# Interval Valued Opposition Intuitionism Fuzzy Sub-Implication Ideals, Sub-Commutative Ideals and Positive Implication Ideals of Subtraction $G$-Algebras

B. Lena, C. Ragavan, A. Iampan and V. Govindan

***Abstract*—The notions of interval valued opposition intuitionism fuzzy sub-implication ideals, positive implication ideals and sub-commutative ideals of subtraction $G$-algebras are introduced. The characterization properties of interval valued opposition intuitionism fuzzy sub-implication ideals, positive implication ideals and sub-commutative ideals are obtained.**

***Index Terms*—Subtraction $G$-algebra, Interval valued opposition intuitionism fuzzy sub-implication ideals, Interval valued opposition intuitionism fuzzy sub-commutative ideals, Interval valued opposition intuitionism fuzzy positive implication ideals.**

## I. Introduction

AFTER the introduction of fuzzy sets by Zadeh [20], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [1] is one among them. Fuzzy sets give a degree of membership of an element in a given set, while intuitionistic fuzzy sets give both degrees of membership and of non- membership. Both degrees belong to the interval $[0, 1]$, and their sum should not exceed. The interval valued intuitionistic fuzzy sets were introduced in 1989 Atanassov [4]. Ragavan and Solairaj [16] some new results on intuitionistic fuzzy H-ideals in BCI-algebras. Senthil Kumar et al. [18] intuitionistic fuzzy translation of antagonistic-intuitionistic fuzzy T-ideals of subtraction BCK/BCI-algebras. A lot of operators were defined and studied in. For BCK-algebras, Jun et al. [9], [10], [11] introduced the notions of fuzzy positive implicative ideals and fuzzy commutative ideals, Liu and Meng [13] sub-implicative ideals and sub-commutative ideals. Liu et al. [14] fuzzy sub-implicative ideals and fuzzy sub-commutative ideals of BCI-algebras. In fact, all these concepts having a good application in other disciplines and real-life problems are now catching momentum, but it is seen that all these theories have their own difficulties.

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In this paper, we have introduced some results in interval valued intuitionism fuzzy sets: interval valued opposition intuitionism fuzzy sub-implication ideals, positive implication ideals and sub-commutative ideals, and show that the results hold in subtraction $G$-algebras. Also, we define their basic operations. Also, some new results along with illustrating examples have been put forward in our work.

## II. Preliminaries

**Definition II.1.** *[5] A subtraction G-algebra we mean a non-empty set $X$ with a binary operation $-$ and a constant $0$ satisfying the following conditions:*

*($F_1$)* $\eta - \eta = 0$,

*($F_2$)* $\eta - (\eta - \varrho) = \varrho$, *for all* $\eta, \varrho \in X$.

**Definition II.2.** *[17] A non-empty subset $S$ of a subtraction G-algebra $X$ is called a subtraction G-subalgebra of $X$ if $\eta - \varrho \in S$ for all $\eta, \varrho \in S$.*

**Definition II.3.** *[18] A fuzzy set $f$ of a universe $X$ is a function from $X$ to the unit closed interval $[0, 1]$, that is $f : X \to [0, 1]$.*

**Definition II.4.** *[1] An intuitionism fuzzy set $A$ in a finite universe of discourse $X = \{\eta_1, \eta_2, \eta_3, \ldots, \eta_n\}$ is given by $A = \{(\eta, \Psi_A(\eta), \Omega_A(\eta)) : \eta \in X\}$, Where $\Psi_A : X \to [0, 1]$ and $\Omega_A : X \to [0, 1]$ such that $0 \leq \Psi_A(\eta) + \Omega_A(\eta) \leq 1$. The number $\Psi_A(\eta)$ and $\Omega_A(\eta)$ denote the degree of membership and non-membership of $\eta \in X$ to A, respectively. For each IFS A in X, if $\pi_A(\eta) = 1 - \Psi_A(\eta) - \Omega_A(\eta)$ for all $\eta \in X$.*

**Definition II.5.** *[4] An interval valued intuitionism fuzzy set $A$ over $X$ is defined as an object of the form: $A = \{(\eta, \Psi_A(\eta), \Omega_A(\eta)) : \eta \in X\}$ where $\Psi_A(\eta) \subset [0, 1]$ and $\Omega_A(\eta) \subset [0, 1]$ are intervals, and for all $\eta \in X, \sup \Psi_A(\eta) + \sup \Omega_A(\eta) \leq 1$.*

**Definition II.6.** *A nonempty subset $I$ of subtraction G-algebra $X$ is called an ideal of $X$ if*

*($I_1$)* $0 \in I$,

*($I_2$)* $\eta - \varrho \in I$ *and* $\varrho \in I$ *imply* $\eta \in I$.

**Definition II.7.** *A fuzzy subset $\Psi_A$ of $X$ is said to be a fuzzy ideal of $X$ if it satisfies*

*($F_3$)* $\Psi_A(0) \geq \Psi_A(\eta)$ *for all* $\eta \in X$,

*($F_4$)* $\Psi_A(\eta) \geq \min\{\Psi_A(\eta - \varrho), \Psi_A(\varrho)\}$ *for all* $\eta, \varrho \in X$.

**Definition II.8.** *An intuitionism fuzzy set $A = \{(\eta, \Psi_A(\eta), \Omega_A(\eta)) : \eta \in X\}$ in $X$ is called an intuitionism fuzzy ideal of $X$ if it satisfies*

*($F_5$)* $\Psi_A(0) \geq \Psi_A(\eta), \Omega_A(0) \leq \Omega_A(\eta)$,
*($F_6$)* $\Psi_A(\eta) \geq \min\{\Psi_A\ (\eta - \varrho), \Psi_A(\varrho)\}$ *and*
*($F_7$)* $\Omega_A(\eta) \leq \max\{\Omega_A\ (\eta - \varrho), \Omega_A(\varrho)\}$ *for all* $\eta, \varrho \in X$.

**Definition II.9.** *An intuitionism fuzzy set* $A = \{(\eta, \Psi_A(\eta), \Omega_A(\eta)) : \eta \in X\}$ *in* $X$ *is called an opposition intuitionism fuzzy ideal of* $X$ *if it satisfies*
*($F_8$)* $\Psi_A(0) \leq \Psi_A(\eta), \Omega_A(0) \geq \Omega_A(\eta)$,
*($F_9$)* $\Psi_A(\eta) \leq \max\{\Psi_A\ (\eta - \varrho), \Psi_A(\varrho)\}$ *and*
*($F_{10}$)* $\Omega_A(\eta) \geq \min\{\Omega_A\ (\eta - \varrho), \Omega_A(\varrho)\}$ *for all* $\eta, \varrho \in X$.

**Definition II.10.** *A nonempty subset* $I$ *of subtraction G-algebra* $X$ *is called a positive implication ideal (i.e., weakly positive implication ideal) of* $X$ *if it satisfies*
*($I_1$)* $0 \in I$ *and*
*($I_3$)* $((\eta - \varsigma) - \varsigma) - (\varrho - \varsigma) \in I$ *and* $\varrho \in I$ *imply* $\eta - \varsigma \in I$ .

**Definition II.11.** *[12] A nonempty subset* $I$ *of subtraction G-algebra* $X$ *is called a sub-implication ideal of* $X$ *if it satisfies*
*($I_1$)* $0 \in I$ *and*
*($I_3$)* $((\eta - (\eta - \varrho)) - (\varrho - \eta)) - \varsigma \in I$ *and* $\varsigma \in I$ *imply* $\varrho - (\varrho - \eta) \in I$.

**Definition II.12.** *[12] A nonempty subset* $I$ *of subtraction G-algebra* $X$ *is called a sub-commutative ideal of* $X$ *if it satisfies*
*($I_1$)* $0 \in I$ *and*
*($I_4$)* $(\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma \in I$ *and* $\varsigma \in I$ *imply* $\eta - (\eta - \varrho) \in I$.

