Finite-time Control of a Class of Engineering System with Input Saturation via Fractional-order Backstepping Strategy

Xiaomin Tian, Xingliu Hu, Juan Gu, Chaoyuan Man, Shumin Fei

***Abstract*—In this paper, the finite-time stabilization of a class of engineering system is investigated, to realize this control objective, the fractional-order backstepping strategy is applied. In the process of design the finite-time controller, the controlled system is assumed to be affected by disturbance, and the upper bound of the disturbance is unknown in advance. Furthermore, the effect of saturated nonlinear input is taken into consideration. Meantime, some special forms of adaption laws are proposed so as to recognize these unknown parameters. The Lyapunov theory in fractional-order form is available to verify that every subsystem is finite-time stable. In the end, simulation example is provided to confirm the efficiency of the given control method.**

***Index Terms*—finite-time control, fractional-order system, backstepping strategy, input saturation.**

I. INTRODUCTION

FRACTIONAL-ORDER calculus as a generation of derivation and integration of integer-order calculus has attracted more and more researchers' attentions in the latest two decades. Some actual systems, such as viscoelastic system [1] and materials and processes with memory and hereditary properties [2-5] are especially suitable for fractional-order calculus to describe them. Applications of fractional-order systems have been found in many fields, for example, a fractional-order dynamic model is applied to manifest the electrical characteristics of fuel cells [6]; A fractional-order capacitor with order within (1, 2) is researched in [7]; Moreover, fractional-order derivatives are used to describe the cardiac tissue-electrode interface in [8], and so on.

At present, the investigation of fractional-order system has become an active research field. Specially, the stabilization and control of fractional-order systems have attracted the attention of researchers in various fields. We has been demonstrate that using fractional-order controllers to fractional-order system can obtain better control result than integer-order controllers, such as fractional-order $\mathrm{PI}^{\lambda}\mathrm{D}^{\mu}$ controller [9], fractional-order nonsingular terminal sliding mode controller [10], fractional-order fuzzy controller [11], fractional-order nonlinear feedback controller [12], and so forth.

However, the methods mentioned above are only for the study of asymptotic stability of controlled systems, the finite-time stabilization of fractional-order nonlinear systems by using backstepping strategy is rarely discussed. While, from the practical point of view, it is very valuable to study the stabilization of fractional-order system in a given time. To realize fast convergence in a given time, finite-time control schemes are known to be the most effective method. Finite-time stability stands for the optimality in settling time. Besides that, finite-time control method has been verified that it has good robustness and anti-interference properties. Consequently, considering theses benefits of finite-time stability, it is very necessary to achieve the stabilization of fractional-order system in time interval given.

For controller design, the backstepping method is a recursive method. Based on designing virtual controllers and partial Lyapunov functions for every subsystem step by step, a comprehensive Lyapunov function of the overall system can be derived according to the above operation. This approach can ensures the global stability, tracking and transient performance of nonlinear systems [13]. In consideration of the excellent performance of backstepping strategy, a growing number of researchers are focusing on this potential matter. On the other side, as a matter of fact, input nonlinearity is often encountered in all kinds of systems, which is one of the causes of instability. Therefore, it is obvious that the influence of nonlinear input must be considered when analyzing and applying a control scheme. As far as we know, it is rarely mentioned in the literatures about the control of fractional-order systems with more complicated input nonlinearity.

Inspired by the above content, it is very challenging and necessary to explore the finite-time stabilization for a class of engineering system with nonlinear input through the use of adaptive backstepping technique. So as to enhance the feasibility of control strategy, in this paper, the influence of system unknown parameters are fully considered, through design virtual controller step by step, the finite-time stability of every subsystem can be proved by using the Lyapunov theory in fractional-order form, then a comprehensive actual finite-time controller can be confirmed.

The rest of the article is structured as follows: In preliminaries section , the relevant definitions and lemmas have been introduced. In system description and main results

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section, the system mathematical model and two situations are all researched. In simulation results section, a simulation example is given to verified the correctness of the proposed control strategy. In conclusion section, the main work and innovation of this paper are summarized.

II. Preliminaries

The Caputo definition is the most frequently used definition for fractional calculus.

