Green Hose-Rectangle Model Approach for Power Efficient Communication Networks*

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***Abstract*—Power efficiency of computer networks is an important issue for green computing. Currently available models such as the *green pipe* model minimizes the power consumption in the networks only for traffic demands which is fixed beforehand. In practice, the ongoing traffic demands can fluctuate due to different reasons, which the green pipe model cannot handle. On the contrary, some other existing models such as the *green hose* model can deal with traffic fluctuations, however with much lower power efficiency compared with the green pipe model. This research presents a robust *green hose-rectangle (green HR)* model that employs the advantages of the mentioned both kinds of models. In one hand, our proposed model improves the power efficiency, on the other hand, allows the traffic demands to fluctuate within some acceptable range. In our model, we use an uncertainty set which is the intersection of the rectangle and hose uncertainty sets to allow errors and traffic fluctuations. Our model is tractable by modern optimization software within a reasonable time although it is in the form of mixed-integer linear programming (MILP) problem. Our experiments show some promising results. The efficiency of our proposed model is improved in terms of power savings, number of deactivating links, and computation time when compared with the green hose model.**

***Index Terms*—robust optimization, green hose-rectangle model, power efficient, traffic fluctuations, green pipe model, uncertainty set, green networking.**

I. Introduction

THE world temperature is increasing day by day which is the main cause of global warming. Nowadays, computers, electronics industry, and large number of high data speed devices use much energy consumption and causes over radiation of green house gas. Furthermore, the internet users are increasing dramatically in this pandemic situation where a big amount of data are broadcasting through the internet. By providing green computing, the power consumption may be reduced from computer networks [1]. In modern communication system, minimizing power consumption is a major concern for environmental and also for economical impact. Too much power consumption negatively affects the global climate. The global warming is a big challenge for our environment and it is very hard to tackle it. Therefore, it is very essential to reduce or minimize power consumption in different sectors specially in communication networks. For green computing, we can manufacture and prepare devices which give out low temperature and gas with an intake of low energy. Low power consumption equipment in communication network design can reduce global warming and hence provide green computing [3]. Research on power-efficient network has earned a noticeable concern over the last decade for economical and environmental reasons. A good number of research on power efficient network have been proposed in the history for green computing. The power consumption by ICT devices is increasing rapidly which creates environmental and financial problems. The devices in the information and communication (ICT) sector absorbs a huge amount of energy. This sector can save energy by 2% to 10% by carefully choosing efficient devices with optimal design [2]-[5]. Nowadays, the network framework is growing and taking a big portion of the power footprint in ICT sector. The traditional approach adopted to design communications networks causes the extensive power consumption. By designing proper infrastructures in networks, providing power efficient devices, and proper routing with optimized methodology can minimize power utilization in communication networks. Power utilization in the ICT sector is rising to a great extent with ever-growing service requirements and plays an important role in global greenhouse gas radiation. Reducing such power utilization in communication networks is vital issue for financial and environmental reasons. This is why, the idea of power efficient networking is proposed. The green technology which is used in computing is referred as green computing. Proceeding the traffic by over-provisioning and link redundancy can ensure robustness against peak hours traffic and also can deter unpredictable facts such as failures. However such approaches will have a good number of power leaching active redundant links which are unused during low-traffic periods [7] and ultimately causes a huge power waste [6]. Our approach tries to deactivate the redundant links to save power.

A. Related Works

In regards of reducing the power consumption, the work of Bianzino et al. [8] is interesting. The authors presented the green pipe model to reduce energy consumption in networks. In that model, the authors fixed the network traffic demands

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and based on this exact traffic demands they formulated their model to minimize the power in communications networks. The aim of their research was to scale down the power by turning off some redundant links for green computing. Due to previously known traffic, the green pipe model obtains high efficiency. However in reality, network traffic regularly fluctuates and it is problematic for operators to estimate the actual traffic. Another limitation is that the green pipe model is not fully suitable in the case where the traffic usually fluctuates due to many reasons. We adopted the power model proposed by Bianzino et al. [8] and integrated trafic-fluctuation flexibility in it. We will discuss this in details in the model formulation section.

Chiaraviglio et al. [9] presented a formulation for internet service provider (ISP) networks to reduce power in communication networks. They minimize the ISP networks power consumption by turning off some nodes and links considering maximum link utilization constraints and guaranteeing full connectivity in the network. Their work is different from ours, they turn off nodes and links dealing with link employment constraints whereas we turn off unnecessary links only by adding upper and lower bounds of traffic demands.

Duffield et al. [10] proposed the hose model which does not need exact traffic demands. The hose model is opposed to the pipe model developed by Wang and Wang [11]. The hose model bounds the traffic with total incoming and outgoing traffic for each node in the network. In the hose model, all possible sets of traffic demands are examined which leads to the lower performance in power saving compared to the pipe model. The hose model is known as more extensible service model. Therefore, in our model, we would like to tighten the range of traffic illustrated by the hose model in order to enhance the routing performance.

Oki and Iwaki [12] proposed the intermediate model to present an optimal routing, which tightens the range of traffic volumes expressed by the hose model. The achievement of the intermediate model is higher than the hose model in terms of minimizing the congestion ratio due to limited range of traffic condition. Similar to hose model, Ouédraogo and Oki [13] presented the green hose model with bound of link traffic (green HLT) model considering traffic uncertainty to gain power savings. The HLT proposed by Oki and Ouédraogo [14] is used to developed the green HLT model, which extends the hose model [15]. In the HLT model, the total volume of traffic deliberate on each link determines the additional traffic bounded for that link. The authors added a parameter in the green HLT model that demonstrates uncertainty to traffic volume deliberated on each link. The authors incorporated this parameter in order to provide the robustness in the model. The green HLT model is different from our proposed model in terms of setting parameters and traffic bound. Our proposed model with rectangle uncertainty set compacts the range of traffic by lower and upper bounds. Also in our model, we do not need any initial routing where the green HLT model [13] needs an initial routing.

B. Robust Optimization

The concept of robust optimization is developed from the field of optimization [16], [17], [18], [19] in the last decades. Some data of an optimization problem are not given directly in robust optimization, but it is said that the data are contained to a predefined set, which is called the *uncertainty set*. In our proposed model we used the *uncertainty set* to allow the flexibility of traffic-fluctuation. In the worst case, optimizing the problem is the objective of robust optimization where some data of the problem belong to the uncertainty set. In general, robust optimization problems are tractable and classified into so-called semi-infinite programming (SDP) problems. A problem is called a *robust counterpart* of an SDP problem if the problem is developed as a tractable optimization problem. A good number of research in real-life field such as communications [13], [20], mechanics [21], finance [22], management [23], and control [24], [25] are using robust optimization technique. For more details about robust optimization readers may refer to Ben-Tal Nemirovski [16] and Boyd Vandenberghe [19].

