Research on Mixed Procurement Decision of Spot and Option for Risk-averse Retailers under Fuzzy Demand

Yanchun Wan, Zhuoxin Qiu and Qiucen Chen

***Abstract*—This paper studies a two-echelon innovative product supply chain composed of two suppliers and one retailer, where one supplier provides a spot purchase based on a wholesale price contract and the other provides an option contract. In terms of the different characteristics of spot and option contracts, a single purchasing model and a mixed purchasing model under fuzzy demand are established respectively. Triangular fuzzy numbers (TFNs) are applied to describe the uncertain demand of innovative products and the conditional value-at-risk (CVaR) method is used to express the profits of risk-averse retailers. The results show that the mixed procurement decision under fuzzy demand can significantly improve the flexibility of procurement execution and the profits of retailers. Interestingly, we find that, the spot order quantity in the mixed procurement has nothing to do with the retail price of the product, but decreases as the wholesale cost of the product increases. Additionally, numerical examples further indicate that, compared with the risk-neutral retailer, the risk-averse retailer's decision-making behavior is more conservative, and its total order quantity is less influenced by the fuzziness of the market demand and other model parameters.**

***Index Terms*—Conditional value-at-risk, fuzzy demand, mixed procurement decision, option contract.**

I. Introduction

In recent years, with the diversification of demand, customers' expectations for products are constantly increasing. At the same time, consumers are having more and more initiative in the product selection, and the competition in the supply chain is gradually becoming fierce. In order to cope with the rising expectations of customers and enhance the competitiveness, retailers need to formulate corresponding procurement strategies according to different products. Based on the types of demand, Fisher [1] divides products into functional products and innovative products. Compared with functional products, innovative products have the characteristics of a short life cycle, unpredictable demand and high profits. For retailers selling innovative products, the most important factor in the procurement process is to improve the matching between supply and demand.

To improve the degree of matching of supply and demand, firms use a variety of procurement contracts to reduce the impact of procurement risk. These contracts include spot contracts and option contracts. The application of supply chain spot contracts effectively mediates the mismatch between supply and demand and promotes the degree of supply chain coordination [2]. Moreover, option contracts, which were originally used in financial markets, have also been introduced into supply chain management to help firms better handle risks. For example, Hewlett-Packard uses a mixed procurement decision with spot and option contracts when purchasing various electronic products, such as memory chips and scanners. Approximately 50% of the procurement funds of Hewlett-Packard are spent on long-term contracts, approximately 35% are spent on options contracts, and the rest are spent on the spot market or online trading [3]. China Telecom buys about 100 billion yuan of electronic products from its upstream suppliers via wholesale prices, call options and put options [4]. Nike signs options contracts with its suppliers to hedge against uncertain future transactions [5].

When studying the procurement decisions of retailers, it is necessary to mathematically express the market demand for innovative products. However, because of the lack of historical data and sufficient information on innovative products, it is difficult to accurately describe the changing market demand in reality, and they can only have a vague understanding of the change of demand. He and Hong [6] studied the two-stage fuzzy supply chain dominated by manufacturer and retailers respectively, and used fuzzy variables to solve the sequential game equilibrium solution. Chang [7] studied the supply chain decision-making problem based on the product return in a fuzzy environment for short-lived products and pointed out that the fuzzy parameters of demand have important impacts on the number of orders and the profits of the members. Therefore, it is effective and more in line with the actual situation to use fuzzy mathematics to deal with the uncertainty of market demand for innovative products [8].

Taking the supply chain of innovative products as the research object and considering their market demand as triangular fuzzy variables, this paper establishes the single

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purchasing model and mixed purchasing model of retailers and uses the CVaR method to describe the retailers' risk aversion attitude. In addition, through analysis of various purchasing models and parameter variables, the optimal procurement decision to maximize the profits of retailers is studied. Different from previous studies, when combining spot contracts with option contracts, the uncertainty and fuzziness of demand are considered rather than assumed to follow a random distribution, and the CVaR method is used to express the degree of risk aversion, which makes the research more in line with the supply chain practice.

This paper is mainly divided into five sections. The first section is the brief introduction. Then, the related literature is reviewed. Next, in Section 3, the retailer purchasing models under fuzzy demand are proposed. In Section 4, a numerical example and some revelations are provided. Finally, the conclusions and some directions for future research are given in Section 5.

II. Literature Review

A. Fuzzy Demand

In 1965, Zadeh first proposed fuzzy theory in the international academic journal *Controland Information*, which marked the birth of fuzzy mathematics [9]. Then, Zadeh creatively proposed the possibility measure theory to measure fuzzy sets in 1978 [10]. However, due to the lack of self-duality, this possibility measure theory has some defects and limitations. In 2002, Liu et al. [11] proposed the credibility measure theory to correct this defect. In 2004, Liu [12] further established an axiomatic system with perfect credibility theory and showed that fuzzy variables are parallel to random variables, which laid a foundation for the following relevant research on fuzzy environmental decision-making.

The newsboy model under the fuzzy demand environment is a problem studied by many scholars [13-15]. This branch of the literature expands the application scope of the newsboy model. Ryu et al. [16] regarded market demand, product wholesale price and product retail price as fuzzy variables and compared the retailer's purchase volumes and profits in three different purchasing decisions under the fuzzy newsboy problem. In terms of fuzzy demand and the cost environment, Zhang et al. [17] obtained the conditions for coordination of the supply chain through analyzing the distributed decision-making and centralized decision-making of supply chain. By considering fuzzy demand and information asymmetry variables, Yu [18] showed that the return policy can help achieve the supply chain coordination and further promote the total benefits of the supply chain. From the manufacturer's point of view, Xu [19] established a coordination model of the revenue-sharing contract, and discovered that under the coordination of the contract, greater profits can be obtained in the supply chain.

Although some studies have already investigated fuzzy demand, the case of combining a mixed procurement decision, the CVaR method, and innovative products has not been considered in previous studies.

B. Risk Aversion and the CVaR Method

At present, many scholars have performed extensive research on what risk measurement method to use to effectively measure the risk aversion attitude. In the current research on supply chain contracts, the main methods used to measure the risk coefficient are as follows: the utility function, the mean-variance, the value-at-risk (VaR) and the conditional value-at-risk (CVaR).

In research on the utility function method, Chen et al. [20] used the loss aversion utility function to describe the degree of loss aversion of retailers and studied the optimal purchasing decision of retailers who coordinated the supply chain in a two-level supply chain. Liu et al. [21] characterized the loss utility function of manufacturers and analyzed the optimal decision of retailers under a single option and single price discount contract for a supply chain of innovative products composed of manufacturers and retailers. In research on the mean-variance method, Basu and Nair [22] developed a method for tracking mean–variance solutions in inventory control. Zhuo et al. [23] built a mean–variance model in a two-stage supply chain and studied option contracts that considered risk.

In addition, some scholars use the VaR method and CVaR method to describe the risk aversion attitude. A method to measure the downside risk is value-at-risk (VaR). However, the VaR method is not a consistent and convex risk measurement index, and the use of the VaR method to describe risk may make the optimization problem lack concavity and convexity, which makes it difficult to solve the problem. Therefore, Rockafller and Uryasev [24] proposed conditional value at risk (CVaR) to measure the degree of risk. The CVaR model, which makes up for the shortcomings of the VaR model to some extent, is a consistent risk measurement model with subadditivity and can better reflect the potential losses of decision makers. Xu et al. [25] studied the optimal decision in the newsboy model under out-of-stock cost by using the mean-CVaR model. Through maximizing the CVaR of utility, Liu et al. [26] analyzed the supply chain coordination decision-making problem of retailers with loss aversion and introduced a combined contract. Xie et al. [27] studied the optimal decision-making of retailers in supply chain contract models, such as the wholesale price contract, repurchase contract and revenue-sharing contract models, using the mean-CVaR method.

However, this paper considers the optimal decision of retailers in the supply chain contract model that combines spot and option contracts under fuzzy demand. The CVaR method is used to solve the proposed model of retailers' procurement decisions since it has been employed in numerous studies on retailers' procurement decisions.

C. Procurement Decision in the Supply Chain

Many studies have studied the application of a variety of procurement methods to procurement decisions to control risks. Some scholars introduce spot contracts and option contracts into the study of supply chains to explore how to design contracts to increase the benefits of both sides. The forms of these contracts include nonlinear pricing, quantity discount, revenue sharing and others. Ma [28] proposed a new spot contract on the basis of a wholesale price contract and determined the optimal decision to make the benefit of supply chain achieve maximum. Through analysis of the two-stage model, Barnes [29] discovered that the option procurement contract can control the risk and promote the return by improving the flexibility. In addition, Wang et al. [30] established a retailer-led model to study channel coordination and proved that the option contract can promote the supply chain's profits. Recently, many scholars have introduced two-way option contracts into supply chain contracts. Chen et al. [31] introduced the two-way option contract into a short life cycle product supply chain with service level demand, and the results showed that the two-way option contract is indeed beneficial to both retailers and suppliers. Zhao et al. [32] introduced the two-way option contract and wholesale price contract into a two-level supply chain system and proved that the hybrid purchasing decision of the wholesale price contract and option contract can effectively improve supply chain coordination.

