Multi-objective Optimization Model of Hydrodynamic Sliding Bearing Based on MOPSO with Linear Weighting Method

Junfeng Zhao, Jian Li and Cuirong Huang

***Abstract*—The machinery manufacturing industry has always been an important pillar of China's economic development. When we solve different problems in mechanical production, people often face the trade-off of having to choose among some desired multiple optimization goals, because many goals cannot often be optimized at the same time, or even conflict with each other. To improve the comprehensive performance of hydrodynamic sliding bearing, the improved multi-objective model is applied to this problem to obtain more reasonable parameter design. We use MOPSO with linear weighting method to solve our proposed model. The results of a numerical example show that our model and algorithm are feasible and more effective than the traditional MOPSO by more reasonable parameter design.**

***Index Terms*—multi-objective optimization, machinery manufacturing, linear weighting method, constrain, objective function**

I. Introduction

Hydrodynamic Sliding Bearing (HSB) has many advantages in engineering, including strong bearing capacity, low power consumption, good impact resistance, high operation accuracy and so on, so it is widely used in a series of rotating machinery. Generally, the parameters of HSB are often selected in a certain range according to the experience of operators, which is unstable, and its optimality is usually not guaranteed. It is often accompanied by problems such as high fuel consumption, rapid temperature rise and poor bearing capacity. To improve the comprehensive performance of HSB, this paper applies the improved multi-objective optimization model to obtain more reasonable parameter design.

On the one hand, some scholars did some research about algorithms. Gui et al. [1] did some researches on partner selection based on NSGA-II. Shahrzad et al. [2] designed a virtual enterprise partner selection algorithm considering customer types. Hossein et al. [3] applied the multi-objective particle swarm optimization (MOPSO) algorithm to evaluate the performance. An alternative MOPSO algorithm was proposed by Farshad [4] for conjunctive water use management. Rajani [5] discussed the influence of parameters on the performance of the MOPSO algorithm.

On the other hand, some scholars put forward some ideas about multi-objective optimization. Li et al. [6] adopted a multi-objective decision-making method in the selection of virtual enterprise partners. Huang et al. [7] solved the problem of multi-criteria partner selection under fuzzy uncertainty. Son et al. [8] innovated the collaborative transportation scheduling optimization method based on genetic algorithm. Zhao et al. [9] studied multi-objective models with different relative advantage parameters based on the MOPSO algorithm. Yu [10] proposed a fuzzy multi-objective multi-period portfolio based on wavelet neural network.

There are some scholars who proposed some achievements on interval optimization fields. Meng et al. [11] did some researches on interval neuromorphic preference relations. Mahmood et al. [12] applied the risk assessment method to the virtual enterprise of small and medium-sized enterprises. Liao et al. [13] analyzed the benefits of enterprise server virtualization and cloud computing. Kohnke et al. [14] studied the risk and rewards of enterprise use of augmented reality and virtual reality. Huang et al. [15] did some research on the partner selection of virtual enterprises under the condition of uncertain candidate information.

In Section II, we explain the notations and basic conceptions. In Section III, we introduce the procedure of MOPSO. Then, we introduce construction of multi-objective optimization model for HSB in Section IV. Moreover, we use numerical examples to compare the similarities and differences of MOPSO and linear weighting method in Section V. Finally, we draw some conclusions in Section VI.

II. Model Construction

A. Model Variables

The design of sliding bearing involves many structural parameters. According to the actual conditions, some parameters are selected to optimize the performance of sliding bearing, including ratio of width to diameter, relative gap size and dynamic viscosity of lubricating oil. Then the model variables can be expressed as

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$$X = [x_1, x_2, x_3]^T = [B/d, \psi, \eta]^T. \quad (1)$$

B. Objective Functions

The objective function is set according to the working requirements of HSB. We want to minimum friction coefficient, minimum calorific value and maximum bearing capacity, which can be transformed into objective functions.

(1) Minimum friction coefficient

In order to ensure the transmission efficiency of sliding bearing transmission, the greater the friction resistance is, the worse the efficiency is. Therefore, the first objective function is to minimize the friction coefficient.

$$\min f_1(X) = f = \frac{\pi\eta\omega}{p\psi} + 0.55\psi\xi = \frac{\pi\omega d^2 x_1 x_3}{F x_2} + 0.55 x_2 x_1^{-1.5}, \quad (2)$$

where ξ is the diameter width ratio of bearing. When $B/d \geq 1$, $\xi = 1$; When $B/d < 1$, $\xi = (d/B)^{1.5}$; η is dynamic viscosity of lubricating oil. It is generally considered that the bearing is under the average working temperature. The unit of η is $Pa \cdot s$; ω is journal angular velocity which unit is rad/s; p is average specific pressure of bearing which unit is Pa; ψ is relative clearance; d is the bearing diameter which unit is m; F is bearing working load which unit is N.

