 D_0(u_0), \\ \dot{x}_2 = D_1(u_1) - x_1 D_0(u_0), \end{cases} \tag{3}$$

where D_j $(j = 0, 1)$ is the dead-zone input nonlinearities described by

$$D_j(u_j) = \begin{cases} m_j(u_j - b_j), & u_j \geq b_j, \\ 0, & -b_j < u_j < b_j, \\ m_j(u_j + b_j), & u_j \leq -b_j, \end{cases} \tag{4}$$

with m_j and b_j being the slopes and the breakpoints of the dead-zone characteristic, respectively.

Remark 1. Consider the system (3) with free of actuator dead-zones. With the traditional discontinuous feedback control technique, one can design $u_0 = -x_0$ to ensure $x_0(t) \neq 0$ (or equivalently, $u_0(t) \neq 0$) for all $t \geq 0$ provided $x_0(0) \neq 0$. Then, by introducing the discontinuous change of coordinates $z_1 = x_1/u_0$ and $z_2 = x_2$ to overcome the obstacle that the x-subsystem is uncontrollable in the case of $u_0 = 0$, one can change the x-subsystem into a strict-feedback-like form

$$\begin{cases} \dot{z}_1 = z_2 + z_1, \\ \dot{z}_2 = u_1 - x_0^2 z_1, \end{cases} \tag{5}$$

and solve the stabilization problem by the well-known 'backstepping' technique or its variations. However, when the actuator dead-zones are involved, it is clear that such transformation fails to work. That is, the traditional discontinuous feedback control technique is inapplicable to the dead-zone constrained nonholonomic systems. Consequently, new control techniques are wanted for solving the problem of global stabilization of the dead-zone constrained system (3).

The following, assumption, definitions and lemmas will serve as the basis of the coming control design and performance analysis.

Assumption 1. There are positive constants $\underline{m}$, $\overline{m}$, $\underline{b}$ and $\overline{b}$ such that for $j = 0,\ 1$, one has $\underline{m} \leq m_j \leq \overline{m}$ and $\underline{b} \leq b_j \leq \overline{b}$.

Definition 1[17]**.** Consider the nonlinear system

$$\dot{x} = f(t, x) \ \ with \ f(t, 0) = 0, \quad x \in R^n, \tag{6}$$

where $f : R^+ \times R^n \to R^n$ is continuous with respect to x. The equilibrium $x = 0$ of the system is globally finite-time stable if it is Lyapunov stable and for any initial condition $x(t_0) \in R^n$ at any given initial time $t_0 \geq 0$, there is a settling time $T > 0$, such that every $x(t, t_0, x(t_0))$ of system (6) satisfies $\lim_{t\to T} x(t, t_0, x(t_0)) = 0$ for $t \in [t_0, T)$ and $x(t, t_0, x(t_0)) = 0$ for any $t \geq T$.

Lemma 1[17]**.** Consider the nonlinear system described in (6). Suppose there is a C^1 function $V(t, x)$ defined on R^n, class K functions π_1 and π_2, real numbers $c > 0$ and $0 < \alpha < 1$, for $t \in [t_0, T)$ and $x \in R^n$ such that

$$\pi_1(|x|) \leq V(t, x) \leq \pi_2(|x|), \forall t \geq t_0, \forall x \in R^n,$$

and

$$\dot{V}(t, x) + cV^{\alpha}(t, x) \leq 0, \forall t \geq t_0 \ , \forall x \in R^n.$$

Then, the origin of (6) is globally finite-time stable with

$$T \leq \frac{V^{1-\alpha}(t_0, x(t_0))}{c(1 - \alpha)}.$$

Definition 2[28]**.** The origin of system (6) is referred to be globally fixed-time stable if it is globally finite-time stable and the settling time function $T(x_0)$ is bounded, that is, there exists a positive constant T_{max} such that $T(x_0) \leq T_{max}$, $\forall x_0 \in \mathbb{R}^n$.