**Definition II.13.** *An opposition fuzzy subset* $\Psi_A$ *of* $X$ *is called an opposition fuzzy sub-implication ideal of* $X$ *if it satisfies*
*($F_{11}$)* $\Psi_A(0) \leq \Psi_A(\eta)$ *for all* $\eta \in X$, *and*
*($F_{12}$)* $\Psi_A(\varrho - (\varrho - \eta)) \leq \max\{\Psi_A(((\eta - (\eta - \varrho)) - (\varrho - \eta)) - \varsigma), \Psi_A(\varsigma)\}$ *for all* $\eta, \varrho, \varsigma \in X$.

**Definition II.14.** *An opposition fuzzy subset* $\Psi_A$ *of* $X$ *is called an opposition fuzzy sub-commutative ideal of* $X$ *if it satisfies* ($F_1$) *and*
*($F_{13}$)* $\Psi_A(\eta-(\eta-\varrho)) \leq \max\{\Psi_A((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Psi_A(\varsigma)\}$ *for all* $\eta, \varrho, \varsigma \in X$.

**Definition II.15.** *An interval valued opposition fuzzy set* $\mu_A$ *of* $X$ *is called an interval valued opposition fuzzy positive implication ideal of* $X$ *if it satisfies*
*($F_{14}$)* $\Psi_A(0) \leq \Psi_A(\eta)$ *and*
*($F_{15}$)* $\Psi_A(\eta-\varsigma) \leq \max\{\Psi_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Psi_A(\varrho)\}$ *for all* $\eta, \varrho, \varsigma \in X$.

**Definition II.16.** *An interval valued opposition fuzzy set* $(\Psi_A, \Omega_A)$ *of* $X$ *is called an interval valued opposition intuitionism fuzzy sub-implication ideal of* $X$ *if it satisfies*
*($F_{16}$)* $\Psi_A(0) \leq \Psi_A(\eta), \Omega_A(0) \geq \Omega_A(\eta)$ *for all* $\eta \in X$, *and*
*($F_{17}$)* $\Psi_A(\varrho - (\varrho - \eta)) \leq \max\{\Psi_A(((\eta - (\eta - \varrho)) - (\varrho - \eta)) - \varsigma), \Psi_A(\varsigma)\}$ *for all* $\eta, \varrho, \varsigma \in X$.
*($F_{18}$)* $\Omega_A(\varrho-(\varrho-\eta)) \geq \min\{\Omega_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Omega_A(\varsigma)\}$ *for all* $\eta, \varrho, \varsigma \in X$.

**Definition II.17.** *An interval valued opposition fuzzy set* $(\Psi_A, \Omega_A)$ *of* $X$ *is called an interval valued opposition intuitionism fuzzy sub-commutative ideal of* $X$ *if it satisfies*
*($F_{19}$)* $\Psi_A(0) \leq \Psi_A(\eta), \Omega_A(0) \geq \Omega_A(\eta)$ *for all* $\eta \in X$, *and*
*($F_{20}$)* $\Psi_A(\eta-(\eta-\varrho)) \leq \max\{\Psi_A((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Psi_A(\varsigma)\}$ *for all* $\eta, \varrho, \varsigma \in X$,
*($F_{21}$)* $\Omega_A(\eta-(\eta-\varrho)) \geq \min\{\Omega_A((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Omega_A(\varsigma)\}$ *for all* $\eta, \varrho, \varsigma \in X$.

**Definition II.18.** *An interval valued opposition fuzzy set* $(\Psi_A, \Omega_A)$ *of* $X$ *is called an interval valued opposition intuitionism fuzzy positive implication ideal of* $X$ *if it satisfies*
*($F_{22}$)* $\Psi_A(0) \leq \Psi_A(\eta), \Omega_A(0) \geq \Omega_A(\eta)$ *for all* $\eta \in X$, *and*
*($F_{23}$)* $\Psi_A(\eta-\varsigma) \leq \max\{\Psi_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Psi_A(\varrho)\}$ *for all* $\eta, \varrho, \varsigma \in X$,
*($F_{24}$)* $\Omega_A(\eta-\varsigma) \geq \min\{\Omega_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Omega_A(\varrho)\}$ *for all* $\eta, \varrho, \varsigma \in X$.

## III. Interval valued opposition intuitionism fuzzy sub-implication ideals

**Theorem III.1.** *If* $A$ *and* $B$ *are interval valued opposition intuitionism fuzzy sub-implication ideals of subtraction G-algebra, then* $A * B$ *is also an interval valued opposition intuitionism fuzzy sub-implication ideal of subtraction G-algebra.*

**Proof:** Given $A$ and $B$ are interval valued opposition intuitionism fuzzy sub-implication ideals of subtraction $G$-algebra $X$.
(1) $\Psi_A(0) \leq \Psi_A(\eta), \Omega_A(0) \geq \Omega_A(\eta)$ for all $\eta \in X$, and
(2) $\Psi_A(\varrho-(\varrho-\eta)) \leq \max\{\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Psi_A(\varsigma)\}$ for all $\eta, \varrho, \varsigma \in X$.
(3) $\Omega_A(\varrho-(\varrho-\eta)) \geq \min\{\Omega_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Omega_A(\varsigma)\}$ for all $\eta, \varrho, \varsigma \in X$.
(4) $\Psi_B(0) \leq \Psi_B(\eta), \Omega_B(0) \geq \Omega_B(\eta)$ for all $\eta \in X$, and
(5) $\Psi_B(\varrho-(\varrho-\eta)) \leq \max\{\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Psi_B(\varsigma)\}$ for all $\eta, \varrho, \varsigma \in X$.
(6) $\Omega_B(\varrho-(\varrho-\eta)) \geq \min\{\Omega_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Omega_B(\varsigma)\}$ for all $\eta, \varrho, \varsigma \in X$.

Case 1:
$\{\{\inf\Psi_A(0) + \inf\Psi_B(0)\}/2.\{\inf\Psi_A(0).\inf\Psi_B(0) + 1\}, \{\sup\Psi_A(0) + \sup\Psi_B(0)\}/2.\{\sup\Psi_A(0).\sup\Psi_B(0) + 1\}\} \leq \{\{\inf\Psi_A(\eta) + \inf\Psi_B(\eta)\}/2.\{\inf\Psi_A(\eta).\inf\Psi_B(\eta) + 1\}, \{\sup\Psi_A(\eta) + \sup\Psi_B(\eta)\}/2.\{\sup\Psi_A(\eta).\sup\Psi_B(\eta) + 1\}\}$.
Thus $(A * B)(0) \leq (A * B)(\eta)$.
$\{\{\inf\Omega_A(0) + \inf\Omega_B(0)\}/2.\{\inf\Omega_A(0).\inf\Omega_B(0) + 1\}, \{\sup\Omega_A(0) + \sup\Omega_A(0)\}/2.\{\sup\Omega_A(0).\sup\Omega_B(0) + 1\}\} \geq \{\{\inf\Omega_A(\eta)+\inf\Omega_B(\eta)\}/2.\{\inf\Omega_A(\eta).\inf\Omega_B(\eta)+1\}, \{\sup\Omega_A(\eta) + \sup\Omega_B(\eta)\}/2.\{\sup\Omega_A(\eta).\sup\Omega_B(\eta) + 1\}\}$.
Thus $(A * B)(0) \geq (A * B)(\eta)$.