(2) Minimum calorific value

In order to control the heating and wear of sliding bearing, the value of bearing should be minimized, thus, the objective function can be obtained by

$$\min f_2(X) = pv = \frac{Fv}{Bd} = \frac{Fv}{(B/d)d^2} = \frac{Fv}{x_1 d^2}, \quad (3)$$

where p is average specific pressure of bearing. Its unit is Pa; F is bearing working load which unit is N; B/d is the diameter width ratio of bearing; d is the bearing diameter which unit is m; v is journal circumferential speed which unit is m/s.

(3) Maximum bearing capacity

Hydrodynamic sliding bearing design needs to have enough bearing capacity, which is directly expressed as the bearing capacity coefficient $C_p = \frac{p\psi^2}{\eta\omega}$. The greater C_p is, the stronger the bearing capacity is. Thus, the objective function can be obtained by

$$\max f_3(X) = C_p = \frac{p\psi^2}{\eta\omega} = \frac{F x_2^2}{\omega d^2 x_1 x_3}, \quad (4)$$

where η is dynamic viscosity of lubricating oil. It is generally considered that the bearing is under the average working temperature. The unit of η is $Pa \cdot s$; ω is journal angular velocity which unit is rad/s; p is average specific pressure of bearing which unit is Pa; ψ is relative clearance; d is the bearing diameter which unit is m; F is bearing working load which unit is N.

At the same time, the design of sliding bearing is limited in several aspects, which can be summarized as the following.

C. Constraints

(1) Minimum film thickness

The larger the minimum oil film thickness $h_{\min}$, the worse the bearing capacity of the model. The roughness of the friction surface, the cleanness of the lubricating oil, the deformation of the shaft and bearing and other factors will also affect it. Thus, we have

$$h_{\min} = 55\frac{\eta n d^3 (B/d)^2}{\psi(B/d+1)F} = 55\frac{\eta d^3 x_3 x_1^2}{F x_2 (x_1+1)} \geq k(R_{z1} + R_{z2}). \quad (5)$$

Namely,

$$g_1(X) = k(R_{z1} + R_{z2}) - 55\frac{n d^3 x_3 x_1^2}{F x_2 (x_1+1)} \leq 0, \quad (6)$$

where v is bearing speed, its unit is $r/\min$; k is safety factor which is generally taken as $k = 2 \sim 3$; R_{z1} and R_{z2} are the surface roughness and bearing hole respectively.

(2) Bearing width diameter ratio

The general requirement of bearing design specification is

$$(B/d)_{\min} \leq B/d \leq (B/d)_{\max}. \quad (7)$$

Namely,

$$g_2(X) = x_1 - (B/d)_{\max} \leq 0 \quad (8)$$

and

$$g_3(X) = (B/d)_{\min} - x_1 \leq 0. \quad (9)$$

(3) Specific pressure

$$\begin{cases} p_{\min} \leq p \leq p_{\max} \\ p = \frac{F}{Bd} = \frac{F}{(B/d)d^2} = \frac{F}{x_1 d^2} \end{cases}. \quad (10)$$

By the above function, we can get

$$g_4(X) = p - \frac{F}{x_1 d^2} \leq 0 \quad (11)$$

and

$$g_5(X) = \frac{F}{x_1 d^2} - p_{\max} \leq 0. \quad (12)$$

(4) Bearing relative clearance

The size of bearing relative clearance ψ affects the bearing capacity, temperature rise and the accuracy of rotation. In design progress, the value range of ψ is usually determined by empirical formula $\psi_{\min} \leq \psi \leq \psi_{\max}$. This is equivalent to the following formula,

$$g_6(X) = \psi_{\min} - x_2 \leq 0 \quad (13)$$

and

$$g_7(X) = x_2 - \psi_{\max} \leq 0. \quad (14)$$

(5) Viscosity of lubricating oil

We choose the type and brand of lubricating oil according to the general oil selection principle. Assuming that the temperature of lubricating oil is $t = 35°C \sim 60°C$, so we can get the constraint function as below,

$$g_8(X) = \eta_{\min} - x_3 \leq 0 \quad (15)$$

and

$$g_9(X) = x_3 - \eta_{\max} \leq 0. \quad (16)$$

III. MOPSO (MULTI OBJECTIVE PARTICLE SWARM OPTIMIZATION ALGORITHM)

Particle swarm optimization (PSO) is an algorithm that simulates the social behavior of birds. In PSO algorithm. The particles will fly over the hyperplane in the search space. The change of particle's position in search space is based on the social psychology trend that particles in the population imitate other successful particles. A population contains a series of particles, each of them represents a solution in the feasible region. Particles will continuously optimize their position based on the position of themselves and other particles.