C. Problem Statement

The green pipe model presented by Bianzino et al. [8] is developed on the basis of fixed traffic demands and it is not robust to traffic fluctuations. Since traffic usually fluctuate due to many reasons and customers need, the green pipe model can not handle the traffic fluctuations. The green hose model is robust to traffic change but its contribution in terms of power savings is much lower than the pipe model. The green HLT model [13] is also robust but their traffic fluctuation considers link-local bounds, which uses traffic amount broadcasting into each link, in addition to the hose bounds. Although the green HLT model achieves a good performance, the weakness is that it can not deal with total amount of fluctuation over the network. Besides, in the green HLT model, they need initial routing which is not desirable to maintain network stability. The green HR model that we propose through current research may address those problems.

D. Contribution

In this paper, we introduce the green HR model to reduce power in communication networks for green computing. The green HR model bounds the traffic with upper and lower bounds and allows fluctuations in traffic demands using robust optimization technique. To do this in green computing, our methodology starts with the idea proposed by Oki and Iwaki [12]. In our model, the network operators can set the lower and upper bounds of traffic from their past traffic data and operational experience. To develop our model, we consider that the real traffic demand is accommodated in an uncertainty set which is the intersection of the rectangle and hose uncertainty sets. The role of rectangle uncertainty set is to tighten the range of traffic demands explained by the hose model and the hose uncertainty set makes fluctuations in the traffic demands.

In our model, we contemplate a subproblem for robust optimization. The subproblem maximizes the link load in the worst-case of traffic scenario for given routing. The green HR model is developed in the form of MILP using the dual transformation of the subproblem.

We use Gurobi [26] optimizer software to track our model numerically. The numerical simulations show that our model is solvable within acceptable time and gains better performance compared to the green hose model with traffic fluctuations. For large and mesh networks, the green HR model shows better performance which is another advantage of our proposed model.

This research work is an extended version of the conference paper presented by Das et al. [2] with various additions. We have provided details background of this research with our problem statement and contributions in the introduction section. We add the graphical representation of the considered power model with description. We conduct numerical experiments considering different networks and update the results with comparisons accordingly in Section III. To describe our contribution compared to previous studies, we also present another comparison between green pipe and green hose models in terms of energy savings in the considered networks. We analyse the effect of our proposed model in details in the experiments section. We also conduct comprehensive evaluations for different values of parameters used in our model and the results are compared with the existing works.

The remaining part of this article is designed as follows. In Section II, first we present the formulation of the *green pipe* model [8] after presenting the power and network models. Note that the green pipe model needs a fixed traffic-demand for every pair in the network. The *green hose* model is also formulated in this section. In Section II-E, we formulate the green HR model after presenting the rectangle uncertainty set. In Section III, we present experiments through numerical simulations. Comprehensive analysis and evaluation are presented there. The achievement of green HR model are also discussed in that section. Finally, concluding remarks are presented in Section IV.

II. Model Formulation

The formulation of our model focuses on improved power saving with an acceptable traffic fluctuation. In this regard, we integrate the green pipe model [8] with a very good power rating and green hose model [15], [29], [30], [31] with large variation of traffic demands.

A. Power Model

It is essential to consider an appropriate power model in order to efficiently reduce power consumption in networks. We look at the *energy aware power model* which is used by Bianzino et al. [8]. In the *energy aware model*, the link power consumption is an affine function of its employment. If a link is on, the power consumption (PC) of that link is represented by

$$PC = (E_M - E_0)x + E_0, \tag{1}$$

and 0 otherwise. Here, x represents the total portion of traffic information broadcasting across the link, E_0 indicates the required power to keep the link on, and E_M is the maximum amount of power when the link is used at its fullest capacity. We assume that E_0 and E_M are depending on the load of link. Therefore, we have $0 \leq x \leq 1$. The power consumption of a link gradually increases from E_0 to E_M when the link is used to its fullest capacity.

The model is called *energy agnostic* if $E_M = E_0$ and the model is known as *fully proportional* if $E_0 = 0$. In this case, the link utilization rate does not effect the power consumption.

Existing literature have different other power models which address different other issues. Among those models, two popular models are *dynamic voltage scaling* [27] and *adaptive link rate* [28]. These two technologies were presented as advertising computing techniques, however it is not easy to produce these kind of devices by the currently available technologies. Our aim is to propose a scheme which can be implemented by currently available technologies, the energy aware model serves that purpose. Therefore, we choose to use the energy aware model.

B. Network Model

If A is the set of links and V is the set of nodes, a network is a directed graph expressed by $G(V, A)$. The set of edge node is indicated by Q and $Q \subseteq V$. Through the edge node, data is admitted into and going outside the network. The edge-node pair is represented by $(p, q) \in W$ where $p \neq q$ and $p \in Q$ and $q \in Q$ in this research. Here W indicates the set of edge-node pairs (p, q). The link from node $i \in V$ to node $j \in V \setminus \{i\}$ is represented as $(i, j) \in A$, $i \neq j$. Here c_{ij} and u_{ij} indicate the capacity and the data flow on link $(i, j) \in A$ respectively. We consider the fully duplex links in this research. The on/off status of a link is represented by the binary variable b_{ij}. If a link (i, j) is deactivated, it means both directions (i, j) and (j, i) are deactivated. E_{0ij} is the constant term and E_{fij} is the slope of the function. When a link (i, j) is kept on but does not flow any traffic then the power consumption of that link is equal to E_{0ij}.

C. Formulation of Green Pipe Model

To reduce the power in network, Bianzino et al. [8] proposed the *green pipe* model as a mixed-integer formulation. In the green pipe model, the authors assume that the traffic-demand matrix, $T = \{d_{pq} : (p, q) \in W\}$ is given. Note that the green pipe model is not robust to traffic fluctuations. The formulation of the green pipe model is as follows:

$$\min \frac{1}{2} \sum_{(i,j)\in A} \left(\frac{u_{ij} + u_{ji}}{c_{ij}} E_{fij} + b_{ij} E_{0ij} \right) \tag{2a}$$

$$\text{s.t.} \sum_{j:(i,j)\in A} x_{ij}^{pq} - \sum_{j:(j,i)\in A} x_{ji}^{pq} = 1, \quad \forall (p, q) \in W, i = p, \tag{2b}$$

$$\sum_{j:(i,j)\in A} x_{ij}^{pq} - \sum_{j:(j,i)\in A} x_{ji}^{pq} = 0, \quad \forall (p, q) \in W, \forall i \in V \setminus \{p, q\}, \tag{2c}$$