The previous literature studies either option procurement or spot procurement, but there are few studies that combine the two. Therefore, this paper combines options and spots to study the mixed procurement decision of retailers in a supply chain.

The innovation of this paper is that this paper takes innovative products as the research object and introduces fuzzy demand and the CVaR method into the two-echelon supply chain model. Moreover, option and spot supply chain contracts are combined to study the optimal purchasing decision to maximize retailers' profits, which provides decision makers with new managerial insights on the procurement decision of retailers in the supply chain.

III. Retailer Purchasing Model under Fuzzy demand

This section takes innovative products as the main object and combines the elements of fuzzy demand, conditional value at risk (CVaR), and spot and option supply chain contracts to establish the retailer purchasing model.

A. Model Description

This paper considers a single-cycle two-echelon supply chain composed of a supplier A, a supplier B and a retailer C to provide and sell innovative products with short sales period. The market demand faced by retailers is uncertain. Supplier A provides a wholesale price contract, and supplier B provides a call option contract.

Before the arrival of the sales season, based on the market information obtained, the retailer signs a wholesale price contract with supplier A to determine the spot purchase quantity, signs a call option contract with supplier B to determine the option purchase quantity, or signs a wholesale price contract and a call option contract with supplier A and supplier B at the same time to determine the mixed spot purchase quantity and the mixed option purchase quantity, respectively. During the sales season, the retailer determines the number of options sold to the supplier according to the observed actual market demand and spot purchases, and sell the products. When the sales season is over, if the market demand cannot be met, then the retailer has to bear the out-of-stock loss; if there are unsold products, then the retailer has to address the unsold products.

The structure of the single-cycle two-echelon supply chain mentioned above is shown in Figure 1, which presents the general procurement process of the retailer purchasing model. The description of the relevant notations is provided in the following paragraphs.

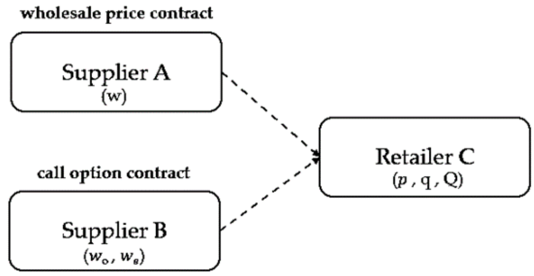

Fig. 1. Purchasing model structure of retailers in a single-cycle two-stage supply chain

B. Notations

The notations involved in the models and their descriptions are shown as follows.

TABLE I
Parameters and variables

Symbol	Parameter description
$\tilde{D}$	Fuzzy market demand for innovative products
p	Retail price of innovative products per unit
w	Wholesale price of innovative products per unit
w_p	Option purchase price per unit
w_e	Option execution price per unit
s	Shortage cost of innovative products per unit
v	Salvage value of innovative products per unit
q	Spot purchase quantity (decision variable)
q_m	Mixed spot purchase quantity (decision variable)
Q	Option purchase quantity (decision variable)
Q_m	Mixed option purchase quantity (decision variable)
u	The value at risk at a given confidence level η, which means the possible profit ceiling
η	Risk aversion coefficient of the retailer, where $\eta \in (0,1]$. The degree of risk aversion of retailers increases with the decrease of η. When $\eta = 1$, retailers behaves as risk-neutral.
E	Expected value of a decision variable
$\tilde{\Pi}_r^k$	Fuzzy profit function of the retailer, where superscript k characterizes the purchasing model (S,O,M)

C. Basic Assumptions

Some basic assumptions are as follows.

1) Suppliers behave as risk-neutral, and retailers behave as risk-averse.
2) Before the beginning of the sales season, supplier A can deliver all the spot purchases of the retailer on time; during the sales season, supplier B can deliver all the options executed by the retailer on time.
3) The information of each member in the supply chain is symmetrical
4) External demand can be roughly expressed by triangular fuzzy numbers. $\tilde{D} = (\underline{\alpha}, \alpha, \overline{\alpha})$, $0 < \underline{\alpha} < \alpha < \overline{\alpha}$ and $\underline{\alpha} < q < \overline{\alpha}$. Here, $\underline{\alpha}$ is the infimum of fuzzy demand, which refers to the minimum possible value of market demand; $\overline{\alpha}$ is the supremum of fuzzy demand, which refers to the maximum possible value of market demand;

and α is called the "fuzzy median", which refers to the most likely value of market demand. The credibility density function $\varphi(x)$ and credibility distribution function $\Phi(x)$ of external demand are, respectively, as follows [8]:

$$\varphi(x)=\begin{cases}\dfrac{1}{2(a-\underline{a})}, & \underline{a}\le x\le a\\ \dfrac{1}{2(\overline{a}-a)}, & a<x\le\overline{a}\\ 0, & else\end{cases};$$

$$\Phi(x)=\begin{cases}0, & x<\underline{a}\\ \dfrac{x-\underline{a}}{2(a-\underline{a})}, & \underline{a}\le x\le a\\ \dfrac{x+\overline{a}-2a}{2(\overline{a}-a)}, & a<x\le\overline{a}\\ 1, & x>\overline{a}\end{cases}.$$

In addition, the expected market demand is as follows: $E(\tilde{D})=\dfrac{\underline{a}+2a+\overline{a}}{4}$.

5) $w<p$. This ensures that retailers can make a profit through spot contracts.
6) $0<v<w$. The salvage value of innovative products should be smaller than the wholesale price; otherwise, the retailer will purchase the product indefinitely.
7) $0<w_p+w_e<p+s$. Only when the retail price and the shortage cost are higher than the sum of the option purchase price and the executive price will retailers have the motivation to execute the option.
8) $0<w<w_p+w_e$. The option purchase price and the executive price are larger than the wholesale cost. For retailers, spot procurement has a price advantage, while option procurement has more procurement flexibility.
9) $v<w_e$. Otherwise, retailers will tend to exercise more options regardless of demand.
10) $w_p<w-v$. If the option purchase price is higher than the difference between the wholesale cost and the salvage value of the product, then the retailer will only buy the spot product and not the option product.

D. Single Spot Purchasing Model under Fuzzy Demand

Aiming at the market demand of newly launched innovative products with great uncertainty and strong ambiguity, this chapter constructs a single spot purchasing model of risk-neutral retailers under fuzzy demand. And on this basis, a single spot purchasing model of risk-averse retailer is built by CVaR method, and the optimal single spot purchase quantity and the maximum profit of retailers are solved.

Spot Purchasing Model for the Risk-neutral Retailer under Fuzzy Demand

According to the centralized decision-making and distributed decision-making methods for supply chain coordination proposed by Giannoccaro [33], the decision-making method used in this supply chain system is distributed decision-making. This decision-making method is suitable for supply chain systems with multiple supply chain links and decision-makers, and the supply chain model in this paper belongs to this case.

In this supply chain purchasing model, the main factors that affect the profits of a single spot purchasing retailer are the following: sales income, salvage income, shortage cost and purchasing cost. Then, the fuzzy sales quantity of the retailer, the fuzzy surplus inventory and the fuzzy shortage quantity are $\min(\tilde{D},q)$, $\max(q-\tilde{D},0)$, and $\max(\tilde{D}-q,0)$, respectively.

The fuzzy profits of the single spot purchasing retailer are as follows:

(1) When $\tilde{D}\le q$,

$\tilde{\prod}_r^S$ = *sales income* + *salvage income* - *purchasing cost*.

(2) When $\tilde{D}>q$,

$\tilde{\prod}_r^S$ = *sales income* - *shortage cost* - *purchasing cost*.

The expected fuzzy sales quantity, expected fuzzy surplus inventory, and expected fuzzy shortage quantity are expressed as $S(q)$, $I(q)$, and $O(q)$. Then, by determining the optimal single spot purchase quantity q, the expected fuzzy profits of the risk-neutral retailer $\tilde{\prod}_r^S$ are maximized, and therefore, they can be expressed as follows:

$$\max E[\tilde{\prod}_r^S]=p\times S(q)+v\times I(q)-s\times O(q)-wq \tag{1}$$

where $\underline{a}<q<\overline{a}$.

In the objective function (1), $p\times S(q)$ is the sales income, $v\times I(q)$ is the salvage income, $s\times O(q)$ is the shortage cost and wq is the purchasing cost.

In addition, $S(q)$, $I(q)$ and $O(q)$ can be calculated as follows:

$$S(q)=E[\min(\tilde{D},q)]=q-\int_{\underline{a}}^{q}(q-x)\varphi(x)dx \tag{2}$$

$$I(q)=E[\max(q-\tilde{D},0)]=q-S(q)=\int_{\underline{a}}^{q}(q-x)\varphi(x)dx \tag{3}$$

$$O(q)=E[\max(\tilde{D}-q,0)]=E(\tilde{D})-\mathrm{S}(q)=\int_{q}^{\overline{a}}(x-q)\varphi(x)dx \tag{4}$$

Then, by substituting equations (2), (3), (4) into (1) at the same time, the objective function value $E[\tilde{\prod}_r^S]$ can be obtained as follows:

$$\begin{aligned}E[\tilde{\prod}_r^S]&=p\times\left(q-\int_{\underline{a}}^{q}(q-x)\varphi(x)dx\right)\\&+v\times\left(\int_{\underline{a}}^{q}(q-x)\varphi(x)dx\right)-s\times\left(\int_{q}^{\overline{a}}(x-q)\varphi(x)dx\right)-wq\end{aligned} \tag{5}$$

where $\underline{a}<q<\overline{a}$.