Case 2:
$\{\{\inf\Psi_A(\varrho-(\varrho-\eta))+\inf\Psi_B(\varrho-(\varrho-\eta))\}/2.\{\inf\Psi_A(\varrho-(\varrho-\eta)).\inf\Psi_B(\varrho-(\varrho-\eta))+1\}, \{\sup\Psi_A(\varrho-(\varrho-\eta)) + \sup\Psi_B(\varrho-(\varrho-\eta))\}/2.\{\sup\Psi_A(\varrho-(\varrho-\eta)).\sup\Psi_B(\varrho-(\varrho-\eta)) + 1\}\} \leq \max\{\{\inf\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Psi_A(\varsigma)\} + \inf\{\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Psi_B(\varsigma)\}/2.\inf\{\Psi_A(1((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Psi_A(\varsigma)\}.\inf\{\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Psi_B(\varsigma)+1\}, \sup\{\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Psi_A(\varsigma)\} + \sup\{\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Psi_B(\varsigma)\}/2.\sup\{\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Psi_A(\varsigma)\}.\{\sup\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma), \Psi_B(\varsigma)+1\}\}$.
$\{\{\inf\Psi_A(\varrho-(\varrho-\eta))+\inf\Psi_B(\varrho-(\varrho-\eta))\}/2.\{\inf\Psi_A(\varrho-(\varrho-\eta)).\inf\Psi_B(\varrho-(\varrho-\eta))+1\}, \{\sup\Psi_A(\varrho-(\varrho-\eta)) +$

$\sup\Psi_B(\varrho-(\varrho-\eta))\}/2.\{\sup\Psi_A(\varrho-(\varrho-\eta)).\sup\Psi_B(\varrho-(\varrho-\eta))+1\}\} \leq \max\{\{\inf\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)+\inf\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)\}2.\{\inf\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma).\inf\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)+1\},\{\sup\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)+\sup\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)\}2.\{\sup\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma).\sup\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)+1\}\},\max\{\{\inf\Psi_A(\varsigma)+\inf\Psi_B(\varsigma)\}/2.\{\inf\Psi_A(\varsigma).\inf\Psi_B(\varsigma)+1\},\{\sup\Psi_A(\varsigma)+\sup\Psi_B(\varsigma)\}/2.\{\sup\Psi_A(\varsigma).\sup\Psi_B(\varsigma)+1\}\}$.

Thus $(A*B)(\varrho-(\varrho-\eta)) \leq \max\{(A*B)(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),(A*B)(\varsigma)\}$.

Case 3:

$\{\{\inf\Omega_A(\varrho-(\varrho-\eta))+\inf\Omega_B(\varrho-(\varrho-\eta))\}/2.\{\inf\Omega_A(\varrho-(\varrho-\eta)).\inf\Omega_B(\varrho-(\varrho-\eta))+1\},\{\sup\Omega_A(\varrho-(\varrho-\eta))+\sup\Omega_B(\varrho-(\varrho-\eta))\}/2.\{\sup\Omega_A(\varrho-(\varrho-\eta)).\sup\Omega_B(\varrho-(\varrho-\eta))+1\}\} \geq \min\{\{\inf\Omega_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Omega_A(\varsigma)\}+\inf\{\Omega_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Omega_B(\varsigma)\}/2.\{(\inf\Omega_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Omega_A(\varsigma)).(\inf\Omega_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Omega_B(\varsigma))+1\},\{\sup\Omega_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Omega_A(\varsigma)\}+\sup\{\Omega_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Omega_B(\varsigma)\}/2.\{(\sup\Omega_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Omega_A(\varsigma)).(\sup\Omega_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Omega_B(\varsigma))+1\}\}$.

$\{\{\inf\Omega_A(\varrho-(\varrho-\eta))+\inf\Omega_B(\varrho-(\varrho-\eta))\}/2.\{\inf\Omega_A(\varrho-(\varrho-\eta)).\inf\Omega_B(\varrho-(\varrho-\eta))+1\},\{\sup\Omega_A(\varrho-(\varrho-\eta))+\sup\Omega_B(\varrho-(\varrho-\eta))\}/2.\{\sup\Omega_A(\varrho-(\varrho-\eta)).\sup\Omega_B(\varrho-(\varrho-\eta))+1\}\} \geq \min\{\{\inf\Omega_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)+\inf\Omega_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)\}2.\{\inf\Omega_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma).\inf\Omega_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)+1\},\{\sup\Omega_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)+\sup\Omega_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)\}2.\{\sup\Omega_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma).\sup\Omega_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)+1\}\},\min\{\{\inf\Omega_A(\varsigma)+\inf\Omega_B(\varsigma)\}/2.\{\inf\Omega_A(\varsigma).\inf\Omega_B(\varsigma)+1\},\{\sup\Omega_A(\varsigma)+\sup\Omega_B(\varsigma)\}/2.\{\sup\Omega_A(\varsigma).\sup\Omega_B(\varsigma)+1\}\}$.

Thus $(A*B)(\varrho-(\varrho-\eta)) \geq \min\{(A*B)(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),(A*B)(\varsigma)\}$.

Therefore, $A*B$ is an interval valued opposition intuitionism fuzzy sub-implication ideal of $X$.

**Example III.1.** *Let $X=\{a_0,a_1,a_2,a_3\}$ be a subtraction G-algebra with the following Cayley table.*

| $-$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|
| $a_0$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
| $a_1$ | $a_0$ | $a_0$ | $a_2$ | $a_3$ |
| $a_2$ | $a_0$ | $a_1$ | $a_0$ | $a_3$ |
| $a_3$ | $a_0$ | $a_1$ | $a_2$ | $a_0$ |

*We define an interval valued opposition fuzzy set A, then $A=<X,\Psi_A,\Omega_A>$ by routine calculation A is an interval valued opposition intuitionism fuzzy sub-implication ideal of X.*

| $X$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|
| $\Psi_A$ | .14, .19 | .24, .22 | .51, .41 | .71, .52 |
| $\Omega_A$ | .72, .66 | .63, .42 | .45, .43 | .22, .40 |

*We define an interval valued opposition fuzzy set B, then $B=<X,\Psi_B,\Omega_B>$ by routine calculation B is an interval valued opposition intuitionism fuzzy sub-implication ideal of X.*

| $X$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|
| $\Psi_B$ | .52, .26 | .54, .34 | .62, .43 | .73, .45 |
| $\Omega_B$ | .46, .61 | .41, .54 | .33, .55 | .22, .43 |

*Then $A*B$ is an interval valued opposition intuitionism fuzzy sub-implication ideals of $X$.*

**Theorem III.2.** *If $A$ and $B$ are interval valued opposition intuitionism fuzzy sub-implication ideals of subtraction G-algebra, then $A+B$ is also an interval valued opposition intuitionism fuzzy sub-implication ideal of subtraction G-algebra.*

**Proof:** Given $A$ and $B$ are interval valued opposition intuitionism fuzzy sub-implication ideals of subtraction $G$-algebra $X$.

(1) $\Psi_A(0)\leq\Psi_A(\eta),\Omega_A(0)\geq\Omega_A(\eta)$ for all $\eta\in X$, and
(2) $\Psi_A(\varrho-(\varrho-\eta))\leq\max\{\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Psi_A(\varsigma)\}$ for all $\eta,\varrho,\varsigma\in X$.
(3) $\Omega_A(\varrho-(\varrho-\eta))\geq\min\{\Omega_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Omega_A(\varsigma)\}$ for all $\eta,\varrho,\varsigma\in X$.
(4) $\Psi_B(0)\leq\Psi_B(\eta),\Omega_B(0)\geq\Omega_B(\eta)$ for all $\eta\in X$, and
(5) $\Psi_B(\varrho-(\varrho-\eta))\leq\max\{\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Psi_B(\varsigma)\}$ for all $\eta,\varrho,\varsigma\in X$.
(6) $\Omega_B(\varrho-(\varrho-\eta))\geq\min\{\Omega_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Omega_B(\varsigma)\}$ for all $\eta,\varrho,\varsigma\in X$.

Case 1:

$[\inf\Psi_A(0)+\inf\Psi_B(0)-\inf\Psi_A(0).\inf\Psi_B(0),\sup\Psi_A(0)+\sup\Psi_B(0)-sup\Psi_A(0).\sup\Psi_B(0)] \leq [\inf\Psi_A(\eta)+\inf\Psi_B(\eta)-\inf\Psi_A(\eta).\inf\Psi_B(\eta),\sup\Psi_A(\eta)+\sup\Psi_B(\eta)-\sup\Psi_A(\eta).\sup\Psi_B(\eta)]$.