Next, some special notations are introduced: $x_i(t)$ represent the position of particle p_i at the moment of t, $v_i(t)$ represent the velocity of particle p_i at the moment of t. The position of p_i is updated by adding a velocity of $v_i(t)$. The position update formula is as follow:

$$x_i(t) = x_i(t-1) + v_i(t). \tag{17}$$

The velocity update formula is as follow:

$$v_i(t) = Wv_i(t-1) + C_1r_1(x_{pbest_i} - x_i(t)) + C_2r_2(x_{gbest} - x_i(t)), \tag{18}$$

where W is the inertia weight, C_1, C_2 are learning factor coefficients which are usually constants, r_1, r_2 are random values in $[0,1]$. To deal with the multi-objective problem, Sierra and Coello [16] created MOPSO algorithm based on PSO algorithm. The main procedure is shown below and summarized in TABLE I.

Step 1: Initialization population POP: for $i=0$ to MAX /*MAX is the total number of particles*/ initialization POP[i];

Step 2: Initialize the velocity vector for each particle: for $i=0$ to MAX, VEL[i]=0;

Step 3: Calculate the value of each particle in POP;

Step 4: The particle position representing the non-dominated vector is stored in warehouse REP;

Step 5: Generating hypercube of search space explored so far, these hypercubes are used as coordinate system to locate particles, and the coordinates of each particle are determined according to the value of the objective function;

Step 6: Initializes the memory of each particle which will serve as a guide to traverse all search fields, and memory will also be stored in the repository: for $i=0$ to MAX, $\text{PBESTS}[\text{i}] = \text{POP}[\text{i}]$;

Step 7: According to the given formula, the velocity and position of particles are updated iteratively;

Step 8: End cycle.

In TABLE I, inertia weight W values 0.4; R_1 and R_2 are random numbers in $[0,1]$; current position of particle i is $\text{POP}[\text{i}]$; PBESTS[i] is the best position for the i particle to pass through; $\text{REP}[\text{h}]$ is a value taken from the warehouse; The serial number h is generated as follows. Each hypercube with more than one particle is assigned a fitness equal to any number divided by the number of particles contained in the hypercube. This is mainly to reduce the fitness of hypercube with more particles, which can be regarded as a method of sharing fitness. Then, we will use the roulette method to select the hypercube and randomly extract particles from it.

IV. CONSTRUCTION OF MULTI-OBJECTIVE OPTIMIZATION MODEL FOR HSB

Combined with the above conditions, a multi-objective optimization model of HSB is proposed as below.

$$\min \quad F(X) = [f_1(X), f_2(X), ..., f_p(X)]^T$$
$$\begin{cases} \text{s.t.} \quad g_u(X) \ge 0, \quad u = 1,2,...,m, \\ \quad h_v(X) = 0, \quad v = 1,2,...,q < n, \\ \quad k_j(X) \subset K_{j\lambda^*}, \quad j=1,2,...,\text{J}, \\ \quad X \in R^n, \end{cases} \tag{19}$$

where n means the dimension of solution variables, p means the number of objective functions, m is the number of inequality constraint functions and n is the number of equality constraints. $k_j(X)$ means the fuzzy constraint, and K_j represents the fuzzy constraint range that $k_j(X)$ can fall into. Noting that $f_3(X)$, the original model is to find its maximum value, here we add a negative sign to convert it into the minimum value of $-f_3(X)$, that is

$$\min \quad F(X) = [f_1(X), f_2(X), f_3(X)]^T$$
$$\begin{cases} \text{s.t.} \quad g_u(X) \ge 0, \quad u = 1,2,...,9, \\ \quad \text{X} \in R^3. \end{cases} \tag{20}$$

In the model (20),

$$f_1(X) = f = \frac{\pi\eta\omega}{p\psi} + 0.55\psi\xi = \frac{\pi\omega d^2 x_1 x_3}{Fx_2} + 0.55x_2x_1^{-1.5}, \tag{21}$$

$$f_2(X) = pv = \frac{Fv}{Bd} = \frac{Fv}{(B/d)d^2} = \frac{Fv}{x_1 d^2}, \tag{22}$$

$$-f_3(X) = -C_p = -\frac{p\psi^2}{\eta\omega} = -\frac{Fx_2^2}{\omega d^2 x_1 x_3}. \tag{23}$$

Next, we can use the MOPSO algorithm introduced in Section III to solve the model to obtain the pareto solution set. Consider the following example: a mine hoist needs to use hydrodynamic sliding bearing device, and known working load $F = 35000N$, which is stable. The diameter of journal is $d = 100mm$, and speed of shaft is $n = 1000r/min$.