Definition 1. The Caputo fractional differentiation of order α is given as

$$ {}_{t_0}D_t^{\alpha} f(t)=\begin{cases} \frac{1}{\Gamma(m-\alpha)}\int_{t_0}^{t}\frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}}d\tau, & m-1<\alpha<m \\ \frac{d^m}{dt^m}f(t), & \alpha=m \end{cases} \quad (1) $$

where m is the smallest integer number, larger than α. In the following part of the article, D^{α} will be used to replace of ${}_0D_t^{\alpha}$.

***Lemma 1** (See [14])*. Assume $a_1, a_2, ..., a_n$ and $0<q<2$ are all real numbers, then the inequality like the following form holds:

$$ |a_1|^q+|a_2|^q+...+|a_n|^q \geq (a_1^2+a_2^2+...+a_n^2)^{q/2} \quad (2) $$

***Lemma 2** (See [15])*. Let $x(t)$ be a continuously differentiable vector value function. Then for any time instant $t \geq t_0$

$$ \frac{1}{2}D^{\alpha}x^T(t)x(t) \leq x^T(t)D^{\alpha}x(t), \quad \alpha\in(0,1) \quad (3) $$

***Lemma 3** (See [16])*. Suppose a continuous, positive definite function $V(t)$ satisfies the following form inequality:

$$ D^qV(t) \leq -cV^{\eta}(t) \quad (4) $$

in which, $c>0$, $0<\eta<q$ are all constants, and then $V(t)$ meets the inequality as follows:

$$ V^{q-\eta}(t) \leq V^{q-\eta}(t_0) - \frac{c\Gamma(1+q-\eta)(t-t_0)^q}{\Gamma(1+q)\Gamma(1-\eta)} \quad (5) $$

where $t_0 \leq t \leq t_1$, and $V(t)=0$ for all $t \geq t_1$, here t_1 is given by

$$ t_1 = t_0 + \Big(\frac{\Gamma(1+q)\Gamma(1-\eta)V^{q-\eta}(t_0)}{c\Gamma(1+q-\eta)}\Big)^{1/q} \quad (6) $$

III. System description and main results

A. System description

A class of engineering system with external perturbation and saturated nonlinear input is shown in the following section:

$$ \begin{aligned} D^{\alpha}x_1 &= x_2+\phi_1^T(x_1)\theta_1 \\ D^{\alpha}x_2 &= x_3+\phi_2^T(x_1,x_2)\theta_2 \\ &\vdots \\ D^{\alpha}x_n &= b\cdot sat(u(t))+\phi_n^T(x)\theta_n+f(x)+d(t) \end{aligned} \quad (7) $$

in which, $\alpha\in(0,1)$ is system order, $x=(x_1,x_2,...,x_n)^T$ is state vector of the above nonlinear system, ϕ_i is the column vector of system, θ_i, $i=1,2,...,n$ is the column vector of system parameters, b is the positive input coefficient, $f(x)$ is the nonlinear section of system, $d(t)$ is the external disturbance, its bounded and satisfied as follows:

$$ |d(t)| \leq \gamma \quad (8) $$

here γ is a positive real.

$sat(u(t))$ is the nonlinear saturated input, it's definition as shown in below [17]:

$$ sat(u(t)) = \begin{cases} u_{max}, & u(t)\geq u_{max} \\ u(t), & u_{min}<u(t)<u_{max} \\ u_{min}, & u(t)\leq u_{min} \end{cases} \quad (9) $$

here $u(t)$ is the input signal of the saturated input nonlinearity, $u_{max}>0$ and $u_{min}<0$ are the unknown coefficients. In (9), the saturation function can be approximated as a smooth piecewise function given in below:

$$ sat(u(t)) = g(u(t)) + \Delta u(t) \quad (10) $$

where

$$ g(u(t)) = \begin{cases} u_{max}\cdot tanh(\frac{u(t)}{u_{max}}), & u(t)\geq 0 \\ u_{min}\cdot tanh(\frac{u(t)}{u_{min}}), & u(t)<0 \end{cases} \quad (11) $$

further, we have

$$ g(u(t)) = \begin{cases} u_{max}\cdot\frac{e^{u(t)/u_{max}}-e^{-(u(t)/u_{max})}}{e^{u(t)/u_{max}}+e^{-(u(t)/u_{max})}}, & u(t)\geq 0 \\ u_{min}\cdot\frac{e^{u(t)/u_{min}}-e^{-(u(t)/u_{min})}}{e^{u(t)/u_{min}}+e^{-(u(t)/u_{min})}}, & u(t)<0 \end{cases} \quad (12) $$

and $\Delta u(t) = sat(u(t)) - g(u(t))$, satisfying

$$ \begin{aligned} |\Delta u(t)| &= |sat(u(t))-g(u(t))| \\ &\leq max\{u_{max}(1-tanh(1)), u_{min}(tanh(1)-1)\} \\ &= \delta \end{aligned} \quad (13) $$

where δ is a positive constant.