Thus $(A+B)(0)\leq(A+B)(\eta)$.

$[\inf\Omega_A(0).\inf\Omega_B(0),\sup\Omega_A(0).\Omega_B(0)] \geq [\inf\Omega_A(\eta).\inf\Omega_B(\eta),\sup\Omega_A(\eta),\sup\Omega_B(\eta)]$.

Thus $(A+B)(0)\geq(A+B)(\eta)$.

Case 2:

$[\inf\Psi_A(\varrho-(\varrho-\eta))+\inf\Psi_B(\varrho-(\varrho-\eta))-\inf\Psi_A(\varrho-(\varrho-\eta)).\inf\Psi_B(\varrho-(\varrho-\eta)),\sup\Psi_A(\varrho-(\varrho-\eta))+\sup\Psi_B(\varrho-(\varrho-\eta))-\sup\Psi_A(\varrho-(\varrho-\eta)).\sup\Psi_B(\varrho-(\varrho-\eta))] \leq \max[\inf\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Psi_A(\varsigma)+\inf\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Psi_A(\varsigma)-\inf\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Psi_A(\varsigma).\inf\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Psi_A(\varsigma),\sup\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Psi_A(\varsigma)+\sup\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Psi_A(\varsigma)-\sup\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Psi_A(\varsigma).\sup\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\Psi_A(\varsigma)]$.

$[\inf\Psi_A(\varrho-(\varrho-\eta))+\inf\Psi_B(\varrho-(\varrho-\eta))-\inf\Psi_A(\varrho-(\varrho-\eta)).\inf\Psi_B(\varrho-(\varrho-\eta)),\sup\Psi_A(\varrho-(\varrho-\eta))+\sup\Psi_B(\varrho-(\varrho-\eta))-\sup\Psi_A(\varrho-(\varrho-\eta)).\sup\Psi_B(\varrho-(\varrho-\eta))] \leq \max[\{\inf\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)+\inf\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)-\inf\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma).\inf\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\sup\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)+\sup\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)-\sup\Psi_A(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma).\sup\Psi_B(((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma),\{[\inf\Psi_A(\varsigma)+\inf\Psi_B(\varsigma)-\inf\Psi_A(\varsigma).\inf\Psi_B(\varsigma),\sup\Psi_A(\varsigma)+\sup\Psi_B(\varsigma)-\sup\Psi_A(\varsigma).\sup\Psi_B(\varsigma)]\}\}]$.

Thus $(A+B)(\varrho-(\varrho-\eta))\leq\max\{(A+B)((((\eta-(\eta-\varrho))-(\varrho-\eta))-\varsigma)),(A+B)(\varsigma)\}$.

Case 3:

$[\inf\Omega_A(\varrho-(\varrho-\eta)).\inf\Omega_B(\varrho-(\varrho-\eta)),\sup\Omega_A(\varrho-(\varrho-\eta)).\sup\Omega_B(\varrho-(\varrho-\eta))] \geq \min\{[\inf(\Omega_A(((\eta-(\eta-$

$\varrho)) - (\varrho - \eta)) - \varsigma), \Omega_A(\varsigma)). \inf(\Omega_B(((\eta - (\eta - \varrho)) - (\varrho - \eta)) - \varsigma), \Omega_B(\varsigma))], [\sup(\Omega_A(((\eta - (\eta - \varrho)) - (\varrho - \eta)) - \varsigma), \Psi_A(\varsigma)). \sup(\Omega_B(((\eta - (\eta - \varrho)) - (\varrho - \eta)) - \varsigma), \Omega_B(\varsigma))]\}$.

$[\inf \Omega_A(\varrho - (\varrho - \eta)). \inf \Omega_B(\varrho - (\varrho - \eta)), \sup \Omega_A(\varrho - (\varrho - \eta)). \sup \Omega_B(\varrho - (\varrho - \eta))] \geq \min\{\{[\inf \Omega_A(((\eta - (\eta - \varrho)) - (\varrho - \eta)) - \varsigma). \inf \Omega_B(((\eta - (\eta - \varrho)) - (\varrho - \eta)) - \varsigma)], [\sup \Omega_A(((\eta - (\eta - \varrho)) - (\varrho - \eta)) - \varsigma). \sup \Omega_B(((\eta - (\eta - \varrho)) - (\varrho - \eta)) - \varsigma)]\}, \{[\inf \Omega_A(\varsigma). \inf \Omega_B(\varsigma), \sup \Omega_A(\varsigma). \sup \Omega_B(\varsigma)]\}\}$.

Thus $(A + B)(\varrho - (\varrho - \eta)) \geq \min\{(A + B)((((\eta - (\eta - \varrho)) - (\varrho - \eta)) - \varsigma)), (A + B)(\varsigma)\}$.

Therefore, $A + B$ is an interval valued opposition intuitionism fuzzy sub-implication ideal of $X$.

**Example III.2.** *Let* $A = \{b_0, b_1, b_2, b_3\}$ *be a subtraction G-algebra with the following Cayley table.*

| $-$ | $b_0$ | $b_1$ | $b_2$ | $b_3$ |
|---|---|---|---|---|
| $b_0$ | $b_0$ | $b_1$ | $b_2$ | $b_3$ |
| $b_1$ | $b_0$ | $b_0$ | $b_2$ | $b_3$ |
| $b_2$ | $b_0$ | $b_1$ | $b_0$ | $b_3$ |
| $b_3$ | $b_0$ | $b_1$ | $b_2$ | $b_0$ |

*We define an interval valued opposition fuzzy set A, then* $A =< X, \Psi_A, \Omega_A >$ *by routine calculation A is an interval valued opposition intuitionism fuzzy sub-implication ideal of* $X$.

| $X$ | $b_0$ | $b_1$ | $b_2$ | $b_3$ |
|---|---|---|---|---|
| $\Psi_A$ | .14, .19 | .24, .22 | .51, .41 | .71, .52 |
| $\Omega_A$ | .72, .66 | .63, .42 | .45, .43 | .22, .40 |

*We define an interval valued opposition fuzzy set B, then* $B =< X, \Psi_B, \Omega_B >$ *by routine calculation B is an interval valued opposition intuitionism fuzzy sub-implication ideal of* $X$.

| $X$ | $b_0$ | $b_1$ | $b_2$ | $b_3$ |
|---|---|---|---|---|
| $\Psi_B$ | .52, .26 | .54, .34 | .62, .43 | .73, .45 |
| $\Omega_B$ | .46, .61 | .41, .54 | .33, .55 | .22, .43 |

*Then* $A + B$ *is an interval valued opposition intuitionism fuzzy sub-implication ideal of* $X$.

## IV. Interval valued opposition intuitionism fuzzy sub-commutative ideals

**Theorem IV.1.** *If A and B are interval valued opposition intuitionism fuzzy sub-commutative ideals of subtraction G-algebra, then* $A \bowtie B$ *is also an interval valued opposition intuitionism fuzzy sub-commutative ideal of subtraction G-algebra.*

**Proof:** Given $A$ and $B$ are interval valued opposition intuitionism fuzzy sub-commutative ideals of subtraction $G$-algebra $X$.

(1) $\Psi_A(0) \leq \Psi_A(\eta), \Omega_A(0) \geq \Omega_A(\eta)$ for all $\eta \in X$, and
(2) $\Psi_A(\eta - (\eta - \varrho)) \leq \max\{\Psi_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Psi_A(\varsigma)\}$ for all $\eta, \varrho, \varsigma \in X$.
(3) $\Omega_A(\eta - (\eta - \varrho)) \geq \min\{\Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Omega_A(\varsigma)\}$ for all $\eta, \varrho, \varsigma \in X$.
(4) $\Psi_B(0) \leq \Psi_B(\eta), \Omega_B(0) \geq \Omega_B(\eta)$ for all $\eta \in X$, and
(5) $\Psi_B(\eta - (\eta - \varrho)) \leq \max\{\Psi_B((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Psi_B(\varsigma)\}$ for all $\eta, \varrho, \varsigma \in X$.
(6) $\Omega_B(\eta - (\eta - \varrho)) \geq \min\{\Omega_B((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Omega_B(\varsigma)\}$ for all $\eta, \varrho, \varsigma \in X$.