TABLE I
ALGORITHM

```
WHILE COUNT<COUNTMAX DO
  FOR i < Max
     Step C1 VEL[i] = W × VEL[i] + R1 × (PBESTS[i] − POP[i])+R2 × (REP[h] − POP[i]), POP[i] = POP[i] + VEL[i].
     Step C2 If POP[i] > the boundries, VEL[i] = −VEL[i] or POP[i] = the boundries.
     Step C3 Calculate FIT[i].
     Step C4 Update REP.
     Step C5 If FIT[i] < any FIT[j] in REP, replace the j-th particle by the i-th particle.
     Step C6 If FIT[i] is best before, PBESTS[i] = POP[i].
  END
 COUNT = COUNT + 1.
```

The lining material is babbitt alloy and the bearing is split type. Other parameters are as follows.

$\psi = 0.001 \sim 0.002$, $B/d = 0.5 \sim 1.2$,

$p = (0.5 \sim 3.5)MPa$, $\eta = (0.02 \sim 0.04)Pa \cdot s$.

$R_{z1} + R_{z2} = 4.8\mu m$, $k = 2$. It can be seen from the above actual working conditions that the value range of x_1 is $[0.5, 1.2]$, x_2 is $[0.001, 0.002]$ and x_3 is $[0.02 \sim 0.04]$.

V. Solutions and Analysis Based on Linear Weighting Method

A. *Solution of Pareto Set ---MOPSO Algorithm*

In the MOPSO algorithm (see Section III), the subalgebra quantity is 200, the warehouse size is 200, the maximum iteration number is 100, the inertia weight is 0.4, the self-confidence coefficient C_1 = 2, the population influence coefficient C_2 = 2, the number of scales in each dimension is 20, and the coefficient of variation is 0.5.

The running results are shown in the TABLE II. We draw the changes of their corresponding objective functions to draw the following Fig. 1. Combined with the Fig. 1, it can be seen that after 200 iterations, the change of x_1 is still large in the space limited by the pareto solution set and the values of x_2 and x_3 are relatively stable, but still have a certain range of changes. At the same time, in order to maximize the objective function $f_3(X)$, we often need to reduce the value of x_1, and at the same time, the value of $f_1(X)$ and $f_2(X)$ will increase to a certain extent. This conclusion is consistent with the definition of pareto solution.