$$\sum_{(p,q)\in W} d_{pq} x_{ij}^{pq} = u_{ij}, \qquad \forall (i, j) \in A, \tag{2d}$$

$$u_{ij} \leq c_{ij}, \qquad \forall (i, j) \in A, \tag{2e}$$

$$M b_{ij} \geq u_{ij} + u_{ji}, \qquad \forall (i, j) \in A, \tag{2f}$$

$$x_{ij}^{pq} \geq 0, \qquad \forall (p, q) \in W, \forall (i, j) \in A, \tag{2g}$$

$$b_{ij} \in \{0, 1\}, \qquad \forall (i, j) \in A. \tag{2h}$$

The variable x_{ij}^{pq} represents the amount of traffic broadcasting from node p to q using the link (i, j). The equation (2b) indicates that the sum of portions of data passing from node $i(= p)$ is equal to 1. The constraints (2b) and (2c) describe the network data flow control constraints at source and intermediate nodes respectively. The equation (2c) represents that the total amount of data entering to node i must be the same as that of passing from node i if node i is neither a source nor a destination node. The equation (2e) express the link capacity constraint and the equation (2d) indicates the total data flow through link $(i, j) \in A$. The constraint (2f) is applied to keep the link on if there is a data in one direction of the link. Here, M is considered as a positive number which is two times greater than the highest capacity of link. The objective function reduces the data flow passing through each link so that the link without any data can be put into sleep mode i.e., the equation (2a) minimizes the network power utilization by deactivating some unnecessary links. In this formulation, we assume that the link capacity for both directions remain same i.e. the values of c_{ij} and c_{ji} are same in both directions. The objective function (2a) is divided by two in order to avoid calculating the power utilization twice for each link.

D. Formulation of Green Hose Model

The hose model is developed on the idea that the network operators can easily determine the total outgoing and incoming amount of traffic rather than actual given traffic. Here, the total outgoing traffic from node p is described as

$$\sum_{q} d_{pq} \leq \alpha_p, \qquad \forall p \in Q, \tag{3}$$

where α_p is the maximum volume of traffic that node p can pass into the network. Similarly, the total incoming traffic to node q is expressed as

$$\sum_{p} d_{pq} \leq \beta_q, \qquad \forall q \in Q, \tag{4}$$

where β_q is the maximum volume of traffic that node q can collect from the network.

The authors in Juttner et al. [15], Duffield et al. [29], Chu and Lea [30], and Liu and Guo [31] formulated their traffic-demand models using these traffic bounds and labeled their models as the hose model.

The uncertainty set used in the hose model can be considered as:

$$\mathcal{H} = \left\{ \mathbf{d} \in R^{|W|} : \begin{array}{l} \sum_{q} d_{pq} \leq \alpha_p, \quad \forall p \in Q, \\ \sum_{p} d_{pq} \leq \beta_q, \quad \forall q \in Q, \\ d_{pq} \geq 0, \quad \forall (p,q) \in W, \end{array} \right\}. \tag{5}$$

In this paper, we call their uncertainty set as the *hose uncertainty set*.

In the green hose model, we think that the true traffic demand $\mathbf{d}$ belongs to $\mathcal{H}$ and we consider that the routing x_{ij}^{pq} for $(i, j) \in A$ and $(p, q) \in W$ is fixed. In the worst case of traffic, the flow on each link is consider as:

$$u_{ij} = \max_{\mathbf{d} \in \mathcal{H}} \sum_{(p,q) \in W} d_{pq} x_{ij}^{pq}, \quad \forall (i,j) \in A. \tag{6}$$

To satisfy the equations (2e) and (2f), we apply the flow u_{ij} in the pipe model. We attain the subsequent problem dealing with x_{ij}^{pq} as a variable:

$$(H) : \min \frac{1}{2} \sum_{(i,j) \in A} \left(\frac{u_{ij} + u_{ji}}{c_{ij}} E_{fij} + b_{ij} E_{0ij} \right) \tag{7a}$$

$$\text{s.t.} \sum_{j:(i,j) \in A} x_{ij}^{pq} - \sum_{j:(j,i) \in A} x_{ji}^{pq} = 1, \quad \forall (p,q) \in W, i = p, \tag{7b}$$

$$\sum_{j:(i,j) \in A} x_{ij}^{pq} - \sum_{j:(j,i) \in A} x_{ji}^{pq} = 0, \quad \forall (p,q) \in W, \forall i \in V \setminus \{p,q\}, \tag{7c}$$

$$\max_{\mathbf{d} \in \mathcal{H}} \sum_{(p,q) \in W} d_{pq} x_{ij}^{pq} = u_{ij}, \quad \forall (i,j) \in A, \tag{7d}$$

$$u_{ij} \leq c_{ij}, \qquad \forall (i,j) \in A, \tag{7e}$$

$$M b_{ij} \geq u_{ij} + u_{ji}, \qquad \forall (i,j) \in A, \tag{7f}$$

$$x_{ij}^{pq} \geq 0, \qquad \forall (i,j) \in A, \forall (p,q) \in W, \tag{7g}$$

$$b_{ij} \in \{0,1\}, \qquad \forall (i,j) \in A. \tag{7h}$$

Here, the left hand side of the equation (7d) is the optimal value of a linear programming (LP) problem. Since we know that the dual of an LP problem has the same optimal value as the primal, we can rewrite the left hand side of (7d) by its dual form. Hence, using the duality theorem [19], we get the following robust problem

$$(H) : \min \frac{1}{2} \sum_{(i,j) \in A} \left(\frac{u_{ij} + u_{ji}}{c_{ij}} E_{fij} + b_{ij} E_{0ij} \right) \tag{8a}$$

$$\text{s.t.} \sum_{j:(i,j) \in A} x_{ij}^{pq} - \sum_{j:(j,i) \in A} x_{ji}^{pq} = 1, \quad \forall (p,q) \in W, i = p, \tag{8b}$$

$$\sum_{j:(i,j) \in A} x_{ij}^{pq} - \sum_{j:(j,i) \in A} x_{ji}^{pq} = 0, \quad \forall (p,q) \in W, \forall i \in V \setminus \{p,q\}, \tag{8c}$$

$$\sum_{p \in Q} \alpha_p \pi_{ij}(p) + \sum_{p \in Q} \beta_p \lambda_{ij}(p) = u_{ij}, \quad \forall (i,j) \in A, \tag{8d}$$

$$x_{ij}^{pq} \leq \pi_{ij}(p) + \lambda_{ij}(q), \quad \forall (i,j) \in A, \forall (p,q) \in W, \tag{8e}$$

$$\pi_{ij}(p), \lambda_{ij}(q) \geq 0, \quad \forall (i,j) \in A, \forall (p,q) \in W, \tag{8f}$$