Case 1:

$\{(2[\inf \Psi_A(0). \inf \Psi_B(0)]/[\inf \Psi_A(0) + \inf \Psi_B(0)]), (2[\sup \Psi_A(0). \sup \Psi_B(0)]/[\sup \Psi_A(0) + \sup \Psi_B(0)])\} \leq \{(2[\inf \ \Psi_A(\eta). \inf \Psi_B(\eta)]/[\inf \Psi_A(\eta) + \inf \Psi_B(\eta)]), (2[\sup \Psi_A(\eta). \sup \Psi_B(\eta)]/[\sup \Psi_A(\eta) + \sup \Psi_B(\eta)])\}$.

Thus $(A \bowtie B)(0) \leq (A \bowtie B)(\eta)$.

$\{(2[\inf \Omega_A(0). \inf \Omega_B(0)]/[\inf \Omega_A(0) + \inf \Omega_B(0)]), (2[\sup \Omega_A(0). \sup \Omega_B(0)]/[\sup \Omega_A(0) + \sup \Omega_B(0)])\} \leq \{(2[\inf \Omega_A(\eta). \ \inf \Omega_B(\eta)]/[\inf \Omega_A(\eta) + \inf \Omega_B(\eta)]), (2[\sup \Omega(\eta). \sup \Omega_B(\eta)]/[\sup \Omega_A(\eta) + \sup \Omega_B(\eta)])\}$.

Thus $(A \bowtie B)(0) \geq (A \bowtie B)(\eta)$.

Case 2:

$\{(2[\inf \Psi_A(\eta - (\eta - \varrho)). \inf \Psi_B(\eta - (\eta - \varrho))]/[\inf \Psi_A(\eta - (\eta - \varrho)) + \inf \Psi_B(\eta - (\eta - \varrho))]), (2[\sup \Psi_A(\eta - (\eta - \varrho)). \sup \Omega_B(\eta - (\eta - \varrho))]/[\sup \Psi_A(\eta - (\eta - \varrho)) + \sup \Psi_B(\eta - (\eta - \varrho))])\} \leq \max\{(2[\inf(\Psi_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Psi_A(\varsigma)). \inf(\Psi_B((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Psi_B(\varsigma))]/[\inf(\Psi_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Psi_A(\varsigma)) + \inf(\Psi_B((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Psi_B(\varsigma))]), (2[\sup(\Psi_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Psi_A(\varsigma)). \sup(\Psi_B((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Psi_B(\varsigma))]/[\sup(\Psi_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Psi_A(\varsigma)) + \sup(\Psi_B((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Psi_B(\varsigma))])$.

$\{(2[\inf \Psi_A(\eta - (\eta - \varrho)) \inf \Psi_B(\eta - (\eta - \varrho))]/[\inf \Psi_A(\eta - (\eta - \varrho)) + \inf \Psi_B(\eta - (\eta - \varrho))], ([\sup \Psi_A(\eta - (\eta - \varrho)). \sup \Omega_B(\eta - (\eta - \varrho))]/[\sup \Psi_A(\eta - (\eta - \varrho)) + \sup \Psi_B(\eta - (\eta - \varrho))]))\} \leq \max\{(2[\inf \Psi_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma). \inf \Psi_B((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma)]/[\inf \Psi_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma) + \inf \Psi_B((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma)]), (2[\sup \Psi_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma). \sup \Psi_B((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma)]/[\sup \Psi_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma) + \sup \Psi_B((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma)]), \{(2[\inf \Psi_A(\varsigma). \inf \Psi_B(\varsigma)]/[\inf \Psi_A(\varsigma) + \inf \Psi_B(\varsigma)]\}, ([\sup \Psi_A(\varsigma). \sup \Psi_B(\varsigma)]/[\sup \Psi_A(\varsigma) + \sup \Psi_B(\varsigma)])\}$.

Thus $(A \bowtie B)(a - (\eta - \varrho)) \leq \max\{(A \bowtie B)((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), (A \bowtie B)(\varsigma)\}$.

Case 3:

$\{(2[\inf \Omega_A(\eta - (\eta - \varrho)). \inf \Omega_B(\eta - (\eta - \varrho))]/[\inf \Omega_A(\eta - (\eta - \varrho)) + \inf \Omega_B(\eta - (\eta - \varrho))])\}, \{(2[\sup \Omega_A(\eta - (\eta - \varrho)). \sup \Omega_B(\eta - (\eta - \varrho))])/([\sup \Omega_A(\eta - (\eta - \varrho)) + \sup \Omega_B(\eta - (\eta - \varrho))])\} \geq \min\{(2[\inf(\Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Omega_A(\varsigma)). \inf(\Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Omega_A(\varsigma))]/([\inf(\Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Omega_A(\varsigma)) + \inf(\Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Omega_A(\varsigma))]), ([\sup(\Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Omega_A(\varsigma)). \sup(\Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Omega_A(\varsigma))])/([\sup(\Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Omega_A(\varsigma)) + \sup(\Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma), \Omega_A(\varsigma))])\}$.

$\{(2[\inf \Omega_A(\eta - (\eta - \varrho)). \inf \Omega_B(\eta - (\eta - \varrho))]/[\inf \Omega_A(\eta - (\eta - \varrho)) + \inf \Omega_B(\eta - (\eta - \varrho))])\}, \{(2[\sup \Omega_A(\eta - (\eta - \varrho)) \sup \Omega_B(\eta - (\eta - \varrho))])/([\sup \Omega_A(\eta - (\eta - \varrho)) + \sup \Omega_B(\eta - (\eta - \varrho))])\} \geq \min\{\{(2[\inf \Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma). \inf \Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma)])/([\inf \Omega_A((\varrho - (\varrho - (\eta - (a - \varrho)))) - \varsigma) + \inf \Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma)]), \sup \Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma). \sup \Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma).([\sup \Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma) + \sup \Omega_A((\varrho - (\varrho - (\eta - (\eta - \varrho)))) - \varsigma)])\}, \{(2[\inf \Omega_A(\varsigma). \inf \Omega_B(\varsigma)]/[\inf \Omega_A(\varsigma) + \inf \Omega_B(\varsigma)]), (2[\sup \Omega_A(\varsigma). \sup \Omega_B(\varsigma)]/[\sup \Omega_A(\varsigma) +$

$\sup \Omega_B(\varsigma)]\}\}$.

Thus $(A \bowtie B)(a-(\eta-\varrho)) \geq \min\{(A \bowtie B)((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma)(A \bowtie B)(\varsigma)\}$.

Therefore, $A \bowtie B$ is an interval valued opposition intuitionism fuzzy sub-commutative ideal of $X$.

**Example IV.1.** *Let $A = \{c_0, c_1, c_2, c_3\}$ be a subtraction G-algebra with the following Cayley table.*

| $-$ | $c_0$ | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|---|
| $c_0$ | $c_0$ | $c_1$ | $c_2$ | $c_3$ |
| $c_1$ | $c_0$ | $c_0$ | $c_2$ | $c_3$ |
| $c_2$ | $c_0$ | $c_1$ | $c_0$ | $c_3$ |
| $c_3$ | $c_0$ | $c_1$ | $c_2$ | $c_0$ |

*We define an interval valued opposition fuzzy set $A$, then $A =< X, \Psi_A, \Omega_A >$ by routine calculation $A$ is an interval valued opposition intuitionism fuzzy sub-commutative ideal of $X$.*

| $X$ | $c_0$ | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|---|
| $\Psi_A$ | $.26, .24$ | $.34, .37$ | $.55, .43$ | $.62, .53$ |
| $\Omega_A$ | $.71, .62$ | $.65, .52$ | $.44, .51$ | $.35, .45$ |

*We define an interval valued opposition fuzzy set $B$, then $B =< X, \Psi_B, \Omega_B >$ by routine calculation $B$ is an interval valued opposition intuitionism fuzzy sub-commutative ideal of $X$.*

| $X$ | $c_0$ | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|---|
| $\Psi_B$ | $.24, .32$ | $.36, .34$ | $.56, .44$ | $.71, .56$ |
| $\Omega_B$ | $.75, .66$ | $.62, .52$ | $.41, .45$ | $.26, .33$ |

*Then $A \bowtie B$ is an interval valued opposition intuitionism fuzzy sub-commutative ideal of $X$.*

**Theorem IV.2.** *If $A$ and $B$ are interval valued opposition intuitionism fuzzy sub-commutative ideals of subtraction G-algebra, then $A@B$ is also an interval valued opposition intuitionism fuzzy sub-commutative ideal of subtraction G-algebra.*

**Proof:** Given $A$ and $B$ are interval valued opposition intuitionism fuzzy sub-commutative ideals of subtraction $G$-algebra $X$.