From equation (5) above,

$\dfrac{d^2E[\tilde{\prod}_r^S]}{dq^2}=-p\varphi(q)+v\varphi(q)-s\varphi(q)<0$, which means that $E[\tilde{\prod}_r^S]$ is concave in q. Therefore, the optimal solution should be satisfied with the following formula.

$$\frac{dE\left[\tilde{\prod}_r^S\right]}{dq}=p-p\Phi(q)+v\Phi(q)+s-s\Phi(q)-w=0$$

Then, the optimal solution q_A^{S*} can be obtained through the first-order necessary condition. The result is obtained in equation (6).

$$q_A^{S*} = \Phi^{-1}\left(\frac{p+s-w}{p+s-v}\right) \tag{6}$$

It is important to note that $\Phi^{-1}(x)$ is the inverse function of $\Phi(x)$, and the maximum profit of the risk-neutral retailer is $E\left[\tilde{\Pi}_r^S\left(\Phi^{-1}\left(\frac{p+s-w}{p+s-v}\right)\right)\right]$.

Proposition 1.

The optimal spot purchase quantity of the risk-neutral retailer q_A^{S*} decreases monotonously with respect to w.

Proof: $\frac{\partial q}{\partial w} = -\frac{\varphi'\left(\frac{p+s-w}{p+s-v}\right)}{p+s-v} < 0$. $\varphi'(x)$ is the first derivative of the credibility distribution function $\Phi^{-1}(x)$, the same as below.

Spot Purchasing Model for the Risk-averse Retailer under Fuzzy Demand

For retailers selling innovative products, they are often risk averse in the procurement decision-making process, and their risk-averse attitude will affect the optimal procurement decision.

The risk aversion behavior of the retailer in this model is described by the CVaR method. According to the general definition of CVaR, the expected fuzzy profits (also known as "conditional value at risk") of the single spot purchasing retailer with risk aversion can be expressed as:

$$\mathrm{CVaR}\left(\tilde{\Pi}_r^S\right) = \max_{q\in(\underline{a},\bar{a}),u\in R}\left\{u + \frac{1}{\eta}E\left[\min\left(\tilde{\Pi}_r^S - u, 0\right)\right]\right\} \tag{7}$$

For the simplicity of calculation, we set function (7) as

$$G(q,u) = u + \frac{1}{\eta}E\left[\min\left(\tilde{\Pi}_r^S - u, 0\right)\right] \tag{8}$$

Then, equation (5) is expressed as

$$E\left[\tilde{\Pi}_r^S\right] = \int_{\underline{a}}^{q}((p-w)q-(p-v)(q-x))\varphi(x)dx + \int_q^{\bar{a}}((p-w)q - s(x-q)\varphi(x)dx \tag{9}$$

By substituting equation (9) into equation (8), equation (8) can be calculated as

$$G(q,u) = u - \frac{1}{\eta}\int_{\underline{a}}^{q}(u-(v-w)q-(p-v)x)^{+}\varphi(x)dx - \frac{1}{\eta}\int_q^{\bar{a}}(u-(p-w)q+s(x-\mathrm{q}))^{+}\varphi(x)dx \tag{10}$$

Through the first order derivation of the function (10), the optimal solutions q_B^{S*} and u^{S*} can be obtained in equations (11) and (12), respectively.

$$q_B^{S*} = \frac{s\Phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right) + (p-v)\Phi^{-1}\left(\frac{p+s-w}{p+s-v}\eta\right)}{p+s-v} \tag{11}$$

$$u^{S*} = \frac{s(v-w)\Phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}{p+s-v} + \frac{(p-w+s)(p-v)\Phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v} \tag{12}$$

Therefore, the maximum profits are as follows:

$$\begin{aligned}\mathrm{CVaR}\left(\tilde{\Pi}_r^S\right) &= \frac{s(v-w)\Phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}{p+s-v} + \frac{(p-w+s)(p-v)\Phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v} \\ &- \frac{p-v}{\eta}\int_{\underline{a}}^{\Phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}\Phi(x)dx \\ &- \frac{s}{\eta}\left(\left(\bar{a} - \Phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)\right) - \int_{\Phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}^{\bar{a}}\Phi(x)dx\right)\end{aligned} \tag{13}$$

Proposition 2.

The optimal spot purchase quantity of the risk-averse retailer q_B^{S*} decreases monotonously with respect to w.

Proof:

$$\frac{\partial q}{\partial w} = -\frac{s\varphi'\left(1+\frac{v-w}{p+s-v}\eta\right)\eta + (p-v)\varphi'\left(\frac{p+s-w}{p+s-v}\eta\right)\eta}{(p+s-v)^2} < 0$$

E. Single Option Purchasing Model under Fuzzy Demand

This chapter builds a single option purchasing model under fuzzy demand, and according to different types of retailers, this single purchasing model includes a single option purchasing model of risk-neutral retailers and a single option purchasing model of risk-averse retailers.

Option Purchasing Model for the Risk-neutral Retailer under Fuzzy Demand

In this supply chain purchasing model, the main factors that affect the profits of a single option purchasing retailer are the following: sales income, shortage cost and purchasing cost. In addition, the retailer's fuzzy sales quantity, fuzzy shortage quantity and option execution quantity are $\min(\tilde{D}, Q)$, $\max(\tilde{D}-q, 0)$, and $\min(\tilde{D}, Q)$, respectively. Here, the fuzzy option execution quantity is equal to the fuzzy sales quantity.

The fuzzy profits of the single option purchasing retailer are as follows.

(1)when $\tilde{D} \le Q$,

$$\tilde{\Pi}_r^O = \textit{sales income} - \textit{option execution cost} - \textit{option purchasing cost}.$$

(2) when $\tilde{D} > Q$,

$$\tilde{\Pi}_r^O = \textit{sales income} - \textit{shortage cost} - \textit{option execution cost} - \textit{option purchasing cost}.$$

Since the expected fuzzy sales volume of the retailer is equal to the expected fuzzy option execution quantity in the single option purchasing decision, this paper assumes that the expected fuzzy sales quantity, the expected fuzzy shortage quantity and the expected fuzzy option execution quantity of the retailer are $S(Q)$, $O(Q)$, and $S(Q)$. Then, the maximum profits of retailers are roughly expressed as follows.

$$\max E\left[\tilde{\Pi}_r^O\right] = p \times S(Q) - s \times O(Q) - w_e \times S(Q) - w_p Q \tag{14}$$

where $\underline{a} < Q < \bar{a}$.

In addition, $S(Q)$ and $O(Q)$ can be calculated as follows:

$$S(Q) = E[\min(\tilde{D}, Q)] = Q - \int_{\underline{a}}^{Q} (Q - x)\varphi(x)dx \tag{15}$$

$$O(Q) = E[\max(\tilde{D} - Q, 0)] = E(\tilde{D}) - \mathrm{S}(Q) = \int_{Q}^{\bar{a}} (x - Q)\varphi(x)dx \tag{16}$$

Then, by substituting equations (15) and (16) into (14), the objective function value $E\left[\tilde{\Pi}_r^O\right]$ can be obtained as follows:

$$\begin{aligned} E\left[\tilde{\Pi}_r^O\right] &= (p - w_e) \times \left(Q - \int_{\underline{a}}^{Q} (Q - x)\varphi(x)dx\right) \\ &- s \times \left(\int_{Q}^{\bar{a}} (x - Q)\varphi(x)dx\right) - w_p Q \end{aligned} \tag{17}$$

where $\underline{a} < Q < \bar{a}$.

From equation (17),

$\frac{d^2 E[\tilde{\Pi}_r^O]}{dq^2} = -(p - w_e)\varphi(Q) - s\varphi(Q) < 0$, which means that $\frac{d^2 E[\tilde{\Pi}_r^O]}{dq^2}$ is negative and $E[\tilde{\Pi}_r^O]$ is concave in Q.

Therefore, the optimal solution should be satisfied with the following formula.

$$\frac{dE\left[\tilde{\Pi}_r^O\right]}{dQ} = (p - w_e)(1 - \Phi(Q)) - s(-1 + \Phi(Q)) - w_p$$

Then, the optimal solution Q_A^{O*} can be obtained as follows.

$$Q_A^{O*} = \Phi^{-1}\left(\frac{p + s - w_e - w_p}{p + s - w_e}\right) \tag{18}$$

The maximum profits can be expressed as

$$E\left[\tilde{\Pi}_r^O\left(\Phi^{-1}\left(\frac{p + s - w_e - w_p}{p + s - w_e}\right)\right)\right].$$

Proposition 3.