Lemma 2[28]**.** Consider the nonlinear system (6). Suppose there exist a C^1, positive definite and radially unbounded function $V(x) : \mathbb{R}^n \to \mathbb{R}$ and real numbers $c > 0$, $d > 0$, $0 < \alpha < 1$, $\gamma > 1$, such that

$$\dot{V}(x) \leq -cV^{\alpha}(x) - dV^{\gamma}(x), \ \ \forall x \in R^n.$$

Then, the origin of system (6) is globally fixed-time stable and the settling time $T(x_0)$ satisfies

$$T(x_0) \leq T_{max} := \frac{1}{c(1 - \alpha)} + \frac{1}{d(\gamma - 1)}, \ \ \forall x_0 \in R^n.$$

Lemma 3[36]**.** For any $x,\ y \in R$, and a constant $a \geq 1$, one has

$$|x + y|^a \leq 2^{a-1}|x^a + y^a|;$$

$$(|x| + |y|)^{1/a} \leq |x|^{1/q} + |y|^{1/a} \leq 2^{(a-1)/a}(|x| + |y|)^{1/a}.$$

Lemma 4[36]**.** If c, d are positive constants, then for any real-valued function $\delta(u, v) > 0$, one has

$$|u|^c|v|^d \leq \frac{c}{c + d}\delta(u, v)|u|^{c+d} + \frac{d}{c + d}\delta^{-c/d}(u, v)|v|^{c+d}.$$

III. Fixed-time control

In this section,a constructive procedure for the finite-time stabilizer design of system (3) is given for any given settling time $T > 0$.

A. Fixed-time stabilization of the x-subsystem

For the x_0-subsystem, take

$$u_0 = u_0^*, \tag{7}$$

where u_0^* is a constant satisfying $u_0^* > \bar{b}$. Then, the x-subsystem in this case is described as

$$\begin{cases} \dot{x}_1 = h_1 x_2, \\ \dot{x}_2 = h_2 u_1 + \Phi_2(\bar{x}_2), \end{cases} \tag{8}$$

with $h_1 = D_0(c_0^*)$, $h_2 = 1$ and $\Phi_2 = -x_1 D_0(c_0^*)$. As a result, it is easily checked from Assumption 1 that

Proposition 1. Under (7), the solution of the x_0-subsystem $x_0(t)$ is well-defined on $[0, +\infty)$ and there are positive constants C, h_{i1} and h_{i2}, $i = 1, 2$ such that $h_{i1} \le h_i \le h_{i2}$ and $|\Phi_2| \le C|x_1|$.

Next, the system (8) will be stabilized within the settling time θT by employing API technique. Before proceeding, we take $r_1 = 1$ and $r_{i+1} = r_i + \tau > 0, \quad i = 1, 2, 3$ with $\tau \in (-\frac{1}{n}, 0)$ being a negative number, and introduce the following coordinate transformation:

$$\begin{array}{l} \xi_i = [x_i]^{\frac{1}{r_i}} - [\alpha_{i-1}]^{\frac{1}{r_i}}, \\ \alpha_i = -g_i^{r_{i+1}}(\bar{x}_i)[\xi_i]^{r_{i+1}}, \quad i = 1, 2, \end{array} \tag{9}$$

where $\alpha_0 = 0$, $\alpha_2 = u_1$ and $g_i(\bar{x}_i) > 0$ is a C^1 function to be specified later.

We further define $W_i : \mathbb{R}^i \to \mathbb{R}$ as follows:

$$W_i(\bar{x}_i) = \int_{\alpha_{i-1}}^{x_i} \left[[s]^{\frac{1}{r_i}} - [\alpha_{i-1}]^{\frac{1}{r_i}} \right]^{2-r_{i+1}} ds. \tag{10}$$

The detailed design procedure is elaborated as follows.

Step 1. For the x_1-subsystem of (8), take x_2 as the virtual control input. Choose $V_1 = W_1$ and $g_1 = ((1 + l_1 + l_2|\xi_1|^p)/h_{11})^{\frac{1}{r_2}}$ with design parameters $l_1 > 0$, $l_2 > 0$ and $p > -\tau$ to be determined later, one has

$$\dot{V}_1 \le -(1 + l_1)|\xi_1|^2 - l_2|\xi_1|^{2+p} + h_1[\xi_1]^{2-r_2}(x_2 - \alpha_1). \tag{11}$$