$$u_{ij} \leq c_{ij}, \qquad \forall (i,j) \in A, \tag{8g}$$

$$M b_{ij} \geq u_{ij} + u_{ji}, \qquad \forall (i,j) \in A, \tag{8h}$$

$$x_{ij}^{pq} \geq 0, \qquad \forall (p,q) \in W, \forall (i,j) \in A, \tag{8i}$$

$$b_{ij} \in \{0,1\}, \qquad \forall (i,j) \in A. \tag{8j}$$

Here, $\pi_{ij}(p)$ and $\lambda_{ij}(p)$ are dual variables used in the dual transformation. $\pi_{ij}(p)$ represents the ratio of traffic on link (i, j) outgoing from node p and $\lambda_{ij}(p)$ indicates the ratio of traffic on link (i, j) incoming to node p. The problem in Eqn. (8) is known as the robust counterpart of the problem in Eqn. (7). Note that the green hose model is robust and it has a big margin to traffic fluctuations since it examine every possible set of traffic demands. The background history of the green hose model [15] describes that this model is robust to traffic variations but its performance in terms of power saving is much lower than the green pipe model.

In this paper, our thinking is to present a model which can improve the performance of the green hose model in terms of power saving.

E. Formulation of Green Hose-Rectangle (green HR) Model

In this research, our goal is to formulate a robust optimization model which can claim the power efficiency of the green pipe model and the flexibility of the green hose model allowing traffic-fluctuations in different context. To do this, we apply the concept of a model introduced by Oki and Iwaki [12], [32]. They used their idea to reduce network congestion ratio. In our work, we propose a robust model that minimizes power consumption in network for green computing. Specifically, first, we look at the following uncertainty set to bind the network traffic by upper and lower bounds:

$$\mathcal{R} = \left\{ \mathbf{d} \in R^{|W|} \ : \ \delta_{pq} \le d_{pq} \le \gamma_{pq} \right\}. \tag{9}$$

The uncertainty set $\mathcal{R}$ is called the *rectangle uncertainty set* in this paper where δ_{pq} and γ_{pq} are lower bound and upper bound of demand d_{pq} respectively. The function of the rectangle uncertainty set is to tighten the range of traffic volume expressed by the hose model.

In our model, we consider the uncertainty set which is the intersection of the rectangle and hose uncertainty sets. Our hypothesis is that the real demand of traffic, $\mathbf{d}$ belongs to this uncertainty set i.e. $\mathbf{d} \in \mathcal{R} \cap \mathcal{H}$. For maximum traffic demand, following the similar approach used in the green hose model, we address u_{ij} as:

$$\max_{\mathbf{d}\in\mathcal{R}\cap\mathcal{H}} \sum_{(p,q)\in W} d_{pq} x_{ij}^{pq} = u_{ij}, \quad \forall (i,j) \in A. \tag{10}$$

In the worst-case of traffic scenario, for every $(i,j) \in A$, we consider the following subproblem:

$$S(\mathbf{x}_{ij}) : \max \sum_{(p,q)\in W} d_{pq} x_{ij}^{pq} \tag{11a}$$

$$\text{s.t.} \sum_{q\in Q} d_{pq} \le \alpha_p, \qquad \forall p \in Q, \tag{11b}$$

$$\sum_{p\in Q} d_{pq} \le \beta_q, \qquad \forall q \in Q, \tag{11c}$$

$$\delta_{pq} \le d_{pq} \le \gamma_{pq}, \quad \forall (p,q) \in W, \tag{11d}$$

$$d_{pq} \ge 0, \qquad \forall (p,q) \in W. \tag{11e}$$

In $S(\mathbf{x}_{ij})$, we think that the routing $\mathbf{x}_{ij} = (x_{ij}^{pq})_{(p,q)\in W}$ is given and the variable $d_{pq}, \forall (p,q) \in W$, is the decision variable. The above subproblem, $S(\mathbf{x}_{ij})$ finds a traffic-demand matrix, $T = \{d_{pq} : (p,q) \in W\}$ for the given routing $\{x_{ij}^{pq}\}$ and it maximizes the link load on $(i,j) \in A$. Here, the optimal value of the optimization problem $S(\mathbf{x}_{ij})$ is expressed by $\text{val}(S(\mathbf{x}_{ij}))$.

Considering the uncertainty sets $\mathcal{R}$ and $\mathcal{H}$, first we introduce the robust optimization model to reduce the power utilization in the following form:

$$\min \frac{1}{2} \sum_{(i,j)\in A} \left(\frac{u_{ij} + u_{ji}}{c_{ij}} E_{fij} + b_{ij} E_{0ij} \right) \tag{12a}$$

$$\text{s.t.} \sum_{j:(i,j)\in A} x_{ij}^{pq} - \sum_{j:(j,i)\in A} x_{ji}^{pq} = 1,$$

$$\forall i = p, \forall (p,q) \in W, \tag{12b}$$

$$\sum_{j:(i,j)\in A} x_{ij}^{pq} - \sum_{j:(j,i)\in A} x_{ji}^{pq} = 0,$$

$$\forall (p,q) \in W, \forall i \in V \setminus \{p,q\}, \tag{12c}$$

$$\text{val}(S(\mathbf{x}_{ij})) = u_{ij}, \quad \forall (i,j) \in A, \tag{12d}$$

$$u_{ij} \le c_{ij}, \qquad \forall (i,j) \in A, \tag{12e}$$

$$M b_{ij} \ge u_{ij} + u_{ji}, \qquad \forall (i,j) \in A, \tag{12f}$$

$$x_{ij}^{pq} \ge 0, \qquad \forall (i,j) \in A, \forall (p,q) \in W, \tag{12g}$$

$$b_{ij} \in \{0,1\}, \qquad \forall (i,j) \in A. \tag{12h}$$

In the constraint (12d), $\text{val}(S(\mathbf{x}_{ij}))$ is the optimal value of an LP problem, so it can be calculated by its dual primal value. Using the duality theorem Ben-Tan et al. [16], the dual modification of the primal problem, $S(\mathbf{x}_{ij})$ is expressed as follows:

$$\min \sum_{p\in Q} \alpha_p \pi_{ij}(p) + \sum_{q\in Q} \beta_q \lambda_{ij}(q) + \sum_{p\in Q}\sum_{q\in Q} [\gamma_{pq}\eta_{ij}(p,q) - \delta_{pq}\theta_{ij}(p,q)] \tag{13a}$$

$$\text{s.t.} \ \pi_{ij}(p) + \lambda_{ij}(q) + \eta_{ij}(p,q) - \theta_{ij}(p,q) - x_{ij}^{pq} \ge 0,$$

$$\forall (p,q) \in W, \tag{13b}$$

$$\pi_{ij}(p) \ge 0, \qquad \forall p \in Q, \tag{13c}$$

$$\lambda_{ij}(q) \ge 0, \qquad \forall q \in Q, \tag{13d}$$

$$\eta_{ij}(p,q) \ge 0, \qquad \forall (i,j) \in A, \forall (p,q) \in W, \tag{13e}$$

$$\theta_{ij}(p,q) \ge 0, \qquad \forall (i,j) \in A, \forall (p,q) \in W. \tag{13f}$$

The dual conversion of the optimal problem $(S(\mathbf{x}_{ij}))$ is described in Appendix.