By the mean value theorem, exist a constant μ, $0<\mu<1$ such that

$$ g(u(t)) = g(u_0) + g_{u_\mu}(u(t)-u_0) \quad (14) $$

where

$$ g_{u_\mu} = \frac{\partial g(u(t))}{\partial u(t)}|_{u(t)=u_\mu}, \ u_\mu = \mu u(t)+(1-\mu)u_0 \quad (15) $$

When selecting $u_0=0$, eq.(14) can be represented as

$$ g(u(t)) = g_{u_\mu}u(t) \quad (16) $$

B. Main results

For improving the validity of the given control scheme, two situations both are considered, that is:

(B.1) Both the bounds of external disturbances and saturation input uncertainty are known, system parameters are known.

This section introduces the finite-time backstepping control strategy for a class of fractional-order engineering system with saturated input nonlinearity and perturbation, for design the comprehensive controller, transformation variables can be selected firstly as

$$ \begin{aligned} z_1 &= x_1 \\ z_i &= x_i - \tau_{i-1}, \quad i=2,3,...,n \end{aligned} \quad (17) $$

where z_i is transformation variable, τ_i is the virtual controller, which can be assigned as

$$ \begin{aligned} \tau_1 &= -(|z_2|+|z_1|^{\rho}+|\phi_1|^T|\theta_1|)sgn(z_1) \\ \tau_j &= -(|z_{j+1}|+|z_j|^{\rho}+|\phi_j|^T|\theta_j|)sgn(z_j)+D^{\alpha}\tau_{j-1} \end{aligned} \quad (18) $$

where $\rho \in (0,1)$, $j = 2,3,...,n-1$, $sgn(\cdot)$ is sign function.

Theorem 1. When the system (7) is affected by saturated nonlinear input, the controller which results in the finite-time stabilization of system can designed as follows

$$u(t) \;=\; -\frac{1}{bg_{u_\mu}}\Big[\big(|z_n|^\rho + b\delta + |\phi_n|^T|\theta_n| + |f(x)| \\ +\gamma\big)sgn(z_n) - D^\alpha\tau_{n-1}\Big] \quad (19)$$

Proof. In step 1: The first new subsystem can be get based on eqs.(7) and (17)

$$D^\alpha z_1 = z_2 + \phi_1^T\theta_1 + \tau_1 \quad (20)$$

for demonstration the finite-time stability of the above system (20), selecting the Lyapunov function as below

$$V_1 = \frac{1}{2}z_1^2 \quad (21)$$

taking the αth order fractional derivative of both sides for eq.(21), and based on Lemma 2, we have

$$D^\alpha V_1 \le z_1 D^\alpha z_1 = z_1(z_2 + \phi_1^T\theta_1 + \tau_1) \quad (22)$$

inserting the first equation of eq.(18) into (22), it yields

$$D^\alpha V_1 \le |z_1||z_2| + |z_1||\phi_1|^T|\theta_1| - |z_1|(|z_2| + |z_1|^\rho + |\phi_1|^T|\theta_1|) \\ = -|z_1|^{\rho+1} = -2^{\frac{\rho+1}{2}}\big(\frac{1}{2}z_1^2\big)^{\frac{\rho+1}{2}} = -cV_1^\eta \quad (23)$$

where $c = 2^{\frac{\rho+1}{2}}$, $\eta = \frac{\rho+1}{2}$, according to Lemma 3, z_1 will converge to zero in finite time.