Step 2 . Consider the second Lyapunov function $V_2 = V_1 + W_2$. It can be deduced from (15) that

$$\begin{aligned} \dot{V}_2 \le & -(1 + l_1)|\xi_1|^2 - l_2|\xi_1|^{2+p} \\ & + h_1[\xi_1]^{2-r_2}(z_2 - \alpha_1) + [\xi_2]^{2-r_3}\Phi_2 \\ & + h_2[\xi_2]^{2-r_3} D_1(u_1) + \frac{\partial W_2}{\partial z_1} h_1 z_2. \end{aligned} \tag{12}$$

First, we observe from Lemmas 3 and 4 that

$$\begin{aligned} h_1[\xi_1]^{2-r_2}(z_2 - \alpha_1) & \le 2h_1|\xi_1|^{2-r_2}|\xi_2|^{r_2} \\ & \le \frac{1}{2}|\xi_1|^2 + \varphi_{21}|\xi_2|^2, \end{aligned} \tag{13}$$

where $\varphi_{21} \ge 0$ is a C^1 function.

Then, by using Lemmas 3 and 4, we have

$$[\xi_2]^{2-r_3}\Phi_2 + \frac{\partial W_2}{\partial z_1} h_1 z_2 \le \frac{1}{2}|\xi_1|^2 + \varphi_{22}|\xi_2|^2, \tag{14}$$

where $\varphi_{22} \ge 0$ is a C^1 function.

Choosing

$$g_2 = \Big(\frac{l_1 + \varphi_{21} + \varphi_{22} + l_2|\xi_2|^p}{h_{21}} \Big)^{\frac{1}{r_3}}, \tag{15}$$

and substituting (13), (14 and (15) into (12), we have

$$\begin{aligned} \dot{V}_2 \le & -l_1 \sum_{j=1}^{2} |\xi_j|^2 - l_2 \sum_{j=1}^{2} |\xi_j|^{2+p} \\ & + h_2[\xi_2]^{2-r_3} \left(D_1(u_1) - \alpha_2 \right). \end{aligned} \tag{16}$$

Thus, from Assumption 2, the control u_1 is designed as

$$u_1 = \begin{cases} \dfrac{\alpha_2}{\underline{m}} + \bar{b}, & \alpha_2 > 0, \\ 0, & \alpha_2 = 0, \\ \dfrac{\alpha_2}{\underline{m}} - \bar{b}, & \alpha_2 < 0, \end{cases} \tag{17}$$

which renders

$$\begin{aligned} & D_1(u_1) - \alpha_2 \\ = & \begin{cases} m_1 \left(\dfrac{\zeta_{n+1}^*}{\underline{m}} + \bar{b} - b_1 \right) - \alpha_2, & \alpha_2 > 0, \\ 0, & \alpha_2 = 0, \\ m_1 \left(\dfrac{\zeta_{n+1}^*}{\underline{m}} - \bar{b} + b_1 \right) + (-\alpha_2), & \alpha_2 < 0. \end{cases} \\ = & \begin{cases} \dfrac{1}{\underline{m}}(m_1 - \underline{m})\alpha_2 + m_1(\bar{b} - b_1) > 0, & \alpha_2 > 0, \\ 0, & \alpha_2 = 0, \\ \dfrac{1}{\underline{m}}(m_1 - \underline{m})\alpha_2 - m_1(\bar{b} - b_1) < 0, & \alpha_2 < 0. \end{cases} \end{aligned} \tag{18}$$

By noting $-[\xi_2]^{2-r_3}\alpha_2 \ge 0$, one gets such that

$$\dot{V}_2 \le -l_1 \sum_{j=1}^{2} |\xi_j|^2 - l_2 \sum_{j=1}^{2} |\xi_j|^{2+p}, \tag{19}$$

where $V_2 = \sum_{j=1}^{2} W_j$.

Consequently, the following result is obtained.

Proposition 2. If the controller u_1 of system (8) is specified by (17) with design parameters $l_1 > 0$, $l_2 > 0$ and $p > -\tau$ satisfying

$$\frac{2(\tau - 2)}{\theta l_1 \tau} + \frac{(2 - \tau)2^{\frac{2+2p+\tau}{2-\tau}}}{\theta l_2(p + \tau)} < T, \tag{20}$$

then the equilibrium $x = 0$ of CLS is globally fixed-time stable and all the trajectories converge to zero before a fixed time θT.