The optimal option purchase quantity of the risk-neutral retailers Q_A^{O*} decreases monotonously with respect to w_p and w_e.

Proof:

$$\frac{\partial Q}{\partial w_p} = -\varphi'\left(\frac{p + s - w_e - w_p}{p + s - w_e}\right)\frac{1}{p + s - w_e} < 0;$$

$$\frac{\partial Q}{\partial w_e} = -\varphi'\left(\frac{p + s - w_e - w_p}{p + s - w_e}\right)\frac{-w_p}{(p + s - w_e)^2} < 0.$$

Option Purchasing Model for the Risk-averse Retailer under Fuzzy Demand

The expected fuzzy profits of the single option purchasing retailer with risk aversion can be expressed as:

$$\mathrm{CVaR}\left(\tilde{\Pi}_r^O\right) = \max_{Q \in (\underline{a}, \bar{a}), u \in R}\left\{u + \frac{1}{\eta} E\left[\min\left(\tilde{\Pi}_r^O - u, 0\right)\right]\right\} \tag{19}$$

For the simplicity of calculation, we set function (19) as

$$G(Q, u) = u + \frac{1}{\eta} E\left[\min\left(\tilde{\Pi}_r^O - u, 0\right)\right] \tag{20}$$

Then, equation (17) can be written as

$$\begin{aligned} E\left[\tilde{\Pi}_r^O\right] &= \left(\int_{\underline{a}}^{Q}\left(-w_p Q + (p - w_e)x\right)\varphi(x)dx\right) \\ &+ \left(\int_{Q}^{\bar{a}}\left((p + s - w_e - w_p)Q - sx\right)\varphi(x)dx\right) \end{aligned} \tag{21}$$

By substituting equation (21) into equation (20), equation (20) can be calculated as

$$\begin{aligned} G(Q, u) &= u - \frac{1}{\eta}\int_{\underline{a}}^{Q}\left(u - \left(-w_p Q + (p - w_e)x\right)\right)^{+}\varphi(x)dx \\ &- \frac{1}{\eta}\int_{Q}^{\bar{a}}\left(u - \left((p + s - w_e - w_p)Q - sx\right)\right)^{+}\varphi(x)dx \end{aligned} \tag{22}$$

Through the first order derivation of the function (22), the optimal solutions Q_B^{O*} and u^{O*} can be obtained in equations (23) and (24), respectively.

$$\begin{aligned} Q_B^{O*} &= + \frac{s\Phi^{-1}\left(1 - \frac{w_p \eta}{p + s - w_e}\right)}{p + s - w_e} \\ &+ \frac{(p - w_e)\Phi^{-1}\left(\frac{(p + s - w_e - w_p)\eta}{p + s - w_e}\right)}{p + s - w_e} \end{aligned} \tag{23}$$

$$\begin{aligned} u^{O*} &= -\frac{w_p s\Phi^{-1}\left(1 - \frac{w_p \eta}{p + s - w_e}\right)}{p + s - w_e} \\ &+ \frac{(p + s - w_e - w_p)(p - w_e)\phi^{-1}\left(\frac{(p + s - w_e - w_p)\eta}{p + s - w_e}\right)}{p + s - w_e} \end{aligned} \tag{24}$$

Therefore, the maximum profits are as follows:

$$\begin{aligned} &\mathrm{CVaR}\left(\tilde{\Pi}_r^O\right) \\ &= \frac{(p + s - w_e - w_p)(p - w_e)\Phi^{-1}\left(\frac{(p + s - w_e - w_p)\eta}{p + s - w_e}\right)}{p + s - w_e} \\ &- \frac{w_p s\Phi^{-1}\left(1 - \frac{w_p \eta}{p + s - w_e}\right)}{p + s - w_e} - \frac{p - w_e}{\eta}\int_{\underline{a}}^{\Phi^{-1}\left(\frac{(p+s-w_e-w_p)\eta}{p+s-w_e}\right)}\Phi(x)dx \\ &- \frac{s}{\eta}\left(\bar{a} - \Phi^{-1}\left(1 - \frac{w_p \eta}{p + s - w_e}\right)\right) - \int_{\Phi^{-1}}^{\bar{a}}\left(1 - \frac{w_p \eta}{p + s - w_e}\right)\Phi(x)dx \end{aligned} \tag{25}$$

Proposition 4.

The optimal option purchase quantity of the risk-averse retailer Q_B^{O*} decreases monotonously with respect to w_p and w_e.

Proof:

$$\frac{\partial Q}{\partial w_p}=\frac{s\eta\varphi'\left(1-\frac{w_p\eta}{p+s-w_e}\right)}{\left(p+s-w_e\right)^2};$$

$$-\frac{\left(p-w_e\right)\eta\varphi'\left(\frac{\left(p+s-w_e-w_p\right)\eta}{p+s-w_e}\right)}{\left(p+s-w_e\right)^2}<0$$

$$\frac{\partial Q}{\partial w_e}=-\frac{w_p\eta}{\left(p+s-w_e\right)^2}\varphi'\left(\eta-\frac{w_p\eta}{p+s-w_e}\right)$$

$$+s\frac{-\frac{w_p\eta}{p+s-w_e}\left(\varphi'\left(1-\frac{w_p\eta}{p+s-w_e}\right)-\varphi'\left(\eta-\frac{w_p\eta}{p+s-w_e}\right)\right)}{\left(p+s-w_e\right)^2}$$

$$+s\frac{-\Phi^{-1}\left(1-\frac{w_p\eta}{p+s-w_e}\right)+\Phi^{-1}\left(\eta-\frac{w_p\eta}{p+s-w_e}\right)}{\left(p+s-w_e\right)^2}<0$$

F. Mixed Purchasing Model of Spot and Option Contracts under Fuzzy Demand

On the basis of Section III.D and Section III.E, this chapter combines the spot contract with the option contract, building a mixed purchasing model under fuzzy demand.

In the process of constructing and solving the model, triangular fuzzy numbers are used to represent the market demand of innovative products, the CVaR method is used to express the conditional risk value of risk-averse retailers, and finally, the credibility theory is combined to solve the model.

Mixed Purchasing Model for the Risk-neutral Retailer under Fuzzy Demand

In this supply chain purchasing model, the main factors that affect the profits of spot and option mixed purchasing retailers are the following: sales income, salvage income, shortage cost and purchasing cost. Additionally, the retailer's fuzzy sales quantity, fuzzy surplus inventory, fuzzy shortage quantity and fuzzy option execution quantity are $\min(\tilde{D},\ q_m+Q_m)$, $\max(\tilde{D}-q_m,\ 0)$, $\max(\tilde{D}-(q_m+Q_m),\ 0)$, and $\max(\min(\tilde{D}-q_m,Q_m),\ 0)$, respectively.

The fuzzy profits of the spot and option mixed purchasing retailer are as follows.

(1) When $\tilde{D}\le q_m$,

$$\begin{aligned}\tilde{\prod}_r^M &= \textit{sales income}+\textit{salvage income}\\ &\textit{-spot purchasing cost - option purchasing cost}\end{aligned}.$$

(2) When $q_m<\tilde{D}\le q_m+Q_m$,

$$\begin{aligned}\tilde{\prod}_r^M &= \textit{sales income - spot purchasing cost}\\ &\textit{-option purchasing cost - option execution cost}\end{aligned}.$$

(3) When $q_m+Q_m<\tilde{D}$,

$$\begin{aligned}\tilde{\prod}_r^M &= \textit{sales income - shortage cost - spot purchasing cost}\\ &\textit{-option purchasing cost - option execution cost}\end{aligned}$$

This paper assumes that the expected fuzzy sales quantity, expected fuzzy surplus inventory, expected fuzzy shortage quantity and expected fuzzy option execution quantity are $S(q_m,Q_m)$, $I(q_m,Q_m)$, $O(q_m,Q_m)$ and $E(q_m,Q_m)$. Then, the maximum profits of the mixed purchasing retailers are roughly expressed as follows.

$$\begin{aligned}E\left[\tilde{\prod}_r^{\mathrm{M}}\right] &= p\times S(q_m,Q_m)+v\times I(q_m,Q_m)\\ &-s\times O(q_m,Q_m)-w_e\times E(q_m,Q_m)-wq_m-w_pQ_m\end{aligned}\tag{26}$$

where $\underline{a}<q_m+Q_m<\bar{a}$.