In step i (i=2, 3, ..., n-1): We continue to investigate the finite-time stability of the i-th new subsystem with transformation variable z_i

$$D^\alpha z_i = D^\alpha x_i - D^\alpha\tau_{i-1} = z_{i+1} + \tau_i + \phi_i^T\theta_i - D^\alpha\tau_{i-1} \quad (24)$$

selecting the Lyapunov function as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 \quad (25)$$

similarly, taking the α-th fractional order derivative of both sides of eq.(25), we obtain

$$D^\alpha V_i \le -\sum_{j=1}^{i-1}|z_j|^{\rho+1} + z_i D^\alpha z_i \\ = -\sum_{j=1}^{i-1}|z_j|^{\rho+1} + z_i(z_{i+1} + \tau_i + \phi_i^T\theta_i - D^\alpha\tau_{i-1}) \quad (26)$$

inserting the second equation of eq.(18) into (26), it has

$$D^\alpha V_i \le -\sum_{j=1}^{i}|z_j|^{\rho+1} \le -\big(\sum_{j=1}^{i}|z_j|^2\big)^{\frac{\rho+1}{2}} \\ \le -2^{\frac{\rho+1}{2}}V_i^{\frac{\rho+1}{2}} = -cV_i^\eta \quad (27)$$

according to Lemma 3, z_i can converge to zero in bounded time.

In step n: In the end, the comprehensive controller is determined. Similarly, the nth subsystem described by transformation variable z_n is presented as

$$D^\alpha z_n = bsat(u(t)) + \phi_n^T(x)\theta_n + f(x) + d(t) - D^\alpha\tau_{n-1} \quad (28)$$

selecting the Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 \quad (29)$$

taking the fractional order derivative of eq.(29), we have

$$D^\alpha V_n \le -\sum_{i=1}^{n-1}|z_i|^{\rho+1} + z_n D^\alpha z_n \\ = -\sum_{i=1}^{n-1}|z_i|^{\rho+1} + z_n\big[bsat(u(t)) + \phi_n^T\theta_n + f(x) \\ + d(t) - D^\alpha\tau_{n-1}\big] \\ \le -\sum_{i=1}^{n-1}|z_i|^{\rho+1} + z_n(bg_{u_\mu}u(t) + b\delta) + |z_n||\phi_n|^T|\theta_n| \\ + |z_n||f(x)| + \gamma|z_n| - z_n D^\alpha\tau_{n-1} \\ \le -\sum_{i=1}^{n}|z_i|^{\rho+1} \le -2^{\frac{\rho+1}{2}}V_n^{\frac{\rho+1}{2}} = -cV_n^\eta \quad (30)$$

that is, z_n will tend to zero in finite time. In view of the above analysis, all transformation variables $z_i, i = 1,2,...,n$ can converge to zero in bounded time, thus the controlled system (7) with saturated nonlinear input can achieve stabilization in finite time. The proof is completed.

(B.2) System parameters are uncertain beforehand, and the bounds of external disturbance and input uncertainty are assumed to be unknown in advance.

Denote $\hat{\theta}$ as the estimation of θ, $\widetilde{\theta}$ is the estimated error of θ, $\hat{\gamma}$ is the estimation of γ, $\widetilde{\gamma}$ is the estimated error of γ, $\hat{\delta}$ is the estimation of δ, $\widetilde{\delta}$ is the estimated error of δ. In order to identify all unknown parameters, we proposed the following fractional-order version of adaptive update rules

$$D^\alpha\hat{\theta}_i = \phi_i z_i \\ D^\alpha\hat{\gamma} = \lambda|z_n| \\ D^\alpha\hat{\delta} = \xi|z_n| \quad (31)$$

where $\lambda > 0$ and $\xi > 0$ are adaptive gains, i=1, 2, ..., n.

In this situation, the virtual controllers are assigned as

$$\tau_1 = -[(|z_2| + |z_1|^\rho)sgn(z_1) + \phi_1^T\hat{\theta}_1] \\ \tau_j = -[(|z_{j+1}| + |z_j|^\rho)sgn(z_j) + \phi_j^T\hat{\theta}_j - D^\alpha\tau_{j-1}] \quad (32)$$

where $j = 2,...,n-1$.

Theorem 2. When the system (7) is affected by saturated input nonlinearity and unknown parameters, the controller which causes the robust stabilization of system in finite time is given below

$$u(t) \;=\; -\frac{1}{bg_{u_\mu}}\Big[\big(|z_n|^\rho + b\hat{\delta} + |f(x)| + \hat{\gamma}\big)sgn(z_n) \\ + \phi_n^T\hat{\theta}_n - D^\alpha\tau_{n-1}\Big] \quad (33)$$

Proof. In step 1: Selecting the following form Lyapunov function to verify the finite-time stability of the first subsystem (20).