Finally, using the dual of the subproblem $S(\mathbf{x}_{ij})$, the green HR model is proposed as follows:

$$(I_1) : \min \frac{1}{2} \sum_{(i,j)\in A} \left(\frac{u_{ij} + u_{ji}}{c_{ij}} E_{fij} + b_{ij} E_{0ij} \right) \tag{14a}$$

$$\text{s.t.} \sum_{j:(i,j)\in A} x_{ij}^{pq} - \sum_{j:(j,i)\in A} x_{ji}^{pq} = 1,$$

$$\forall (p,q) \in W, i = p, \tag{14b}$$

$$\sum_{j:(i,j)\in A} x_{ij}^{pq} - \sum_{j:(j,i)\in A} x_{ji}^{pq} = 0,$$

$$\forall (p,q) \in W, \forall i \in V \setminus \{p,q\}, \tag{14c}$$

$$\sum_{p\in Q} \alpha_p \pi_{ij}(p) + \sum_{p\in Q} \beta_p \lambda_{ij}(p) + \sum_{p\in Q}\sum_{q\in Q} [\gamma_{pq}\eta_{ij}(p,q) - \delta_{pq}\theta_{ij}(p,q)] = u_{ij},$$

$$\forall (i,j) \in A, \tag{14d}$$

$$x_{ij}^{pq} \le \pi_{ij}(p) + \lambda_{ij}(q) + \eta_{ij}(p,q) - \theta_{ij}(p,q),$$

$$\forall (p,q) \in W, \forall (i,j) \in A, \tag{14e}$$

$$\pi_{ij}(p), \lambda_{ij}(q), \theta_{ij}(p,q), \eta_{ij}(p,q) \ge 0,$$

$$\forall (p,q) \in W, \forall (i,j) \in A, \tag{14f}$$

$$u_{ij} \le c_{ij}, \qquad \forall (i,j) \in A, \tag{14g}$$

$$M b_{ij} \ge u_{ij} + u_{ji}, \qquad \forall (i,j) \in A, \tag{14h}$$

$$x_{ij}^{pq} \ge 0, \qquad \forall (i,j) \in A, \forall (p,q) \in W, \tag{14i}$$

$$b_{ij} \in \{0,1\}, \qquad \forall (i,j) \in A. \tag{14j}$$

where $\pi_{ij}(p)$, $\lambda_{ij}(p)$, $\theta_{ij}(p,q)$, and $\eta_{ij}(p,q)$ are new variables obtained by the dual of the subproblem.

The model (I_1) represents a robust optimizaton model in the form of mixed-integer LP formulation. To develop the green HR model, we change the left hand side of the equation (2d) by the optimal value of the dual of subproblem $S(\mathbf{x}_{ij})$. The equations (13b), (13c), (13d), (13e), (13f) are also added to the green pipe model to convert it in robust optimization.

In dual transformation of the subproblem, the variables $\pi_{ij}(p)$ and $\lambda_{ij}(p)$ are introduced in the green hose model while the variables $\theta_{ij}(p,q)$ and $\eta_{ij}(p,q)$ are newly presented with parameters γ_{pq} and δ_{pq} in our proposed green HR model. That is the difference between the green hose model and our proposed model in terms of variable and parameter setting in the subproblem.

III. Experimental Result and Evaluation

A. Experiment Settings

We conduct numerical experiments to find out whether the presented model can be run by any software tools or not. We also compute the performances numerically. For this purpose we choose four popular backbone networks used by many other researchers of the same fields such as Ouédraogo and Oki [13], Oki and Iwaki [33], and Das et al. [20]. The details of networks features are demonstrated in Figure 1. We compare the achievement of our model with the green hose and green pipe models. For the green pipe model, we need exact traffic demands, $\bar{d}_{pq}$ which are arbitrarily produced in the range of (1,150) with a homogeneous distribution. The links capacities are determined in the range of (800, 2800). To explain the power utilization of 1 Gbps links, the values of E_{fij} and E_{0ij} are taken from Bianzino et al. [8] which are 0.3 and 1.7 respectively.

The power savings are obtained using the following rule for each model:

$$\text{power savings ratio} = \frac{P_{\mathrm{T}} - P_{\mathrm{R}}}{P_{\mathrm{T}}}, \tag{15}$$

where P_{T} is the total absorbed energy of all considered links when they are kept on and P_{R} is the required energy of links after the optimal solution.

The parameters α_p and β_p are calculated as follows: $\alpha_p = \sum_{q \in Q} \bar{d}_{pq},\ \forall p \in Q$ and $\beta_q = \sum_{p \in Q} \bar{d}_{pq},\ \forall q \in Q$.

For each pair in the network, the green HR model needs the upper and lower bounds of demands to allow fluctuation in traffic. Based on the estimated traffic demand $\bar{d}_{pq}$, we calculate the upper bound as: $\gamma_{pq} = \frac{1}{\phi}\bar{d}_{pq}$ where $0 < \phi \leq 1$ and lower bound as: $\delta_{pq} = \xi \bar{d}_{pq}$ where $0 \leq \xi \leq 1$. $(\phi, \xi) \to (0,0)$ and $(\phi, \xi) \to (1,1)$ indicate that the green HR model approaches the green hose and green pipe models respectively.

For the experiments, we use a windows based computer with Intel(R) Core(TM) i7-4790 CPU @ 3.60 GHz and 16 GB memory. Using different type of processing system may vary the computation time rather than amount of power savings achieves by the considered models. We have coded our computer programs with the Python programming language. The professional optimization software tool, Gurobi (version 7.0.1) [26] is used to solve the optimization problems considered in the experiments.