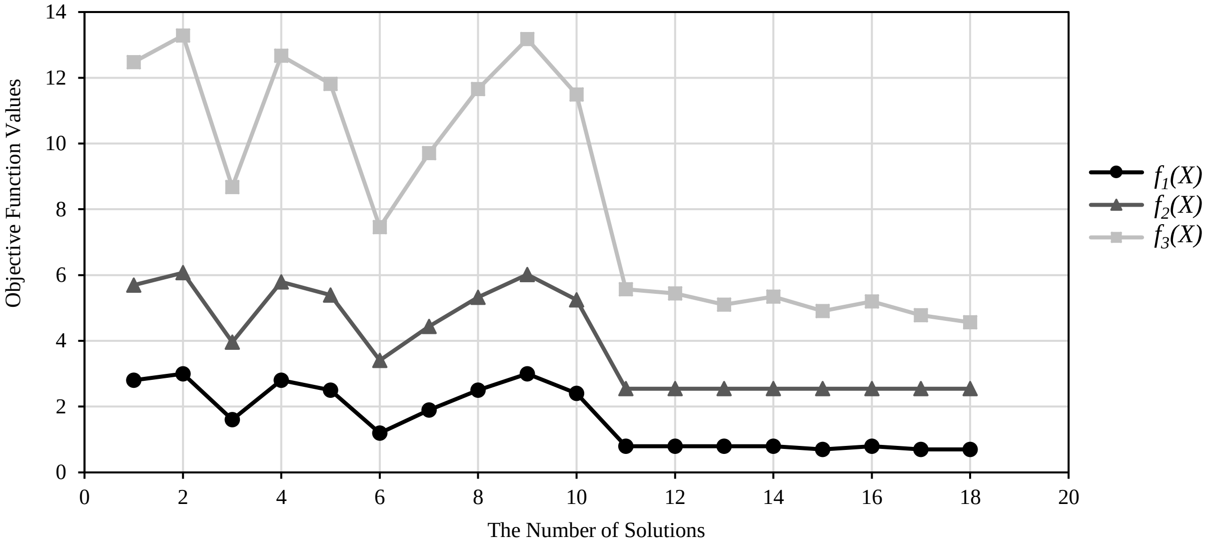

Fig. 1. The change of each objective function corresponding to each solution

TABLE II
RUNNING RESULTS

Solution Number	$x_1 = B/d$	$x_2 = \psi$	$x_3 = \eta$	$f_1(X) = f$	$f_2(X) = pv$ ($\times 10^5$)	$f_3(X) = C_p$
1	0.5357	0.0020	0.0200	0.0028	5.7006	12.4765
2	0.5031	0.0020	0.0200	0.0030	6.0715	13.2873
3	0.7701	0.0020	0.0200	0.0016	3.9663	8.68044
4	0.5270	0.0019	0.0200	0.0028	5.7959	12.6661
5	0.5660	0.0020	0.0200	0.0025	5.3958	11.8091
6	0.8958	0.0020	0.0200	0.0012	3.4095	7.46197
7	0.6884	0.0020	0.0200	0.0019	4.4365	9.70958
8	0.5732	0.0020	0.0200	0.0025	5.3280	11.6605
9	0.5071	0.0020	0.0200	0.0030	6.0224	13.1804
10	0.5817	0.0020	0.0200	0.0024	5.2499	11.4898
11	1.2000	0.0020	0.0200	0.0008	2.5452	5.5704
12	1.2000	0.0019	0.0200	0.0008	2.5452	5.4466
13	1.2000	0.0019	0.0200	0.0008	2.5452	5.1068
14	1.2000	0.0019	0.0200	0.0008	2.5452	5.3489
15	1.2000	0.0018	0.0201	0.0007	2.5452	4.9059
16	1.2000	0.0019	0.0202	0.0008	2.5452	5.1969
17	1.2000	0.0018	0.0201	0.0007	2.5452	4.7778
18	1.2000	0.0018	0.0200	0.0007	2.5452	4.5638