(1) $\Psi_A(0) \leq \Psi_A(\eta), \Omega_A(0) \geq \Omega_A(\eta)$ for all $\eta \in X$, and
(2) $\Psi_A(\eta-(\eta-\varrho)) \leq \max\{\Psi_A((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Psi_A(\varsigma)\}$ for all $\eta, \varrho, \varsigma \in X$.
(3) $\Omega_A(\eta-(\eta-\varrho)) \geq \min\{\Omega_A((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Omega_A(\varsigma)\}$ for all $\eta, \varrho, \varsigma \in X$.
(4) $\Psi_B(0) \leq \Psi_B(\eta), \Omega_B(0) \geq \Omega_B(\eta)$ for all $\eta \in X$, and
(5) $\Psi_B(\eta-(\eta-\varrho)) \leq \max\{\Psi_B((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Psi_B(\varsigma)\}$ for all $\eta, \varrho, \varsigma \in X$.
(6) $\Omega_B(\eta-(\eta-\varrho)) \geq \min\{\Omega_B((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Omega_B(\varsigma)\}$ for all $\eta, \varrho, \varsigma \in X$.

Case 1:

$\{[\inf \Psi_A(0) + \inf \Psi_B(0)]/2, [\sup \Psi_A(0) + \sup \Psi_B(0)]/2\} \leq \{[\inf \Psi_A(\eta) + \inf \Psi_B(\eta)]/2, [\sup \Psi_A(\eta) + \sup \Psi_B(\eta)]/2\}$.

Thus $(A@B)(0) \leq (A@B)(\eta)$.

$\{[\inf \Omega_A(0) + \inf \Omega_B(0)]/2, [\sup \Omega_A(0) + \sup \Omega_B(0)]/2\} \geq \{[\inf \Omega_A(\eta) + \inf \Omega_B(\eta)]/2, [\sup \Omega_A(\eta) + \sup \Omega_B(\eta)]/2\}$.

Thus $(A@B)(0) \geq (A@B)(\eta)$.

Case 2:

$\{[\inf \Psi_A(\eta-(\eta-\varrho)) + \inf \Psi_B(\eta-(\eta-\varrho))]/2, [\sup \Psi_A(\eta-(\eta-\varrho)) + \sup \Psi_B(\eta-(\eta-\varrho))]/2\} \leq \max\{[\inf(\Psi_A((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Psi_A(\varsigma)) + \inf(\Psi_B((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Psi_B(\varsigma))]/2, [\sup(\Psi_A((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Psi_A(\varsigma)) + \sup(\Psi_B((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Psi_B(\varsigma))]/2\}$.

$\{[\inf \Psi_A(\eta-(\eta-\varrho)) + \inf \Psi_B(\eta-(\eta-\varrho))]/2, [\sup \Psi_A(\eta-(\eta-\varrho)) + \sup \Psi_B(\eta-(\eta-\varrho))]/2\} \leq \max\{[\inf \Psi_A((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma) + \inf \Psi_B((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma)]/2, [\sup \Psi_A((\varrho-(\varrho-(a-(\eta-\varrho)))) - \varsigma) + \sup \Psi_B((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma)]/2\}, \{[\inf \Psi_A(\varsigma) + \inf \Psi_B(\varsigma)]/2, [\sup \Psi_A(\varsigma) + \sup \Psi_B(\varsigma)]/2\}\}$.

Thus $(A@B)(a-(\eta-\varrho)) \leq \max\{(A@B)((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), (A@B)(\varsigma)\}$.

Case 3:

$\{[\inf \Omega_A(\eta-(\eta-\varrho)) + \inf \Omega_B(\eta-(\eta-\varrho))]/2, [\sup \Omega_A(\eta-(\eta-\varrho)) + \sup \Omega_B(\eta-(\eta-\varrho))]/2\} \geq \min\{[\inf(\Omega_A((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Omega_A(\varsigma)) + \inf(\Psi_B((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Omega_B(\varsigma))]/2, [\sup(\Omega_A((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Omega_A(\varsigma)) + \sup(\Omega_B((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), \Omega_B(\varsigma))]/2\}$.

$\{[\inf \Omega_A(\eta-(\eta-\varrho)) + \inf \Omega_B(\eta-(\eta-\varrho))]/2, [\sup \Omega_A(\eta-(\eta-\varrho)) + \sup \Omega_B(\eta-(\eta-\varrho))]/2\} \geq \min\{[\inf \Omega_A((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma) + \inf \Omega_B((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma)]/2, [\sup \Omega_A((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma) + \sup \Omega_B((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma)]/2\}, \{[\inf \Omega_A(\varsigma) + \inf \Omega_B(\varsigma)]/2, [\sup \Omega_A(\varsigma) + \sup \Omega_B(\varsigma)]/2\}\}$.

Thus $(A@B)(\eta-(\eta-\varrho)) \geq \min\{(A@B)((\varrho-(\varrho-(\eta-(\eta-\varrho)))) - \varsigma), (A@B)(\varsigma)\}$.

Therefore, $A@B$ is an interval valued opposition intuitionism fuzzy sub-commutative ideal of $X$.

**Example IV.2.** *Let $A = \{d_0, d_1, d_2, d_3\}$ be a subtraction G-algebra with the following Cayley table.*

| $-$ | $d_0$ | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|---|
| $d_0$ | $d_0$ | $d_1$ | $d_2$ | $d_3$ |
| $d_1$ | $d_0$ | $d_0$ | $d_2$ | $d_3$ |
| $d_2$ | $d_0$ | $d_1$ | $d_0$ | $d_3$ |
| $d_3$ | $d_0$ | $d_1$ | $d_2$ | $d_0$ |

*We define an interval valued opposition fuzzy set $A$, then $A =< X, \Psi_A, \Omega_A >$ by routine calculation $A$ is an interval valued opposition intuitionism fuzzy sub-commutative ideal of $A$.*

| $X$ | $d_0$ | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|---|
| $\Psi_A$ | $.26, .24$ | $.34, .37$ | $.55, .43$ | $.62, .53$ |
| $\Omega_A$ | $.71, .62$ | $.65, .52$ | $.44, .51$ | $.35, .45$ |

*We define an interval valued opposition fuzzy set $B$, then $B =< X, \Psi_B, \Omega_B >$ by routine calculation $B$ is an interval valued opposition intuitionism fuzzy sub-commutative ideal of $X$.*

| $X$ | $d_0$ | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|---|
| $\Psi_B$ | $.24, .32$ | $.36, .34$ | $.56, .44$ | $.71, .56$ |
| $\Omega_B$ | $.75, .66$ | $.62, .52$ | $.41, .45$ | $.26, .33$ |

*Then $A \bowtie B$ is an interval valued opposition intuitionism fuzzy sub-commutative ideal of $X$.*

## V. Interval valued opposition intuitionism fuzzy positive implication ideals

**Theorem V.1.** *If $A$ and $B$ are interval valued opposition intuitionism fuzzy positive implication ideals of subtraction G-algebra, then $A\#B$ is also an interval valued opposition intuitionism fuzzy positive implication ideal of subtraction G-algebra.*

**Proof:** Given $A$ and $B$ are interval valued opposition intuitionism fuzzy positive implication ideals of subtraction $G$-algebra $X$.