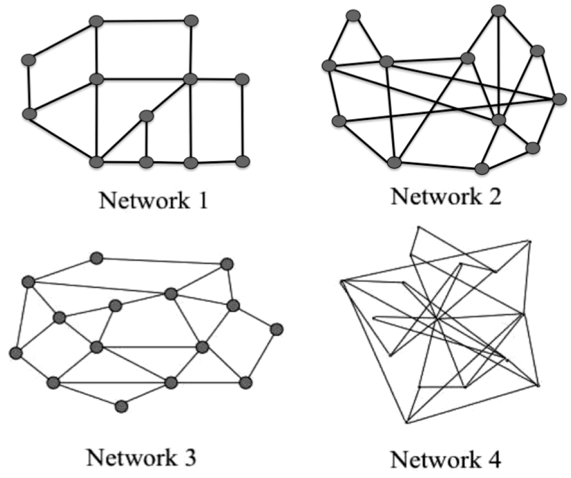


Fig. 1. Sample Networks

TABLE I
Networks features presented in Figure 1

Network	No. of nodes considered	No. of links considered
Network 1	12	18
Network 2	12	22
Network 3	15	27
Network 4	16	32

B. Comparison of Green Pipe and Green Hose Models

In the sample networks, first we compare the power saving determined by the green hose and green pipe models. The green pipe model obtains the first place in terms of power savings due to fixed traffic demand which we discussed in the related work section. Figure 2 indicates that the green pipe model can save 5 -14% more energy than the green hose model. This is because, the green hose model have a wide range of traffic specification that demands more power which we discussed before. Note that the Network 4 has the largest number of nodes and links among the considered networks. The green hose model considers every possible combinations which makes an inundated computation overhead. Therefore, the problem in Network 4 is infeasible for the green hose model. Figure 2 also shows that the green pipe model performs better in large networks compared to the green hose model. This is because the green pipe model has no uncertainty set to allow traffic variations whereas the green hose model has the hose uncertainty set for traffic variations. The hose uncertainty set permits every pair in the network to fluctuate independently.

C. Performance Comparison

The power saving obtain by the green HR model depends on the parameters ϕ and ξ which we use to compute the upper and lower bounds of traffic demands. We compare the power efficiency of our proposed green HR (G. HR) model with the green hose (G. Hose) and green pipe (G. Pipe) models for

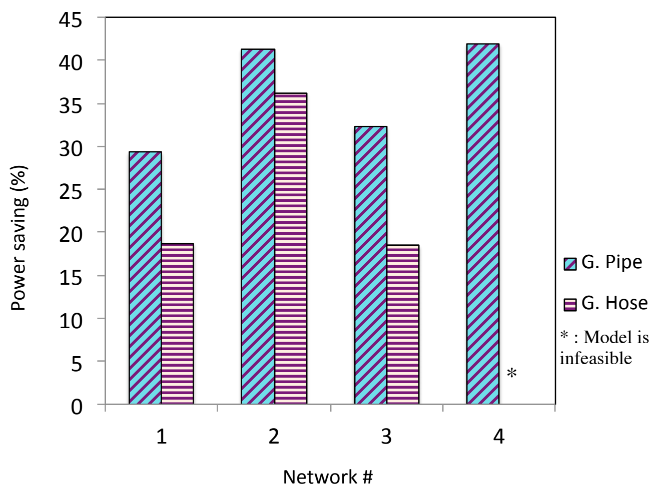


Fig. 2. Comparisons of Power Saving Between G. Pipe and G. Hose Models Based on Eqn. (15)

different values of ϕ and ξ. In Figure 3, the comparisons of power savings by the considered models are described for Networks 1, 2, 3, and 4. As explained before, the G. Pipe model obtains the best power savings, the results also show the same behaviour in all the networks in Figure 1. The results also demonstrate that the power saving for G. Hose and G. Pipe models do not depend on ϕ and ξ. The power savings by the G. Pipe model is ranking at 29.31%, 41.26%, 32.23%, and 41.86% in Networks 1, 2, 3, and 4 respectively, whereas these values for the G. Hose model is only 16.13%, 34.23%, and 16.45% respectively in first three networks. In Network 4, the G. Hose model is infeasible for the considered problem whereas the proposed G.HR model performs with 36.4% to 41.86% power saving in the same problem for different values of ϕ and ξ. The figures also show that the performance of our proposed model always lies between the range of the performances obtained by the G. Pipe and G. Hose models in all cases and in all considered networks.

The results in Figure 3 demonstrate that the power saving for the G. HR model approaches to the G. Pipe model when (ϕ, ξ) is closed to $(1, 1)$ and on the other hand when (ϕ, ξ) is closed $(0, 0)$ the G. HR model approaches to the G. Hose model. The results also confirm that power saving for G. HR model does not depend on smaller values of ϕ. This indicates that the traffic-demand matrix is bounded by only α_p and β_q, not by γ_{pq}.

D. Effect of G. HR Model:

The G. HR model tightens the range of traffic demands expressed by the G. Hose model. The proposed model also avoid the G. Pipe model's requirements of predicting and measuring the accurate traffic demand matrix. The G. HR model minimizes the power utilization compared to the G. Hose model in every considered networks and shows the similar characteristics in all considered networks for every value of ϕ and ξ. The key advantage of the G. HR model is that if operators can define a proper set of (ϕ, ξ) so that they can tighten the range of traffic demand matrix, the G. HR model can minimize the power utilization compared to the G. Hose model. For example, the power consumption for G. HR model is 12.23% less than the G. Hose model when $(\phi, \xi) = (0.8, 0.8)$ is set in Network 3 of Figure 4. In Network 4 of Figure 4, the G. Hose model provides infeasible solution for the considered problem, however our proposed G.HR model provides an effective solution with power savings as high as 41.29% in the same problem. It can also be noted from the figure that for mesh and large networks, for example as in Network 4 of Figure 4, the G.HR performs better in terms of power saving which is also another contribution of the G.HR model.

TABLE II
NUMBER OF DEACTIVATED LINKS FOR $\phi = 0.8$ AND $\xi = 0.8$

Network type	Number of deactivated links		
	G. Pipe	G. HR	G. Hose
Network 1	6	6	4
Network 2	10	10	9
Network 3	10	9	6
Network 4	15	15	*

* The model is infeasible.

TABLE III
COMPARISON OF COMPUTATION TIME FOR $\phi = 0.8$ AND $\xi = 0.8$

Network type	Computation time [sec.]		
	G. Pipe	G. HR	G. Hose
Network 1	63	535	1837
Network 2	83	763	2547
Network 3	96	1342	3567
Network 4	432	3757	*

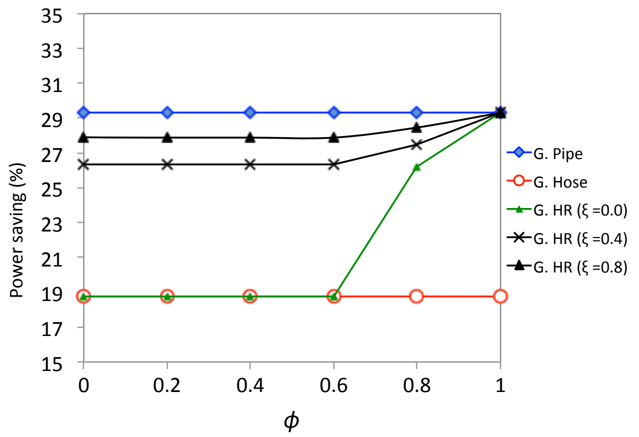
* The model is infeasible.