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\widetilde{\theta}_1^T\widetilde{\theta}_1 \quad (34)$$

computing the αth fractional derivative of eq.(34), we have

$$D^\alpha V_1 \le z_1 D^\alpha z_1 + \widetilde{\theta}_1^T D^\alpha\hat{\theta}_1 \\ = z_1(z_2 + \tau_1 + \phi_1^T\theta_1) + \widetilde{\theta}_1^T\phi_1 z_1 \\ \le -|z_1|^{\rho+1} \le -2^{\frac{\rho+1}{2}}\big(\frac{1}{2}z_1^2\big)^{\frac{\rho+1}{2}} \quad (35)$$

define the auxiliary function

$$V_{1a} = \frac{1}{2} z_1^2 \leq V_1 \tag{36}$$

according to the research results of [18], exist a positive constant $\zeta_1 > 0$, so as to $\zeta_1 V_1^{\frac{\rho+1}{2}} \leq V_{1a}^{\frac{\rho+1}{2}}$, according to eq.(35), it yields

$$D^\alpha V_1 \leq -\zeta_1 2^{\frac{\rho+1}{2}} V_1^{\frac{\rho+1}{2}} = -k_1 V_1^\eta \tag{37}$$

where $k_1 = \zeta_1 2^{\frac{\rho+1}{2}} > 0$, $\eta = \frac{\rho+1}{2}$, according to Lemma 3, z_1 will tends to zero in given time.

In step i (i=2, 3, ..., n-1): We continue to construct the following form Lyapunov function to prove the finite-time stability of the ith new subsystem.

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2} \widetilde{\theta}_i^T \widetilde{\theta}_i \tag{38}$$

taking the α-th fractional order derivative of

$$\begin{aligned} D^\alpha V_i \leq & -\sum_{j=1}^{i-1} |z_j|^{\rho+1} + z_i D^\alpha z_i + \widetilde{\theta}_i^T D^\alpha \hat{\theta}_i \\ = & -\sum_{j=1}^{i-1} |z_j|^{\rho+1} + z_i (z_{i+1} + \tau_i + \phi_i^T \theta_i \\ & - D^\alpha \tau_{i-1}) + \widetilde{\theta}_i^T \phi_i z_i \\ \leq & -\sum_{j=1}^{i} |z_j|^{\rho+1} \leq -(\sum_{j=1}^{i} |z_j|^2)^{\frac{\rho+1}{2}} \\ = & -2^{\frac{\rho+1}{2}} (\frac{1}{2} \sum_{j=1}^{i} z_j^2)^{\frac{\rho+1}{2}} \end{aligned} \tag{39}$$

constructing the auxiliary function

$$V_{ia} = \sum_{j=1}^{i} \frac{1}{2} z_j^2 \leq V_i \tag{40}$$

according to the research results of [18], exist a positive constant $\zeta_i > 0$, so as to $\zeta_i V_i^{\frac{\rho+1}{2}} \leq V_{ia}^{\frac{\rho+1}{2}}$, according to eq.(39), it yields

$$D^\alpha V_i \leq -\zeta_i 2^{\frac{\rho+1}{2}} V_i^{\frac{\rho+1}{2}} = -k_i V_i^\eta \tag{41}$$

where $k_i = \zeta_i 2^{\frac{\rho+1}{2}} > 0$, $\eta = \frac{\rho+1}{2}$, according to Lemma 3, z_i will tends to zero in finite time.

In step n: Similar to the section B.1, the last Lyapunov stability function is established to verify the finite-time stability of the last subsystem with transformation variable z_n

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2} \widetilde{\theta}_n^T \widetilde{\theta}_n + \frac{1}{2\lambda} \widetilde{\gamma}^2 + \frac{b}{2\xi} \widetilde{\delta}^2 \tag{42}$$