Proof. According to $(x_i - \alpha_{i-1})([z_i]^{\frac{1}{r_i}} - [\alpha_{i-1}]^{\frac{1}{r_i}}) \ge 0$, we easily verify that $V_2 = \sum_{j=1}^{2} W_j$ is positive definite and radially unbounded. Moreover, we have the following estimation for V_2.

$$V_2 = \sum_{j=1}^{2} W_j \le 2 \sum_{j=1}^{2} |\xi_j|^{2-\tau}. \tag{21}$$

Letting $\alpha = 2/(2 - \tau)$, it is not difficult to obtain that

$$-\sum_{j=1}^{2} |\xi_j|^2 \le -\frac{1}{2} V_2^{\alpha}. \tag{22}$$

On the other hand, taking (21) into account, it can be deduced that

$$\begin{aligned} -\sum_{j=1}^{2} |\xi_j|^{2+p} = & -\sum_{j=1}^{2} \left(|\xi_j|^{2-\tau} \right)^{\frac{2+p}{2-\tau}} \\ \le & -2^{1-\frac{2+p}{2-\tau}} \Big(\sum_{j=1}^{n} |\xi_j|^{2-\tau} \Big)^{\frac{2+p}{2-\tau}} \\ \le & -2^{-\gamma} 2^{1-\gamma} V_2^{\gamma}, \end{aligned} \tag{23}$$

where $\gamma = (2+p)/(2-\tau)$.

Therefore, by considering (19), (22) and (23), it follows that

$$\dot{V}_2 \le -\frac{1}{2} l_1 V_2^{\alpha} - l_2 2^{-\gamma} 2^{1-\gamma} V_2^{\gamma}. \qquad (24)$$

Since $\alpha < 1$ and $\gamma > 1$, from Lemma 2, we conclude that the equilibrium $z = 0$ of the closed-loop system is globally fixed-time stable and the settling time function T_1 satisfies

$$\begin{aligned} T_1 &\le \frac{2}{l_1(1-\alpha)} + \frac{2^{\gamma} n^{\gamma-1}}{l_2(\gamma-1)} \\ &= \frac{2(\tau-2)}{l_1 \tau} + \frac{(2-\tau) 2^{\frac{2+p}{2-\tau}} n^{\frac{p+\tau}{2-\tau}}}{l_2(p+\tau)} \\ &< \theta T. \end{aligned} \qquad (25)$$

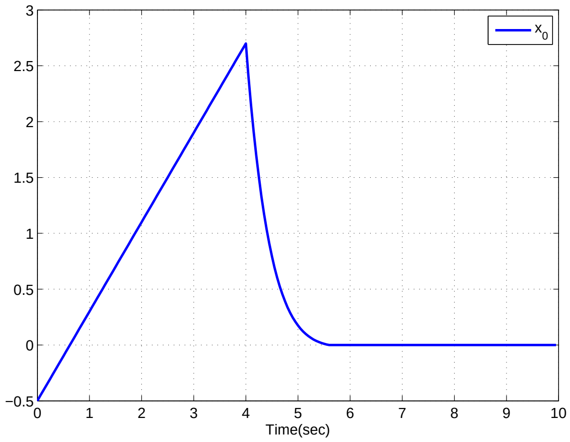

Fig. 2. x_0.

B. Fixed-time stabilization of the x_0-subsystem

From Proposition 2, it si known that $x(t) \equiv 0$ when $t \ge \theta T$. Since $\dot{x}(t) = 0$, $x(t) = 0$ holds for $t \ge \theta T$ in spite that a new controller will be designed for u_0 when $t \ge \theta T$. Hence, it is just needed to stabilize the x_0-subsystem in a fixed time θT. In this case, for the x_0-subsystem, the control α_0 is taken as

$$\alpha_0 = -(m_0 + m_1 |x_0|^q) [x_0]^{\sigma}, \qquad (26)$$

where

$$u_0 = \begin{cases} \dfrac{\alpha_0}{\underline{m}} + \overline{b}, & \alpha_0 > 0, \\ 0, & \alpha_0 = 0, \\ \dfrac{\alpha_0}{\underline{m}} - \overline{b}, & \alpha_0 < 0, \end{cases} \qquad (27)$$

and $0 < \sigma < 1$, $m_0 > 0$, $m_1 > 0$ and $q > 1 - \sigma$ are the design parameters to be determined later.