In addition, $S(q_m,Q_m)$, $I(q_m,Q_m)$, $O(q_m,Q_m)$ and $E(q_m,Q_m)$ can be calculated as follows:

$$\begin{aligned}S(q_m,Q_m) &= E[\min(\tilde{D},q_m+Q_m)]\\ &= q_m+Q_m-\int_{\underline{a}}^{q_m+Q_m}(q_m+Q_m-x)\varphi(x)dx\end{aligned}\tag{27}$$

$$\begin{aligned}I(q_m,Q_m) &= E[\max(q_m-\tilde{D},0)]\\ &= -E[\min(q_m,\tilde{D})]+q_m=\int_{\underline{a}}^{q_m}(q_m-x)\varphi(x)dx\end{aligned}\tag{28}$$

$$\begin{aligned}O(q_m,Q_m) &= E[\max(\tilde{D}-(q_m+Q_m),0)]\\ &= \int_{q_m+Q_m}^{\bar{a}}(x-q_m-Q_m)\varphi(x)dx\end{aligned}\tag{29}$$

$$\begin{aligned}E(q_m,Q_m) &= E[\max(\min(\tilde{D}-q_m,Q_m),0)]\\ &= I(q_m,Q_m)+S(q_m,Q_m)-q_m\\ &= Q_m+\int_{\underline{a}}^{q_m}(q_m-x)\varphi(x)dx-\int_{\underline{a}}^{q_m+Q_m}(q_m+Q_m-x)\varphi(x)dx\end{aligned}\tag{30}$$

Then, by substituting equations (27), (28), (29), and (30) into (26) at the same time, the objective function value $E\left[\tilde{\prod}_r^{\mathrm{M}}\right]$ can be obtained as follows:

$$\begin{aligned}E\left[\tilde{\prod}_r^{\mathrm{M}}\right] &= p\times\left(q_m+Q_m-\int_{\underline{a}}^{q_m+Q_m}(q_m+Q_m-x)\varphi(x)dx\right)\\ &+v\times\int_{\underline{a}}^{q_m}(q_m-x)\varphi(x)dx-s\times\int_{q_m+Q_m}^{\bar{a}}(x-q_m-Q_m)\varphi(x)dx\\ &-w_e\times\left(\begin{aligned}&Q_m+\int_{\underline{a}}^{q_m}(q_m-x)\varphi(x)dx\\ &-\int_{\underline{a}}^{q_m+Q_m}(q_m+Q_m-x)\varphi(x)dx\end{aligned}\right)\\ &-wq_m-w_pQ_m\end{aligned}\tag{31}$$

where $\underline{a}<q_m+Q_m<\bar{a}$.

Since the Hesse Matrix of $E\left[\tilde{\prod}_r^{\mathrm{M}}\right]$ can be proved to be negative definite, $E\left[\tilde{\prod}_r^{\mathrm{M}}\right]$ is concave in q_m and Q_m, and the optimal solutions q_{mA}^{M*} and Q_{mA}^{M*} can be obtained by solving the first derivative of the function (31). However, there are two kinds of optimal solutions, which are as follows.

(1) When $\frac{p+s-w_e-w_p}{p+s-w_e}>\frac{w_e+w_p-w}{w_e-v}$, the results are obtained in equations (32) and (33).

$$q_{mA}^{M*}=\Phi^{-1}\left(\frac{w_e+w_p-w}{w_e-v}\right)\tag{32}$$

$$Q_{mA}^{M*}=\Phi^{-1}\left(\frac{p+s-w_e-w_p}{p+s-w_e}\right)-q_m\tag{33}$$

The maximum profits are expressed as

$$E\left[\begin{array}{l}\tilde{\prod}_r^{\mathrm{M}}\left(\Phi^{-1}\left(\frac{w_e+w_p-w}{w_e-v}\right)\right),\\ \tilde{\prod}_r^{\mathrm{M}}\left(\Phi^{-1}\left(\frac{p+s-w_e-w_p}{p+s-w_e}\right)\right)-\Phi^{-1}\left(\frac{w_e+w_p-w}{w_e-v}\right)\end{array}\right] \quad (34)$$

(2) When $\frac{p+s-w_e-w_p}{p+s-w_e} \le \frac{w_e+w_p-w}{w_e-v}$, the results are obtained in equations (35) and (36).

$$q_{mA}^{M*}=\Phi^{-1}\left(\frac{p+s-w}{p+s-v}\right) \quad (35)$$

$$Q_{mA}^{\mathrm{M}*}=0 \quad (36)$$

In this kind of situation, the retailer will refuse to adopt the option contract, and the model is the same as the "single spot purchasing model under fuzzy demand" presented in Section III.D. Therefore, the solution of this group is not discussed here.

Proposition 5.

The optimal mixed spot purchase quantity of the risk-neutral retailers q_{mA}^{M*} decreases monotonously with respect to w and increases monotonously with respect to w_p and w_e. The optimal mixed option purchase quantity of the risk-neutral retailers $Q_{mA}^{\mathrm{M}*}$ increases monotonously with respect to w and decreases monotonously with respect to w_p and w_e.

Proof: $\frac{\partial q_m}{\partial w}=-\varphi'\left(\frac{w_e+w_p-w}{w_e-v}\right)\frac{1}{w_e-v}<0$,

$$\frac{dq_m}{dw_p}=\varphi'\left(\frac{w_e+w_p-w}{w_e-v}\right)\frac{1}{w_e-v}>0,$$

$$\frac{\partial q_m}{\partial w_e}=\varphi'\left(\frac{w_e+w_p-w}{w_e-v}\right)\frac{w-v-w_p}{(w_e-v)^2}>0,$$

$$\frac{\partial Q_m}{\partial w}=\varphi'\left(\frac{w_e+w_p-w}{w_e-v}\right)\frac{1}{w_e-v}>0,$$

$$\frac{dQ_m}{dw_p}=-\varphi'\left(\frac{p+s-w_e-w_p}{p+s-w_e}\right)\frac{1}{p+s-w_e}-\varphi'\left(\frac{w_e+w_p-w}{w_e-v}\right)\frac{1}{w_e-v}<0,$$

$$\frac{\partial Q_m}{\partial w_e}=-\varphi'\left(\frac{p+s-w_e-w_p}{p+s-w_e}\right)\frac{w_p}{(p+s-w_e)^2}-\varphi'\left(\frac{w_e+w_p-w}{w_e-v}\right)\frac{w-v-w_p}{(w_e-v)^2}<0.$$

Proposition 6.

For risk-neutral retailers, the main factor that affects the ratio of spot to option purchases in the supply chain is the numerical relationship in the contract parameters. If the supplier who provides the option contract wants the retailer to sign the option contract, then the contract parameters need to be set reasonably to make $\frac{p+s-w_e-w_p}{p+s-w_e}>\frac{w_e+w_p-w}{w_e-v}$.

Proof: See the two kinds of optimal solutions of the mixed purchasing risk-neutral retailer.

Proposition 7.

In this paper, the sum of the mixed spot purchases and mixed option purchases is called the "mixed total purchase quantity". The total mixed purchase volume of risk-neutral retailers is $\Phi^{-1}\left(\frac{p+s-w_e-w_p}{p+s-w_e}\right)$. In the risk-neutral case, the total mixed purchase is equal to the single option purchase. In other words, the mixed purchasing decision and the single option purchasing decision can provide the same level of matching of supply and demand for risk-neutral retailers.

Proof: See the optimal solutions of the single option purchasing model in Section III.E and the mixed purchasing model for the risk-neutral retailer in Section III.F.

Mixed Purchasing Model for the Risk-averse Retailer under Fuzzy Demand

In the same way, the expected fuzzy profits of the mixed purchasing retailer with risk aversion can be expressed as:

$$\mathrm{CVaR}\left(\tilde{\prod}_r^{\mathrm{M}}\right)=\max_{q_m+Q_m\in(\underline{a},\bar{a}),u\in R}\left\{u+\frac{1}{\eta}E\left[\min\left(\tilde{\prod}_r^{\mathrm{M}}-u,0\right)\right]\right\} \quad (37)$$

For the simplicity of calculation, we set function (37) as

$$G(q_m,Q_m,u)=u+\frac{1}{\eta}E\left[\min\left(\tilde{\prod}_r^{\mathrm{M}}-u,0\right)\right] \quad (38)$$

Equation (31) can be expressed as

$$\begin{aligned}E\left[\tilde{\prod}_r^{\mathrm{M}}\right]&=\int_{\underline{a}}^{q_m}\left((v-w)q_m-w_pQ_m+(p-v)x\right)\varphi(x)dx\\&+\int_{q_m}^{q_m+Q_m}\left((w_e-w)q_m-w_pQ_m+(p-w_e)x\right)\varphi(x)dx\\&+\int_{q_m+Q_m}^{\bar{a}}\left((p+s-w)q_m+(p+s-w_e-w_p)Q_m-sx\right)\varphi(x)dx\end{aligned} \quad (39)$$

Through substituting equation (39) into equation (38), equation (38) can be expressed as

$$\begin{aligned}&G(q_m,Q_m,u)=\\&u-\frac{1}{\eta}\int_{\underline{a}}^{q_m}\left(u-\left((v-w)q_m-w_pQ_m+(p-v)x\right)\right)^+\varphi(x)dx\\&-\frac{1}{\eta}\int_{q_m}^{q_m+Q_m}(u-\left((w_e-w)q_m-w_pQ_m+(p-w_e)x\right))^+\varphi(x)dx\\&-\frac{1}{\eta}\int_{q_m+Q_m}^{\bar{a}}\left(u-\left((p+s-w)q_m+(p+s-w_e-w_p)Q_m-sx\right)\right)^+\\&\varphi(x)dx\end{aligned} \quad (40)$$

Then, by calculating the first order derivation of the function (40), the optimal solutions q_{mB}^{M*}, $Q_{mB}^{\mathrm{M}*}$ and $u^{\mathrm{M}*}$ can be obtained. However, there are two kinds of optimal solutions, which are as follows.