taking the α-th fractional order derivative of eq.(42), we have

$$\begin{aligned} D^\alpha V_n \leq & -\sum_{i=1}^{n-1} |z_i|^{\rho+1} + z_n D^\alpha z_n + \widetilde{\theta}_n^T D^\alpha \hat{\theta}_n + \frac{1}{\lambda} \widetilde{\gamma} D^\alpha \hat{\gamma} + \frac{b}{\xi} \widetilde{\delta} D^\alpha \hat{\delta} \\ = & -\sum_{i=1}^{n-1} |z_i|^{\rho+1} + z_n (bsat(u(t)) + \phi_n^T \theta_n + f(x) \\ & + d(t) - D^\alpha \tau_{n-1}) + \widetilde{\theta}_n^T \phi_n z_n + \widetilde{\gamma} |z_n| + b\widetilde{\delta} |z_n| \\ \leq & -\sum_{i=1}^{n-1} |z_i|^{\rho+1} + z_n b g_{u_\mu} u(t) + |z_n| b\delta + z_n \phi_n^T \theta_n \\ & + |z_n||f(x)| + \gamma |z_n| - z_n D^\alpha \tau_{n-1} + \widetilde{\theta}_n^T \phi_n z_n \\ & + \widetilde{\gamma} |z_n| + b\widetilde{\delta} |z_n| \\ \leq & -\sum_{i=1}^{n} |z_i|^{\rho+1} \leq -2^{\frac{\rho+1}{2}} (\frac{1}{2} \sum_{i=1}^{n} z_i^2)^{\frac{\rho+1}{2}} \end{aligned} \tag{43}$$

similarly, define the auxiliary function

$$V_{na} = \frac{1}{2} \sum_{i=1}^{n} z_i^2 \leq V_n \tag{44}$$

that is, exist a positive constant $\zeta_n > 0$, so as to $\zeta_n V_n^{\frac{\rho+1}{2}} \leq V_{na}^{\frac{\rho+1}{2}}$, according to eq.(43), it yields

$$D^\alpha V_n \leq -\zeta_n 2^{\frac{\rho+1}{2}} V_n^{\frac{\rho+1}{2}} = -k_n V_n^\eta \tag{45}$$

where $k_n = \zeta_n 2^{\frac{\rho+1}{2}} > 0$, $\eta = \frac{\rho+1}{2}$, then z_n will tends to zero in bounded time. According to the above demonstration, all transformation variables $z_i, i = 1, 2, ..., n$ can converge to zero in given time, that is the controlled system (7) with saturated nonlinear input and unknown parameters can achieve stabilization in finite time. The proof is completed.

IV. Simulation Results

A simulation example is shown to demonstrate the effectiveness and feasibility of the proposed control method. In this example, a fractional-order discontinuous oscillator model is considered, which is described as follows

$$\begin{aligned} D^\alpha x_1 &= x_2 \\ D^\alpha x_2 &= sat(u(t)) - x_1 - 2a_1 x_2 + \frac{x_1}{\sqrt{x_1^2 + a_2^2}} + f_0 cos\omega t + d(t) \end{aligned} \tag{46}$$

where $\alpha = 0.98$, $a_1 = 0.01\sqrt{2}$, $a_2 = 0.5$, $f_0 = 0.8$, $\omega = 0.75\sqrt{2}$, $d(t) = 0.01cos(x_2)$, according to system form (7), $b = 1$, $\phi_1(x) = 0$, $\theta_1 = 0$, $\phi_2(x) = (x_1, x_2)^T$, $\theta_2 = (-1, -2a_1)^T$, $f(x) = \frac{x_1}{\sqrt{x_1^2 + a_2^2}} + f_0 cos\omega t$. The input saturation $sat(u(t))$ is presented as

$$sat(u(t)) = \begin{cases} 5, & u(t) \geq 5 \\ u(t), & -5 < u(t) < 5 \\ -5, & u(t) \leq -5 \end{cases} \tag{47}$$

Selecting the initial conditions $x(0) = (-1, 1)^T$, other parameter is chosen as $\rho = 0.5$, the system (46) behave chaotically, the strange attractor of system (46) is shown in Fig.1, and the system's state trajectories without controller are presented in Fig.2.

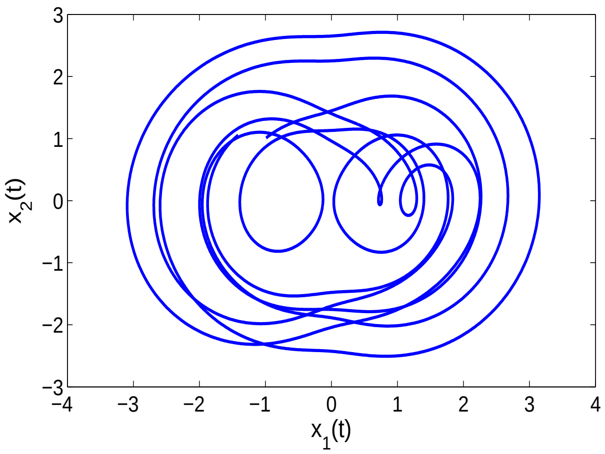


Figure 1. Strange attractor of system (46)