(1) $\Psi_A(0) \leq \Psi_A(\eta), \Omega_A(0) \geq \Omega_A(\eta)$ for all $\eta \in X$, and
(2) $\Psi_A(\eta-\varsigma) \leq \max\{\Psi_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Psi_A(\varrho)\}$ for all $\eta, \varrho, \varsigma \in X$.
(3) $\Omega_A(\eta-\varsigma) \geq \min\{\Omega_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Omega_A(\varrho)\}$ for all $\eta, \varrho, \varsigma \in X$.
(4) $\Psi_B(0) \leq \Psi_B(\eta), \Omega_B(0) \geq \Omega_B(\eta)$ for all $\eta \in X$, and
(5) $\Psi_B(\eta-\varsigma) \leq \max\{\Psi_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Psi_B(\varrho)\}$ for all $\eta, \varrho, \varsigma \in X$.
(6) $\Omega_B(\eta-\varsigma) \geq \min\{\Omega_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma))\Omega_B(\varrho)\}$ for all $\eta, \varrho, \varsigma \in X$.

Case 1:
$[\inf\Psi_A(0).\inf\Psi_B(0), \sup\Psi_A(0).\sup\Psi_B(0)] \leq [\inf\Psi_A(\eta).\inf\Psi_B(\eta), \sup\Psi_A(\eta).\sup\Psi_B(\eta)]$.
Thus $(A\#B)(0) \leq (A\#B)(\eta)$.
$\{[\inf\Omega_A(0) + \inf\Omega_B(0) - \inf\Omega_A(0).\inf\Omega_B(0)], [\sup\Omega_A(0) + \sup\Omega_B(0) - \sup\Omega_A(0).\sup\Omega_B(0)]\} \geq \{[\inf\Omega_A(\eta) + \inf\Omega_B(\eta) - \inf\Omega_A(\eta).\inf\Omega_B(\eta)], [\sup\Omega_A(\eta) + \sup\Omega_B(\eta) - \sup\Omega_A(\eta).\sup\Omega_B(\eta)]\}$.
Thus $(A\#B)(0) \geq (A\#B)(\eta)$.
Case 2:
$[\inf\Psi_A(\eta-\varsigma).\inf\Psi_B(\eta-\varsigma), \sup\Psi_A(\eta-\varsigma).\sup\Psi_B(\eta-\varsigma)] \leq \max[\inf\Psi_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Psi_A(\varrho).\inf\Psi_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Psi_A(\varrho), \sup\Psi_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Psi_A(\varrho).\sup\Psi_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Psi_A(\varrho)]$.
$[\inf\Psi_A(\eta-\varsigma).\inf\Psi_B(\eta-\varsigma), \sup\Psi_A(\eta-\varsigma).\sup\Psi_B(\eta-\varsigma)] \leq \max\{[\inf\Psi_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)).\inf\Psi_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \sup\Psi_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)).\sup\Psi_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma))], [\inf\Psi_A(\varrho).\inf\Psi_B(\varrho), \sup\Psi_A(\varrho).\sup\Psi_B(\varrho)]\}$.
Thus $(A\#B)(\eta-\varsigma) \leq \max\{((A\#B)((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), (A\#B)(\varrho)\}$.
Case 3:
$\{[\inf\Omega_A(\eta-\varsigma)+\inf\Omega_B(\eta-\varsigma)-\inf\Omega_A(\eta-\varsigma).\inf\Omega_B(\eta-\varsigma)], [\sup\Omega_A(\eta-\varsigma)+\sup\Omega_B(\eta-\varsigma)-\sup\Omega_A(\eta-\varsigma).\sup\Omega_B(\eta-\varsigma)]\} \geq \{[\inf(\Omega_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Omega_A(\varrho)) + \inf(\Omega_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Omega_A(\varrho)) - \inf(\Omega_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Omega_A(\varrho)).\inf(\Omega_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Omega_A(\varrho))], [\sup(\Omega_A(((a-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Omega_A(\varrho)) + \sup(\Omega_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Omega_A(\varrho)) - \sup(\Omega_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Psi_A(\varrho)).\sup(\Psi_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Psi_A(\varrho))]\}$.
$\{[\inf\Psi_A(\eta-\varsigma)+\inf\Psi_B(\eta-\varsigma)-\inf\Psi_A(\eta-\varsigma).\inf\Psi_B(\eta-\varsigma)], [\sup\Psi_A(\eta-\varsigma)+\sup\Psi_B(\eta-\varsigma)-\sup\Psi_A(\eta-\varsigma).\sup\Psi_B(\eta-\varsigma)]\} \geq \min\{\{[\inf\Psi_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)) + \inf\Psi_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)) - \inf\Psi_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)).\inf\Psi_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma))], [\sup\Psi_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)) + \sup\Omega_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)) - \sup\Omega_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)).\sup\Omega_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma))]\}, \{[\inf\Omega_A(\varrho) + \inf\Omega_B(\varrho) - \inf\Omega_A(\varrho).\inf\Omega_B(\varrho), \sup\Omega_A(\varrho) + \sup\Omega_B(\varrho) - \sup\Omega_A(\varrho).\sup\Omega_B(\varrho)]\}\}$.
Thus $(A\#B)(\eta-\varsigma) \geq \min\{(A\#B)((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma), (A\#B)(\varrho)\}$.
Therefore, $A\#B$ is an interval valued opposition intuitionism fuzzy positive implication ideal of $X$.

**Example V.1.** *Let $A = \{e_0, e_1, e_2, e_3\}$ be a subtraction G-algebra with the following Cayley table.*

| $-$ | $e_0$ | $e_1$ | $e_2$ | $e_3$ |
|---|---|---|---|---|
| $e_0$ | $e_0$ | $e_1$ | $e_2$ | $e_3$ |
| $e_1$ | $e_0$ | $e_0$ | $e_2$ | $e_3$ |
| $e_2$ | $e_0$ | $e_1$ | $e_0$ | $e_3$ |
| $e_3$ | $e_0$ | $e_1$ | $e_2$ | $e_0$ |

*We define an interval valued opposition fuzzy set $A$, then $A =< X, \Psi_A, \Omega_A >$ by routine calculation $A$ is an interval valued opposition intuitionism fuzzy positive implication ideal of $X$.*

| $X$ | $e_0$ | $e_1$ | $e_2$ | $e_3$ |
|---|---|---|---|---|
| $\Psi_A$ | .27, .25 | .35, .37 | .56, .52 | .65, .50 |
| $\Omega_A$ | .70, .65 | .63, .55 | .42, .40 | .33, .35 |

*We define an interval valued opposition fuzzy set $B$, then $B =< X, \Psi_B, \Omega_B >$ by routine calculation $B$ is an interval valued opposition intuitionism fuzzy positive implication ideal of $X$.*

| $X$ | $e_0$ | $e_1$ | $e_2$ | $e_3$ |
|---|---|---|---|---|
| $\Psi_B$ | .25, .34 | .35, .41 | .55, .44 | .61, .53 |
| $\Omega_B$ | .73, .62 | .63, .51 | .40, .47 | .32, .35 |

*Then $A\#B$ is an interval valued opposition intuitionism fuzzy positive implication ideal of $X$.*

**Theorem V.2.** *If $A$ and $B$ are interval valued opposition intuitionism fuzzy positive implication ideals of subtraction G-algebra, then $A \cap B$ is also an interval valued opposition intuitionism fuzzy positive implication ideal of subtraction G-algebra.*

**Proof:** Given $A$ and $B$ are interval valued opposition intuitionism fuzzy positive implication ideals of $X$.

(1) $\Psi_A(0) \leq \Psi_A(\eta), \Omega_A(0) \geq \Omega_A(\eta)$ for all $\eta \in X$, and
(2) $\Psi_A(\eta-\varsigma) \leq \max\{\Psi_A(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma))\Psi_A(\varrho)\}$ for all $\eta, \varrho, \varsigma \in X$.
(3) $\Omega_A(\eta-\varsigma) \geq \min\{\Omega_A(((\eta-\varsigma)-c)-(\varrho-\varsigma))\Omega_A(\varrho)\}$ for all $\eta, \varrho, \varsigma \in X$.
(4) $\Psi_B(0) \leq \Psi_B(\eta), \Omega_B(0) \geq \Omega_B(\eta)$ for all $\eta \in X$, and
(5) $\Psi_B(\eta-\varsigma) \leq \max\{\Psi_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma))\Psi_B(\varrho)\}$ for all $\eta, \varrho, \varsigma \in X$.
(6) $\Omega_B(\eta-\varsigma) \geq \min\{\Omega_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma))\Omega_B(\varrho)\}$ for all $\eta, \varrho, \varsigma \in X$.