(1) When

$$\frac{(p-w_e)\Phi^{-1}(n_1)+s\Phi^{-1}(n_2)}{p+s-w_e}-\Phi^{-1}\left(\frac{(w_e-w)n_1+(p+s-w)(1-n_2)}{w_e-v}\right)>0 \text{ and}$$

$(p+s-w)w_p \neq (p+s-w_e-w_p)(w-v)$, the results can be obtained in equations (41), (42), and (43). ($n_1 = \frac{\left(p+s-w_e-w_p\right)\eta}{p+s-w_e}$, $n_2 = 1-\frac{w_p\eta}{p+s-w_e}$)

$$q_{mB}^{M*} = \Phi^{-1}\left(\frac{\left(w_e+w_p-w\right)\eta}{w_e-v}\right) \quad (41)$$

$$Q_{mB}^{M*} = \frac{\left(p-w_e\right)\Phi^{-1}\left(n_1\right)+s\Phi^{-1}\left(n_2\right)}{p+s-w_e} - q_m$$
$$= \frac{\left(p-w_e\right)\Phi^{-1}\left(n_1\right)+s\Phi^{-1}\left(n_2\right)}{p+s-w_e} - \Phi^{-1}\left(\frac{\left(w_e+w_p-w\right)\eta}{w_e-v}\right) \quad (42)$$

$$u^{M*} = \left(p-w_e\right)\Phi^{-1}\left(n_1\right) - w_pQ_m + \left(w_e-w\right)q_m$$
$$= \frac{\left(p+s-w_e-w_p\right)\left(p-w_e\right)\Phi^{-1}\left(n_1\right) - w_ps\Phi^{-1}\left(n_2\right)}{p+s-w_e} \quad (43)$$
$$+\left(w_e-w+w_p\right)\Phi^{-1}\left(\frac{\left(w_e+w_p-w\right)\eta}{w_e-v}\right)$$

Then, the maximum profits of the risk-averse retailer can be expressed as

$$G(q_m,Q_m,u) =$$
$$u-\frac{1}{\eta}\int_{\underline{a}}^{q_m}\left(u-\left((v-w)q_m-w_pQ_m+(p-v)x\right)\right)\varphi(x)dx$$
$$-\frac{p-w_e}{\eta}\left(\int_{q_m}^{\Phi^{-1}(n_1)}\Phi(x)dx - \left(\Phi^{-1}\left(n_1\right)-q_m\right)\Phi(q_m)\right) \quad (44)$$
$$-\frac{s}{\eta}\left(\left(\bar{a}-\Phi^{-1}\left(n_2\right)\right)\Phi(\bar{a}) - \int_{\Phi^{-1}(n_2)}^{\bar{a}}\Phi(x)dx\right)$$

(2) When

$$\frac{\left(p-w_e\right)\Phi^{-1}\left(n_1\right)+s\Phi^{-1}\left(n_2\right)}{p+s-w_e} - \Phi^{-1}\left(\frac{\left(w_e-w\right)n_1+(p+s-w)\left(1-n_2\right)}{w_e-v}\right) \leq 0$$

or $(p+s-w)w_p = (p+s-w_e-w_p)(w-v)$, the results can be obtained in equations (45), (46), and (47).

$$q_{mB}^{M*} = \frac{s\Phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right) + (p-v)\Phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v} \quad (45)$$

$$Q_{mB}^{M*} = 0 \quad (46)$$

$$u^{M*} = \frac{s(v-w)\Phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}{p+s-v} + \frac{(p-w+s)(p-v)\Phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v} \quad (47)$$

In this situation, the retailer will refuse to adopt the option contract, and the model is the same as the "single spot purchasing model under fuzzy demand" presented in Section III.D. Therefore, the solution of this group is not discussed here.

Proposition 8.

The optimal mixed spot purchase quantity of the risk-averse retailers q_{mB}^{M*} decreases monotonously with respect to w and increases monotonously with respect to w_p and w_e. The optimal mixed option purchase quantity Q_{mB}^{M*} increases monotonously with respect to w and decreases monotonously with respect to w_p and w_e. Therefore, the higher the option purchasing price and the executive price, the more that retailers tend to make spot purchases. The higher the wholesale cost of the product, the less the retailer relies on spot purchases, and then, more options are purchased.

Proof: $\frac{\partial q_m}{\partial w} = -\varphi'\left(\frac{\left(w_e+w_p-w\right)\eta}{w_e-v}\right)\frac{\eta}{w_e-v} < 0$,

$$\frac{dq_m}{dw_p} = \varphi'\left(\frac{\left(w_e+w_p-w\right)\eta}{w_e-v}\right)\frac{\eta}{w_e-v} > 0,$$

$$\frac{\partial q_m}{\partial w_e} = \varphi'\left(\frac{\left(w_e+w_p-w\right)\eta}{w_e-v}\right)\frac{w-v-w_p}{\left(w_e-v\right)^2}\eta > 0,$$

$$\frac{\partial Q_m}{\partial w} = \varphi'\left(\frac{\left(w_e+w_p-w\right)\eta}{w_e-v}\right)\frac{\eta}{w_e-v} > 0,$$

$$\frac{dQ_m}{dw_p} = \frac{-\left(p-w_e\right)\eta\varphi'\left(n_1\right) - s\eta\varphi'\left(n_2\right)}{\left(p+s-w_e\right)^2} - \varphi'\left(\frac{\left(w_e+w_p-w\right)\eta}{w_e-v}\right)\frac{\eta}{w_e-v} < 0,$$

$$\frac{\partial Q_m}{\partial w_e} = -\frac{w_p\eta}{\left(p+s-w_e\right)^2}\varphi'\left(n_1\right) + s\frac{-\frac{w_p\eta}{p+s-w_e}\left(\varphi'\left(n_2\right)-\varphi'\left(n_1\right)\right)}{\left(p+s-w_e\right)^2} + s\frac{-\Phi^{-1}\left(1-\frac{w_p\eta}{p+s-w_e}\right)+\Phi^{-1}\left(n_1\right)}{\left(p+s-w_e\right)^2} - \varphi'\left(\frac{\left(w_e+w_p-w\right)\eta}{w_e-v}\right)\frac{\eta}{w_e-v} < 0.$$

Proposition 9.

If the supplier providing the option contract wants the retailer to sign the option contract, then the contract parameters need to be set reasonably to make

$$\frac{\left(p-w_e\right)\Phi^{-1}\left(n_1\right)+s\Phi^{-1}\left(n_2\right)}{p+s-w_e} - \Phi^{-1}\left(\frac{\left(w_e-w\right)n_1+(p+s-w)\left(1-n_2\right)}{w_e-v}\right) > 0$$

and $(p+s-w)w_p \neq (p+s-w_e-w_p)(w-v)$.

Proof: See the two kinds of optimal solutions of the mixed purchasing risk-averse retailer.

Proposition 10.

The total mixed purchase volume of risk-averse retailers is

$$\frac{(p-w_e)\Phi^{-1}\left(\frac{(p+s-w_e-w_p)\eta}{p+s-w_e}\right)+s\Phi^{-1}\left(1-\frac{w_p\eta}{p+s-w_e}\right)}{p+s-w_e}.$$

In the case of risk aversion, the total mixed purchase quantity is equal to the single option purchase quantity. In other words, the mixed purchasing decision and single option purchasing decision can provide the same level of matching of supply and demand for risk-averse retailers.

Proof: See the optimal solutions of the single option purchasing model in Section III.D and the mixed purchasing model for the risk-averse retailer in Section III.F.

IV. Numerical Examples

Based on the model proposed in the previous section, this section conducts a numerical analysis of the single spot, single option, spot and option mixed procurement models of risk-averse retailers under fuzzy demand. Then, the following chapter shows the influence of different purchasing decisions on the retailer's purchasing behavior and profits under fuzzy demand. Moreover, the impact of model parameters such as the degree of risk aversion and fuzziness of market demand and contract parameters on the behavior and profits of retailers are also presented.

By combining the characteristics of innovative products and investigating a newly launched electronic product in the market, this paper sets the initial value of the model parameters as follows:

$\eta = 0.8$, $p = 1000$, $w = 400$, $w_p = 200$, $w_e = 250$, $s = 200$, $v = 100$, and $\tilde{D} = (1000, 5000, 8000)$.

A. Analysis of Retailer Purchase Quantity

This section calculates the retailer's optimal decision by using Matlab; the influence of model parameters such as risk aversion, market demand fuzziness and other contract parameters on the purchase quantity is analyzed; the retailer's purchasing behaviors under different purchasing decisions are compared.