Table II shows the total deactivated links earned by the examined models. Network can save power consumption by deactivating the unnecessary links [9]. The more redundant links are deactivated the more power saving is achieved. The table describes that the G. Pipe and our G. HR models deactivate the same number of links in Networks 1, 2, and 4. In Network 3, the number of deactivated link by the G. HR is differed by 1 unit. Nonetheless, for the G. Pipe and G. Hose model, this value differs by 2, 1, and 4 units in Networks 1, 2, and 3 respectively. In each and every case our model deactivates more number of unnecessary links than the G. Hose model.

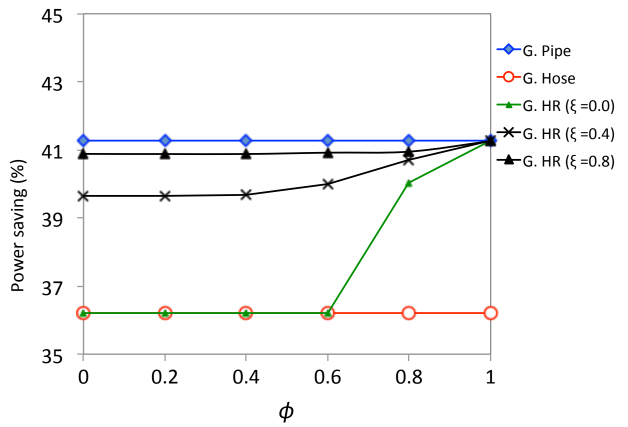
Table III describes the computation times of the examined models. Among the considered models, the G. HR model has the largest number of constraints and variables and we know that the computation time depends on number of constraints and variables in the model. That is why the table shows the G. HR model is a bit slower than the G. Pipe model, however considerably faster than the G. Hose model in terms of computation time. This is because the feasible region of the G. Hose model is bigger than the G. HR model. The G. Pipe model has the lowest number of constraints and variables and it has no uncertainty set which make it faster than the others model. Therefore, the proposed the G. HR model also have another achievement in terms of computation time compared to the G. Hose model.

IV. CONCLUSION AND FUTURE DIRECTION

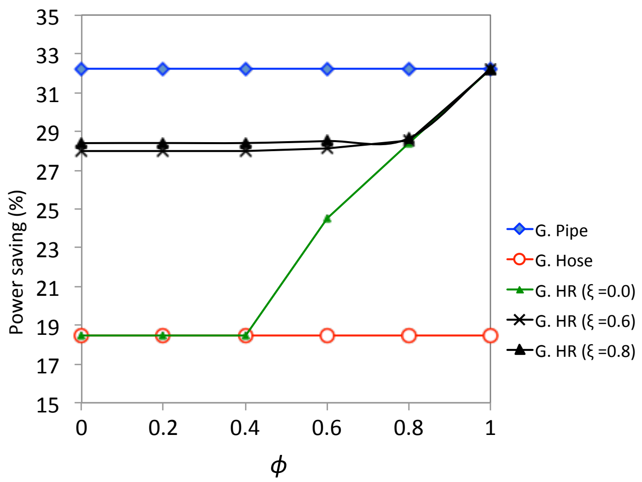
This paper introduced the green HR model which is robust in traffic variations to reduce networks power consumption.



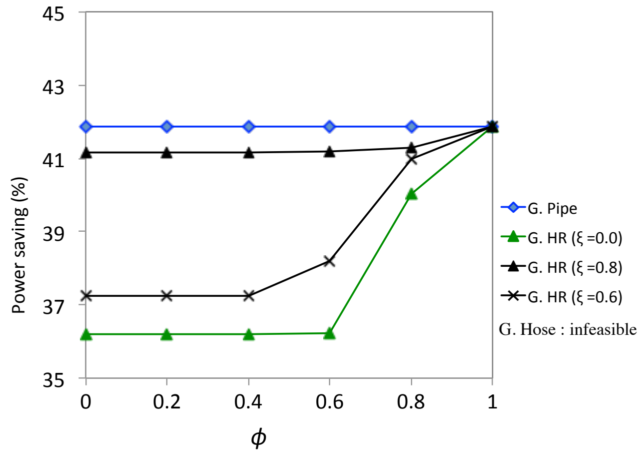
(a) Network 1



(b) Network 2



(c) Network 3



(d) Network 4

Fig. 3. Comparisons of Power Saving for ϕ and ξ

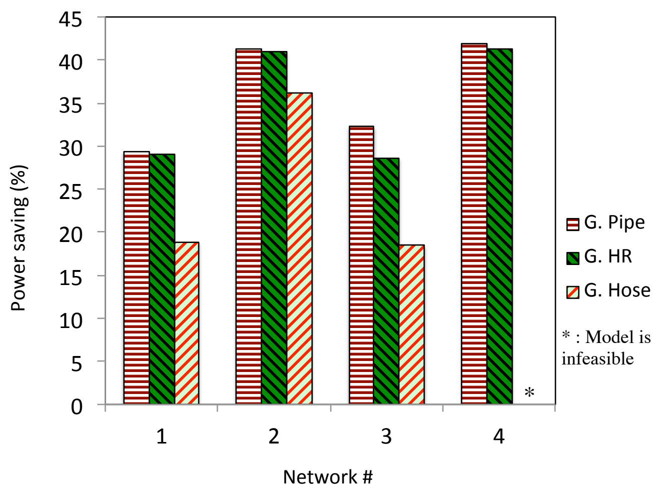


Fig. 4. Comparisons of Power Saving for $\phi = 0.8$ and $\xi = 0.8$

The green HR model allows traffic fluctuations and it is formulated in the form of mixed-integer LP formulation using the robust optimization technique. The proposed green HR model resolves the difficulty of the green pipe model which need known traffic information and tightens the range of traffic amount expressed by the hose model.