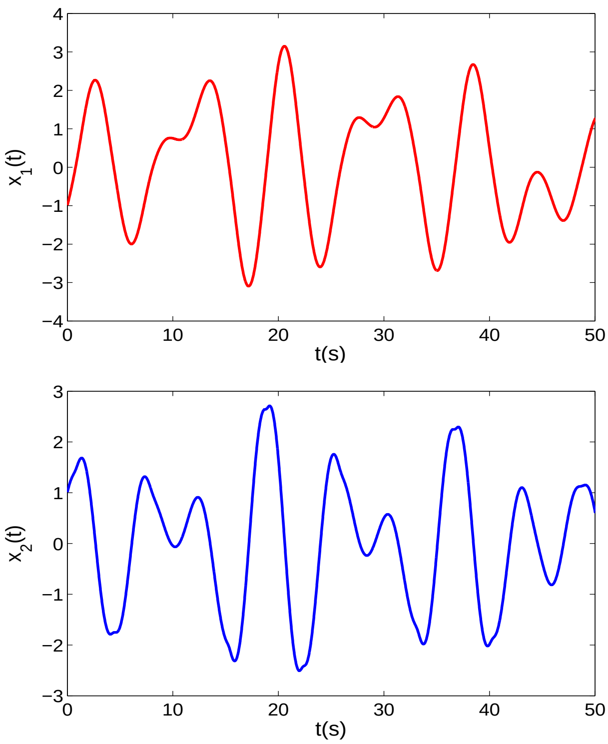
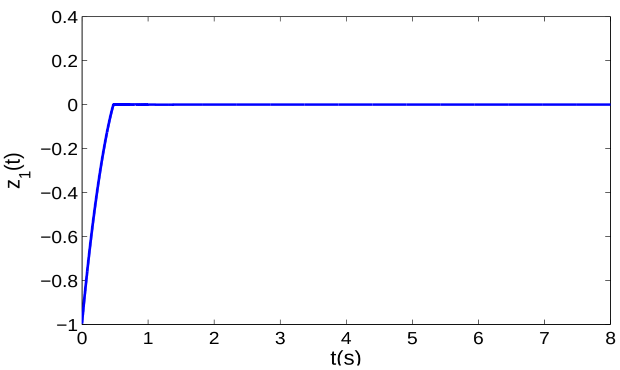


Figure 2. Time responses of system (46) without controller

When the controller is activated, the simulation experiment results are depicted in Fig.3, which reveal the time response of transformation variables. Obviously, all variables tend to zero in finite time.



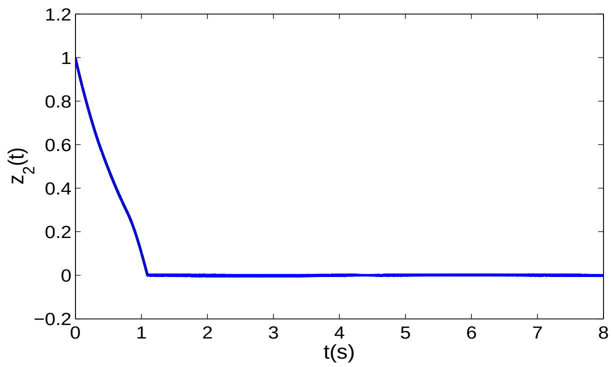


Figure 3. Trajectories of transformation variables when controller activated

All above simulation results sufficiently demonstrate that the proposed finite-time control strategy is valid in stabilizing this type of fractional-order engineering system with saturated nonlinear inputs.

V. Conclusion

In this paper, a finite-time backstepping control scheme for stabilizing this type of fractional-order engineering system is researched. The system is affected by bounded perturbation, and the situations of system parameters are known or unknown both are considered in advance. The impact of saturated input nonlinearity is taken into account in designing the comprehensive controller. To deal with the unknown parameters and external disturbance, a fractional-order version of adaptive update laws is proposed. For reasons of demonstrating the finite-time stability of the controlled system, the fractional-order finite-time stability theory is used. Further, simulation results proved that the control method is feasible and efficacious.

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