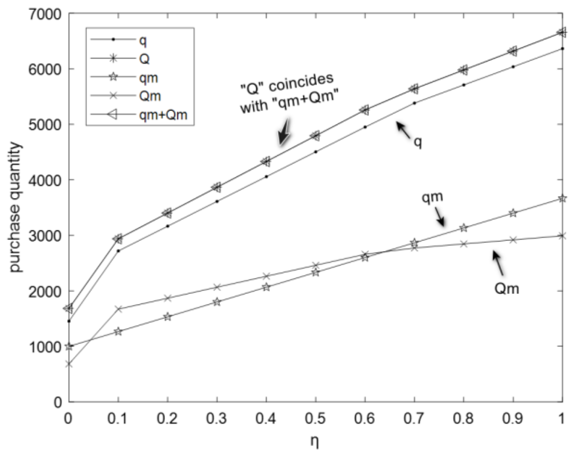

Fig. 2. Influence of the risk aversion coefficient η on the purchase quantity

From Figure 2, we can obviously see that the curve "Q" coincides with the curve "q_m+Q_m". It represents that the purchase quantity of the single option decision is the same as the total purchase quantity of the mixed procurement. This situation is the same in the following parts. Moreover, as the value of η changes, the retailer's purchasing decision has the following characteristics:

1) Regardless of the purchasing decision of the retailer (single purchase or mixed purchase), the purchase quantity of the risk-neutral retailer (when η=1) is always higher than that of the risk-averse retailer.
2) With the increase of retailers' risk aversion, retailers tend to purchase fewer products. This occurs because when facing fuzzy market demand, the greater the risk aversion of retailers, the more conservative their decision-making behavior.
3) Regardless of the degree of risk aversion, the total purchase quantity of the option decision is always higher than that of single spot purchases because options provide retailers with quantity flexibility and reduce the market demand risk, which make retailers tend to purchase more products.

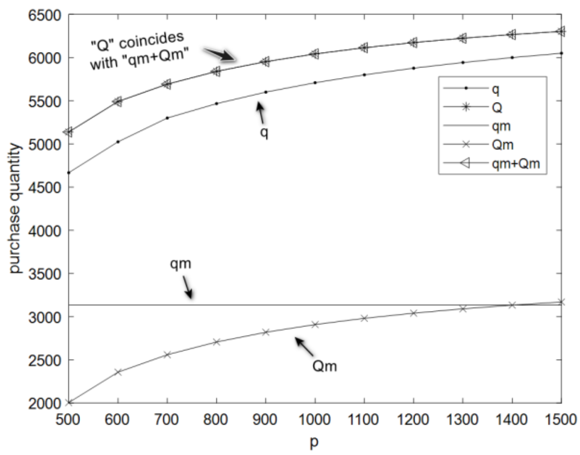

Fig. 3. Influence of the product retail price p on the purchase quantity

Figure 3 shows the retailer's purchasing behavior with different p values, and we find the following:

1) As the product retail price increases, the total purchase quantity of single spot purchases, single option purchases and mixed purchases increases, and the total purchase quantity under different purchase decisions gradually approaches. In other words, the change range of the single spot purchase volume is greater than that with the option decision. This is because the purchase quantity under the flexible procurement decision better matches the market demand, and the flexible procurement decision is not easily affected by the change of the retail price of the product; in addition, the nonflexible procurement contract tends to increase the purchase quantity when facing the temptation of a larger market to improve the matching of the market demand.
2) As the product retail price increases, the amount of mixed spot purchases remains the same and the amount of mixed option purchases increases. In other words, the mixed spot purchase volume has nothing to do with the retail price of innovative products; however, as the retail price of innovative products increases, retailers will tend to buy more options and increase their quantity flexibility to increase their profits.

3) No matter how the product retail price changes, the total purchase quantity of the retailer under the purchase decision of introducing options is always higher than that of the retailer under a single purchase decision.

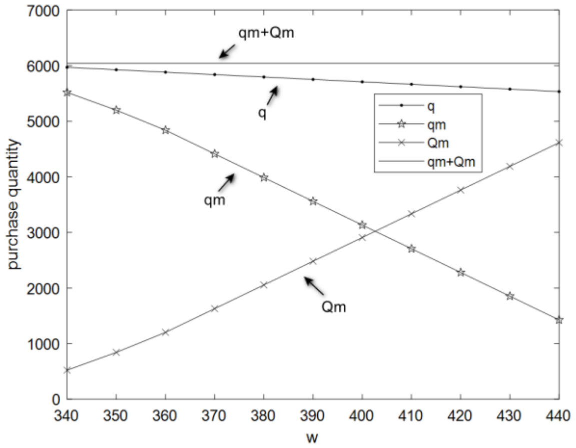

Fig. 4. Influence of the product wholesale cost w on the purchase quantity

In Figure 4, the single option decision is not involved since the product wholesale cost w has no effect on the single option procurement. From Figure 4, when the product wholesale cost increases, the quantity of single spot purchases decreases. Meanwhile, as the product wholesale cost increases, the mixed spot purchase quantity decreases and the mixed option purchase quantity increases to ensure the original degree of matching of supply and demand; therefore, the total mixed purchase quantity remains unchanged. This is because as the product wholesale cost increases, the single spot purchase cost increases while the option purchase cost remains unchanged. Retailers tend to reduce their mixed spot purchase quantity, which increases the mixed option purchase cost.

Therefore, in comparison with the single spot decision, the mixed decision reduces the retailers' dependence on the spot decision and make the purchasing behavior of retailers less affected by the spot cost to ensure a stable level of matching between supply and demand. When formulating the supply chain contract, we can change the proportion of the mixed spot purchase quantity and the mixed option purchase quantity in the mixed purchasing decision by adjusting the wholesale cost of the product.

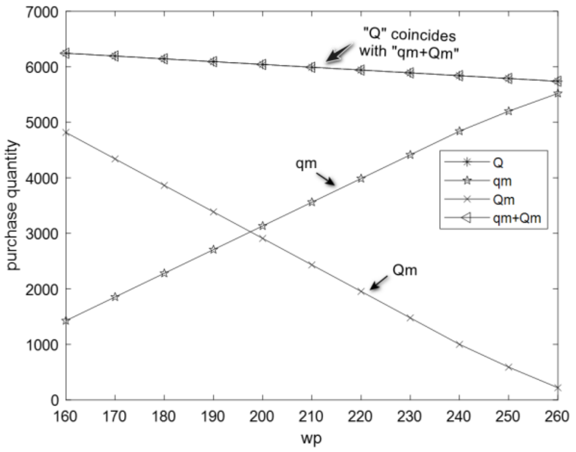

Fig. 5. Influence of the option purchase price w_p on the purchase quantity

Similarly, the single spot decision is still not involved since the option purchase price w_p has no effect on single spot procurement. Figure 5 indicates the impact of the change of w_p on the purchase quantity of retailers with different procurement decisions. Our insights include the following:

1) With the increase of the option purchase price, the purchase quantity of the single option decision and the total purchase quantity of the mixed purchase decision both decrease. As the option contract price increases, the procurement cost increases, and at this time, the retailer is faced with fuzzy demand. To obtain the most profit, retailers are likely to reduce the option purchase quantity.
2) When the option purchase price is at a low level, the mixed spot purchase quantity is lower than the mixed option purchase quantity. As the option purchase price increases, the mixed spot purchase quantity gradually increases and exceeds the mixed option purchase quantity, and the mixed option purchase quantity gradually decreases. This is because when the unit cost of spot procurement remains unchanged and the option procurement cost increases, retailers tend to choose lower-cost procurement methods, that is, they buy more spot contracts and fewer options.
3) Combined with Figure 4, the option price has more influence on the level of matching of supply and demand than the product wholesale cost. When formulating the supply chain contract, we can change the proportion of the mixed spot purchase quantity and mixed option purchase quantity of mixed purchasing decision retailers by adjusting the product's wholesale cost, the option purchase price and the executive price.

Generally, compared with the single option decision, when the total purchase quantity is the same, the mixed decision can make retailers less dependent on options and reduce the impact of option costs on retailer decisions.

B. Analysis of Retailer's Expected Fuzzy Profits

By substituting the relevant variables into the functions of the expected fuzzy profits of the retailers in the three risk-aversion retailer models under fuzzy demand, the corresponding images are drawn in Matlab, and the influences of different parameters on the retailer's profits while keeping the other parameters unchanged are analyzed.