Proposition 3. If design parameters $0 < \sigma < 1$, $m_0 > 0$, $m_1 > 0$ and $q > 1 - \sigma$ in (27) satisfy

$$\frac{2}{m_0(1-\sigma)(1-\theta)} + \frac{2}{m_1(\sigma+q-1)(1-\theta)} < T, \qquad (28)$$

then the state x_0 is regulated to zero within a fixed settling time $(1-\theta)T$.

Proof. The proof of Proposition 3 follows the same line of the proofs of Proposition 2.

Consequently, the following theorem is obtained to summarize the main result of the paper.

Theorem 1. If the following switching control strategy with the appropriate chosen design parameters is applied to system (3), then the states of the CLS are regulated to zero within any given settling time T.

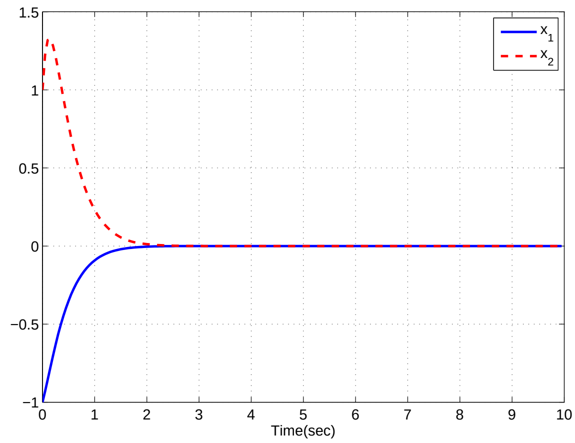

Fig. 3. x_1 and x_2.

IV. Simulation Results

In this section, we illustrate the effectiveness of the proposed approach by taking the dead-zone input nonlinearities D_j, $j = 0, 1$ described by (4) with $m_j = 1 + 0.2 \sin t$, $b_j = 0.3 + 0.1 \cos t$ respectively. In this situation, it is clear that Assumptions 1 holds with $\underline{m} = 0.8$, $\overline{m} = 1.2$, $\underline{b} = 0.2$ and $\overline{b} = 0.4$. Furthermore, by setting $u_0 = c_0^*$ with $c_0^* > \overline{b}$ being a positive constant, it is easily verified that Assumption 2.1 is satisfied with $\nu = 0$, $\lambda_1 = \lambda_2 = \lambda_3 = 1$ and $\varphi_2 = \overline{m}(c_0^* - \underline{b})$.

Taking $\tau = -1/3$ and following the design procedure given in Section III, a state feedback controller of from (17) is constructed such that the states of the x-subsystem of (3) are globally regulated to zero within a fixed settling time θT.

Then, when $t \ge \theta T$, for the x_0-subsystem, switch the control input u_0 to (26) such that the state x_0 is regulated to zero within a fixed settling time $(1-\theta)T$.

In the simulation, by choosing the fixed time $T = 10$ and the gains for the control laws as $u_0^* = 1$, $l_1 = 4$, $l_2 = 5$, $p = 2$, $\theta = 0.8$, $q_1 = q_2 = 1$, $\underline{p} = 0.8$, $\overline{p} = 1.2$, $\sigma = 0.5$ and $m_0 = m_1 = q = 2$, Figs. 2 and 3 is obtained to exhibit the responses of the closed-loop system with $(x_0(0), x_1(0), x_2(0)) = (-0.5, -1, 1)$. From the figures, it can be seen that the states of the closed-loop switched system converge to zero in a given fixed time, which demonstrates the effectiveness of the control method proposed in this paper.

V. Conclusion

This paper has studied the problem of fixed-time stabilization by state feedback for nonholonomic WMR with actuator dead-zones. By employing the API technique, a constructive state feedback design procedure is given, which together with a novel switching control strategy, ensures that the states of the CLS are regulated to zero for any given fixed time.

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REVISION

It is an updated version made on November 16, 2022. Compared with the original version, this version has no content changes, but only indicates that the corresponding author of this paper is Fangzheng Gao.