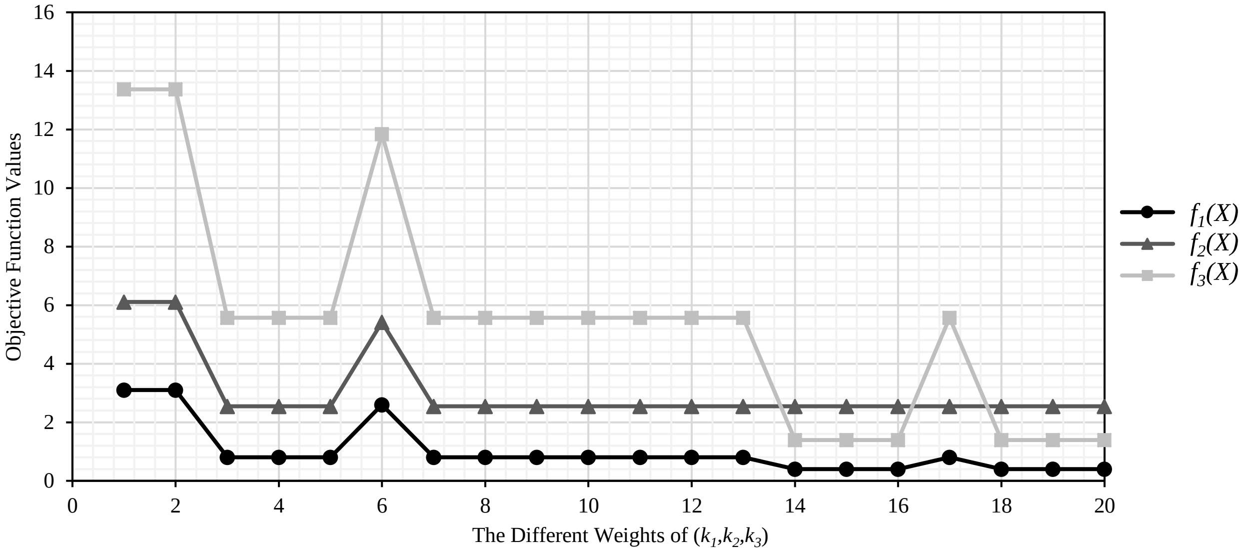

Fig. 2. Change of optimal solution with value

TABLE III
RESULTS UNDER 20 DIFFERENT WEIGHT ALLOCATION METHODS

Number	(k_1,k_2,k_3)	$x_1 = B/d$	$x_2 = \psi$	$x_3 = \eta$	$f_1(X) = f$	$f_2(X) = pv$ ($\times 10^5$)	$f_3(X) = C_p$
1	(0.1,0.1,0.8)	0.5000	0.0020	0.0200	0.0031	6.1087	13.3690
2	(0.1,0.2,0.7)	0.5000	0.0020	0.0200	0.0031	6.1087	13.3690
3	(0.2,0.2,0.6)	0.5643	0.0020	0.0200	0.0026	5.4126	11.8456
4	(0.2,0.3.0.5)	1.2000	0.0020	0.0200	0.0008	2.5453	5.5704
5	(0.3,0.3,0.4)	1.2000	0.0020	0.0200	0.0008	2.5453	5.5704
6	(0.3,0.4,0.3)	1.2000	0.0020	0.0200	0.0008	2.5453	5.5704
7	(0.4,0.4,0.2)	1.2000	0.0020	0.0200	0.0008	2.5453	5.5704
8	(0.5,0.4,0.1)	1.2000	0.0010	0.0200	0.0004	2.5453	1.3926
9	(0.5,0.3,0.2)	1.2000	0.0010	0.0200	0.0004	2.5453	1.3926
10	(0.6,0.2,0.2)	1.2000	0.0010	0.0200	0.0004	2.5453	1.3926
11	(0.6,0.1,0.3)	1.2000	0.0020	0.0200	0.0008	2.5453	5.5704
12	(0.7,0.2,0.1)	1.2000	0.0010	0.0200	0.0004	2.5453	1.3926
13	(0.7,0.1,0.2)	1.2000	0.0010	0.0200	0.0004	2.5453	1.3926
14	(0.8,0.1,0.1)	1.2000	0.0010	0.0200	0.0004	2.5453	1.3926
15	(0.1,0.8,0.1)	1.2000	0.0020	0.0200	0.0008	2.5453	5.5704
16	(0.2,0.7,0.1)	1.2000	0.0020	0.0200	0.0008	2.5453	5.5704
17	(0.2,0.6,0.2)	1.2000	0.0020	0.0200	0.0008	2.5453	5.5704
18	(0.1,0.7,0.2)	1.2000	0.0020	0.0200	0.0008	2.5453	5.5704
19	(0.2,0.5,0.3)	1.2000	0.0020	0.0200	0.0008	2.5453	5.5704
20	(0.1,0.5,0.4)	1.2000	0.0020	0.0200	0.0008	2.5453	5.5704

B. Fuzzy Evaluation Method----Linear Weighting Method

Next, we use the linear weighting method to obtain the ideal solution of the model. They can be obtained by the following formula.

$$\tilde{F}(X^*) = \min \tilde{F}(X) = \min\{\sum_{i=1}^{p} k_i [\frac{f_i(X) - m_i}{M_i - m_i}]^t\}, \tag{24}$$

where X^* is the satisfactory solution, M_i, m_i represent the maximum and minimum values of the first objective function in all the discrete solutions. t is constant, which determines the shape of the satisfaction function (especially, when t is taken as 1, the shape of the function is trapezoid). k_i is the undetermined coefficient, which can be taken by the relative importance of different objective functions. The influence of different values of t and k_i on the final solution will be discussed by different combination.

Next, we use the linear weighted minimum deviation method to evaluate the solution. In order to observe the sensitivity of the pareto solution to each objective function, we tried 20 different weight distribution methods to explore the pareto solution set. The results are shown in TABLE III.

In the same way, the value which reflects the obvious change of optimal solution with k_i is selected. We use the following data to draw a graph.

when $K_1 = (k_1, k_2, k_3)^T = (0.1, 0.2, 0.7)^T$, then f_1=0.0031, f_2=6.1087, f_3=13.3690;

when $K_2 = (k_1, k_2, k_3)^T = (0.1, 0.8, 0.1)^T$, then f_1=0.0008, f_2=2.5453, f_3=5.5704 and so on.

The changes of four groups of solutions and objective function values are plotted as shown in Fig. 2.