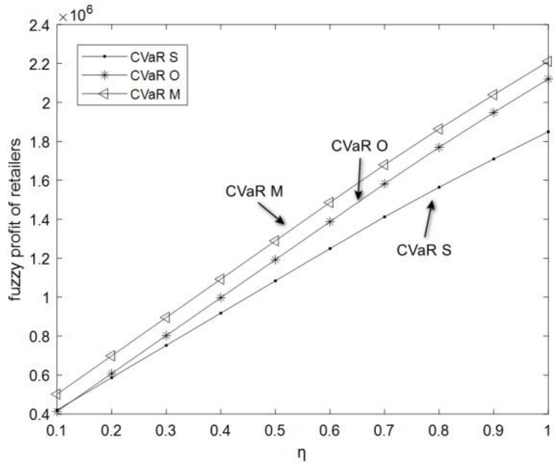

Fig. 6. Influence of the risk aversion coefficient η on the fuzzy profits of retailers

From Figure 6, as the value of η changes, the retailer's expected fuzzy profits have the following characteristics:

1) As the retailers' risk aversion decreases, the expected fuzzy profits of retailers under the three decisions are all increasing, which means that regardless of the decision, the expected fuzzy profits of the risk-neutral retailer (η=1) are higher those that of the risk-averse retailer.
2) As the retailers' risk aversion decreases, the increase of the expected fuzzy profits of the retailers under the flexible procurement decision with options is greater than that of the retailers with a single spot decision. In addition, regardless of the changes of retailer's risk aversion, the retailer's profits with the mixed decision are always higher than the retailer's profits with the single decision, which shows that when a lower degree of risk aversion exists, flexible decision-making can result in greater profits for retailers, and flexible decision-making combined with the initial stock volume can further improve the profits of retailers.

Combined with Figure 2 and Figure 6, it can be seen that the procurement decision with the option contract improves the matching of supply and demand, and reduces the impact of retailers' risk aversion on the profits.

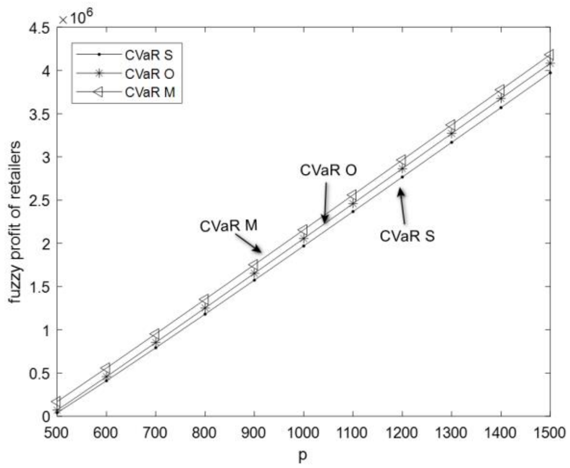

Fig. 7. Influence of the product retail price p on the fuzzy profits of retailers

In Figure 7, the retailers' expected fuzzy profits under the three decisions increase significantly when the retail price of the product increases.

Additionally, we find that the fuzzy profits under the mixed decision are always higher than those under the single decision, while the change of p does not significantly change the profit gap under the three decisions. When only p changes, the gap between the mixed decision profits and the single option decision profits remains unchanged, while the gap between the mixed decision profits and the single spot decision profits increases. In other words, compared to the decision with options, the profits of the single spot decision with lower supply and demand matching is gradually magnified as the product retail price p increases. This is because compared with the single spot decision, the decision with options provides a higher matching of supply and demand, reducing the product surplus and effectively controlling the risk of market price fluctuations.

Therefore, the introduction of option contracts can increase flexibility for retailers' purchasing decisions. When implementing the purchasing decision of innovative products, the comprehensive use of spot contract and option contract can effectively deal with the fuzzy uncertainty of market demand and improve profits.

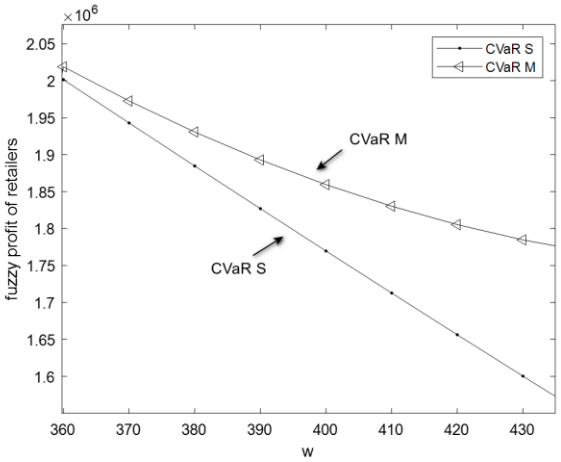

Fig. 8. Influence of the product wholesale cost w on the fuzzy profits of retailers

In Figure 8, we do not discuss the single option decision as the product wholesale cost w has no effect on the single option procurement. As can be seen from Figure 8, with the change of the value of the product wholesale cost w, the retailer's expected fuzzy profit has the following characteristics:

1) As the product wholesale cost increases, the expected fuzzy profits of retailers with a single spot decision and mixed purchasing decision decreases. As the spot purchasing cost increases, the retailer's purchase quantity decreases and the purchase cost increases under the single spot decision; therefore, the total profits decrease. Under the mixed purchasing decision, the retailer's total purchase quantity remains the same while the unit purchase cost increases, and therefore the profit decreases.
2) As the product wholesale cost increases, the gap between the fuzzy profits with single spot decision and the fuzzy profits with mixed decision is increasing, which shows that the expected fuzzy profits of the retailer with a single decision are more easily affected by the product wholesale cost w, while the expected fuzzy profits of the retailer with a mixed decision are more stable.
3) Combined with Figure 4, one of the key factors affecting the numerical relationship between mixed spot purchases and mixed options purchases is the product wholesale cost w. When the other parameters are fixed, a higher wholesale product cost w can reduce retailers' profits in spot purchases. In the case that option contracts can be selected, retailers will reduce their dependence on mixed spot procurement, thus improving the substitution of mixed option purchases. In other words, compared with a single spot contract, mixed decision-making can reduce retailers' dependence on spot purchases, and retailers expect fuzzy profits to be less affected by spot costs.

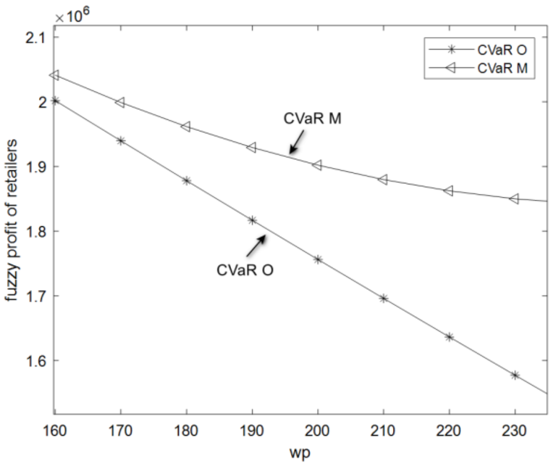

Fig. 9. Influence of the option purchase price w_p on the fuzzy profits of retailers

Again, we do not discuss the single spot decision here since the option purchase price w_p has no effect on single spot procurement. Figure 9 conveys several results:

1) As the option purchase price increases, the fuzzy profits of the retailer with a single option decision and the fuzzy profits with a mixed purchasing decision both decrease.
2) As the option purchase price increases, the gap between the fuzzy profits with a single option decision and the expected fuzzy profits with a mixed purchasing decision is increasing. This is because as the option contract cost increases, the mixed procurement decision reduces the procurement cost and improves the profit margin through the spot contract.
3) Compared with the single option decision, the mixed procurement decision reduces the influence of the option procurement cost on the profits of retailers.

In practice, when the uncertainty of market fuzzy demand is high, we should consider the spot and option mixed design supply chain procurement decision, which makes the retailer's procurement more flexible.

V. CONCLUSIONS AND FUTURE RESEARCH

In actual supply chain procurement decision research, due to the short life cycle and large fluctuations, the market demand of innovative products is often difficult to describe with probability theory. Therefore, retailers often show risk-averse behavior in purchasing innovative products. How to study the demand uncertainty of innovative products and determine the optimal purchasing decision of retailers has become a hot topic for all walks of life. This paper uses the fuzzy mathematics to express the market demand of innovative products, and studies the optimal purchasing decision of retailers in the presence of spot contract and option contract.

The research work and conclusions are as follows:

1) Considering the fuzzy demand and the risk aversion attitude of retailers, this paper builds the procurement decision model, and compares the effects of different degrees of risk aversions, contract parameters and market demand fuzziness on retailers' orders and profits under a single spot contract, a single option contract and a mixed contract.
2) The triangular fuzzy number is applied to the depiction on the uncertain market demand of innovative products. No matter how other model parameters change, the retailer's optimal mixed total purchase quantity always fluctuates around the fuzzy median of market demand. Namely, the retailer's mixed procurement decision can reduce the risk caused by market uncertainty.
3) For retailers selling innovative products, the mixed procurement decision of spot and option under fuzzy demand can significantly improve retailers' profits, and have the advantage of handling the risk of uncertainty in the market.

However, in the actual situation, supply chain members compete with each other and the market environment is complex and changeable. There are three aspects for further research in the future.

1) In this paper, the triangular fuzzy number is applied to the description on the fuzzy market demand. When using fuzzy mathematics to study market demand, some other fuzzy number forms can be used to express the market demand.
2) The influence of the innovation coefficient of innovative products on the market demand under fuzzy demand can be further considered, and the impact of the innovation coefficient on retailers' decision-making and profits can also be considered.
3) This paper takes a newly launched electronic product as the numerical example. In fact, this model can be further extended to fashion, toys and other innovative products and is suitable for situations with a short life cycle, fast product update, high demand volatility and lack of sufficient historical data on the market demand.

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