The achievement of the presented green HR model in terms of power savings is compared with the green hose and green pipe models with computation time and total number of deactivated links in the networks. The green pipe model attains the highest achievement due to known traffic. The disadvantage of the green pipe model is that it has no option to handle the traffic fluctuations. Generally, networks traffic fluctuates on demand. Our proposed model, using mixed-integer LP formulation in the background, can deal with such traffic fluctuations. To reduce power in communication networks for green computing, our proposed green HR model can be an interesting direction. The green HR model can be used as a substitute method to reduce network power where traffic demands vary in a given range. The efficiency of the proposed green HR model is examined by conducting numerical experiments using software tools which show useful results. The power efficiency of our proposed model is also interesting. Though it could not beat the green pipe model in this regards, however was very close to it. Our model clearly shows promising performance when compared with the green hose model. Our proposed green HR model minimizes 6.75%

to 12.23% better power consumption than the green hose model for $\phi = 0.8$ and $\xi = 0.8$ in the considered networks. Our introduced model also have achievements in terms of improved number of deactivated links and computation time.

Mixed-integer programming problem is an NP-hard problem and sometimes difficult to handle with software tools. The experiments validate that although our proposed green HR model is a mixed-integer LP formulation, it can be run within an acceptable time even for large networks.

To compute the power consumption of a link, we used the *energy aware* model which is an affine function of its uses. An affine function is a composition of a linear function with translation. Therefore, the results of this research is limited by the limitations of linear functions. In future, it will be interesting to see the results using different other non-linear power models such as a model with a quadratic function. Our experiments are conducted on some of the currently available popular networks used by other researchers. Applying our model to some other kind of networks could be another future work.

Appendix A
Dual transformation of the the subproblem $S(\mathbf{x}_{ij})$

The problem $S(\mathbf{x}_{ij})$ in (11) can be written as

$$\max \; \mathbf{x}_{ij}^T \mathbf{d} \tag{16a}$$

$$\text{s.t. } \mathbf{A}\mathbf{d} \leq \mathbf{c} \tag{16b}$$

$$\mathbf{d} \geq \mathbf{0}, \tag{16c}$$

where

$$\mathbf{d}^T = [d_{11} d_{12} \dots d_{1n} d_{n1} d_{n2} \dots d_{nn}] \tag{17a}$$

$$\mathbf{x}_{ij}^T = [x_{ij}^{11} x_{ij}^{12} \dots x_{ij}^{1n} x_{ij}^{n1} x_{ij}^{n2} \dots x_{ij}^{nn}] \tag{17b}$$

$$\begin{aligned}\mathbf{c} = [&\alpha_1 \alpha_2 \dots \alpha_n \beta_1 \beta_2 \cdots \beta_n \gamma_{11} \gamma_{12} \cdots \gamma_{1n} \gamma_{n1} \gamma_{n2} \cdots \gamma_{nn} \\ &- \delta_{11} - \delta_{12} \cdots - \delta_{1n} - \delta_{n1} - \delta_{n2} \cdots - \delta_{nn}]\end{aligned} \tag{17c}$$

n is the number of nodes. $\mathbf{x}_{ij}$ is an $nn \times 1$ matrix, $\mathbf{d}$ is an $nn \times 1$ matrix , $\mathbf{c}$ is an $(2n + 2nn) \times 1$ matrix, and $\mathbf{A}$ is an $(2n + 2nn) \times nn$ matrix.

Therefore, for all $(i, j) \in A$, the dual of the problem (16) can be written as

$$\min \; \mathbf{C^T y_{ij}} \tag{18a}$$

$$\text{s.t. } \mathbf{A^T y_{ij}} \geq \mathbf{x_{ij}}, \tag{18b}$$

$$\mathbf{y}_{ij} \geq \mathbf{0}, \tag{18c}$$

where

$$\begin{aligned}\mathbf{y}_{ij}^T =& [\pi_{ij}(1)\pi_{ij}(2) \cdots \pi_{ij}(n) \lambda_{ij}(1) \lambda_{ij}(2) \cdots \lambda_{ij}(n) \\ & \eta_{ij}(1,1)\eta_{ij}(1,2) \cdots \eta_{ij}(1,n) \cdots \eta_{ij}(n,1)\eta_{ij}(1,2) \cdots \\ & \eta_{ij}(n,n) \\ & \theta_{ij}(1,1)\theta_{ij}(1,2) \cdots \theta_{ij}(1,n) \cdots \theta_{ij}(n,1)\theta_{ij}(n,2) \cdots \\ & \theta_{ij}(n,n)]\end{aligned} \tag{19}$$

$\mathbf{y}_{ij}^T$ is a $(2n + 2nn) \times 1$ matrix. Here the equations (18a)-(19) and equations (17b)-(20) are matrix representation of equations (13)-(13f).

$$\mathbf{A} = \left(\begin{array}{cccc|cccc|cccc|c|cccc}
1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & \cdots & 0 & 0 & \cdots & 0 \\
 & & \cdots & & & & \cdots & & & & \cdots & & \cdots & & & \cdots & \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 1 & 1 & \cdots & 1 \\
\hline
1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & \cdots & 1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 & 0 & 1 & \cdots & 0 & 0 & 1 & \cdots & 0 & \cdots & 0 & 1 & \cdots & 0 \\
 & & \cdots & & & & \cdots & & & & \cdots & & \cdots & & & \cdots & \\
0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 1 & \cdots & 0 & 0 & \cdots & 1 \\
\hline
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
 & & \cdots & & & & \cdots & & & & \cdots & & \cdots & & & \cdots & \\
0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\hline
 & & \cdots & & & & \cdots & & & & \cdots & & \cdots & & & \cdots & \\
\hline
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 1 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 1 & \cdots & 0 \\
 & & \cdots & & & & \cdots & & & & \cdots & & \cdots & & & \cdots & \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 1 \\
\hline
-1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & -1 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
 & & \cdots & & & & \cdots & & & & \cdots & & \cdots & & & \cdots & \\
0 & 0 & \cdots & -1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\hline
 & & \cdots & & & & \cdots & & & & \cdots & & \cdots & & & \cdots & \\
\hline
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & -1 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & -1 & \cdots & 0 \\
 & & \cdots & & & & \cdots & & & & \cdots & & \cdots & & & \cdots & \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & -1
\end{array}\right) \tag{20}$$

The first two rows of $\mathbf{A}$ and $\mathbf{C}$ correspond to equations (11b) and (11c) and these rows are same as that of the green hose model. The next $2nn$ rows are newly announced and these rows corresponds to the equation (11d). In $\mathbf{y}_{ij}$, the next $2nn$ rows are newly announced and first $2n$ rows are same asthe green hose model formulation. The term $\sum_{p\in Q}\sum_{q\in Q}[\gamma_{pq}\eta_{ij}(p,q) - \delta_{pq}\theta_{ij}(p,q)]$ in equation (14d) and that of $\eta_{ij}(p,q) - \theta_{ij}(p,q)$ in (14e) are produced by the next $2nn$ rows in $\mathbf{y}_{ij}$.

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