It can be seen from the above chart that the value of x_3 is relatively stable. No matter how the weight of the objective function changes, its value is basically unchanged around 0.0200. The change of weight mainly affects the distribution of x_1 and x_2. It is also found that the change of the spatial position of the solution vector is often caused by the large change of k_3 in the weight vector. We observe the value of k_3 in the first 10 rows of TABLE III, its value decreases row by line. The result is that the gradual increase of x_1 and the decrease of x_2 (the amplitude is small), and the value of each objective function is consistent with the change of corresponding coefficient.

It is not difficult to find out from the TABLE III that when the weight distribution is more uniform, $X=(x_1,x_2,x_3)^T =(1.2000,0.0020,0.0200)^T$ has strong stability. Therefore, this paper regards it as the optimal solution.

The above optimal solution is obtained based on the linear weighted minimum deviation method when $t=1$. Similarly, we choose the strategy of maintaining the weight $(k_1,k_2,k_3)^T=(0.3,0.3,0.4)^T$ to explore the influence of different values of t on the optimal solution. The results are shown in TABLE IV. The corresponding results are plotted as shown in Fig. 3. Then we have the following findings.

(1) When t is in the interval $(0,1]$, the distribution of the solution is relatively stable, x_1 and x_3 are basically unchanged, and x_2 is floating upward. For the objective function, $f_1(X)$ decreases, $f_2(X)$ remains unchanged, and $f_3(X)$ increases.

(2) When t gradually approaches 1, each objective function corresponding to the solution is optimized, and when $t=1$, the optimization effect of the model is the best.

(3) For the case of $t>1$, with the increase of t, x_1 gradually decreases, while x_2 and x_3 remain stable. At the same time, the objective function $f_1(X)$ has a small increase, the value of $f_2(X)$ also increases, but $f_3(X)$ also changes in the direction of optimization. (because we want to maximize $f_3(X)$, so the value of $f_3(X)$ also gradually increases.

This phenomenon is due to the normalization of the objective function value. Therefore, the larger the index is, the smaller the value of the multiplication part corresponding to each weight is. That is, the overall change brought by the difference between the membership degrees of different solutions becomes smaller due to the existence of the index. So, it is easier to tend to the solution with larger weight which makes the overall objective function produce a larger percentage change. For example, when $t=1$ is compared with that $t=3$, we choose to sacrifice the growth of $f_1(X)$ and $f_2(X)$ in exchange for the increase of heavier $f_3(X)$, which is consistent with the change of solution when $t=1$ and weight k_3 increases. Although the underlying principles may have some similarities, the new solutions brought by the change of t value still give us some reference for our final choice.

TABLE IV
OPTIMAL SOLUTION DISTRIBUTION UNDER DIFFERENT T VALUES

t	$x_1=B/d$	$x_2=\psi$	$x_3=\eta$	$f_1(X)=f$	$f_2(X)=pv\ (\times10^5)$	$f_3(X)=C_p$
1/3	1.2000	0.0010	0.0200	4.2066	2.5453	1.3926
1/2	1.2000	0.0010	0.0200	4.2066	2.5453	1.3926
1	1.2000	0.0020	0.0200	0.0008	2.5453	5.5704
2	0.9161	0.0020	0.0200	0.0013	3.3342	7.2971
3	0.8515	0.0020	0.0200	0.0014	3.5869	7.8501