Case 1:
$[\min(\inf\Psi_A(0), \inf\Psi_B(0)), \min(\sup\Psi_A(0), \sup\Psi_B(0))]$.
$\leq [\min(\inf\Psi_A(\eta), \inf\Psi_B(\eta)), \min(\sup\Psi_A(\eta), \sup\Psi_B(\eta))]$.
Thus $(A \cap B)(0) \leq (A \cap B)(\eta)$.
$[\max(\inf\Omega_A(0), \inf\Omega_B(0)), \max(\sup\Omega_A(0), \sup\Omega_B(0))] \geq [\max(\inf\Omega_A(\eta), \inf\Omega_B(\eta)), \max(\sup\Omega_A(\eta), \sup\Omega_B(\eta))]$.
Thus $(A \cap B)(0) \geq (A \cap B)(\eta)$.
Case 2:
$[\min(\inf\Psi_A(\eta-\varsigma), \inf\Psi_B(\eta-\varsigma)\min(\sup\Psi_A(\eta-\varsigma), \sup\Psi_B(\eta-\varsigma) \leq \max[\min(\inf(\Psi_A(((\eta-\varsigma)-$

$\varsigma) - (\varrho - \varsigma)), \Psi_A(\varrho)), \inf\ (\Psi_B(((\eta - \varsigma) - \varsigma) - (\varrho - \varsigma)), \Psi_B(\varrho))), \min(\sup(\Psi_A(((\eta - \varsigma) - \varsigma) - (\varrho - \varsigma)), \Psi_A(\varrho)), \sup(\Psi_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), \Psi_B(\varrho)))]$.

$[\min\{(\inf\Psi_A(\eta - \varsigma), \inf\Psi_B(\eta - \varsigma), \min(\sup\Psi_A(\eta - \varsigma), \sup\Psi_B(\eta - \varsigma)))\}] \leq \max[\min\{\{(\inf\Psi_A(((\eta - \varsigma) - \varsigma) - (\varrho - \varsigma)), \inf\Psi_B(((\eta - \varsigma) - \varsigma) - (\varrho - \varsigma)))\}, \{\min(\sup\Psi_A(((\eta - \varsigma) - \varsigma) - (\varrho - \varsigma)), \sup\Psi_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)))\}, \{\min[\inf\Psi_A(\varrho), \inf\Psi_B(\varrho)], \min[\sup\Psi_A(\varrho), \sup\Psi_B(\varrho)]\}\}]$.

Thus $(A\cap B)(\eta-\varsigma) \leq \max\{(A\cap B)(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), (A\cap B)(\varrho)\}$.

Case 3:

$[\max\{(\inf\Omega_A(\eta - \varsigma), \inf\Omega_B(\eta - \varsigma)), \max(\sup\Omega_A(\eta - \varsigma), \sup\Omega_B(\eta - \varsigma))\}] \geq \min[\max\{\{(\inf(\Omega_A(((\eta - \varsigma) - \varsigma) - (\varrho - \varsigma)), \Omega_A(\varrho)), \inf(\Omega_B(((\eta - \varsigma) - \varsigma) - (\varrho - \varsigma)), \Omega_B(\varrho)))\}, \{\max(\sup(\Omega_A(((\eta - \varsigma) - \varsigma) - (\varrho - \varsigma)), \Omega_A(\varrho)), \sup(\Omega_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma))\Omega_B(\varrho)))\}\}]$.

$[\max\{(\inf\Omega_A(\eta - \varsigma), \inf\Omega_B(\eta - \varsigma)), \max(\sup\Omega_A(\eta - \varsigma), \sup\Omega_B(\eta - \varsigma))\}] \geq \min[\max\{\{(\inf\Omega_A(((\eta - \varsigma) - \varsigma) - (\varrho - \varsigma)), \inf\Omega_B(((\eta - \varsigma) - \varsigma) - (\varrho - \varsigma)))\}, \{\max(\sup\Omega_A(((\eta - \varsigma) - \varsigma) - (\varrho - \varsigma)), \sup\Omega_B(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)))\}, \{\max[\inf\Omega_A(\varrho), \inf\Omega_B(\varrho)], \max[\sup\Omega_A(\varrho), \sup\Omega_B(\varrho)]\}\}]$.

Thus $(A\cap B)(\eta-\varsigma) \geq \min(A\cap B)(((\eta-\varsigma)-\varsigma)-(\varrho-\varsigma)), (A\cap B)(\varrho)$.

Therefore, $A \cap B$ is an interval valued opposition intuitionism fuzzy positive implication ideal of $X$.

**Example V.2.** *Let $A = \{f_0, f_1, f_2, f_3\}$ be a subtraction G-algebra with the following Cayley table.*

| $-$ | $f_0$ | $f_1$ | $f_2$ | $f_3$ |
|---|---|---|---|---|
| $f_0$ | $f_0$ | $f_1$ | $f_2$ | $f_3$ |
| $f_1$ | $f_0$ | $f_0$ | $f_2$ | $f_3$ |
| $f_2$ | $f_0$ | $f_1$ | $f_0$ | $f_3$ |
| $f_3$ | $f_0$ | $f_1$ | $f_2$ | $f_0$ |

*We define an interval valued opposition fuzzy set A, then $A =< X, \Psi_A, \Omega_A >$ by routine calculation $A$ is an interval valued opposition intuitionism fuzzy positive implication ideal of $X$.*

| $X$ | $f_0$ | $f_1$ | $f_2$ | $f_3$ |
|---|---|---|---|---|
| $\Psi_A$ | .27, .25 | .35, .37 | .56, .52 | .65, .50 |
| $\Omega_A$ | .70, .65 | .63, .55 | .42, .40 | .33, .35 |

*We define an interval valued opposition fuzzy set B, then $B =< X, \Psi_B, \Omega_B >$ by routine calculation $B$ is an interval valued opposition intuitionism fuzzy positive implication ideal of $X$.*

| $X$ | $f_0$ | $f_1$ | $f_2$ | $f_3$ |
|---|---|---|---|---|
| $\Psi_B$ | .25, .34 | .35, .41 | .55, .44 | .61, .53 |
| $\Omega_B$ | .73, .62 | .63, .51 | .40, .47 | .32, .35 |

*Then $A \cap B$ is an interval valued opposition intuitionism fuzzy positive implication ideal of $X$.*

## VI. Conclusion

A $G$-algebra is an important class of logical algebras. Many logical algebras can be represented in $G$-algebras. For example, Boolean algebras are equivalent to the bounded implication $G$-algebras. In this paper, some new operations, i.e., $@, +, *, \cap$ and $\#$ are introduced, and some basic properties are investigated. Also, we define necessity and possibility operations on interval valued intuitionism fuzzy sets: interval valued opposition intuitionism fuzzy sub-implication ideals, positive implication ideals and sub-commutative ideals, and study their basic properties and some results in subtraction $G$-algebras.

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