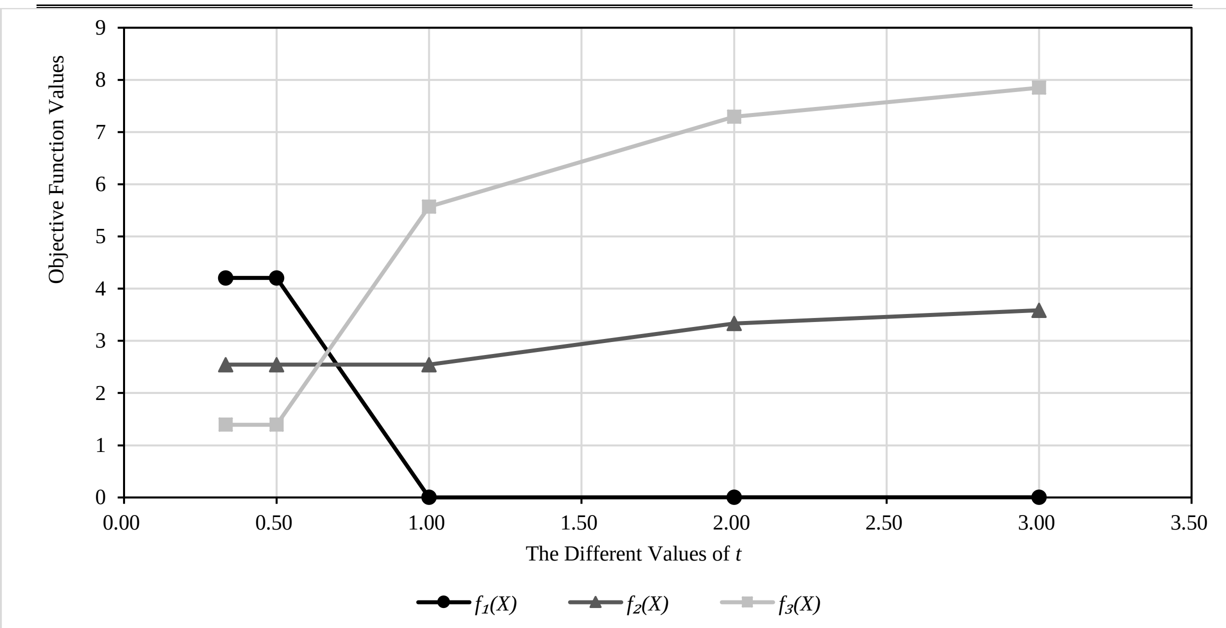

Fig. 3. Change of optimal solution with value t

TABLE V
COMPARISON OF CONVENTIONAL AND OPTIMIZED DESIGN

Design Method	x_1	x_2	x_3	$f_1(X)$	$f_2(X)$	$f_3(X)$
Conventional method	1.0000	0.0010	0.0180	0.0024	18.4376	1.8623
Optimization method	1.2000	0.0020	0.0200	0.0008	25.4980	5.5719

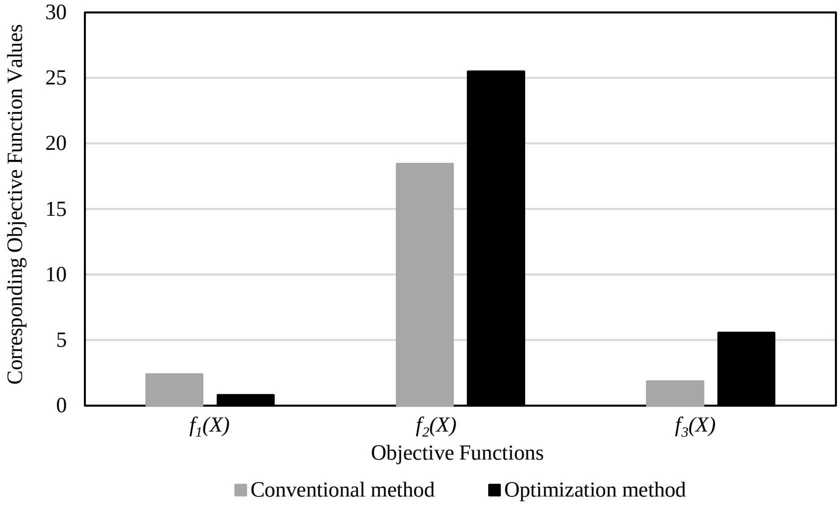

Fig. 4. Comparison of conventional method and optimization method

Finally, we choose the optimal result of $t=1$, $(k_1,k_2,k_3)^T=(0.3,0.3,0.4)^T$ as the final optimization result of the model, and make a simple comparison with the conventional optimization design results, the results are shown in the TABLE V and Fig. 4.

It can be seen from Fig. 4 that using MOPSO linear weighting method model to optimize the design of hydrodynamic sliding bearing can reduce the friction coefficient and significantly improve the bearing capacity. But, at the same time it also can increase a certain amount of heat of the model. From the numerical point of view, the friction coefficient is reduced by 66.6%, the bearing capacity is increased by 199.5%, but the calorific value is also increased by 38.0%. Overall, the friction coefficient of the model is greatly reduced, the bearing capacity is greatly improved, and the calorific value is slightly increased. However, compared with the overall improvement of the model is still within the acceptable range, so we can think that the optimization results of this model have considerable rationality and guiding value - in the subsequent bearing design, the enhancement of heat dissipation capacity is mainly considered to obtain smaller friction coefficient and significantly increased bearing capacity.

VI. CONCLUSIONS

In this paper, the physical background of hydrodynamic sliding bearing model is briefly explained. The variables, objective functions and constraint functions are determined, and a complete multi-objective optimization model is constructed. The pareto solution set is obtained by MOPSO algorithm. In the framework of linear weighting method, the influence of weight and index change on the optimal solution obtained by linear weighting method is discussed, and the optimization result is compared with the conventional design result.

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